Relay-Aided Communication in Large Interference Limited Wireless Networks

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M.Sc. Daniel Papsdorf
geboren am 14. Dezember 1983 in Wiesbaden, Deutschland

Referent:	Prof. Dr.-Ing. Anja Klein
Korreferent:	Prof. Dr.-Ing. habil. Tobias Weber
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Kurzfassung

In den letzten Jahren ist die Anzahl der drahtlosen Geräte exponentiell gestiegen und es ist daher zu erwarten, dass auch die Interferenz steigt. Die Interferenz zwischen Kommunikationsverbindungen ist der maßgebliche limitierende Faktor in heutigen Kommunikationsnetzen und der Umgang mit dieser daher eine der größten Herausforderungen in drahtlosen Kommunikationsnetzen der Zukunft. Wenn die Interferenzsignale im Vergleich zum Nutzsignal schwach sind, so können diese einfach als Rauschen betrachtet werden. Sind die Interferenzsignale im Vergleich zum Nutzsignal stark, so können diese zuverlässig dekodiert und an den Empfängern vom Empfangssignal subtrahiert werden. In Mehrbenutzer-Kommunikationsnetzen sind die Interferenzsignale und das Nutzsignal jedoch oft von vergleichbarer Signalstärke. Der herkömmliche Ansatz zur Verarbeitung dieser Interferenzsignale besteht darin, das Nutzsignal und die Interferenzsignale zu orthogonalisieren, z.B. durch Zeitmultiplexverfahren (Time Division Multiple Access, TDMA) oder Frequenzmultiplexverfahren (Frequency Division Multiple Access, FDMA). Anstelle der Orthogonalisierung wurde in den letzten Jahren Interference Alignment (IA) als effiziente Technik zur Verarbeitung von Interferenzsignalen entwickelt, insbesondere im Bereich eines hohen Signal-zu-Rausch-Verhältnisses (SNR). Die Grundidee von IA besteht darin, an jedem Empfänger mehrere Interferenzsignale in einen bestimmten Teilraum mit reduzierter Dimension zu bündeln. Ziel ist es, die durch Interferenzsignale belegten Signaldimensionen an jedem Empfänger zu minimieren. Um IA durchzuführen, wird der Empfangsraum in zwei Unterräume aufgeteilt, den Nutzsignal-Unterraum und den Interferenzsignal-Unterraum. Jeder Sendeknoten gestaltet seine Sendefilter so, dass sich an jedem Empfangsknoten alle Interferenzsignale innerhalb des Interferenzsignal-Unterraums befinden und sich lediglich das Nutzsignal im interferenzfreien Nutzsignal-Unterraum befindet.

Der Schwerpunkt dieser Arbeit liegt auf großen, interferenzbegrenzten, drahtlosen Kommunikationsnetzen. Im Gegensatz zum herkömmlichen Gebrauch von Relais zur Erhöhung der Reichweite werden in dieser Arbeit die Relais dazu verwendet, den effektiven Ende-zu-Ende Kanal zwischen den Sendern und Empfängern zu beeinflussen, um IA im Netzwerk durchzuführen. Da die Relais lediglich zur Unterstützung des IA Prozesses verwendet werden, sind die von den Knoten übertragenen Datenströme für die Relais irrelevant. Somit reichen Verstärkungs- und Weiterleitungsrelais aus, um den IA Prozess zu unterstützen. Der Hauptfokus dieser Arbeit liegt daher auf Verstärkungs- und Weiterleitungsrelais. Es gilt die Annahme, dass alle Knoten und Relais Mehrantennengeräte sind, welche im Halbduplex-Modus arbeiten. Werden große Netzwerke betrachtet, so kann aufgrund physikalischer Ausbreitungsphänomene, wie z.B. hoher Pfadverlust und Abschattung, nicht angenommen werden, dass alle Knoten mit allen

Relais verbunden sind. In solch großen Netzwerken können die Abstände zwischen den einzelnen Knoten stark variieren, was zu Verbindungen mit deutlich unterschiedlichen Signalstärken führt. Ausreichend schwache Verbindungen können hierbei vernachlässigt werden. Somit sind große Netzwerke im Allgemeinen nur teilweise verbunden. In dieser Arbeit werden folgende drei wichtige, interferenzlimitierte, relaisgestützte, drahtlose Netzwerktopologien untersucht: das teilweise verbundene, relaisgestützte Mehrpaar-Paarweise-Kommunikationsnetzwerk, das vollständig verbundene Mehrgruppen-Mehrwege-Relaisnetzwerk und das teilweise verbundene Mehrgruppen-Mehrwege-Relaisnetzwerk. Für jede dieser Topologien werden in dieser Arbeit neue Algorithmen zur Durchführung von IA entwickelt.

Zunächst wird ein großes, teilweise verbundenes, relaisgestütztes Kommunikationsnetzwerk betrachtet, in welchem paarweise kommuniziert wird. Es wird ein Konzept zur geeigneten Aufteilung dieses Netzwerks in Teilnetzwerke vorgestellt, die wiederum vollständig verbunden sind. Jedes dieser Teilnetzwerke enthält ein einzelnes Relais und alle Knoten, die mit diesem Relais verbunden sind. Einige Knoten oder sogar Kommunikationspaare können mit mehreren Relais verbunden sein. Die bidirektionale, paarweise Kommunikation zwischen den Knoten erfolgt über die Relais, unter Verwendung des Zweiwege-Relaisprotokolls. Da nur Relais, welche mit beiden Knoten eines Kommunikationspaares verbunden sind, dieses Paar bedienen können, wird angenommen, dass alle Kommunikationspaare im gesamten Netzwerk von mindestens einem Relais bedient werden. Der Umgang mit Knoten, die mit mehreren Relais verbunden sind, stellt in einem solch teilweise verbundenen Netzwerk die größte Herausforderung dar. Daher werden die als Simultaneous Signal Alignment (SSA) und Simultaneous Channel Alignment (SCA) bezeichneten Verfahren vorgeschlagen, um Signal Alignment (SA) und Channel Alignment (CA) mit mehreren Relais gleichzeitig durchzuführen zu können. Bei SA senden alle Knoten derart an das Relais, dass die Signale jedes Kommunikationspaares am Relais paarweise ausgerichtet werden. Für CA, dem dualen Problem zu SA, wird das Empfangsfilter jedes Knotens so entworfen, dass die effektiven Kanäle zwischen den Relais und beiden Knoten eines Kommunikationspaares den gleichen Teilraum aufspannen. In dieser Arbeit wird eine Lösung in geschlossener Form zur Durchführung von IA in dieser Netzwerktopologie erzielt und die Voraussetzungen für SSA und SCA werden hergeleitet. Es wird gezeigt, dass lokale Kanalzustandsinformation (Channel State Information, CSI) ausreicht, um IA in teilweise verbundenen Netzwerken durchzuführen, während in vollständig verbundenen, relaisgestützten Netzwerken im Allgemeinen globale CSI erforderlich ist. Durch Simulationen wird veranschaulicht, dass die vorgeschlagene geschlossene Lösung mehr Freiheitsgrade (DoF) erreicht als die Referenzalgorithmen sowie eine höhere Summenrate erzielt, insbesondere im hohen SNR-Bereich. Vor allem in großen, drahtlosen Netzwerken kann es vorkommen, dass nicht beide Knoten eines Kommunikationspaares mit den gleichen Relais verbunden sind. Wenn ein einzelner Knoten eines Kommunikationspaares eine zusätzliche Verbindung zu einem Relais aufweist, kann dieses Relais die Kommunikation nicht unterstützen. Der betreffende Knoten empfängt demzufolge nur Interferenzen und kein Nutzsignal von diesem Relais; er unterliegt somit Inter-Subnetzwerkinterferenz. In dieser Arbeit wird ein Algorithmus in geschlossener Form vorgeschlagen, der die Inter-Subnetzwerkinterferenzleistung im gesamten, teilweise verbundenen Netzwerk minimiert und dessen Voraussetzungen werden hergeleitet. Es werden Voraussetzungen formuliert, unter denen mit dem Algorithmus sogar eine interferenzfreie Kommunikation erreicht werden kann. Des Weiteren wird gezeigt, dass der vorgeschlagene Algorithmus zur Minimierung der Inter-Subnetzwerkinterferenz im Vergleich zum betrachteten Referenzalgorithmus eine höhere Summenrate erreicht.

Anschließend wird ein vollständig verbundenes Mehrgruppen-Mehrwege-Relaisnetzwerk betrachtet. In einem solchen Netzwerk bilden mehrere Knoten eine Gruppe und jeder Knoten möchte seine Nachricht über das Relais mit allen anderen Knoten seiner Gruppe teilen. Die gruppenweise Kommunikation zwischen den Knoten innerhalb einer Gruppe erfolgt über das Relais unter Verwendung einer Übertragungsstrategie, die im Allgemeinen mehrere Multiple Access (MAC) Phasen und mehrere Multicast (MC) Phasen verwendet. In dieser Arbeit wird ein Multicast IA Algorithmus zur Verarbeitung der in einem solchen Netzwerk auftretenden Interferenz vorgeschlagen. Die Idee beruht darauf, dass in jeder der MC-Phasen ein Multiple-Input-Multiple-Output (MIMO)-Interferenz-Multicast-Kanal erzeugt wird. Dazu werden die Antennen des Relais in so viele Cluster aufgeteilt, wie Gruppen im Netzwerk vorhanden sind. Jedes dieser Cluster bedient eine bestimmte Gruppe von Knoten und sendet derartig, dass die von verschiedenen Clustern übertragenen Signale an den Empfangsknoten der nicht beabsichtigten Multicast-Gruppen gleich ausgerichtet sind. Es wird gezeigt, dass die mindestens erforderliche Anzahl von Antennen am Relais unabhängig von der Anzahl der Knoten pro Gruppe ist. Dies ist eine wichtige Eigenschaft, da die Anzahl der am Relais verfügbaren Antennen im Allgemeinen begrenzt ist. Darüber hinaus werden die Voraussetzungen für den vorgeschlagenen Multicast IA-Algorithmus hergeleitet. Es wird gezeigt, dass der vorgeschlagene Multicast-Algorithmus den Referenzalgorithmus für einen weiten SNR-Bereich übertrifft, während er gleichzeitig weniger Antennen am Relais benötigt.

Abschließend wird ein großes, teilweise verbundenes Mehrgruppen-Mehrwege-Relaisnetzwerk betrachtet. Im Gegensatz zum vollständig verbundenen Mehrgruppen-Mehrwege-Relaisnetzwerk werden in diesem teilweise verbundenen Netzwerk mehrere Relais berücksichtigt. Ein solches Netzwerk kann in Teilnetze unterteilt werden, die selbst vollständig verbunden sind. Jedes dieser Teilnetzwerke enthält dann ein einzelnes Relais und alle Gruppen von Knoten, die mit diesem Relais verbunden sind. Jede Gruppe von Knoten kann mit einem oder mehreren Relais verbunden sein. Dies bedeutet, dass nicht alle Gruppen von Knoten mit allen Relais im Netzwerk verbunden sind. Jede Gruppe ist jedoch mit mindestens einem Relais verbunden, welches diese Gruppe von Knoten bedient. Der gruppenweise Datenaustausch zwischen den Knoten innerhalb einer Gruppe erfolgt über das Multi-Way-Relaying-Protokoll. Der herausforderndste Teil eines solchen teilweise verbundenen Netzwerks ist die Behandlung der Knoten innerhalb von Gruppen, die mit mehreren Relais verbunden sind. Um diese Herausforderung zu bewältigen, werden neue Verfahren vorgestellt, die als Simultaneous Group Signal Alignment (SGSA) und Simultaneous Group Channel Alignment (SGCA) bezeichnet werden, um SA und CA in teilweise verbundenen Mehrgruppen-Mehrweg-Relaisnetzwerken durchzuführen. Für diese Netzwerktopologie wird eine IA Lösung in geschlossener Form erzielt und die Voraussetzungen für die Lösbarkeit von SGSA und SGCA werden hergeleitet. Es wird gezeigt, dass der vorgeschlagene IA-Algorithmus den Referenzalgorithmus in Bezug auf die Summenrate und die DoF übertrifft.

