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Full-Link AoI Analysis of Uplink Transmission in Next-Generation FTTR WLANs

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***Abstract*—Fiber-to-the-room (FTTR) wireless local area networks (WLANs) are a promising sixth-generation (6G) technology for extreme broadband low-latency indoor wireless communications. With dense deployment of access points (APs), namely optical network units (ONUs), and efficient spatial frequency reuse across the ONUs, FTTR WLANs enable the mobile devices to flexibly access any ONU in its communication range and reduce the collisions during data packet transmissions. However, FTTR WLANs share a passive optical network (PON) for time division multiplexing (TDM) based backhauling, which may incur long delays for scheduling packet transmissions over the PON. In this paper, the full-link age of information (FL-AoI) is proposed as a new performance metric to analyze the timeliness of indoor communications in FTTR WLANs, taking into account both the carrier sense multiple access with collision avoidance (CSMA/CA) based wireless transmission and the TDM based packet scheduling over the PON. The FL-AoI of FTTR WLANs is analyzed using stochastic geometry and stretched exponential path-loss (SEPL) based indoor wireless channel model. We show that rather than accessing the nearest ONUs, the FTTR WLAN also enable the mobile devices to access further ONUs to reduce the average FL-AoI. Meanwhile, there exists an optimal transmission distance to achieve minimal average FL-AoI in the FTTR WLAN, whose value depends on the deployment density of ONUs.**

***Index Terms*—Fiber-to-the-rooms (FTTR), Full-link Age of Information, Stochastic Geometry.**

I. Introduction

Emerging indoor applications such as online education, interactive online gaming, and augmented/virtual reality (AR/VR) usually require not only extremely high data rate but also ultra-low end-to-end network latency [1]. How to satisfy these stringent requirements is a significant challenge for designing the sixth-generation (6G) indoor wireless networks.

Recently, the International Telecommunication Union (ITU) has identified Fiber-to-The-Room (FTTR) wireless local area networks (WLANs) as a novel holistic solution to enable extreme user experience for indoor communications [2]. The FTTR WLAN enables dense deployment of access points (APs), referred to as optical network units (ONUs), which are interconnected using a high-capacity passive optical network (PON) [3], [4], to serve indoor mobile devices. By employing the basic service set (BSS) coloring technique as introduced in the IEEE 802.11ax WLAN [5], FTTR WLANs enable full reuse of the available frequency band across all ONUs for wireless channel access and communication. Consequently, rather than accessing only APs in the nearest distance or with the (biased) maximal received power as in conventional WLANs [6], the mobile devices in FTTR WLANs can flexibly access multiple ONUs in a probabilistic manner to mitigate collisions during carrier sense multiple access with collision avoidance (CSMA/CA) based data transmission [7]. Meanwhile, thanks to the reduced communication distances between mobile devices and the ONUs and the efficient spatial frequency reuse, FTTR WLANs are expected to significantly improve the data rate, spectral and energy efficiency of indoor wireless networks.

Despite the aforementioned advantages, dense deployment of ONUs in FTTR WLANs may increase the delay incurred for data packet transmissions over the PON. This is because the PON is shared by the ONUs and typically operates in a time-division multiplexing (TDM) manner. Hence, a trade-off between wireless and optical communication delays exists in FTTR WLANs. This needs to be analyzed for optimizing indoor network services and management, but faces several challenges. First, analysis of the flexible ONU access policy adopted by FTTR WLANs depends critically on the deployment of the ONUs, the complex indoor propagation environment, and the interference in the physical layer. Second, the delay of FTTR WLANs can be impacted by both the CSMA/CA based wireless transmission and the TDM based packet scheduling, including the access probability and collision probability of wireless medium access control (MAC) layer and packet scheduling of optical network layer. Last but not least, the timeliness requirements of the extreme broadband low-latency applications needs to be explicitly included.

To address these research challenges, this paper presents a holistic cross-layer delay analysis for FTTR WLANs. In particular, we propose a new performance metric referred to as the average full-link AoI (FL-AoI) to evaluate the timeliness of broadband low-latency wireless applications in FTTR WLANs. The FL-AoI is defined based on the age of information (AoI) metric [8], which has been widely adopted in cognitive radio based Internet of Things (IoT) systems [9], satellite-IoT systems [10], ultra-reliable low-latency vehicular networks [11], [12], and WLANs [13], [14]. In our context, the FL-AoI can capture the timeliness of both wireless and optical communications involved in the FTTR WLAN. To analyze the FL-AoI, we consider the uplink of a dense FTTR WLAN with randomly distributed ONUs and mobile devices. A channel model based on the stretched exponential path loss (SEPL) model is adopted to capture the characteristics of the

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indoor wireless communications over short distances. With the aid of stochastic geometry theory and queueing theory, the probabilities of accessing different ONUs and the average FL-AoI are then derived taking into account the both the CSMA/CA based wireless transmission to the ONUs and the TDM based packet scheduling over the PON. Our results show that there exists an optimal transmission distance to minimize the average FL-AoI, whose value depends on the density of ONUs. Moreover, dense deployment of ONUs can significantly lower the average FL-AoI, particularly when the mobile device has small transmission distances.

