Kilian Kiekenap, and Anja Klein, ”Optimum Sensor Value Transmission Scheduling for Linear Wireless Networked Control Systems,” in *2020 IEEE 92nd Vehicular Technology Conference (VTC2020-Fall) 2020*, November 2020.

Optimum Sensor Value Transmission Scheduling for Linear Wireless Networked Control Systems

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Abstract—**In this work, an optimum communication resource allocation for sensor value transmission of wireless networked control systems consisting of independent linear subsystems is calculated. To use new service types of upcoming mobile radio standards in an optimal way, the communication resources used by each subsystem have to be adapted to the current individual subsystem requirements. In our case, the scheduling of the transmission of fresh sensor values to the controller of the subsystems is optimized. The quality of scheduling is directly related to the deviation of a subsystem from its control goal. A proof for the optimality of a regular updating scheme for minimizing this deviation is given. Based on this regularity, the optimum communication resource allocation for a limited amount of resources and a set of control systems with different given characteristics is derived. Using the resulting resource shares for the subsystems from this optimization, an algorithm to schedule the actual transmissions during runtime is given. This split approach saves computational effort during system operation. To simplify the evaluation of different scheduling policies, a deterministic calculation of the expected cost with mean absolute error cost function for each subsystem is presented, which removes the need of Monte-Carlo experiments for evaluation.**

Index Terms—**Cyber-Physical Systems, Communication and Control, Networked Control Systems, Internet of Things**

I. Introduction

The current roll-out of 5G mobile networks will greatly influence the types of services used over mobile networks. While the growth of the number of personal devices is already slowing down, the fraction of traffic of autonomous devices communicating with each other is rapidly growing [1]. While previous generations of mobile networks focused on human users and high data rates, 5G networks will also provide communication channels particularly suited for industrial communication and control applications [2], as well as connected cars and autonoumous driving [3]. Despite the ever-growing number of devices per cell and increased data rates, also latency and reliability guarantees can be established [4]. This allows for new services, especially from the control domain, which heavily depend on latency guarantees [5]. One important application from the control domain is autonomous driving, where information from sensors in the vehicles and along the road has to be transmitted between vehicles and central entities to manage traffic. Obviously, the amount of data to be exchanged varies heavily, depending on the number of vehicles per area, the weather conditions etc.

For Industry 4.0, a shift to wireless instead of the current wired connections is desired. Wireless systems are fastly reconfigurable and can be easily adapted to current production requirements. The real-time requirements of industrial control systems imply challenges on the communication system design very different from previous generations of communication systems. The control counterpart also has to be adapted to the specific characteristics of wireless connections. Current control systems rely on high data rates, low latency, and low error probabilities. The combination of control and wireless communication systems, called Wireless Networked Control Systems (WNCS), is an important area of current research, especially the joint optimization of both, to adapt either part to possibly varying conditions of each other. If, for example, an autonomous vehicle drives along a straight and empty road, the communication can be reduced. In a crowded city scenario with a lot of intersections, the communication effort is much higher. At the same time, the speed of the vehicle can be adapted to the available communication resources.

WNCS are composed of subsystems which communicate over a shared wireless communication channel to exchange sensor values and control commands. The subsystems either work cooperatively to achieve a common task, or they compete for communication and other resources.

WNCS can generally be divided into two types, centralized and decentralized, as shown in [6]. The subsystems of decentralized WNCS compete for communication resources in a shared communication medium such as a frequency band. Centralized WNCS, however, have a central scheduler (CS) which allocates resources to the individual subsystems. In this paper, the second type is considered. The allocation of communication resources to subsystems with different control parameters is optimized by a CS. The goal is to schedule measurement transmissions from sensors to controllers to minimize the plant state estimation error at the controller. The CS can request measurements from the distributed sensors and transmit them to the corresponding controller. Examples for sensors remote from the plants might be cameras, infrared thermometers or ultrasonic distance sensors. In the well-known inverted pendulum example from control theory, the sensor could be a camera observing the pendulum inclination and position.