Abstract

In recent years, the number of active wireless devices increases exponentially and it is, therefore, to expect that the interference increases as well. Interference between communication links is the major performance limiting factor in today's communication networks. Hence, the handling of the overall interference in a network is one major challenge in wireless communication networks of the future. If the interference signals are weak in comparison to the useful signal, they can be simply treated as noise. If the interference signals are strong in comparison to the useful signal, they can be reliably decoded and subtracted from the received signal at the receivers. However, in multiuser communication networks, the interference and the useful signal are often of comparable signal strength. The conventional approach to handle these interference signals is to orthogonalize the useful signal and the interference signals using, e.g., time division multiple access (TDMA) or frequency division multiple access (FDMA). In the past few years, instead of orthogonalization, interference alignment (IA) has been developed as an efficient technique to handle interference signals, especially in the high signal to noise ratio (SNR) region. The basic idea of IA is to align multiple interference signals in a particular subspace of reduced dimension at each receiver. The objective is to minimize the signal dimensions occupied by interference at each receiver. In order to perform IA, the receive space is divided into two disjoint subspaces, the useful signal subspace and the interference signal subspace. Each transmitting node designs its transmit filters in such a way that at each receiving node, all interference signals are within the interference subspace and only the useful signal is in the useful subspace.

In this thesis, the focus is on large interference limited wireless communication networks. In contrast to the conventional use of relays, for extending the coverage, in this thesis, the relays are used to manipulate the effective end-to-end channel between the transmitters and receivers to perform IA in the network. Since the relays are used to assist the process of IA and not interested in the data streams transmitted by the nodes, amplify-and-forward relays are sufficient to support the process of IA. Therefore, the main focus of this thesis is on amplify-and-forward relays. Throughout this thesis, it is assumed that all nodes and relays are multi-antenna half-duplex devices. When considering large networks, the assumption that all nodes are connected to all relays does not hold due to physical propagation phenomena, e.g., high path loss and shadowing. In such large networks, the distances between different nodes may differ a lot, leading to links of considerably different signal strengths, where sufficiently weak links may be neglected. Hence, large networks are in general partially connected. In this thesis, three important interference-limited relay aided wireless network topologies are investigated, the partially connected relay aided multi-pair pair-wise communication network, the fully con-

nected multi-group multi-way relaying network and the partially connected multi-group multi-way relaying network. For each of these topologies, new algorithms to perform IA are developed in this thesis.

First, a large partially connected relay aided pair-wise communication network is considered. The concept of an appropriate partitioning of a partially connected network into subnetworks which are themselves fully connected is introduced. Each of these subnetworks contains a single relay and all nodes being connected to this relay. Some nodes or even communication pairs may be connected to multiple relays. The bidirectional pair-wise communication between the nodes takes place via the intermediate relays, using the two-way relaying protocol. Only relays which are connected to both nodes of a communication pair can serve this pair. Hence, it is assumed that all communication pairs in the entire network are served by at least one relay. The most challenging part of such a partially connected network is the handling of nodes which are connected to multiple relays. Hence, techniques called simultaneous signal alignment (SSA) and simultaneous channel alignment (SCA), are proposed to perform signal alignment (SA) and channel alignment (CA) with multiple relays simultaneously. SA means that all nodes transmit to the relay in such a way that the signals of each communicating pair are pair-wise aligned at the relay. For CA, which is dual to SA, the receive filter of each node is designed such that the effective channels between the relay and both nodes of a communicating pair span the same subspace. A closed-form solution to perform IA in this network topology is obtained and the properness conditions for SSA and SCA are derived. It is shown that local channel state information (CSI) is sufficient to perform IA in partially connected networks, whereas in fully connected relay aided networks, global CSI is required in general. Through simulations, it is shown that the proposed closed-form solution achieves more degrees of freedom (DoF) than the reference algorithms and has better sum-rate performance, especially in the high SNR-region. Especially in large wireless networks, it may happen that not both nodes of a communication pair are connected to the same relays. If a single node of a communication pair is in addition connected to a relay which, therefore, cannot assist the communication, this node receives only interference and no useful signal from this relay. Such a node suffers from inter-subnetwork interference, due to the connection by an inter-subnetwork link to the additional relay. Hence, in this thesis, a closed form algorithm which minimizes the inter-subnetwork interference power in the whole partially connected network is proposed and the properness conditions are derived. The condition under which an interference free-communication can be achieved by the proposed inter-subnetwork interference power minimization algorithm is derived. Further, it is shown that the proposed inter-subnetwork interference power minimization algorithm achieves a higher sum rate in comparison to the considered reference algorithm.

Secondly, a fully connected multi-group multi-way relaying networks is considered. In such a network, multiple nodes form a group and each node wants to share its message with all other nodes in its group via an intermediate relay. The group-wise communication between the nodes inside a group takes place via the intermediate relay, using a transmission strategy considering several multiple access (MAC) phases and several multicast (MC) phases, in general. In this thesis, a multicast IA algorithm to handle the interference in such a network is proposed. The idea of the proposed algorithm is that in each of the MC phases, a multiple input multiple output (MIMO) interference multicast channel is created by separating the antennas of the relay into as many clusters as groups in the network. Each of these clusters serves a specific group of nodes and transmits in such a way that the signals transmitted from different clusters are aligned at the receiving nodes of the non-intended multicast groups. It is shown that the minimum required number of antennas at the relay is independent of the number of nodes per group, which is an important property since the number of antennas available at the relay is limited in general. Furthermore, the properness conditions for the proposed multicast IA algorithm are derived. It is shown that the proposed multicast algorithm outperforms a reference algorithm for a broad range of SNR values, while still requiring less antennas at the relay.

Finally, a large partially connected multi-group multi-way relay network is considered. In contrast to the fully connected multi-group multi-way relaying network, multiple relays are considered in this partially connected network. Such a partially connected network can be partitioned into subnetworks that are themselves fully connected. Hence, such a partially connected network consists of multiple subnetworks, where each of these contains a single relay and all groups of nodes which are connected to this relay. Each group of nodes may be connected to one or multiple relays. This means that not all groups of nodes are connected to all relays in the network. However, any group is connected to at least one relay which serves this group of nodes. The group-wise exchange of data between the nodes inside a group is performed via the multi-way relaying protocol. The most challenging part of such a partially connected network is the handling of the nodes inside groups which are connected to multiple relays. To overcome this challenge, new techniques called simultaneous group signal alignment (SGSA) and simultaneous group channel alignment (SGCA) are introduced to perform SA and CA in partially connected multi-group multi-way relaying networks. A closed-form IA solution for this network topology is obtained and the properness conditions for the solvability of SGSA and SGCA are derived. It is shown that the proposed IA algorithm outperforms the reference algorithm in terms of sum rate and DoF.

Contents

Chapter 1

Introduction

1.1 Motivation and Objectives

The evolution of wireless communication technologies has brought many advantages in comparison to wired connections. Typical wireless communication applications of our daily life are satellite television, cellular telephony or the GPS-service. The number of wireless applications is growing continuously, and as a consequence, the wireless data traffic will grow. As an example, the mobile data traffic has been growing exponentially in recent years and it is expected, that the mobile data traffic will increase by a factor of 5 over the next 6 years [Eri18; Cis17]. The growing traffic is mainly driven by the user demand, which has shifted from simple voice service or text messages to web-browsing, video-streaming, and file-sharing in recent years. Primarily, the increased viewing time of videos, embedded videos on websites and the evolution toward higher resolution are responsible for the growth of mobile data traffic [Eri18]. Hence, the required bandwidth demand for data and video content increases. Because of the high bit rate of video content in contrast to other applications, video content will generate much more data traffic than other applications [Cis17]. Additionally, the number of active network devices increases as well, this leads also to an increase of network traffic.

Furthermore, machine to machine (M2M) communication and the internet of things (IoT) will play an important role in the future. M2M and IoT refer both to devices communicating with each other, i.e., there is no substantial difference between IoT and M2M [ANP13]. In contrast to IoT, M2M systems typically have homogeneous machine types within a network. Nodes of an M2M network can only communicate via an M2M gateway with external nodes in other M2M networks. In contrast to M2M networks, in IoT networks, computing is one of the main aspects. Hence, cloud services and cloud computing play an important role in IoT [ANP13]. The increasing number of smart devices and the growing number of M2M connections is a clear indicator for the growing number of IoT devices [Cis17]. M2M communication can be considered as a subset of the IoT of the future [ANP13]. The M2M mobile connections are growing really fast and will reach more than a quarter of the total devices and connections by 2021 [Cis17]. The success of IoT will mainly depend on the evolution of the next generation of wireless communication. Hence, 5G is a step in the right direction in order to support millions of wireless devices and allow access to any content on any device from anywhere [Bro16].

The emergence of wearables plays a crucial role in the growing number of IoT devices [Cis17]. Wearables, as the name suggests, are devices, worn by people, which have the capability to communicate with other devices.

The theoretical, as well as the technical progress in recent years, has enabled a higher data rate in wireless networks. Many modern applications like wireless internet, video streaming, IoT, etc., of our daily life, are profiting from these advances. Mobile communication has become essential for many applications, a lot of people consider video and audio streaming or file-sharing as necessary or even indispensable services.

In wireless communication networks, the transmitters utilize electromagnetic waves in order to transmit data to the desired receivers. These electromagnetic waves of wireless communication networks suffer from several impairments such as shadowing, interference, attenuation, multi-path fading, and noise [TV05; Pro01]. Interference between communication links is a fundamental complex limitation and phenomenon of wireless communication networks. The interference is the result of the broadcast nature of wireless communication if multiple users share the same resources. The simultaneously transmitted signals will superimpose at each receiver and therefore cause interference [TV05; LJ16].

As the total number of active devices in a network increases, it is to expect, that the interference in the network will also increase. With this growing number of wireless devices, the bottleneck effect of the interference becomes apparent. Hence, the handling of the overall interference in a network is one major challenge in wireless communication networks of the future [JMLZ18; JZD17]. Therefore, interference management is very important in order to fulfill the higher throughput requirements in 5G.

Conventional well known techniques to handle interference in wireless communication networks are [TV05; CJ08b; LJ16; JMLZ18]:

- treating interference signals as noise,
- decoding interference first,
- orthogonalization in time, frequency or space.

Treating the interference signal as noise is one of the simplest strategies of interference management for wireless networks. This strategy can be applied if the interference signal is weak in comparison to the useful signal [AV09]. In addition, this strategy is robustness to channel uncertainty [AV09; CJ08b]. If the interference signals are strong in comparison to the useful signal, they can be reliably decoded and subtracted from the received signal at each receiver [CJ08b; SVJS08]. The orthogonalization strategy is conventionally applied if the interference and the useful signal are of comparable signal strength, e.g., time division multiple access (TDMA) or frequency division multiple access (FDMA) [CJ08b; Jaf14]. Hence, every source node can only utilize a fraction of the total resources interference-free. Hence, such a transmission scheme is termed "cake-cutting" scheme in the literature and in general suboptimal in terms of data streams that can be transmitted free of interference [JMLZ18]. This number of data-streams that can be transmitted simultaneously free of interference is characterized by the degrees of freedom (DoF) metric [CJ07; JV10; JMLZ18].

In order to overcome the drawback of the orthogonalization approaches, interference alignment (IA) has been developed in recent years [LJ16; CJ08b; Jaf11; ZWQW18]. This new scheme is a promising technique to mitigate interference if the useful signal and the interference signal are of comparable signal strength. The basic idea of IA is to align multiple interference signals in a particular subspace of reduced dimension at each receiver, i.e., the objective is to minimize the signal dimensions occupied by interference at each receiver. The signals can be aligned in any dimension, time [JS08], frequency [SHV08], or space [GCJ08; PH09; SLTL10].

Applying IA on an interference channel each transmitter is simultaneously able to transmit at a data rate equal to half of his interference-free channel capacity to its desired receiver regardless of the number of nodes in the entire network, i.e., each node could get "half of the cake" [CJ08b]. This cake cutting example well-known in the context of IA was introduced in [JF07]. In this descriptive example, each user can get half of the cake, only if the cake can be made large enough, i.e., the bandwidth can be made arbitrarily large and infinite time extensions can be used [JF07; GCJ11].