The remainder of the paper is organized as follows. The system model of FTTR WLAN in the uplink is presented in detail in Section II. Section III analyzes the access probability of FTTR WLAN based on stochastic geometry theory and Section IV provides the average FL-AoI performance for uplink transmission in FTTR WLAN. Section V provides the simulation results and finally, Section VI concludes this paper.

II. System Model

A. Network Model

As shown in Fig. 1, we consider an FTTR WLAN consisting of an optical line terminal (OLT) and N ONUs. The OLT is connected with all ONUs over a shared PON such as Ethernet PON (EPON) or Gigabit PON (GPON) [4]. The OLT is further connected to the Internet gateway. Meanwhile, the ONUs are equipped with both wireless and optical transceivers to provide indoor communication services for K mobile devices.

Without loss of generality, in this paper, we focus on the uplink scenario, where each mobile device aims to timely transmit its data packets to the Internet via the OLT. Let $\mathcal{N} = \{1, ..., n, ..., N\}$ and $\mathcal{K} = \{1, ..., k, ..., K\}$ be the index sets of the ONUs and the mobile devices, respectively. We assume that the FTTR WLAN is deployed with high density in indoor hotspots such as shopping malls, airports and corporate offices. Our aim is to evaluate the asymptotic performance of the considered FTTR WLAN, when N and K are relatively large. For a tractable analysis, we assume that the locations of the ONUs, denoted by $\Phi_g = \{g_n, n \in \mathcal{N}\}$, follow a Poisson Point Process (PPP) with density λ_g [15], [16]. Meanwhile, the locations of the mobile devices, denoted as $\Phi_l = \{l_k, k \in \mathcal{K}\}$, follow a homogeneous PPP with density μ_l, independent of Φ_g. Note that the PPP has been widely applied for accurate performance analysis of large-scale wireless networks [17], [18].

B. Wireless and Optical Communication Protocols

1) ONU Selection: The ONUs utilize the same bandwidth of B Hz to provide wireless access services to the mobile devices. Due to the dense deployment of ONUs, a mobile device may locate in the communication range of multiple ONUs and can flexibly choose an ONU from them for accessing the FTTR WLAN. We assume that each mobile device employs a probabilistic policy for determining the ONU to be accessed. Particularly, mobile device k randomly accesses ONU n with probability $C^A_{n,k} \in [0, 1]$, where $\sum_{n\in\mathcal{N}} C^A_{n,k} = 1$. The mobile device will keep accessing the selected ONU

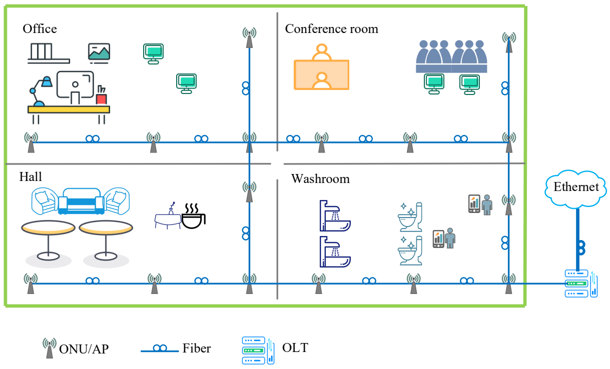

Fig. 1. System model of an FTTR WLAN.

within a relatively long time period, but can switch to other ONUs in different time periods.

Let $C(r_{n,k})$ be the probability that the typical mobile device k can successfully access ONU n. A well-known probabilistic selection policy is to choose the access probability $C^A_{n,k}$ in proportion to the access probability $C(r_{n,k})$, that is

$$C^A_{n,k} = \frac{C(r_{n,k})}{\sum_{\zeta=1}^{N_s} C(r_{\zeta,k})}, \tag{1}$$

where N_s is the number of ONUs that are in the communication range of mobile device k.

2) CSMA/CA based Wireless Transmission: The mobile devices adopt the CSMA/CA protocol as in IEEE 802.11ax WLANs [5] for reliable and efficient uplink communications with the selected ONUs. On the one hand, CSMA/CA enables efficient spatial reuse of the adopted frequency spectrum across all ONUs of the dense FTTR WLAN. To mitigate co-channel interference and collisions within the BSSs, where a BSS includes an ONU and its connected mobile devices, CSMA/CA employs the BSS coloring mechanism [5], [19]. Thereby, intra-BSS transmissions are indicated by the same information field of BSS color. If the same color is detected among overlapping BSSs, the ONUs will initiate a color change process [19].