II. Related Work

A very general solution to minimize the overall time passed since the last update of the sensor value at the controller is investigated in [7]. This ensures a timely update of the state information for all subsystems, but does not consider the individual subsystem dynamics. If the system has a CS to schedule the updates, which is aware of the dynamics, the scheduling decision can be further optimized. This system layout is investigated in [8], where the CS transmits the sensor values to the subsystems according to a precalculated schedule, which is based on the uncertainty about the current subsystem states. In each time step, a fixed number of transmission slots is available. The required optimization is a mixed-integer problem, which only gives a schedule for a fixed time horizon and is considerably hard because of its nonconvex structure. The restriction of the time limited schedule is relieved in [9], where the time passed since the last transmission of the sensor value for a subsystem is used to find the most outdated values. This approach is compared to a scheduling based on the error covariance of the current state estimate at the controller. The variance-based approach considers different dynamics of the subsystems. In [10], the problem is modeled as a Markov Decision Process (MDP) with the time passed since the last transmission for each subsystem as the state and the scheduling decision for the current time step as action. The model also incorporates packet-loss probabilities for the communication links, which results then in a deterministic scheduling policy. In [11], the optimum scheduling is done for WNCSs with lossy links, where the loss probability depends also on the number of sending subsystems.

In this paper, the focus is on the deterministic case without packet loss. There are multiple control loops and a CS, which is aware of the subsystems characteristics and schedules the transmissions from sensors and to controllers. The optimality of a fixed update frequency scheme for a minimum mean-square estimation error at the controller is shown. For derivation of the frequencies and the actual scheduling, we propose a two-step approach. In the first step, the individual update frequency for each subsystem is determined based on the system noise power and the subsystem dynamics. We show in Section IV, that this resource allocation problem is in fact a convex problem, which can be solved existing optimization frameworks like [12]. For the second step, we developed an algorithm in Section V, which schedules the available communication resources in each time step fairly to the subsystems according to the derived update frequencies from the first step. The main advantage of this two-step approach is the reduced effort during runtime, because only the second step has to be carried out during runtime. Finally, we show the advantage of our approach with numerical results in Section VI.

III. System Model

A. Notations

The expected value of a random variable is denoted by $\mathbb{E}(\cdot)$, the variance by $\mathrm{Var}(\cdot)$. Vectors and matrices are set in boldface, matrices with capital letters and vectors with small letters. The ith element of vector $\boldsymbol{x}$ is denoted by x_i. $\boldsymbol{x}(t)$ denotes the value of a vector time instant t.

B. Control System Model

The overall control system consists of N_{sys} independent subsystems as shown in Fig. 1. Each subsystem i consists

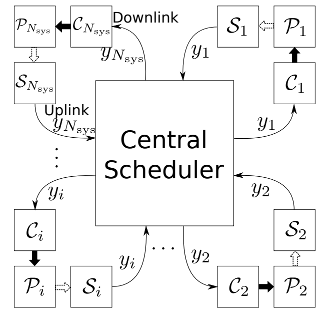

Fig. 1: System model of the WNCS

of a plant $\mathcal{P}_i$, a controller $\mathcal{C}_i$ and a sensor $\mathcal{S}_i$. A central scheduler polls measurements y_i from the sensor $\mathcal{S}_i$ (uplink) and forwards them to the controller $\mathcal{S}_i$ (downlink). Each subsystem $i \in 1, \ldots, N_{\text{sys}}$ is modeled as a discrete time linear system with a scalar state $x_i(k)$ at time instant k, a system coefficient a_i, measurement noise $w_i(k)$ and the control variable is $u_i(k)$. Each subsystem i follows a linear system equation

$$x_i(k+1) = a_i x_i(k) + u_i(k) + w_i(k). \tag{1}$$

The system noise $w_i(k)$ is assumed to be zero-mean Gaussian i.i.d. for all times k and all systems i with variance W_i. The subsystems can be observed according to the observation equation