An IA algorithm does not care about the useful signal power at the receivers and aims in general at maximizing the DoF [CJ08b]. The goal of IA in contrast to other approaches is the minimization of the interference space at the undesired receivers, the useful signal power is not considered in IA algorithms. The received signal will be projected onto the null-space of the interference signal in order to decode the desired signal without interference at each of the receivers [CJ08b]. Conventional interference avoidance strategies are often selfish approaches, i.e., each of the users want to maximize his own signal to noise plus interference ratio (SINR). Hence, IA can be termed as a cooperative approach because the transmitter's design their transmit filters in order to nullify the interference in the entire network. If IA is achieved, the interference at each receiver can completely nullified and the achievable data rate at each receiver does not saturate as the transmit power increases. IA algorithms are only optimal in the high SNR-region because in the low SNR-region noise and not the interference is the dominant factor.

Different IA schemes have been proposed and investigated in the literature, such as IA using time extensions [CJ08b], IA considering multiple antennas [GCJ11], and relays aided IA [GCJ11; NMK10]. Performing IA in the spatial dimension, considering multiple antennas at the transmitters and the receivers, is the in the literature mostly investigated system. Such a system with multiple transmit and receive antennas is called a multiple-input multiple-output (MIMO) system [TV05]. One reason for this extensive investigation is that MIMO systems are a promising technique to increase the performance in wireless communication networks of the future [Jaf13; ZWQW18]. There are only few publications on relay aided IA, although relays can support the process of IA [NMK10]. Hence, relay aided IA is still a promising research topic.

Conventionally, relays are deployed in wireless networks as an effective and economical method to improve the performance in noise-limited scenarios [WY13; HBG+13]. Relaying techniques in wireless communication networks can be used to extend the coverage and to enhance the capacity, if the transmitting nodes are power limited [BKW+09]. A simple relaying example is shown in Figure 1.1. In this example, the nodes of the same color want to exchange data bidirectionally. The direct links between the nodes are assumed to be zero. In contrast to this conventional use of relays in wireless networks, in this thesis, the relays are used to manipulate the effective channel between the transmitters and receivers in order to perform IA in the network. Two well known relaying protocols are one-way relaying and two-way relaying.

One-way relaying requires four phases to achieve a bidirectional pair-wise exchange of data, two phases for the transmission from the source to the destination via the intermediate relay and two additional phases for the reverse communication direction [BKW+09]. In contrast to this, two-way relaying allows a bidirectional pair-wise exchange of data in two phases and overcomes therefore the duplex loss of other relay schemes [RW07]. In the first phase of two-way relaying, all nodes simultaneously transmit to the relay. This phase is called multiple access (MAC) phase in literature. In the second phase of two-way relaying, the so-called broadcast (BC) phase, the relay retransmit a linearly processed version of the signal received in the MAC phase back to the nodes. Since two-way relaying is spectrally more efficient for a bidirectional communication than one-way relaying, two-way relaying is considered in this thesis.

Half-duplex devices are more realistic and easier to realize than full-duplex devices. The problem with full-duplex devices is the high dynamic range between the transmit and receive signal, this makes it complicated to cancel out the self-interference. Therefore, only

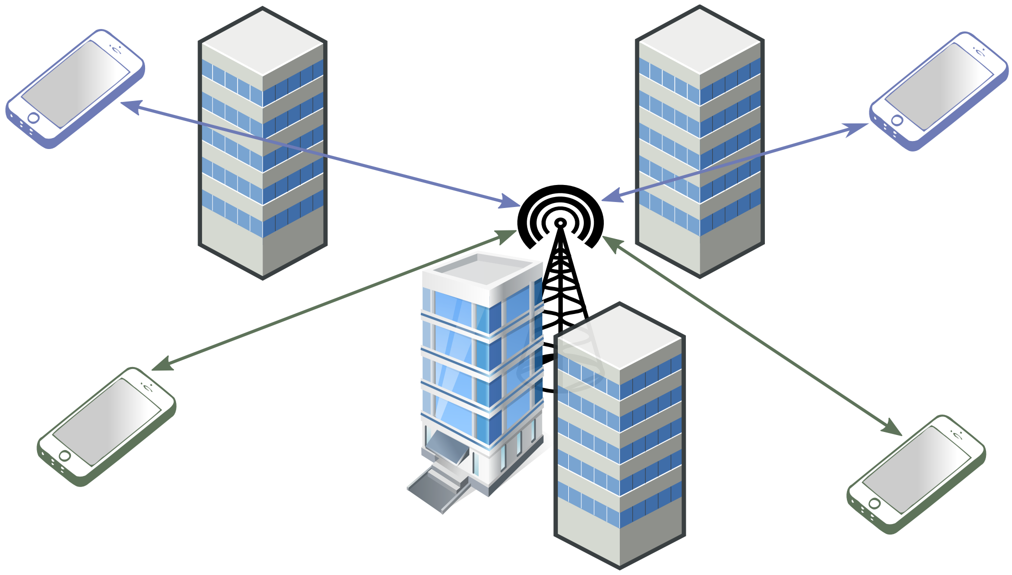


Figure 1.1. Two-pair bidirectional relaying

half-duplex devices are considered in following. Since half-duplex nodes cannot transmit and receive simultaneously a communication via a direct link in only one time-slot is not possible. Two well-known signal processing techniques which can be applied at the relay are amplify-and-forward as well as decode-and-forward. The main disadvantage of amplify-and-forward relaying is that the received noise is retransmitted, which is less efficient in terms of transmit power. The main advantage of amplify-and-forward relaying is that at the relay, no decoding and re-encoding is necessary. Decode-and-forward relays are more complex and have higher latency in comparison to amplify-and-forward relaying. Since the relays are used to assist the process of IA, the relays are not interested in the data streams transmitted by the nodes. Hence, amplify-and-forward is sufficient in order to support the process of IA.

Besides the theoretical investigation of IA, another important research field is the development of IA algorithms, i.e., the development of closed-form solutions and the derivation of the feasibility conditions for realistic networks. This is another important advantage of relay aided IA because many relay aided IA problems can be solved linearly. This ends up in a closed form-solution, for such a problem.

In large networks, which are in general more realistic than small networks, it is appropriate to assume that such a network consists of multiple relays and nodes. An example is shown in Figure 1.2. When considering such large communication systems, the assumption that all nodes in a network are connected to all relays with similar signal strengths does not hold. Usually the received signals have quite different power levels due to physical phenomena, e.g., path loss, shadowing, or fading. Therefore, the receivers receiving in general signals via three different kinds of channels, the desired channel, the significant interference channel, and the weak interference channel [Jaf14]. Hence, the whole received signal comprises of the desired signal, strong interference signals, and weak interference signals. Sufficiently weak links may even be neglected, i.e. can be approximated by zero, which results in a partially connected network [Jaf14]. Partially connectivity has been taken into account for a MIMO interference channel without relay in [GG11]. However, partial connectivity, especially in combination with relaying, is still an open challenging problem.

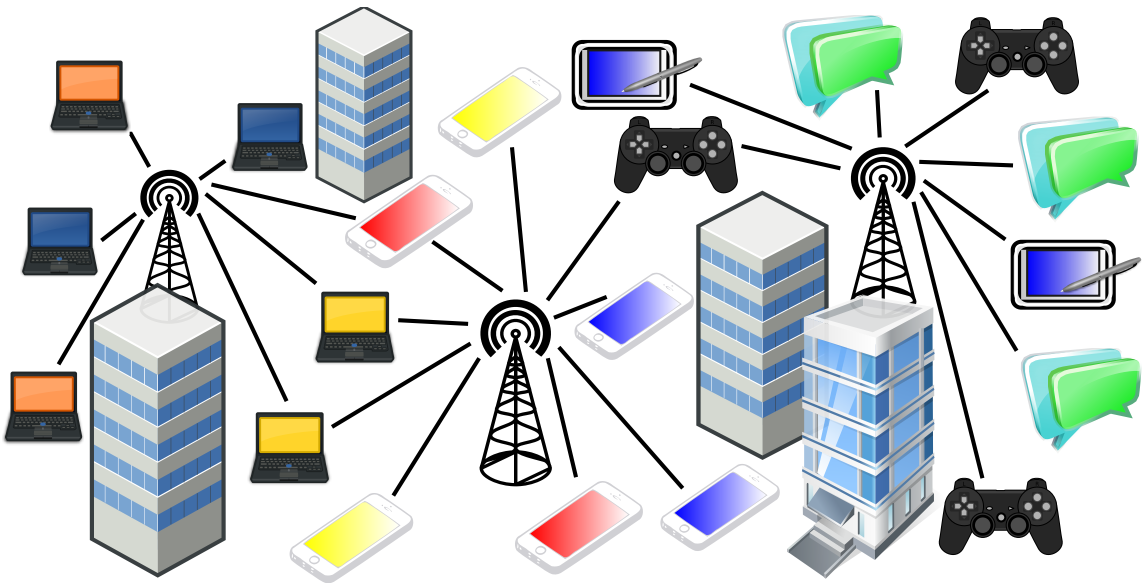


Figure 1.2. Large partially connected network

In a fully connected network, which means that all nodes are connected to all relays via a no zero link, relays that assist the communication cannot increase the DoF [CJ08a]. However, it is conjectured in [CJ08a] that relays can improve the DoF in partially connected networks. In order to keep the achieved DoF in a fully connected network constant, if the number of node pairs increases, the number of antennas at each node has to be increased as well. In contrast to such a fully connected network in a partially connected network, the number of antennas has not necessarily to be increased if the network is only partially connected [Jaf13]. Besides the other mentioned reasons, this is a further important reason to investigate relay-aided IA, especially in partially connected networks.

1.2 Concept of MIMO-IA using an Example

In this section, an example of MIMO IA is introduced and explained in order to illustrate the basic idea of IA. As already briefly introduced in Section 1.1, IA is a promising technique to mitigate interference in wireless communication networks of the future. Figure 1.3 shows the concept of MIMO interference alignment in a network with 3 communication pairs considering a MIMO interference channel. Each of the in total 6 nodes is equipped with 2 antennas. Hence, two spatial dimensions or spatial resourses are available at each node, i.e., the transmit as well as the receive spaces are of dimension two. It is assumed that each transmitting node wants to transmit a signal data stream, T1 to D1, T2 to D2 and T3 to D3. The colored vectors in Figure 1.3 indicate the orienta-

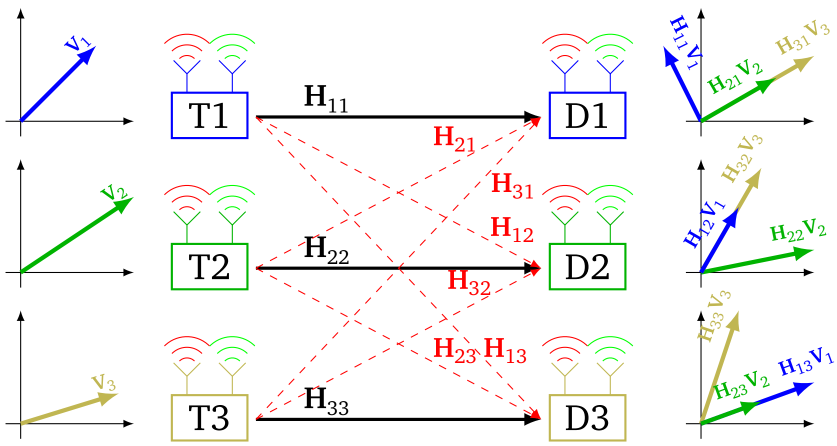


Figure 1.3. Interference alignment in a three communication pair MIMO interference channel with 2 antennas at each node

tion of the signals in the two dimensional transmit and receive space. The length of the vectors is related to the energy of the transmitted symbols. According to the idea of IA, the transmit filters $\mathbf{V}_k$, $\forall k \in \{1,2,3\}$ are designed such that at each receiving node, the interference signals are aligned within a one-dimensional interference signal subspace (ISS) and only the useful signal in a linearly independent one-dimensional useful signal subspace (USS). The signal of T1 needs to align with the signal transmitted by T2 at D3, for example. This ends up in a chain of alignment conditions. Obviously, the ISS at each receiving node is defined by all transmit filters. Hence, the transmit filters have to be designed jointly. The aligned interference can be suppressed by a projection of the received signal onto the null space of the ISS. Therefore, the receive filters have to be designed accordingly. It is important to know, that the transmitted signals will be scaled and rotated by the corresponding channel matrices. Hence, these matrices have to be considered for the design of the transmit filters. Consequently, global channel knowledge is necessary at all the nodes [CJ08b].