On the other hand, to avoid collision during channel access, CSMA/CA requires each mobile device to first sense the channel activity associated with its intended ONU. When the channel of the intended ONU is sensed to be idle, the mobile device sends a Request-To-Send (RTS) message to the ONU. We assume that the mobile device transmits signals with power $P_{\max}$. The ONU measures the power of the received signals, particularly that of the received 802.11ax preamble symbols, to determine whether this connection is feasible [20]. If the signal power received at the ONU exceeds a given signal threshold, denoted by ρ_c, the ONU replies to the mobile device with a Clear-To-Send (CTS) message to allow the start of data transmission. The RTS/CTS exchange enables the channel of the intended ONU to be reserved for the mobile device during the transmission of an entire data packet.

However, if the channel is sensed to be busy or a collision occurs when multiple mobile devices contend to communicate with the same ONU at the same time during RTS/CTS exchange, an exponential backoff mechanism is activated at the mobile device to mitigate competition between the mobile devices. Thereby, the mobile device has to wait for a time period of length T_{bt} before starting transmission. T_{bt} is randomly selected from the contention window defined as $[0, 2^m\Xi - 1]$, provided $m \leq m_{\max}$. m and Ξ are the number of collisions and the minimum value of the contending window, respectively, and $m_{\max}$ is the "maximum backoff stage" [12]. During waiting, the mobile device keeps sensing the channel. If the channel is sensed to be idle, T_{bt} is reduced by one; otherwise, it remains unchanged. When T_{bt} vanishes, the mobile device can attempt the packet transmission again, using the same procedure as have presented above.

3) TDM Scheduling over PON: While conventional APs can promptly transmit received data packets to the Ethernet, in the FTTR WLAN, the ONUs have to store/buffer the data packets received from the mobile devices before converting and conveying them for optical transmission to the OLT over the PON. Due to the shared nature, the PON typically operates in a TDM manner. Particularly, each ONU informs the OLT about the volume of its stored data packets. Accordingly, the OLT allocates orthogonal time slots to the ONUs, where the duration of time slot can be optimized for each ONU.

In this paper, we assume that the data packets are generated at each mobile device according to a Poisson distribution with arrival rate ξ_1. Each data packet contains D nats and is encoded into Z symbols for wireless transmission to the ONUs, incurring a transmission delay of $\tau_B = Z/B$. The resulting packet error rate is given by [12]

$$v_{n,k} = Q\left(\frac{\sqrt{Z}\left(1+\gamma_{n,k}\right)\left(\ln\left(1+\gamma_{n,k}\right) - D/Z\right)}{\sqrt{\gamma_{n,k}\left(\gamma_{n,k}+2\right)}}\right), \quad (2)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ is the Q-function and $\gamma_{n,k}$ is the instantaneous signal-to-interference-plus-noise ratio (SINR) at ONU n for receiving from typical mobile device k. Moreover, let $C_{k,b0}$ be the probability that the mobile device k can transmit, implying that T_{bt} has reduced to zero, in a randomly chosen time. Then the probability that ONU n successfully receives a packet from mobile device k in the FTTR within time τ_B is given by $\kappa_{n,k} = C_{k,b0}(1 - v_{n,k})$. Hence, the data packets arriving at ONU n from mobile device k follow a Poisson process with arrival rate of $\xi_1\kappa_{n,k}$.

Since the scheduling of the OLT can be an arbitrary deterministic or stochastic function of the packet arrivals, in this paper, we model the data packet transmission over the PON for each ONU as an $M/G/1$ first-come first-served (FCFS) queueing system [21].

C. Average FL-AoI

Based on Section II-B, due to the dense deployment of ONUs and the resulting spatial frequency reuse for CSMA/CA, the FTTR WLAN can reduce the time required for data packet (re-)transmission from the mobile devices to the ONUs. However, data packet buffering and TDM scheduling may unfavorably penalize the time required for transmitting the data packets over the TDM-based PON. For a comprehensive evaluation of packet transmission latency in the FTTR WLAN, we considered the average FL-AoI as defined blow.

Let us first recall the definition of AoI. Assume that mobile device k transmits its data packets to a given selected ONU. Let t be the current time and $t_{g(t)}$ be the generation time stamp of the packet that is most recently received at the OLT. The instantaneous AoI at time t, denoted by $\Delta(t)$, can be calculated as $\Delta(t) = t - t_{g(t)}$. The instantaneous AoI keeps increasing with time and only decreases upon reception of a new data packet at the OLT. In the latter case, the age is reset to the difference of the current time instant and the generation time stamp of the received update.

Taking into account the randomness of ONU access, the average FL-AoI is defined as

$$\overline{\Delta}_{\tau,k} = \sum\nolimits_{\zeta=1}^{N_s} C_{\zeta,k}^{A} \Delta_{\zeta,k}. \quad (3)$$

where $\Delta_{\zeta,k}$ is the conditional average AoI, on the condition that the typical mobile device k accesses ONU ζ, and $\Delta_{\zeta,k} = \lim\limits_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \Delta(t)dt$. In the following, we analyze the access probability and the conditional average AoI in Sections III and IV, respectively. Based on the derived results, the FL-AoI can be promptly obtained according to (3).