$$y_i(t) = x_i(k) + v_i(t) \tag{2}$$

with the measurement noise $v_i(k) \sim \mathcal{N}(0, V_i)$. Since the system is linear with Gaussian noise, the Kalman filter [13] gives the minimum-mean-square error (MMSE) estimate $\hat{x}$ of the system state x based on the observation y. The estimation error is $e_i(k) = x_i(k) - \hat{x}_i(k)$. The control variable u_i is then calculated according to a deadbeat law, i.e. $u_i(k) = -a_i \hat{x}_i(k)$, to achieve the control goal $x_i = 0$ for all subsystems. A cost function

$$J(k) := |\boldsymbol{x}(k)| \tag{3}$$

for deviating from this goal is assumed. The limited communication resources only allow for sensor readings of scheduled time slots. The Kalman filter is modified to predict the intermediate values. The availability of a new value is described by a binary scheduling decision variable $\pi_i(k)$, which is set to 1, if the system i is scheduled in time slot k and to 0 otherwise. The expression $x(a|b)$ is used to denote quantity x at time instant a with the knowledge from time instant b. The modified Kalman filter with the estimation error covariance P_i of e_i, and the Kalman gain g_i is then given as

$$\hat{x}_i(k|k-1) = a_i \hat{x}_i(k-1|k-1) + u_i(k-1) \tag{4}$$

$$P_i(k|k-1) = a_i^2 P_i(k-1) + W_i \tag{5}$$

$$g_i(k) = \pi_i(k) \frac{P_i(k|k-1)}{P_i(k|k-1) + V_i} \tag{6}$$

$$\hat{x}_i(k|k) = \hat{x}_i(k|k-1) + g_i(k)\left(y_i(k) - \hat{x}_i(k)\right) \tag{7}$$

$$P_i(k|k) = \left(1 - g_i(k)\right) P_i(k|k-1) \tag{8}$$

Because the instantaneous value $x_i(k)$ is not known at the CS, π_i can only be based on P_i. The scheduling should minimize the uncertainty about the system state at the controller, which is induced by the system noise and results in the estimation error $e_i(k)$, which is also Gaussian distributed with zero-mean and variance P_i. So minimizing the uncertainty corresponds to minimizing $\sum_{i=1}^{N_{\text{sys}}} P_i(k) \quad \forall k$. The transmission of the current observation reduces the uncertainty. Henceforth, the measurement noise is assumed to be negligible, i.e. $V_i = 0$. This results in $g_i(t)$ being either 1 or 0 and the system state $x_i(k)$ as well as the error covariance P_i is set to zero every time a transmission to the controller is scheduled. After this, the system noise $w_i(k)$ adds uncertainty in every system time step k. If the system has been scheduled last in time step $l \leq k$, the variance P_i is

$$P_i = \operatorname{Var}(e_{i,k}) = \begin{cases} W_i(k-l) & \text{if } |a_i| = 1 \\ W_i \frac{1-a_i^{2(k-l)}}{1-a_i^2} & \text{else.} \end{cases} \tag{9}$$

Since only the time duration $d = k - l$ between consecutive transmissions at time steps k and l determines the error covariance P_i, a function $f_i(d)$ to calculate the sum of variances P_i after d timeslots without transmission can be written, as

$$f_i(d) := \sum_{m=l}^{k} P_i(m) = \begin{cases} W_i \frac{d(d+1)}{2} & \text{if } a_i = 1 \\ W_i \frac{1}{1-a_i^2}\left(1 - \frac{1-a^{2d}}{1-a_i^2}\right) & \text{else.} \end{cases} \tag{10}$$