In Figure 1.3, IA allows that all three transmitting nodes are transmitting simultaneously without interfering the useful signal at the destination nodes. All three nodes are able to utilize half of the total resources and achieve, therefore, 3 DoF in total, whereas conventional schemes that orthogonalize the signal transmitted by the nodes cannot achieve more than 2 DoF in total.

In a conventional orthogonalisation scheme, e.g., TDMA each transmitting node can send two symbols on a 2×2 MIMO channel. Hence, in order to mitigate interference, only one node can be active in a time slot. Consequently, only 2 DoF can be achieved. Applying IA, all nodes are active simultaneously and each node can send one data symbol. The IA problem becomes difficult if the number of communication pairs increases beyond three or if the number of antennas at each node is reduced to one.

1.3 State-of-the-Art

1.3.1 Interference Alignment

This section will give an overview of the state of the art related to IA without relays. IA was proposed as a novel tool to achieve higher multiplexing gain in [MMK08]. The terminology IA was introduced by Jafar in [JS08]. IA algorithms without relays are designed for unidirectional communication in general. If a bidirectional communication is desired all published IA algorithms without relays required two time-slots to make this possible. As already mentioned, IA can be performed in the dimension time, frequency or space. The focus of this thesis is on IA in the spatial dimension. This section is structured as follows: First, publications related to a K node-pair communication considering an interference channel are given. Secondly, the generalization of the K node-pair communication to group communication is discussed. Thirdly, publications related to the X-channel are given to highlight the differences to group communication. Finally, publications considering the feasibility of IA are presented.

In [CJ08b] Cadambe and Jafar proposed, an asymptotic IA scheme to align an arbitrary number of interferers using time extensions. They considered a single-input-single-output (SISO) interference channel with K transmitters and K receivers. This is an extension of the example in Figure 1.3 to the K node-pair case. This work was further extended to the three-user MIMO interference channel, where all nodes are equipped with N antennas, in the same paper [CJ08b]. The scheme proposed in [CJ08b] is based on beamforming over multiple symbol extensions of the time-varying Gaussian interference channel. The K node-pair interference channel investigated in [CJ08b] has in general $K/2$ DoF. Hence, the scheme which was proposed in [CJ08b] guarantees that every transmitter can utilize a fraction of 1/2 of the total resources. [KRWY11] investigates the K node-pair N antenna interference channel and generalizes thus the algorithm proposed in [CJ08b]. The K node-pair $M \times N$ MIMO interference channel has been considered and investigated in [GJ10; GMK10; NAH11]. In [GJ10] it was shown, that if K transmitting nodes are equipped with M antennas and each of the K receivers with only one antenna in total $\frac{MK}{M+1}$ DoF can achieved if $K > M$. In [JMLZ18] a linear space-time IA scheme for a K node-pair MIMO interference channel to reduce the number of antennas required to eliminate the entire system interference is proposed. An iterative IA algorithm based on alternating minimization has been proposed in [PH09]. The algorithm proposed in [PH09] minimizes the interference power considering a K node-pair MIMO interference channel. The minimization of the interference power is in general a non-convex problem. Hence, such an iterative algorithm may converge to a local minimum, i.e., the residual interference power will be non-zero [PH09]. Three other iterative IA algorithms for the K node-pair MIMO interference channel are proposed in [POJ+17]. The optimization criteria in [POJ+17] are based on minimizing interference leakage, Max-SINR, and Max-sum-rate. For some carefully chosen system parameters, it is possible to determine a closed form solution, e.g., 3 transmitting an 3 receiving nodes, each equipped with 2 antennas, transmitting 1 data stream each, see Figure 1.3.

Besides the in the previous paragraph discussed pairwise communication there are some other interesting topologies, like group communication. Group communication without relays and considering IA has been investigated in [GJW11; KRWY11]. In [GJW11], a single source transmits a common message to all J signal antenna receivers. In [KRWY11] a network with K transmitters and J receivers each equipped with M antennas is considered. Each of the transmitting nodes transmits an independent message and each receiver is interested in an arbitrary subset of messages. This generalizes the K node-pair M antenna interference channel, where each receiving node is only interested in the message from one unique transmitting node. These single antenna receivers can be considered as a group.

Another channel that was considered several times is the X-channel. In an X-channel each transmitting node wants to transmit an independent message to each receiving node. Hence, the X-channel is quid different from group communication, considered in this thesis. The X-channel with two transmitters, two receivers and four independent messages, one form each transmitter to each receiver, was investigated in [JS06]. The DoF of a MIMO X-channel with two transmitters and two receivers is investigated in [JF07; JS08; JS06]. The DoF of an X-channel with M single antenna transmitting nodes, M

single antenna receiving nodes and in total MN independent messages was determined in [CJ09]. It was shown that in total $\frac{MN}{M+N+1}$ DoF can achieved. Each transmitting node transmits an independent message to each receiving node.

The problem of IA becomes in general much more complicated if the number of communication pairs increases beyond three or if the number of antennas at each nodes is reduced to only one. Therefore, it is important to investigate the feasibility of IA. The feasibility of IA for MIMO interference channel was investigated in detail in [TGR09; YGJK09; NSSG09; GCJ11; BCT11]. The feasibility of an IA problem is equivalent to the existence of a non-trivial solution of a system of polynomial equations [YGJK09]. The system of polynomial equations of a generic system is solvable if and only if the number of equations is less than the number of variables in the polynomial. In [YGJK09; NSSG09] the terminology properness condition is used in order to define an IA problem as proper or improper depending on the number of variables and equations of a system of linear equations. It is worth to mention, that proper systems are likely to be feasible and improper systems are likely to be infeasible. A system is called to be proper if and only if the number of independent variables M_v is equal to or greater than the number of equations M_e, i.e., $M_\mathrm{v} \geq M_\mathrm{e}$ [YGJK09]. The properness condition of a system with N antennas at each of the K transmitters and each of the K receivers, where each transmitter wants to transmit d independent data streams is given by

$$N \geq \frac{(K+1)d}{2} \tag{1.1}$$

[YGJK09]. From the field of algebraic geometry, the Bernstein's theorem was used to verify if a proper system is feasibly by evaluating the mixed volume of the polynomials [YGJK09]. Improper systems are in general infeasible.

1.3.2 Relaying and Relay Aided Interference Alignment

This section presents an overview of the publications related conventional relaying and relay aided IA. Conventional relaying means in this context relaying without IA. Two relaying schemes are well known from the field of conventional relaying, namely one-way relaying [BKW+09] and two-way relaying [RW07]. Since this thesis focuses on two-way relaying, publications for one-way relaying are not listed in this section. In the following, first selected publications related to conventional relaying are shown. Secondly, publications covering relay aided IA are investigated, single and multiple relays as well as fully and partially connected networks are considered. Fully connected means that all nodes are connected to all relays via a no zero link. Partially connected, in contrast, means that not all nodes are connected to all relays. However, some nodes may be connected to multiple relays. Finally, literature related to multi-group multi-way relaying with and without IA is introduced.

Conventional relay aided communication with one or multiple relays and K communication pairs has been investigated in a lot of publications [BKW+09; UK08; DK12b; DK12a; RW07; JS10; AK10a; LDLG11]. In [BKW+09], the relay filter is designed in such a way that the interference is nullified by performing zero-forcing. In [BKW+09] the relay needs $R \geq 2K$ antennas in order to spatially separate the data streams. If the receiving nodes are able to perfectly cancel the back-propagated self-interference $R \geq 2K-1$ antennas at the relay are sufficient in order to cancel the whole interference [LDLG11; AK11b; DK12a]. In [DK12b] and [DK12a] multi-antenna nodes are considered and the relay filters are designed in order to minimize the mean squared error (MSE). Each node is able to transmit $d=1$ data stream and requires $R \geq 2K-1$ antennas at the relay in [DK12a] and transmits $d=N$ data streams in [DK12b] and requires to achieve this $R \geq (2K-1)d$ antennas at the relay. In [JS10], the relay filters are based on zero-forcing and MMSE criteria, the relay requires $R \geq 2Kd$ antennas.

IA in a fully connected network has been investigated among others in [NLL10; GCJ11; NMK10; GWK11; GAK+13; LT15; LLC10]. Each of the K multi-antenna communication pairs [GWK11; GLA+13; GAK+13; LT15] wants to transmit d data streams and is equipped with N antennas. A single multi-antenna amplify-and-forward relay equipped with R antennas is considered in [GWK11]. To perform IA with a single relay, the number of antennas at the relay needs to be larger than or equal to the number of communication pairs times the number of transmitted data streams, i.e, $R \geq Kd$. The special case $R = Kd$ has been considered in [GWK11]. The process of IA is decoupled into three linearly independent steps which are termed signal alignment (SA), channel alignment (CA) and transceive zero-forcing (TRxZF) in [GWK11]. It is shown, that signal alignment (SA) is necessary in order to achieve IA at the receivers in a bidirectional MIMO relay aided interference network [GWK11; LT15]. SA means that all nodes transmit their signal to the relay in such a way that the signals of each communication pair are aligned in a d-dimensional subspace at the relay [GWK11]. CA is a dual problem to SA and means that the receive filters of each node are designed such that the effective channels between the relay and both nodes of a communication pair span the same subspace of the relay transmit space [GWK11]. SA in MIMO relay aided networks is among others investigated in [GWK11; LLC10; LT15; LTXL14]. It is shown in [GWK11] that relaying does not decrease the achievable DoF compared to a conventional interference channel without relay. In [GAK+13; LT15; GLA+13] the general case $R \geq Kd$ is considered. The additional antennas, at the relay, are utilized to maximize the SNR at the receivers in [GAK+13]. For this reason, a pair-aware interference alignment (PAIA) method was proposed in [GAK+13].

In [LT15] the DoF for the case $R \geq Kd$ are derived. A closed form solution has been proposed in [GLA+13] for $R \geq Kd$.

Fully connected relay aided networks with K multi-antenna communication pairs equipped with N antennas each, and Q multi-antenna amplify-and-forward relays equipped with R antennas each have been considered in [GAWK13; GWKA13]. In [GAWK13] an iterative IA algorithm as well as an MMSE algorithm are proposed. The properness conditions have been derived as well in [GAWK13].

Relay aided IA in partially connected networks is an open challenging problem. One such challenge is to investigate the benefits of relays in partially connected networks. The development of suitable IA algorithms for partially connected relay aided networks and the derivation of the corresponding properness conditions is another open problem.

The full-duplex multi-group multi-way relay channel has been investigated in [GYGP09; GYGP13]. In multi-group multi-way relaying, each node of a certain group wants to share its information with all other group-members via an intermediate relay. An amplify-and-forward half-duplex multi-way relay network considering a single group was investigated in [AK10b]. The multi-antenna relay spatially separates the received data stream and the communication takes place via several MAC and BC-phases [AK10b]. A single-group multi-way relaying network where each of the K nodes are equipped with a single antenna and the whole communication takes place via the multi antenna relay in 1 MAC and $(K-1)$ BC-phases has been investigated in [LD18]. Multi-group multi-way relaying utilizing a decode-and-forward relay was considered and investigated in [AK11a]. In [AK11a] different BC-strategies has been proposed in order to spatially separate the different groups and enable a multi-way communication within each group. Multi-antenna nodes where each node is equipped with N antennas and wants to transmit $d = N$ data-streams simultaneously is considered in [DK13]. The relay filter in [DK13] is designed in order to minimize the MSE.

IA in a fully connected MIMO multi-group multi-way relaying network with L groups and K nodes in each of the group has been investigated in [GAL+14]. In this network $L(K-1)d$ DoF can be achieved if each node transmits d data streams [GAL+14]. The number of required antennas at the relay scales linearly with the number of nodes per group. The development of suitable algorithms to overcome this problem is still an open problem.