III. Analysis of Access Probability

In this section, we analyze the access probability based on the stochastic geometry theory, taking into account the impact of dense deployment of ONUs, the resulting indoor channel characteristics, and the CSMA/CA protocol in FTTR WLANs.

A. SEPL based Wireless Channel Model

We consider both small- and large-scale propagation effects in the indoor wireless channel of the dense FTTR WLAN. The ONU n located at $g_n \in \Phi_g$ receives power $P_{n,k}$ from mobile device k transmiting at power P_k from location $l_k \in \Phi_l$ is given by $P_{n,k} = P_k H_{n,k} e^{-\alpha r_{n,k}^{\beta}}$, where $H_{n,k}$ captures the small-scale channel fading cause by multi-path indoor signal propagation from mobile device k to ONU n. We consider Rayleigh fading, i.e., $H_{n,k} \sim \exp(1)$ is exponentially distributed with unit mean power. Besides, $r_{n,k} = \|g_n - l_k\|$ and $e^{-\alpha r_{n,k}^{\beta}}$ denote the Euclidean distance and the resulting path loss between mobile device k and AP n, respectively. Here, the SEPL model has been adopted for characterizing the path loss of indoor propagation, where α and β are the multiplicative attenuation parameter and the scaling parameter, respectively, to be flexibly tuned according to the propagation environment. For example, we set $\beta = 1$ when the number of obstacles grows linearly with the distance between the mobile device and the ONU [22]. For a tractable analysis, identical path-loss model parameters α and β are assumed within the FTTR WLAN.

B. Analysis of Transmission Distance

For a typical mobile device located at the origin o and the ONU located at g_0, let $d(o, g_0) = H_{g_0,o} \exp\left(-\alpha \|g_0\|^{\beta}\right)$ be the resulting attenuation of wireless signals during uplink transmission, where $H_{g_0,o}$ is the channel fading and $\exp\left(-\alpha \|g_0\|^{\beta}\right)$ is the path loss. The associated propagation

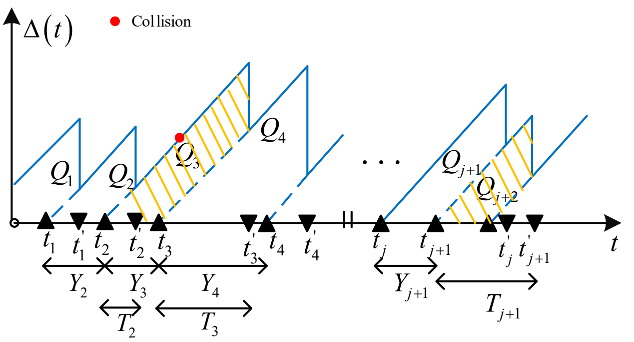

Fig. 2. Evolution of AoI over time.

process is defined as $U_{n,k} \triangleq \{d(o,g_0), g_0 \in \Phi_g\}$, which is obtained by mapping each point of the PPP Φ_n according to function $d(o,g_0)$. Hence, $U_{n,k}$ is also a homogeneous PPP, where the probability density function (PDF) of the distance for the n-th nearest ONU is given by $f_R(r) = \frac{2}{\Gamma(n)}(\lambda_g\pi)^n r^{2n-1}e^{-\lambda_g\pi r^2}$ [16]. Considering the transmit power of mobile devices $P_{\max}$ and the receiving signal threshold of ONUs, the maximum tolerable path loss for typical mobile device is given by $r_{\max} \triangleq \left(\frac{1}{\alpha}\ln\frac{P_{\max}}{\rho_c}\right)^{\frac{1}{\beta}}$ under the SEPL model. Then, the average number of ONUs that can serve the mobile device in the considered FTTR WLAN can be given as

$$N_s = \min\left\{N, \mathrm{Floor}(\lambda_g\pi r_{\max}^2)\right\}, \tag{4}$$

where $\mathrm{Floor}(\cdot)$ is the integer-rounding function.

Moreover, the PDF of the path loss between the mobile device and its n-th nearest ONU, conditioning on $l \le r_{\max}$, is given as

$$\begin{aligned} f_{l\le r_{\max}}(l) &= \frac{f_R(l)}{\Pr(l \le r_{\max})} \\ &= \frac{\frac{2}{\Gamma(n)}(\lambda_g\pi)^n l^{2n-1}e^{-\lambda_g\pi l^2}}{\int_0^{r_{\max}}\frac{2}{\Gamma(n)}(\lambda_g\pi)^n l^{2n-1}e^{-\lambda_g\pi l^2}dl} \\ &= \frac{2(\lambda_g\pi)^n l^{2n-1}e^{-\lambda_g\pi l^2}}{\Gamma(n)-\Gamma(n,\lambda_g\pi r_{\max}^2)}. \end{aligned} \tag{5}$$