The control cost function (3) can be minimized for each time step individually to minimize the overall cost. Because of the deadbeat control, the state x_i is always zero-mean Gaussian distributed. The remaining deviation from the control goal $x_i = 0$ after application of the control variable u_i derived from $\hat{x_i}$ is almost equal to the estimation error e_i, i.e. $\operatorname{Var}(x_i(k)) \approx \operatorname{Var}(e_i(k)) = P_i(k)$. The approximation comes from the fact that the measurement is received with delay due to latency induced by the communication. It is possible that more recent information is available at the CS, reducing the estimation error there, but it could not be used for better control, since it has not been transmitted to the controller yet. The expected value of the cost function J can then be calculated from the standard normal distribution function as

$$\mathbb{E}\{|x_i(k)|\} = \sqrt{\frac{2}{\pi}}\sqrt{\operatorname{Var}(x_i(k))}. \tag{11}$$

The overall control objective with respect to the scheduling is

$$\arg\min_{\pi} \textstyle\sum_{i=1}^{N_{\text{sys}}} \sum_{k=1}^{T_{\text{sim}}} \operatorname{Var}(x_i(k)). \tag{12}$$

C. Communication System Model

The communication time slots, denoted by t, are shorter than the control time slots, denoted by k. In each of the control system time slots k for subsystem i, T_i^s communication time slots t take place. Additionally, a sampling offset T_i^0 between the subsystems is used. The relation between t and k is then given like in [9] as $k_i(t) = \lfloor \frac{t-T_i^0}{T_i^s} \rfloor$. The communication system is based on the scheduling decisions π_i made by a CS and is used to transmit measurements from sensors to controllers, which are directly attached to the plants. Transmission takes place in a packet based fashion with equally sized packets. In one communication time step t, R_{UL} packets can be transmitted in the uplink and R_{DL} packets in the downlink. The system is assumed to have no packet loss. The scheduling decision is modeled by the variable $\boldsymbol{\pi}_{\text{UL}(t)} \in \mathbb{R}^{N_{\text{sys}} \times 1}$ and $\boldsymbol{\pi}_{\text{DL}(t)} \in \mathbb{R}^{N_{\text{sys}} \times 1}$ for uplink and downlink, respectively.

IV. Optimum Resource Allocation

A. Influence of Long-Term evaluation of subsystem variances

In [9], the error covariance P_i is used as Value of Information (VoI) to derive the scheduling π_i in a greedy fashion. This minimizes the uncertainty about subsystem i in the current time step, but (12) rather asks for the minimization of the overall sum of uncertainty. This means, a greedy scheduling decision might be suboptimal. A subsystem i with a system coefficient a_i can accumulate a large P_i over time, while another subsystem j with system constant a_j and $a_i < a_j$ has a lower P_j. The greedy policy will then schedule subsystem i first, but in the next time step, P_j might increase more than P_i of subsystem i is reduced. This results in an increased $\sum_i \sum_t P_i(t)$, which leads to larger estimation errors.

The functions f_i are used to consider this in the scheduling. Their exponential shape makes a regular scheduling of each subsystem desirable. The individual slope of f_i suggests an update rate depending on the system constant a_i, the system noise covariance W_i, and the available communication resources. Stable systems with $|a_i| < 1$ get a low or zero rate, while system with large a_i are scheduled more often.

B. Optimality of a Regular Update Scheme

To show the optimality of a regular update scheme, we consider a single subsystem i with system constant $a_i \neq 1$, a system noise variance of W_i, and a finite operation time horizon T_{sim}, i.e. $k = 1, \ldots, T_{\text{sim}}$. During the operation time, $N + 1$ sensor values are transmitted, where the first and last transmission take place at $k = 0$ and $k = T_{\text{sim}}$, respectively. The time durations between two consecutive transmissions are denoted by $d_{i,1}, \ldots, d_{i,N}$ with $d_{i,n} \geq 0, n = 1, \ldots, N$. Now, the scheduling minimizing the sum of error variances over k up to T_{sim} is to be found. Using (10), this leads to the optimization problem

$$\arg\min_{\boldsymbol{d}_i} \textstyle\sum_{n=1}^{N} f_i(d_{i,n}) \tag{13}$$

$$\text{s.t.} \quad \textstyle\sum_{n=1}^{N} d_{i,n} = T_{\text{sim}} \tag{13a}$$