Relay aided IA in large and partially connected multi-group communication networks is a completely open research field. The derivation of the properness conditions and the development of new IA algorithms are an open problem.

1.4 Open issues

In this section, the open issues addressed in this thesis are described. This thesis deals with three important interference limited network topologies:

- In **partially connected relay aided multi-pair pair-wise communication networks**, multiple communication pairs want to exchange data in a bidirectional manner, generally aided by multiple relays, considering partially connectivity.
- In **fully connected multi-group multi-way relaying networks**, multiple nodes form a group and each node wants to share its message with all other nodes in its group via an intermediate relay.
- In **partially connected multi-group multi-way relay networks**, multiple nodes form a group and each node wants to share its message with all other nodes in its group via multiple relays, considering partial connectivity.

The first investigated network topology is the partially connected relay aided pair-wise communication network. Such a network consists of multiple communication pairs and in general also of multiple relays assisting the communication in order to achieve IA at the receivers. In large interference-limited networks partial connectivity, which means that not all nodes are connected to all relays, plays an important role. In such a network, the received signals have quite different power levels due to physical phenomena. Sufficiently weak links may be neglected which results in a partially connected network. The part of the entire network formed by a relay and all nodes connected to this relay is defined as a subnetwork. In this thesis, the two-way relaying protocol is employed to achieve a bidirectional communication between the half-duplex nodes of a communication pair via an intermediate half-duplex relay. Hence, the direct links cannot be utilized. Only relays which have a connection to both nodes of a communication pair can assist the communication. Throughout the thesis, it is assumed that all nodes are served by at least one relay.

The open issues, referring to the first topology of this thesis, are the following:

Q1. What is the impact of partial connectivity on the feasibility conditions for IA?

Q2. How to perform IA in a large partially connected interference limited network, if both nodes of a communication pair are connected to the same relay?
Q3. How many antennas are required at the nodes, if the relays are equipped with the minimum required the number of antennas to perform IA, in a large partially connected network?
Q4. How to minimize the inter-subnetwork interference power, if only a single node of a communication pair is in addition connected to a relay which cannot assist the communication?
Q5. Which channel state information (CSI) has to be available at each node and each relay so that IA becomes feasible?

The second and the third topology deal with a multi-group multi-way relaying network. In group communication networks, not only pairs of nodes communicate with each other, but rather nodes belonging to the same group want to exchange information with all other group members. In this thesis, multiple half-duplex nodes form a group and each node wants to share its message with all other nodes in its own group.

The second investigated topology is the fully connected multi-group multi-way relaying network. Fully connected means that all nodes of the entire network are connected to a single relay. Hence, nodes belonging to a certain group communicate via this intermediate half-duplex relay.

The open issues addressed for the second topology of this thesis are the following:

Q6. How to perform IA in a large multi-group multi-way relaying network?
Q7. What is the relationship between the number of antennas at the nodes, the number of antennas at the relay, the number of groups and the number of nodes inside the groups to perform IA, in a large multi-group multi-way relaying network?
Q8. How achieve that the required number of antennas at the relay and the nodes become independent of the number of nodes per group?
Q9. How to improve the sum rate performance in the low and medium SNR region?

The third investigated topology is a partially connected multi-group multi-way relaying network. A network consisting of multiple groups and multiple half-duplex relays is considered. Partially connected means that not all groups of nodes are connected to all relays in the entire network. However, each group is connected to at least one relay which enables the communication of the node belonging to this group.

The open issues, addressed for the third topology of this thesis, are the following:

Q10. How to perform IA in a partially connected multi-group multi-way relaying network?

Q11. How many antennas are required at the nodes, if the relays are equipped with the minimum required the number of antennas to perform IA, in a partially connected multi-group multi-way relaying network?

1.5 Contributions and thesis overview

In this section, an overview of the thesis and the contributions which solve the open issues mentioned in Section 1.4 are presented. In the following, the contents along with the main contributions of each chapter are briefly described.

In Chapter 2, the considered network topologies are introduced and briefly described. All important assumptions which are made throughout the thesis are introduced and explained.

In Chapter 3, the system model of a large partially connected multi-pair two-way relaying network is introduced and this topology is investigated. First, the case when both nodes of a communication pair are connected to the same relays is considered. The IA conditions for a partially connected multi-pair two-way relaying network are derived (Q1). It is shown that the IA problem can be decoupled into independent subproblems which can be solved in closed form (Q2). The minimum required number of antennas at the nodes to perform IA is derived (Q3). Secondly, the case when a single node of a communication pair is in addition connected to a relay which cannot assist the communication is considered. Such a node suffers from inter-subnetwork interference. A closed form solution which minimizes the interference power is developed. The minimum required number of antennas at the nodes to perform IA and to minimize the inter-subnetwork interference is derived (Q4). Furthermore, it is shown in this chapter that partially connected relaying network requires less channel state information (CSI) at each node and each relay to perform IA than fully connected networks (Q5).

In Chapter 4, a fully connected multi-group multi-way relaying network is investigated after introducing the underlying system model. The case where each node wants to share the same information with all other nodes in its own group via an intermediate relay is considered. In such a scenario, each node is interested in all messages being transmitted by the other members of its group while the signals from the other groups are treated as interference. A transmission scheme with several multiple access phases and serveral multicast phases is considered. In each of the multicast phases, a MIMO interference channel is created, by separating the antennas of the relays into clusters (Q6). The properness conditions and the DoF of this multicast IA algorithm are investigated and derived (Q7). Each of the antenna clusters at the relay serves a specific group of nodes and transmits in such a way that the signals transmitted from different clusters are aligned at the nodes of the non-intended multicast groups. By applying this algorithm, the minimum required number of antennas at the relay becomes independent of the number of nodes per group. This constitutes a scalable and practical solution if one deals with large networks (Q8). To improve the sum rate performance in the low and medium SNR region, a multicast IA algorithm aided by a decode-and-forward relay is developed. Since the relay decodes and re-encodes the received data stream, the receive noise at the relay is not forwarded to the intended receiving nodes (Q9).

In Chapter 5, a partially connected multi-group multi-way relaying network is considered. Instead of a single relay like considered in Chapter 4, multiple relays are taken into account in this chapter. Partially connected means that not all groups of nodes are connected to all relays. However, each group is connected to at least one relay. Such a network topology can be represented as multiple subnetworks, where each of these contains one relay and all connected groups. It is shown that the IA problem can be decoupled into independent subproblems which can be solved in closed form (Q10). The minimum required number of antennas at the nodes to achieve IA at each receiver is derived (Q11).

Finally, the main conclusions of the thesis are summarized and a brief outlook is given in Chapter 6.

Chapter 2

Network Topologies and Assumptions

2.1 Overview of the Network Topologies

In this chapter, the different network topologies which are considered in this thesis are briefly described. Throughout the entire thesis, it is assumed that the nodes and the relays cannot transmit and receive simultaneously, i.e., half-duplex nodes and relays are considered. This assumption is appropriate due to the high dynamic range between the receive and transmit signal.

The deployment of relays is an effective and common solution to improve the performance of wireless communication networks of the next generations. Relaying techniques are commonly used to extend the coverage, if the transmitting nodes are power limited. In this thesis, the relays are mainly utilized in order to help the process of interference minimization or even cancellation by applying IA algorithms. A well known relaying protocol to overcome the duplex loss of conventional relaying schemes, like one-way relaying, in pair-wise communication networks is the two-way relaying protocol [RW07]. If more than two nodes want to exchange information within a certain group via an intermediate relay, this is called multi-way relaying, established in [GYGP13].

The communication nodes in the considered network topologies want to communicate via at least one intermediate relay. In a pair-wise communication network, only two nodes want to exchange data bidirectionally, in the group communication network, all nodes inside a group want to exchange data group-wise. Hence, the pair-wise communication is a special case of the group communication. As compared to the case where only pairs of nodes exchange information, the difference in the group network is that each node has to receive information being transmitted by more than one other node in its own group.

In this thesis, it is assumed that all nodes and relays which want to exchange information have a radio connection of significant strength. An obvious drawback of relay aided IA in large networks, which are fully connected, is that a large number of antennas at the relay and the nodes may be required. Hence, this thesis is, in particular, focused on large partially connected networks consisting of multiple subnetworks that are themselves fully connected.

The first topology is described in Section 2.2.1 and considers pair-wise communication in large partially connected relay aided networks, i.e., the nodes of a communication pair exchange information in a bidirectional manner. In Section 2.2.2, the second topology, a relay aided fully connected multi-group communication network is investigated, i.e., nodes belonging to the same group exchange information group-wise. In Section 2.2.3, the third topology, a partially connected multi-group communication network is investigated, i.e., nodes belonging to the same group exchange information group-wise via multiple relays.

2.2 Network Topologies

2.2.1 Pair-Wise Communication in Partially Connected Relay Aided Networks

The first investigated topology is the partially connected relay aided pair-wise communication network topology, which is illustrated in Figure 2.1. The communication pairs are indicated by different symbols and the affiliation to a relay by different colors of the lines and the nodes. In a pair-wise wireless communication network, each node wants to transmit data to and receive data from a specific node. Hence, typical applications of relay aided pair-wise communication are data transfer between two devices, controlling a machine and transmitting private messages. In large networks, the distance between different nodes my differ a lot, leading to communication links of considerably different strengths. Sufficiently weak communication links in such a large network may even be neglected. This results in a partially connected network, i.e., a large network consists in general of multiple fully connected networks which are mutually connected via a limited number of communication links. This scenario will be investigated in detail in Chapter 3.

This topology has been considered and published by the author of this thesis in [PGL+14; PLWK15; PLWK16; LAG+14; LPKW15].

In particular, we assume that each relay is connected only to a subset of nodes. The part of the network formed by a relay and the nodes being connected to this relay is defined as a subnetwork, illustrated by the lines in Figure 2.1. There will be interference between the different subnetworks due to nodes which are belonging to multiple subnetworks, i.e., being connected to multiple relays. These node are located in the so called intersection area of multiple subnetworks. The most challenging part of such a partially connected network is the handling of the nodes which are located inside the intersection

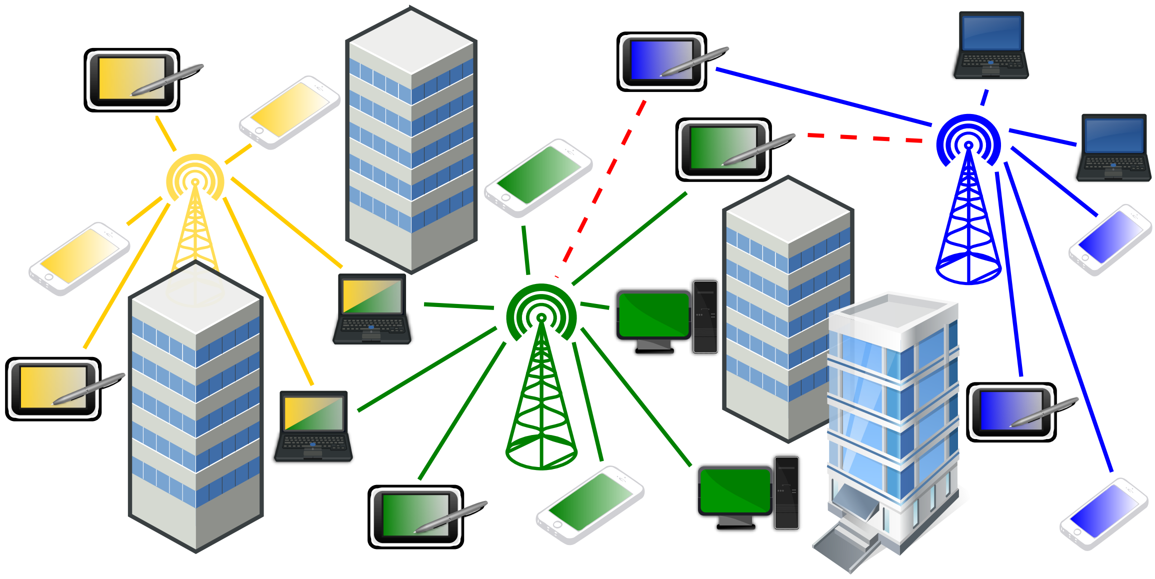


Figure 2.1. Partially connected pair-wise communication network containing three subnetworks and nine communication pairs in total

area. One the one hand, limited connectivity of a network results in less interference in the whole network. On the other hand, due to the limited connectivity of the relays, they can only provide limited assistance in handling the interference in the whole network. Hence, there is always a tradeoff between the possibility of interference handling and interference caused in the whole network.