C. Analysis of Access Probability

Assume that the typical mobile device connects to the n-th nearest ONU, denoted as ONU n with a slight abuse of notation. For uplink transmission from the mobile device, the resulting received SINR of ONU n is given as

$$\gamma_{n,k} = \frac{P_{\max}H_{n,k}e^{-\alpha r_{n,k}^\beta}}{N_0+\sigma_I}, \tag{6}$$

where N_0 is noise power and $\sigma_I = \sum_{i\in\mathcal{K}, i\ne k}P_{\max}H_{n,i}e^{-\alpha r_{n,i}^\beta}$ is the interference power caused by mobile devices transmitting to other ONUs. Note that as each ONU allocates channel independently, the typical mobile device having broadband and low-latency wireless applications will occupy broad spectrum and hence, we assume all active mobile device can interfere with typical mobile device. Then $C(r_{n,k})$ can be calculated in (7), where (a) is due to $H_{n,k} \sim \exp(1)$. γ_0 is the SINR threshold for ONUs to successfully receive and decode the mobile device's signal. $\mathcal{L}_{H_{n,i}}(\cdot)$ is the Laplace transform of PDF of the addition interference σ_I, and is given as

$$\begin{aligned} &\mathcal{L}_{H_{n,i}}\left(\gamma_0 e^{\alpha r_{n,k}^\beta}\right) \\ &= \exp\left(\mathbb{E}_{r_{n,i}}\left[-2\pi\lambda_g\int_0^\infty\left(1-\exp\left(-\gamma_0 e^{\alpha r_{n,k}^\beta}H_{n,i}e^{-\alpha z^\beta}\right)\right)zdz\right]\right) \\ &\overset{(b)}{=} \exp\left(\frac{2\pi\lambda_g}{\beta(-\alpha)^{\frac{2}{\beta}}}\int_0^1\frac{\mathbb{E}_{r_{n,i}}\left[1-\exp\left(-\gamma_0 e^{\alpha r_{n,k}^\beta}H_{n,i}x\right)\right]}{x}\ln(x)^{\frac{2-\beta}{\beta}}dx\right) \\ &\overset{(c)}{=} \exp\left(-\frac{2\pi\lambda_g}{\beta\alpha^{\frac{2}{\beta}}}\int_0^1\frac{\gamma_0 e^{\alpha r_{n,k}^\beta}}{1+\gamma_0 e^{\alpha r_{n,k}^\beta}x}(-\ln(x))^{\frac{2-\beta}{\beta}}dx\right) \\ &= \exp\left(\frac{2\pi\lambda_g\alpha^{-\frac{2}{\beta}}\Gamma\left(\frac{2}{\beta}\right)\mathrm{Li}_{\frac{2}{\beta}}\left(-\gamma_0 e^{\alpha r_{n,k}^\beta}\right)}{\beta}\right), \end{aligned} \tag{8}$$

where (b) follows by substituting $x = \exp(-\alpha z^\beta)$ and (c) is due to averaging over the channel fading power, $H_{n,i} \sim \exp(1)$. $\Gamma(x)$ and $\mathrm{Li}_z(x)$ denote the Gamma and Ploylog functions [23], respectively, where $\mathrm{Li}_{z+1}(x) = \int_0^x \frac{\mathrm{Li}_z(t)}{t}dt$ and $\mathrm{Li}_1(x) = -\ln(1-x)$.

Substituting (7) and (8) into (1), the access probability of typical mobile device for connecting ONU n is given in (9). From (9) we know that the access probability can have a significant impact on the average FL-AoI. This is because, according to (7), the path loss affects the access probability of each mobile device.

IV. Analysis of Average FL-AoI

Based on the derivation results of Section III, in this section, we further analyze the average FL-AoI, taking into account both the wireless and optical communication processes in the FTTR WLAN.

A. Reformulation of FL-AoI

Assume that the mobile device k accesses the ONU n for data packet transmission. The time evolution of AoI for transmitting a sequence of data packets from the mobile device is shown in Fig. 2. Let t_j and t_j' denote the time instants at which the jth data packet was generated at mobile device k, and arrived at the accessed ONU, respectively. Then, the time spent in the system by the jth packet is a random variable given by

$$T_j = t_j' - t_j. \tag{10}$$

Meanwhile, let the random variable Y_j represent the jth interarrival time of data packets generated mobile device k,