with the Langragian

$$L(\boldsymbol{d}, \mu) = \inf_{\mu} \sum_{n=1}^{N} \frac{W_i}{1-a_i^2}\left(1 - \frac{1-a_i^{2d_{i,n}}}{1-a_i^2}\right) + \mu\left(d_{i,n} - T_{\text{sim}}\right) \tag{14}$$

and its partial derivatives with respect to the d_i:

$$\frac{\partial L}{\partial d_{i,n}} = \frac{W_i}{\left(1-a_i^2\right)^2}\left(-2\log\left(a_i\right) a_i^{2d_{i,n}}\right) + \mu. \tag{15}$$

Since μ, W_i and a_i are not depending on n, they are equal for all partial derivatives of $L(\mu, \boldsymbol{d}_i)$. This results in equal $d_{i,n}$ to bring all components of the gradient to zero, i. e., equal durations between the transmissions for a single subsystem are optimal for reducing the sum of error variances over time, but are not necessarily integer multiples of the time slot duration. This is also applicable in case of multiple subsystems. Henceforth, scalar d_i will be used for the durations between consecutive transmissions of subsystem i.

C. The rate optimization problem

To find the optimum duration d_i for each subsystem, the optimization problem (13) is modified to use an update rate $r_i = \frac{1}{d_i}$. The results from [9] suggest a resource allocation, which is defined by the $\min\{R_{\text{DL}}, R_{\text{UL}}\}$, since there is no gain in receiving information at the CS, which cannot be forwarded to the systems. On the other hand, if no data was received from the sensors, the downlink capacity R_{DL} cannot be fully used. Since the data itself does not influence the scheduling decision, but rather the calculated error variance, the scheduling for uplink and downlink is always the same. Then, the sum of the average variances per timeslot k is minimized:

$$\arg\min_{\boldsymbol{r}} \textstyle\sum_{i=1}^{N_{\text{sys}}} r_i \max\left(W_i, f_i(1/r_i)\right) \tag{16}$$

$$\text{s.t.} \quad 0 \leq r_i, i = 1, \ldots, N_{\text{sys}} \tag{16a}$$

$$R = T^{\text{s}} \min\{R_{\text{UL}}, R_{\text{DL}}\} \tag{16b}$$

$$R \geq \textstyle\sum_{i=1}^{N_{\text{sys}}} r_i. \tag{16c}$$

The maximum in the objective (16) sets the lower bound of uncertainty to the system noise covariance W_i.Constraint (16a) ensures positive rates, while (16c) limits the sum rate to the available communication resources given by (16b).

V. Scheduling Algorithm

After calculating the rates, the actual scheduling is derived. As discussed in Subsection IV-C, the uplink and downlink scheduling is equal. The primary goal of the scheduling algorithm for the possibly non-integer duration values d_i is to bring the individual durations as close to the desired values as possible. As shown in Subsection IV-B, the duration between consecutive transmissions is more important than the average, so the Alg. 1 only considers the time since the last scheduling. The first transmission for all subsystems is assumed to take place at $t = 0$. Then, in each communication time slot t, Alg. 1 is run to find the R_{UL} or R_{DL} subsystems, which have the longest time passed since their respective last transmission, relative to their desired duration d_i. The resulting vector $\boldsymbol{\pi}$ has elements for every subsystem.