The bidirectional communication is carried out in two phases via the intermediate relays, named MAC phase and BC phase. In the MAC phase, all nodes simultaneously transmit to all connected relays. Hence, the relays receive signals from all nodes in their subnetwork. In the BC phase, the relays retransmit a linearly processed version of the received signals back to the connected nodes. Hence, each node which belongs to a single subnetwork receives, besides the useful signal and self-interference, only interference from this subnetwork. Nodes inside the intersection area receive interference from several subnetworks in the BC phase. If a communication pair is located inside the intersection area, each node of this pair receives in general, besides the interference, also the useful signal via several relays. In general, a communication pair can only be served by the relays which are connected to both nodes of the communication pair. If only one node of a communication pair is connected to multiple relays, illustrated by the red lines in Figure 2.1, this link is a pure interference link and the node can only receive interference via this link. The direct links between the half-duplex nodes cannot be utilized, because all nodes are transmitting or receiving simultaneously.

In such a partially connected network, one can distinguish between three different types of nodes.

- Nodes of the first type are connected to a single relay and therefore belong to a single subnetwork, e.g., blue laptops in Figure 2.1. Nodes of this type receive the useful as well as interference signal only via one relay.
- Nodes of the second type are connected to multiple relays, thus belong to multiple subnetworks, e.g., yellow-green laptops in Figure 2.1. Nodes of this type receive the useful as well as interference signal via multiple relays.
- Nodes of the third type have at least one common relay and are connected in addition to a relay which cannot assist the communication, e.g. green tablet-PC in Figure 2.1. Nodes of this type receive the useful as well as interference signal via at least one relay and an additional interference signal from a relay which cannot assist the communication.

2.2.2 Fully Connected Multi-Group Multi-Way Relaying

The second investigated scenario is the fully connected multi-group multi-way relaying communication network, which is illustrated in Figure 2.2. The affiliation of the nodes to the groups is indicated by different symbols and the communication links of a group are indicated by lines of different color. A multi-way relaying network is a group communication network where each node within a certain group wants to share its own data stream with all the other nodes in its group. I.e., each node is interested in all messages being transmitted by the other members of its group while the signals from other groups are treated as interference. As compared to the first scenario where only pairs of nodes exchange information, the difference is that each destination node has to receive information being transmitted by more than one source node. Typical applications of a multi-group multi-way relaying network are video conferences, area monitoring, text based chats and multiplayer gaming. This scenario will be investigated in detail in Chapter 4.

This topology has been considered and published by the author of this thesis in [PSK17].

Due to physical propagation phenomena, e.g., path loss and shadowing, it is not realistic to assume that all nodes inside a group are connected via a direct link. Hence, in this thesis, a network topology in which the whole communication takes place via an intermediate relay is considered. The group-wise exchange of data between the nodes inside a group is performed via the multi-way relaying protocol.

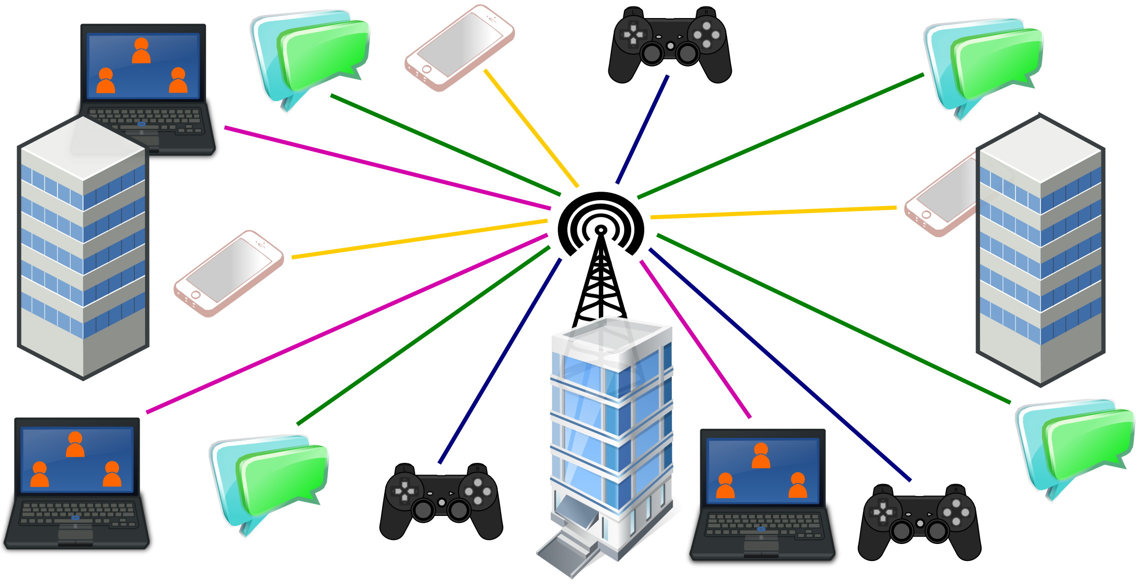


Figure 2.2. Fully connected multi-group multi-way relaying network containing three groups

Since every node wants to receive the signals from all other nodes of its own group, there is no intra-group interference. The different groups receive inter-group interference in each transmission phase of the relay. In general, the nodes can also receive self interference if the relay is transmitting.

In order to exchange information between the nodes belonging to the same group group-wise, a transmission scheme with multiple MAC phases and multiple multicast MC phases is utilized. In each of the MAC phases, a subset of nodes transmits their data simultaneously to the relay. Hence, the relay receives in general signals from multiple nodes of different groups in each MAC phase. In each of the multicast MC phases, the relay retransmits a processed signal to multiple nodes in the different groups.

2.2.3 Partially Connected Multi-Group Multi-Way Relaying

The third investigated scenario is the partially connected multi-group multi-way relaying communication network, which is illustrated in Figure 2.3. Partially connected means in this case that not all groups of nodes are connected to all relays in the entire network. However, any group is connected to at least one relay which serves this group of nodes. Similar to the fully connected multi-group multi-way relaying communication network, each node wants to exchange data streams with all other nodes in its group, but not with nodes of other groups. In Figure 2.3, the affiliation of the nodes to the groups is indicated by different symbols and the communication links of a group are indicated by lines of different colors. The typical applications of a partially connected multi-group multi-way relaying network are in general the same as in the fully connected case but in larger networks, in which the assumption of full connectivity is not realistic due to physical phenomena. This scenario will be investigated in detail in Chapter 5.

This topology has been considered and published by the author of this thesis in [PVK19].

Figure 2.3. Partially connected multi-group multi-way relaying network consisting of two subnetworks and in total fife groups

The part of the network formed by a relay and all groups being connected to this relay is defined as a subnetwork. A group which is connected to multiple relays, i.e., belonging to multiple subnetworks, is located in the so called intersection area of multiple subnetworks.

In order to exchange information between the nodes belonging to the same group groupwise, a transmission scheme with one MAC phase and multiple BC phases is utilized. In the MAC phase, all nodes transmits their data simultaneously to all connected relays. In each of the BC phases, the relays retransmits a processed signal to all connected nodes in the different groups.

2.3 Assumptions

In this section, the assumptions which are valid throughout the entire thesis unless otherwise stated are described. This thesis is a continuation and an extension of [Gan15]. Hence, few assumptions are based on [Gan15]. In contrast to [Gan15], large partially connected networks and group-communication networks are investigated. The objective of this thesis is to design the transmit, relay and receive filters such that the interference at the receivers is minimized. Hence, this thesis looks mostly for interference alignment solutions.

The following assumptions on the system and the communication protocol are valid throughout the entire thesis unless otherwise stated in the particular sections:

- The relay and the nodes are assumed to be half-duplex devices. Hence, they cannot transmit and receive simultaneously.
- The direct links between the half-duplex nodes are irrelevant, because all nodes are transmitting or receiving simultaneously.
- In order to guarantee that the d data streams transmitted by node k can be reliably received and decoded at the corresponding receiving node j, it is assumed that the number of antennas at node j is greater than or equal to d, i.e., $N_j \geq d$.
- It is assumed that the data streams transmitted by the different nodes are linear independent.
- The channels are assumed to be constant for at least one communication cycle, consisting, in general, of multiple MAC phases and multiple BC/MC phases.
- For simplicity, it is assumed that the received signals at the relays are synchronized.
- The relays and the nodes perform linear signal processing.
- Additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2_{\mathrm{n},k}$ and $\sigma^2_{\mathrm{r},q}$ is assumed at the receive antennas of the nodes and at the receive antennas of the relay, respectively.
- The discrete equivalent low pass signal model in considered. Hence, signals and channels are represented by their complex valued samples in the frequency domain.
- It is assumed that the nodes can perfectly cancel the self-interference.

- The relays have always enough antennas so that the interference and the useful signal can be separated at each receiver.
- Perfect global CSI is assumed to be available at each node and each relay. However, in the considered partially connected networks this assumption can be relaxed.
- Each node and each relay in the entire network has an individual maximum power constraint. The term maximum power implies that the nodes as well as the relays can transmit with a power level less than or equal to the maximum power constraint.
- The input signals are assumed to be zero mean circular symmetric complex Gaussian distributed.
- The nodes and the relays are equipped with multiple antennas.

Throughout this thesis, lower case letters represent scalars, lower case bold letters represent vectors, and upper case bold letters represent matrices. $\mathbb{R}$ and $\mathbb{C}$ represents the set of real numbers and complex numbers, respectively. $(.)^{\mathrm{T}}$, $(.)^{*}$ and $(.)^{\mathrm{H}}$ denote the transpose, the complex conjugate and the complex conjugate transpose of the matrix inside the brackets, respectively. The inverse of a square matrix is denoted by $(.)^{-1}$. $\mathbb{E}[.]$ denotes the expectation of the element inside the brackets. $|\mathcal{K}|$ denotes the cardinality of $\mathcal{K}$. $\mathbf{I}_N$ denotes an $N \times N$ identity matrix. The trace, i.e., the sum of the main diagonal elements of a matrix is denoted by $\mathrm{tr}(.)$. The Frobenious norm of $\mathbf{A}$ is denoted by $||\mathbf{A}||_{\mathrm{F}} = \sqrt{\mathrm{tr}\left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)}$. $\nu_{\mathrm{min},d}(.)$ denotes an operation delivering a matrix containing the eigenvectors corresponding to the d smallest eigenvalues of the matrix within the brackets, as its columns. The null space of a matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$ is given by $\mathrm{null}(\mathbf{A}) = \{\mathbf{x} \in \mathbb{C}^{m} : \mathbf{A}\mathbf{x} = \mathbf{0}\}$. The span of a matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$ is denoted by $\mathrm{span}(\mathbf{A}) = \{\mathbf{A}\mathbf{x} : \mathbf{x} \in \mathbb{C}^{m}\}$.

The term achievable sum-rate used in this thesis does not mean the information-theocratic capacity or any boundary. The achievable sum rate in this thesis is defined as the sum of data rates achieved by all nodes in the entire network that can be obtained by transmit, receive and relay filters designed with the corresponding algorithm.

Chapter 3

Interference Alignment in Large Partially Connected Pair-Wise Communication Networks

3.1 Introduction

In this chapter, IA in a large partially connected relay aided pair-wise communication network is considered. In such a large partially connected network, not all nodes are necessarily connected to all relays in the entire network. This leads to multiple partially connected subnetworks, where each subnetwork includes a single relay and all nodes connected to this relay. The most challenging part of such a partially connected network is the handling of the nodes which are connected to multiple relays, i.e., which are located inside the intersection area of at least two subnetworks. Hence, new algorithms to handle this type of nodes are proposed in this chapter. It is assumed that the relays do not have enough antennas to spatially separate the data streams transmitted by the nodes. The relays are used to manipulate the effective channels from the transmitters to the receivers to achieve an IA solution at each receiver.