$$
\begin{aligned}
C(r_{n,k}) = \mathbb{P}\left\{\gamma_{n,k} > \gamma_0\right\} &= \int_0^{r_{\max}} \mathbb{P}\left\{ H_{n,k} > \frac{e^{\alpha r_{n,k}^{\beta}}(N_0+\sigma_I)\gamma_0}{P_{\max}} \right\} \cdot f_{r_{n,k}\le r_{\max}}(r_{n,k})\mathrm{d}r_{n,k}, \\
&= \frac{2(\lambda_g\pi)^n}{\Gamma(n)-\Gamma(n,\lambda_g\pi r_{\max}^2)} \cdot \int_0^{r_{\max}} r_{n,k}^{2n-1} e^{-\lambda_g\pi r_{n,k}^2} \cdot \mathbb{P}\left\{ H_{n,k} > e^{\alpha r_{n,k}^{\beta}}\gamma_0 (\frac{N_0}{P_{\max}} + \sum_{i\in\mathcal{K}, i\neq k} H_{n,i}e^{-\alpha r_{n,i}^{\beta}}) \right\} \mathrm{d}r_{n,k} \\
&\overset{(a)}{=} \frac{2(\lambda_g\pi)^n}{\Gamma(n)-\Gamma(n,\lambda_g\pi r_{\max}^2)} \cdot \int_0^{r_{\max}} r_{n,k}^{2n-1} e^{-\lambda_g\pi r_{n,k}^2} \exp(-e^{\alpha r_{n,k}^{\beta}}\frac{\gamma_0 N_0}{P_{\max}}) \cdot \mathcal{L}_{H_{n,i}}\left(\gamma_0 e^{\alpha r_{n,k}^{\beta}}\right)\mathrm{d}r_{n,k}.
\end{aligned} \tag{7}
$$

$$
C_{n,k}^{A} = \frac{\frac{2(\lambda_g\pi)^n}{\Gamma(n)-\Gamma(n,\lambda_g\pi r_{\max}^2)} \cdot \int_0^{r_{\max}} r_{n,k}^{2n-1} e^{-\lambda_g\pi r_{n,k}^2} \exp(-e^{\alpha r_{n,k}^{\beta}}\frac{\gamma_0 N_0}{P_{\max}}) \cdot \mathcal{L}_{H_{n,i}}\left(\gamma_0 e^{\alpha r_{n,k}^{\beta}}\right)\mathrm{d}r_{n,k}}{\sum_{\zeta=1}^{N_s} \frac{2(\lambda_g\pi)^{\zeta}}{\Gamma(\zeta)-\Gamma(\zeta,\lambda_g\pi r_{\max}^2)} \cdot \int_0^{r_{\max}} r_{\zeta,k}^{2\zeta-1} e^{-\lambda_g\pi r_{\zeta,k}^2} \exp(-e^{\alpha r_{\zeta,k}^{\beta}}\frac{\gamma_0 N_0}{P_{\max}}) \cdot \mathcal{L}_{H_{\zeta,i}}\left(\gamma_0 e^{\alpha r_{\zeta,k}^{\beta}}\right)\mathrm{d}r_{\zeta,k}}. \tag{9}
$$

i.e., the time elapsed between the generation of packets $j-1$ and j, where

$$Y_j = t_j - t_{j-1}. \tag{11}$$

As shown in Fig. 2, the jth local maximum/peak value of the AoI is given by $\Delta_{P_j} = Y_j + T_{j-1}$. Moreover, let Ω_t be the number of successfully received data packets, referred to as effective data packets, till time t. That is, $\Omega_t = \max\{j \mid t'_j \le t\}$. Then, the average AoI from device k to the ONU of the considered FTTR WLAN is defined as

$$\Delta_P = \lim_{t\to\infty} \frac{1}{\Omega_t}\sum_{j=1}^{\Omega_t} \Delta_{P_j} \overset{(d)}{=} \mathbb{E}(Y) + \mathbb{E}(T), \tag{12}$$

where $\mathbb{E}(\cdot)$ is the expectation operator. In (d), the AoI evolution is assumed to be an ergodic process, such that the time average of AoI converges to its expected value when time t and Ω_t are sufficiently large. Moreover, let Δ_G be the average age of scheduling from ONUs to the OLT. We have

$$\Delta_{\zeta,k} = \mathbb{E}(\Delta_P) + \mathbb{E}(\Delta_G). \tag{13}$$

In the remainder of this section, we derive $\mathbb{E}(\Delta_P)$ and $\mathbb{E}(\Delta_G)$ separately.

B. Derivation of AoI from Mobile Device to ONU

For the considered CSMA/CA protocol with RTS/CTS exchange, the probability that the mobile device k transmits in a randomly chosen time slot is given by [24]

$$C_{k,b0} = \frac{2\left(1-2C_{k,a}\right)}{\left(1-2C_{k,a}\right)\left(\Xi+1\right) + C_{k,a}\Xi\left[1-\left(2C_{k,a}\right)^{m_{\max}}\right]}, \tag{14}$$

where $C_{k,a}$ is the probability of collision when mobile device k starts the transmission, given as

$$C_{k,a} = \frac{2\Xi(K_s-1)}{(\Xi+1)^2 + 2\Xi(K_s-1)}. \tag{15}$$

Moreover, K_s is the average number of mobile devices that can access ONU n and is approximated as

$$K_s \approx \min\left\{\mathrm{Floor}(K/N), \mathrm{Floor}(\mu_l\pi r_{\max}^2)\right\}. \tag{16}$$

Since data packets arrive at the ONUs following the Poisson process with intensity $\xi_1\kappa_{n,k}$, we have

$$\mathbb{E}(Y) = \frac{1}{\xi_1\kappa_{n,k}}, \tag{17}$$

where $\kappa_{n,k} = C_{k,b0}(1-\upsilon_{n,k})$.