VI. Numerical Results

In this section, numerical results are derived for four system classes, each of them containing one quarter of the subsystems, with $a_i \in \{0.75, 1, 1.25, 1.5\}$. The noise variance for all systems is set to $W_i = 1$. The number of communication time slots per system time slot is set to $T_{\text{s,i}} = 10$. First, we want to investigate, how this constraining environment

Algorithm 1 Transmission scheduling

$R_{\text{remain}} \leftarrow \min\{R_{\text{DL}}, R_{\text{UL}}\}$ {Remaining resources for this time slot}
$\boldsymbol{\pi}(t) \leftarrow \mathbf{0}$
while $R_{\text{remain}} \neq 0$ **do**
  $\boldsymbol{\delta_n} \leftarrow \frac{1}{\delta_r}(t\mathbf{1} - \boldsymbol{t_l})$ {Normalized time since last TX}
  $n_i \leftarrow \arg\max_i \boldsymbol{\delta_n}(i)$
  $\pi_{n_i}(t) \leftarrow 1$ {Schedule subsystem with longest duration since last transmission}
  $\boldsymbol{t}_{\text{l},n_i} \leftarrow t$ {Save current transmission time step}
  $R_{\text{remain}} \leftarrow R_{\text{remain}} - 1$
end while
return $\boldsymbol{\pi}(t)${Return the scheduling for the current time slot}

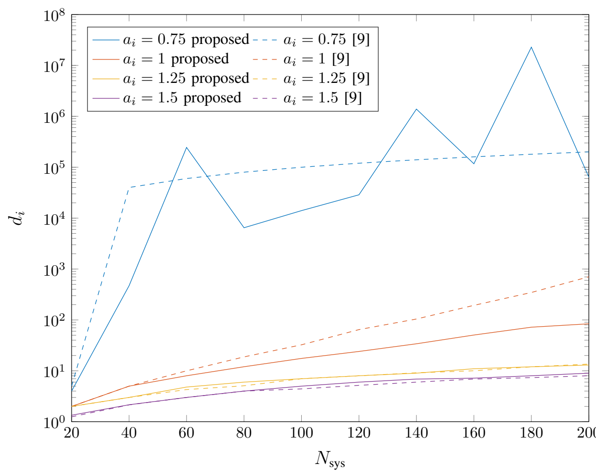

Fig. 2: Duration between transmissions for $R_{\text{UL}} = 1$

influences the time duration between two transmissions of the different subsystem classes. By increasing N_{sys} while keeping R_{UL} and R_{DL} constant, the resources per subsystem are reduced, which leads to increasing durations between transmissions. Additionally, the share of resources changes between the subsystem classes; subsystems with smaller a_i get a smaller share. Fig. 2 shows the average duration between transmissions for different numbers of subsystems N_{sys} with resources $R_{\text{UL}} = R_{\text{DL}} = 1$. The optimization shown in (16) reduces the resources assigned to the systems with a low system constant, when N_{sys} increases, resulting in longer times between two subsequent transmissions. For the stable subsystems with $a_i = 0.75$, almost no transmissions are scheduled for $N_{\text{sys}} \geq 60$. For small r_i, even small changes result in great changes of the corresponding d_i and lead to fluctuations. For comparison, the results from [9] are shown. The difference between the two schemes are significant especially for the subsystems with $a_i = 1$, which are scheduled more often with the results from (16).

Next, the control performance of the different scheduling algorithms is compared in terms of the mean absolute estimation error $\Sigma_{\text{e}}(k) = \frac{1}{N_{\text{sys}}} \sum_{i=1}^{N_{\text{sys}}} \alpha \sqrt{P_i(k)}$. Fig. 3 shows the results for $R_{\text{UL}} = R_{\text{DL}} = 1$ and $R_{\text{UL}} = R_{\text{DL}} = 3$. With