In Section 3.2, the system model of the considered relay aided pair-wise communication in a partially connected networks is introduced. The corresponding network topology was already introduced in Section 2.2.1. The assumptions which are made regarding the considered scenario have been introduced in Section 2.3.

In Section 3.3, the achievable sum rate expression of the considered partially connected relay aided network is derived. Throughout this thesis, the achievable sum rate is the main performance measure.

In Section 3.4, the degrees of freedom metric of the considered network topology is introduced and briefly described.

In Section 3.5, the case where both nodes of a communication pair are connected to the same multiple relays is considered. First, an IA scheme is proposed, mainly to handle the nodes inside the intersection area. Further, it will be shown that the IA problem can be decoupled into linearly independent subproblems. The properness condition which determines the number of required antennas at the nodes and at the relay will be derived.

A closed form solution to achieve IA is proposed. Finally, the performance of the proposed algorithm is analyzed.

In Section 3.6, the case where a single node of a communication pair is in addition connected to a relay which cannot assist the communication, because the relay has no connection to the second node of the communication pair, is considered. Such a node receives only interference and no useful signal from the relay which cannot assist the communication. These nodes suffer from so called inter-subnetwork interference. An algorithm to minimize the inter-subnetwork interference power at a relay which has an additional connection to a single node of a communication pair is proposed. The properness condition for the proposed algorithm is derived as well. It is worth to mention, that the proposed algorithm is a closed-form solution. Finally, the performance of the proposed algorithm is evaluated.

Several parts of this chapter have been originally published be the author of this thesis in [PGL+14; PLWK15; PLWK16].

3.2 System Model

In this section, the system model of the considered pair-wise communication in partially connected relay aided networks is introduced. The entire network consists of Q subnetworks which intersect. Each of the Q subnetworks contains a single amplify-and-forward half-duplex multi-antenna relay, i.e, the number of subnetworks is equal to the number of relays. The K multi-antenna communication pairs are distributed over the Q subnetworks and communicate bidirectionally via at least one of the intermediate relays. Figure 3.1 shows an example scenario of such a partially connected pair-wise communication network consisting of $Q = 3$ subnetworks. One can see, that not all nodes are connected to all Q relays, i.e., the depicted network is partially connected.

Let $q \in \mathcal{Q} = \{1, ..., Q\}$ denote the relay index or the subnetwork index, respectively. The q-th relay in the q-th subnetwork is equipped with $R_q \geq 1$ antennas. Let (j, k), $j, k \in \mathcal{K} = \{1, ..., 2K\}$, without loss of generality, denote a communication pair, where the communication partner index is given by the index function

$$k = \Pi(j) = \begin{cases} j + K, & \forall j \leq K, \\ j - K, & \forall j > K. \end{cases} \tag{3.1}$$

In Figure 3.1 the tree subnetworks intersect in such a way that several nodes are connected to multiple relays. The communication pair $(3, 11)$ is connected and served by

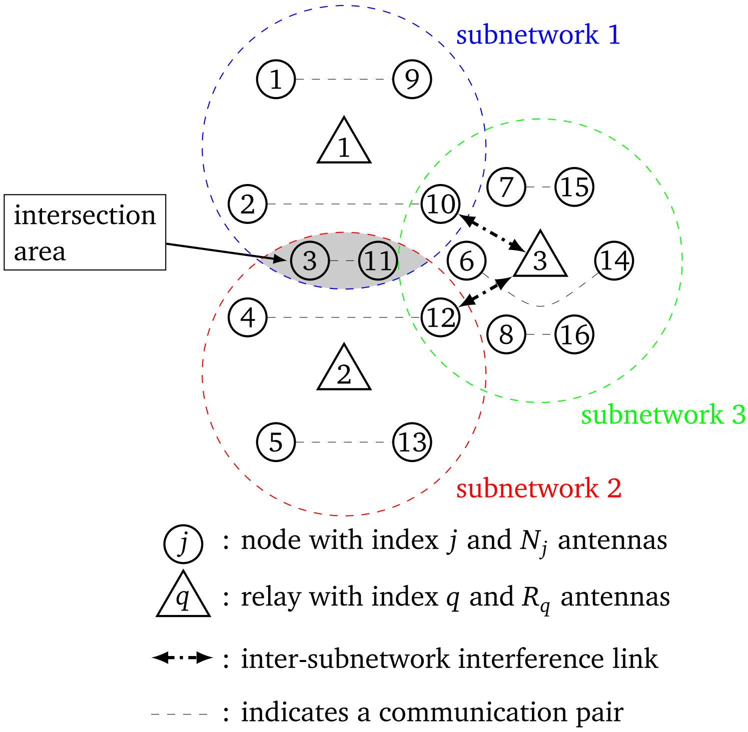


Figure 3.1. Example scenario of a partially connected pair-wise communication network consisting of $Q = 3$ subnetworks and $K = 8$ communication pairs [PLWK16].

the relays 1 and 2. The communication pair $(2, 10)$ is connected and served by relay 1 and in addition, node 10 is connected to relay 3 which cannot assist the communication.

Each node $k \in \mathcal{K}$ of the $2K$ nodes in the whole network is equipped with N_k antennas. For simplicity, it is assumed that all nodes in the network want to transmit the same number of data streams. Hence, each node wants to transmit $d \leq N_k$ data streams to its intended communication partner. To guarantee that node j can receive the d data streams transmitted from node k, it is necessary that $N_j \geq d$. Hence, in the following it is assumed that for all communication pairs (j, k), $N_j \geq d$.

In large, partially connected networks, it is appropriate to introduce sets, in order to present the connected network members in a clear manner. Let $\mathcal{K}(q)$ denote the set of nodes which are connected to relay q and $\mathcal{R}(k)$ the set of relays which are connected to node k. Hence, two example sets in Figure 3.1 are $\mathcal{K}(1) = \{1, 2, 3, 9, 10, 11\}$ and $\mathcal{R}(11) = \{1, 2\}$. Therefore, the sets of all nodes and relays in the entire partially connected network are given by

$$\mathcal{K} = \bigcup_{q \in \mathcal{Q}} \mathcal{K}(q), \tag{3.2}$$

$$\mathcal{Q} = \bigcup_{k \in \mathcal{K}} \mathcal{Q}(k), \tag{3.3}$$

respectively. The set $\mathcal{K}^{\wedge}(q)$ denotes the set of nodes which are only connected to relay q and not to any other relay, e.g., $\mathcal{K}^{\wedge}(1) = \{1,2,9\}$ in Figure 3.1. The set $\mathcal{K}^{\cap}(q1,q2) = \mathcal{K}(q1) \cap \mathcal{K}(q2)$ denotes the set of nodes inside the intersection area between q_1 and q_2, e.g., $\mathcal{K}^{\cap}(1,2) = \{3,11\}$ in Figure 3.1, respectively.

$\mathcal{R}^{\cap}(j,k) = \mathcal{R}(j) \cap \mathcal{R}(k)$ denotes the set of relays which are connected to the communication pair (j,k). All relays in this set are able to serve the communication pair (j,k), e.g., $\mathcal{R}^{\cap}(3,11) = \{1,2\}$ in Figure 3.1. If both nodes of a communication pair (j,k) are connected to the same relays $\mathcal{R}(k) = \mathcal{R}(j)$ holds.

To exchange information between the two nodes of a communication pair in a bidirectional manner, the two-way relaying protocol [RW07] is exploited. In the MAC phase, all nodes simultaneously transmit to all connected relays. Consequently, during the MAC phase, each relay receives signals from all nodes in its subnetwork. In the BC phase, the relays retransmit a linearly processed version of the received signals back to all connected nodes. Hence, each node which belongs to a single subnetwork receives, besides its useful signal and self-interference, only interference from this subnetwork, i.e., only intra-subnetwork interference. For instance, node 1 in Figure 3.1 receives interference only from a single relay, namely relay 1. Nodes which belong to multiple subnetworks receive, besides self-interference, interference from several subnetworks, i.e., intra- and inter-subnetwork interference. For instance, node 11 in Figure 3.1 receives interference from multiple relays, namely relays 1 and 2. Due to the considered two-way relaying protocol, communication pairs can only be served by relays which are connected to both nodes of a communication pair. A relay which is only connected to a single node j of the communication pair (j,k) cannot assist the communication of the communication pair (j,k). Hence, nodes which are connected to multiple relays can receive the useful signal from these relays if and only if both nodes of the communication pair are connected to these relays. If a single node of a communication pair is connected to multiple relays, this node receives inter-subnetwork interference from all relays which cannot serve the pair to which this single node belongs. For instance, node 10 in Figure 3.1 receives inter-subnetwork interference from relay 3.

Figure 3.2 shows the equivalent low pass signal model of a generalized partially connected relay aided pair-wise communication network. Generalized means in this case that the low pass signal model is valid for pairs connected to a single or to several relays as well as for single nodes connected to several relays. The direct links between the half-duplex nodes are irrelevant, because all nodes are transmitting or receiving simultaneously.

Let the column vector $\mathbf{d}_j$ denote the data vector that node j wants to transmit to node k.

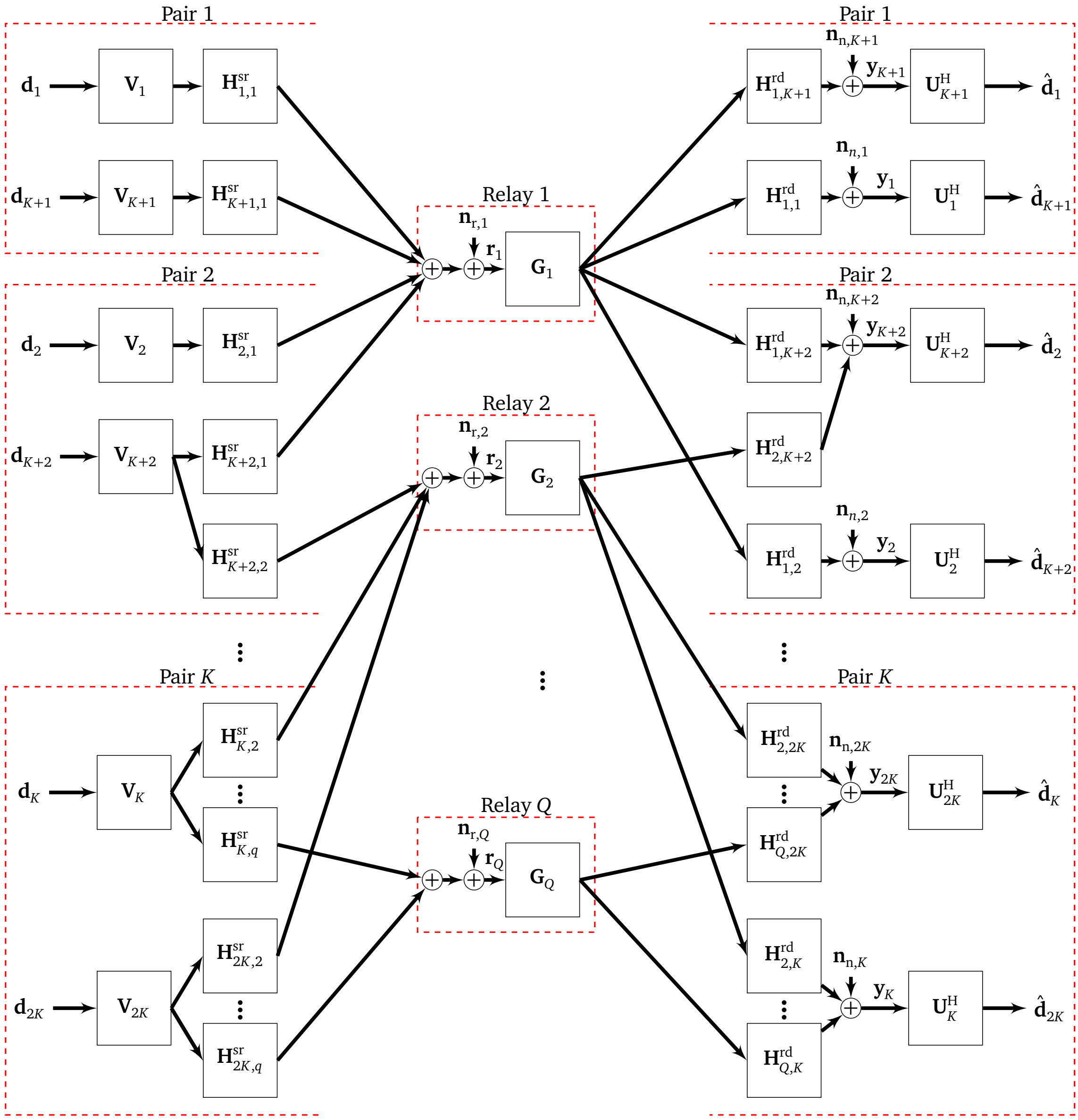


Figure 3.2. System model of a partially connected multi-pair relay aided pair-wise communication network