Meanwhile, as $\mathbb{E}(T_{j-1})$ composes the time required for successfully transmitting the $(j-1)$th effective data packet from the mobile device to the ONU, we have

$$\mathbb{E}(T) = \mathbb{E}(T_{j-1}) = \tau_B \tag{18}$$

Hence, Δ_P can be rewriten as

$$\mathbb{E}(\Delta_P) = \frac{1}{\xi_1\kappa_{n,k}} + \tau_B. \tag{19}$$

C. Derivation of AoI from ONU to OLT

Recall that the buffer at ONU n is modeled as a single-source $M/G/1$ FCFS queueing model, where the mobile device k generates data packets according to a Poisson process. We assume that the expectation and variance of the service time of a data packet over the PON, denoted by $\mathbb{E}(\tilde{T})$ and $\mathrm{Var}(\tilde{T})$, respectively, are finite and known. Then, the average AoI is given by [25]

$$
\begin{aligned}
\mathbb{E}(\Delta_G) = \mathbb{E}(\tilde{T}) \ &+ \frac{\xi_1\kappa_{n,k}[\mathbb{E}^2(\tilde{T}) + \mathrm{Var}(\tilde{T})]}{2(1-q)} \\
&+ \frac{1-q}{\xi_1\kappa_{n,k}\mathcal{L}(\xi_1\kappa_{n,k})}.
\end{aligned} \tag{20}
$$

where $q = \mathbb{E}(\tilde{T})\xi_1\kappa_{n,k}$ and $\mathcal{L}(\xi_1\kappa_{n,k}) = \mathbb{E}[e^{-\xi_1\kappa_{n,k}\tilde{T}}]$ is the Laplace transform of the service time. Finally, the average FL-AoI of the considered FTTR WLAN is given as

$$\overline{\Delta}_{\tau,k} = \sum_{\zeta=1}^{N_s} \frac{C(r_{\zeta,k})}{\sum_{\zeta=1}^{N_s} C(r_{\zeta,k})}[\mathbb{E}(\Delta_P) + \mathbb{E}(\Delta_G)]. \tag{21}$$

V. Simulation Analysis

In this section, the derived analytical results are validated using Monte Carlo simulations, where the impact of system parameters on the system performance is evaluated. Unless

TABLE I: Simulation Parameters.

Total number of channel uses, Z	300
System bandwidth, B	5 MHz
Maximum transmission power, $P_{\max}$	30 dBm
Signal threshold, ρ_c	-80 dBm
Minimum value of the contending window, Ξ	32
Maximum number of backoff stages, $m_{\max}$	5
SINR threshold, γ_0	-10 dBm
Packet arrival rate, ξ_1	10
Density of ONUs, λ_g	0.05
SEPL parameter, α	1
SEPL parameter, β	1/2
Size of data packet, D	300 Nats

otherwise specified, the simulation parameters are set according to Table I [6].

Fig. 3 shows the access probabilities to different ONUs in the FTTR WLAN as a function of the maximal transmission distance, $r_{\max}$. From Fig. 3 we observe that the when $r_{\max}$ is small, the mobile devices will access the ONUs in close proximity with a high probability. This is because these ONUs have a high receiving SINR and hence, a high likelihood of successfully receiving from the mobile device, i.e., $C(r_{n,k})$ has a large value for small n. As $r_{\max}$ increases, the access probability decreases as the mobile devices will access ONUs located far away with an increased probability. When $r_{\max}$ becomes large, the mobile devices can access all deployed ONUs with an equal probability. This is because $C(r_{n,k})$ will approach a constant independent of n, cf. (7).

Fig. 4 evaluates the average FL-AoI as a function of the maximal transmission distance $r_{\max}$ for different densities of ONUs. We observe from Fig. 4 that for all considered densities of ONUs, the average FL-AoI decreases with $r_{\max}$ when $r_{\max}$ is small, but increases with $r_{\max}$ and even saturates when $r_{\max}$ becomes large. This is because, when $r_{\max}$ is small, the mobile devices will access the nearest ONUs with high probability. Hence, as $r_{\max}$ increases, fewer mobile devices would contend to communicate with the same ONU, which decreases the collision probability and re-transmission delay. By contrast, when $r_{\max}$ is large, the collision probability increases as the mobile devices can access more deployed ONUs.

From Fig. 4 we also observe that dense deployment of ONUs achieves the lowest FL-AoI when $r_{\max}$ is small, whereas the opposite behavior is observed when $r_{\max}$ is large. This is because the mobile device tends to access the nearest ONUs due to the dense deployment of ONUs. However, as deployment density of ONUs increases, the increase of collision probability will lead to larger FL-AoI. Hence, a trade-off between the number of ONUs that the mobile device can access N_s and collision probability exists. Interestingly, the average FL-AoI tends to saturate to the same value for different considered densities of ONUs. This is because when $r_{\max}$ is large, in addition to the equalized access probabilities and collision probabilities to all ONUs, cf. (9) and (15) , the transmission of the packets queued at the ONUs over the PON also assumes the same average service rate.