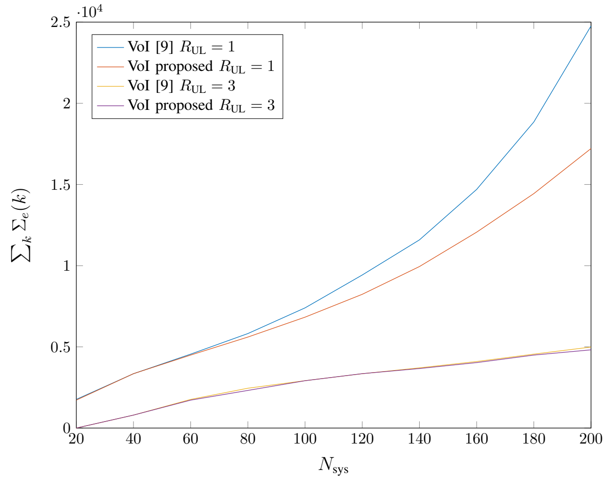

Fig. 3: $\sum_k \Sigma_{\mathrm{e}}$ for $R_{\mathrm{UL}} = R_{\mathrm{DL}} = 1$ and $R_{\mathrm{UL}} = R_{\mathrm{DL}} = 3$

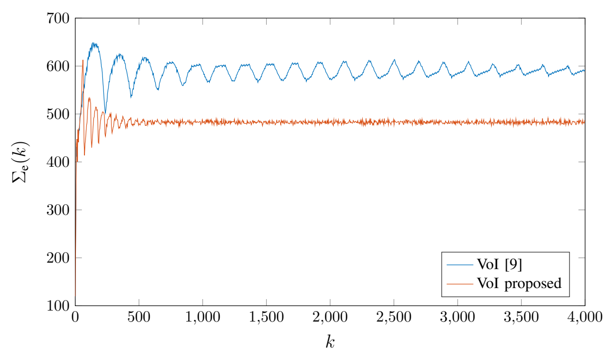

Fig. 4: Σ_{e} for each k for $R_{\mathrm{UL}} = 1$ and $N_{\mathrm{sys}} = 160$

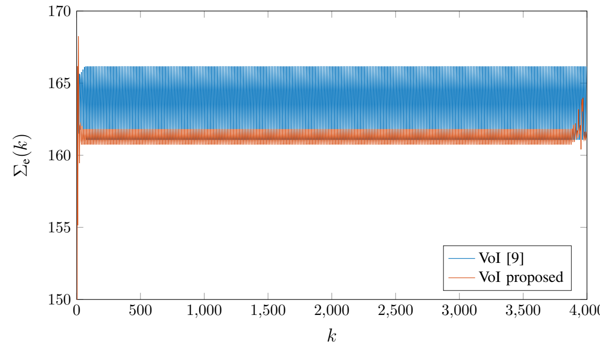

Fig. 5: Σ_{e} for each k for $R_{\mathrm{UL}} = 3$ and $N_{\mathrm{sys}} = 160$

$R_{\mathrm{UL}} = R_{\mathrm{DL}} = 3$, both algorithms achieve the same Σ_{e}. In the more constrained case ($R_{\mathrm{UL}} = R_{\mathrm{DL}} = 1$), the reduction of Σ_{e} of the proposed method compared to [9] becomes apparent.

Next, the behavior of $\Sigma_{\mathrm{e}}(k)$ of the subsystems over time is investigated. Exemplarily, Σ_{e} for $N_{\mathrm{sys}} = 160$ and $R_{\mathrm{UL}} = R_{\mathrm{DL}} = 1$ is shown in Fig. 4 for each control system time step k. Since the subsystems are assumed to have been reset at $k = 0$, multiple systems have to be scheduled in the first time slots, which results in a transient phase for both algorithms. Our algorithm is very stable after this phase, while the one from [9] fluctuates more as described in Subsection IV-A.

In Fig. 5 the same experiment is conducted with $R_{\mathrm{DL}} = R_{\mathrm{UL}} = 3$. While in Fig. 3 both, the proposed and the method from [9], seem almost equivalent, Fig. 5 reveals the lower variance of the proposed method. The small spike around $k = 3900$ of the proposed method comes from the fact, that at this time the subsystems with $a_i = 0.75$ are scheduled the first time, resulting in a small disturbance, which can be viewed as a very long transient phase. The much smaller fluctuation in Σ_{e} of the proposed method is also clearly visible.

Acknowledgement

This work has been performed in the context of the DFG Collaborative Research Center (CRC) 1053 MAKI and the LOEWE Center emergenCITY.

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