Finally, Fig. 5 evaluates the average FL-AoI of FTTR WLAN as a function of the number of channel uses or the blocklength, Z, for different path loss parameters. From Fig. 5

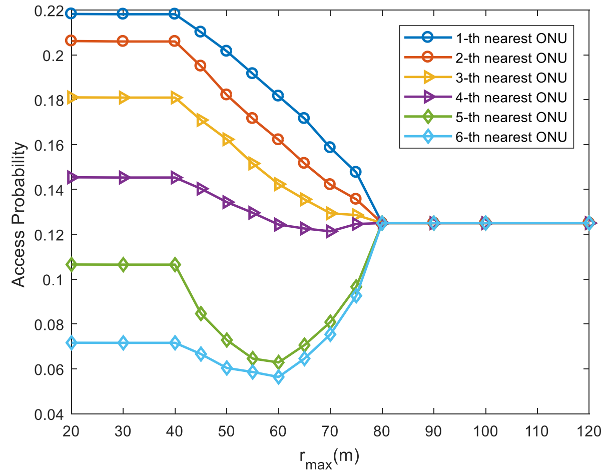

Fig. 3. Access probability $C^A_{n,k}$ vs. the maximal transmission distance $r_{\max}$.

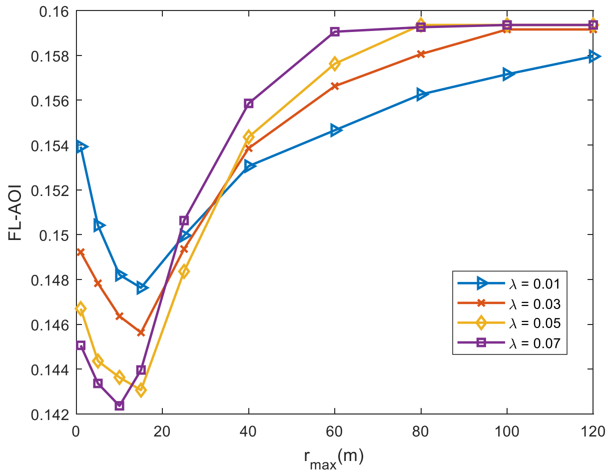

Fig. 4. FL-AoI vs. the maximal transmission distance $r_{\max}$.

we observe that for all considered path loss parameters, the average FL-AoI of FTTR WLAN decreases with Z in the small blocklength regime. This is because when Z increases , the packet error rate decreases as shown in (2) and hence, the re-transmission delay decreases which makes the average FL-AoI decreases. Meanwhile, Fig. 5 also shows that the average FL-AoI increases with Z in the large blocklength regime. This is because the PON will consume more time to schedule the data packets in the buffers, which increases the average FL-AoI. Therefore, there exists an optimal blocklength for minimizing the average FL-AoI, due to the tradeoff between the access delay and queueing delay in the PON. From Fig. 5 we further observe that the average FL-AoI decreases with β. This is because, as β increases, the mobile devices can more easily access the nearest ONUs, reducing the retransmission of data packets.

VI. Conclusion

In this paper, the access probability and average FL-AoI of FTTR WLAN are analyzed for uplink mobile devices. We

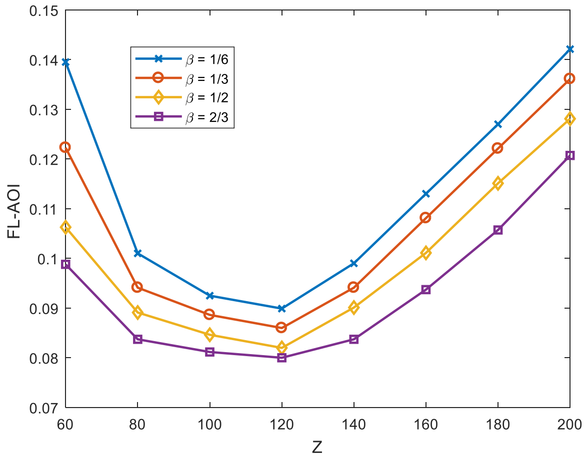

Fig. 5. FL-AoI vs. the number of channel uses Z.

derive the closed-form expressions of access probability under the SEPL based indoor wireless channel model. We show that when the maximal transmission distance is large enough, the mobile device will have the same access probability to all ONUs in the FTTR WLAN. Meanwhile, FTTR WLAN can significantly reduce the collision probability and hence, the delay for mobile devices. Moreover, the path loss parameters of the adopted SEPL model have significant impact on the average FL-AoI. These results provide guidelines for practical deployment and operation of low-latency FTTR WLANs.

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