The covariance matrix of $\mathbf{d}_j$ is given by

$$\mathbf{R}_{\mathbf{d}_j} = \mathbb{E}[\mathbf{d}_j \mathbf{d}_j^{\mathrm{H}}]. \tag{3.4}$$

It is assumed that the transmit symbols are independent and identically distributed (i.i.d.), so that $\mathbb{E}[\mathbf{d}_k \mathbf{d}_k^{\mathrm{H}}] = \mathbf{I}_d$, $\forall k \in \mathcal{K}$ holds. The $2K$ nodes are transmitting independent data streams, i.e., $\mathbb{E}[\mathbf{d}_k \mathbf{d}_j^{\mathrm{H}}] = \mathbf{0}$, $\forall k \neq j$. Due to the linear signal processing at the nodes and at the relay, the filtering can be modeled by a matrix multiplication with the input signal vector [LS03]. Hence, $\mathbf{V}_j \in \mathbb{C}^{N_j \times d}$ denotes the linear filtering matrix of node j. For simplicity of the notation, it is assumed that each of the $2K$ nodes has a maximum transmit power, denoted by $P_{\mathrm{n,max}}$, $\forall k \in \mathcal{K}$. To satisfy the maximum transmit power constraint, the precoders are normalized. This normalization is given by

$$||\mathbf{V}_j||_F^2 \leq P_{\mathrm{n,max}}, \quad \forall j \in \mathcal{K}. \tag{3.5}$$

In the equvivalent low pass signal model, the MIMO communication channel between the transmit and receive antennas can be expressed by complex coefficients in matrix-algebraic form [LS03]. In a frequency non-selective or flat fading channel, these complex coefficients represent the transfer function in the frequency domain [GS05]. Hence, the channels between each node and each relay and vice versa is given by an $R_q \times N_j$ and $N_j \times R_q$ matrix, respectively.

In order to create independent signal paths, the antenna spacing has to be sufficiently large, so that the channel gains between the different antenna pairs fade more or less independently. The required antenna separation depends on the scattering environment as well as on the carrier frequency. Typically, an antenna separation of half to one carrier wavelength of the electromagnetic waves is sufficient to get uncorrelated channels [TV05]. Thus, the resulting channel matrices are mutually independent and of full rank with probability 1.

Let $\mathbf{H}_{k,q}^{\mathrm{sr}} \in \mathbb{C}^{R_q \times N_k}$ denote the frequency-flat, quasi-static MIMO channel matrix between node k and relay q in the MAC phase. Further, let $\mathbf{n}_{\mathrm{r},q} = \mathcal{CN}(0, \sigma_{\mathrm{r},q}^2 \mathbf{I}_{R_q}) \in \mathbb{C}^{R_q \times 1}$ denote the noise at relay q. The components of the noise vector $\mathbf{n}_{\mathrm{r},q}$ are assumed to be i.i.d. zero-mean complex Gaussian random variables with variance $\sigma_{\mathrm{r},q}^2$. In the MAC phase, all $2K$ nodes transmit their signals to the Q relays simultaneously. The received signal at relay q is given by

$$\mathbf{r}_q = \sum_{k \in \mathcal{K}(q)} \mathbf{H}_{k,q}^{\mathrm{sr}} \mathbf{V}_k \mathbf{d}_k + \mathbf{n}_{\mathrm{r},q}, \quad \forall q \in \mathcal{Q}. \tag{3.6}$$

Before relay q retransmits the received signal to all connected nodes, the relay processes this received signal linearly. Hence, the linear signal processing at relay q can be modeled as a matrix multiplication of the received signal vector $\mathbf{r}_q$ and the linear processing matrix of relay q, denoted by $\mathbf{G}_q$. For simplicity of the notation, it is assumed that all Q relays have the same maximum transmit power, denoted by $P_{\mathrm{r,max}}$, $\forall q \in \mathcal{Q}$. The relay processing matrix $\mathbf{G}_q$ is normalized such that the maximum transmit power constraint

$$\mathbb{E}\left\{\left\|\beta_q \tilde{\mathbf{G}}_q \mathbf{r}_q\right\|_F^2\right\} \leq P_{\mathrm{r,max}} \tag{3.7}$$

with

$$\mathbf{G}_q = \beta_q \cdot \tilde{\mathbf{G}}_q \tag{3.8}$$

is fulfilled, where $\tilde{\mathbf{G}}_q$ denotes the unnormalized precoder and β_q the normalization factor related to relay q. (3.7) can be rewritten as

$$\mathbb{E}\left\{\left\|\beta_q \tilde{\mathbf{G}}_q \left(\sum_{k\in\mathcal{K}(q)} \mathbf{H}^{\mathrm{sr}}_{k,q}\mathbf{V}_k\mathbf{d}_k + \mathbf{n}_{\mathrm{r},q}\right)\right\|_F^2\right\} \leq P_{\mathrm{r,max}} \tag{3.9}$$

which leads to the normalization factor

$$\beta_q = \sqrt{\frac{P_{\mathrm{r,max}}}{\left\|\tilde{\mathbf{G}}_q \sum_{k\in\mathcal{K}(q)} \mathbf{H}^{\mathrm{sr}}_{k,q}\mathbf{V}_k\right\|_F^2 + \left\|\tilde{\mathbf{G}}_q\right\|_F^2 \sigma^2_{\mathrm{r},q}}}. \tag{3.10}$$

In the second time slot, the BC phase, the Q relays broadcast a linearly processed version of the received signal back to all connected nodes. Let $\mathbf{H}^{\mathrm{rd}}_{q,k} \in \mathbb{C}^{N_k\times R_q}$ denote the frequency-flat, quasi-static MIMO channel matrices between relay q and node k in the BC phase. Let $\mathbf{n}_{\mathrm{n},k} = \mathcal{CN}(0, \sigma^2_{\mathrm{n},k}\mathbf{I}_{N_k}) \in \mathbb{C}^{N_k\times 1}$ denote the noise at node k. The components of the noise vector $\mathbf{n}_{\mathrm{n},k}$ are assumed to be i.i.d. complex Gaussian random variables with variance $\sigma^2_{\mathrm{n},k}$. The received signal at node k in the BC phase is given by

$$\begin{aligned}
\mathbf{y}_k \quad = \quad & \sum_{q\in\mathcal{R}^{\cap}(k,j)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{j,q}\mathbf{V}_j\mathbf{d}_j + \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{k,q}\mathbf{V}_k\mathbf{d}_k \\
& + \sum_{q\in\mathcal{R}^{\cap}(k,j)} \sum_{\substack{i\in\mathcal{K}(q),\\ i\neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i\mathbf{d}_i \\
& + \sum_{q\in\mathcal{R}(j)\setminus\mathcal{R}(k)} \sum_{\substack{i\in\mathcal{K}(q),\\ i\neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i\mathbf{d}_i \\
& + \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k},
\end{aligned} \tag{3.11}$$

where nodes j and k are communication partners. The first and the second term of (3.11) are the useful signal and the self-interference signal, respectively. The third and the fourth term of (3.11) represent the intra-subnetwork and inter-subnetwork interference, respectively. The last two terms represent the effective noise at node k.

Since node k knows its precoding matrix $\mathbf{V}_k$ and its data vector $\mathbf{d}_k$, node k can subtract the backpropagated self-interference from the received signal $\mathbf{y}_k$, if the backpropagation channel $\mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{k,q}\,\forall q \in \mathcal{R}(k)$ is also known at node k [RW07]. In absence of perfect backpropagation channel information, pilot symbols can be used to estimate the backpropagation channel. Throughout this thesis, it is assumed that the self-interference can be perfectly canceled.

Let $\mathbf{U}_k^{\mathrm{H}} \in \mathbb{C}^{d\times N_k}$ denote the receive zero-forcing filter at node k. The estimated data vector at node k is given by

$$\begin{aligned}
\hat{\mathbf{d}}_j \;=\;& \mathbf{U}_k^{\mathrm{H}} \sum_{q\in\mathcal{R}^{\cap}(k,j)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{j,q}\mathbf{V}_j\mathbf{d}_j \\
&+\mathbf{U}_k^{\mathrm{H}} \sum_{q\in\mathcal{R}^{\cap}(k,j)} \sum_{\substack{i\in\mathcal{K}(q),\\ i\neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i\mathbf{d}_i \\
&+\mathbf{U}_k^{\mathrm{H}} \sum_{q\in\mathcal{R}(j)\setminus\mathcal{R}(k)} \sum_{\substack{i\in\mathcal{K}(q),\\ i\neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i\mathbf{d}_i \\
&+\mathbf{U}_k^{\mathrm{H}}\left(\sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k}\right).
\end{aligned} \tag{3.12}$$

In (3.12), the self interference has been subtracted before applying the receive filter.

3.3 Achievable Sum Rate

In this section, the achievable sum rate of a relay aided MIMO pair-wise communication network is derived. The achievable sum rate of the entire network is the sum of data rates achieved at each receiving node. The data vector $\mathbf{d}_j,\ \forall j \in \mathcal{K}$ is a circular symmetric Gaussian random vector [Gal08; NM93], as described in Section 3.2. Since the distribution of $\mathbf{d}_j,\ \forall j \in \mathcal{K}$ is assumed to be Gaussian, the sum rate expression derived in this section is the maximum achievable data rate in the network, for a given channel realization and the transmit, relay and receive filters designed with the corresponding algorithm.

The achievable data rate at a MIMO receiver, in the presence of interference, is given by [BCC+07; GJJV03]

$$R_{\mathrm{MIMO}} = \log_2\left(|\mathbf{I} + \mathbf{SINR}|\right). \tag{3.13}$$

The abbreviation **SINR** denotes the ratio of the useful signal covariance matrix to the interference plus noise signal covariance matrix [Kay93]. The useful signal at node k

transmitted by node j is given by

$$\boldsymbol{\chi}^{\mathrm{U}}_{jk} = \sum_{q\in\mathcal{R}^{\cap}(k,j)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{j,q}\mathbf{V}_j\mathbf{d}_j, \tag{3.14}$$

therefore the useful signal covariance matrix is given by

$$\boldsymbol{\Xi}^{\mathrm{U}}_{jk} = \mathbb{E}\left[\left(\boldsymbol{\chi}^{\mathrm{U}}_{jk}\right)\left(\boldsymbol{\chi}^{\mathrm{U}}_{jk}\right)^{\mathrm{H}}\right]. \tag{3.15}$$

The intra-subnetwork interference, i.e., the interference caused by nodes in the same subnetwork, at node k is given by

$$\boldsymbol{\chi}^{\mathrm{I1}}_{k} = \sum_{q\in\mathcal{R}^{\cap}(k,j)} \sum_{\substack{i\in\mathcal{K}(q),\\ i\neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i\mathbf{d}_i. \tag{3.16}$$

The inter-subnetwork interference, i.e., the interference caused by nodes from neighboring subnetworks, at node k is given by

$$\boldsymbol{\chi}^{\mathrm{I2}}_{k} = \sum_{q\in\mathcal{R}(k)\setminus\mathcal{R}(j)} \sum_{\substack{i\in\mathcal{K}(q),\\ i\neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i\mathbf{d}_i. \tag{3.17}$$

Therefore the overall interference covariance matrix is given by

$$\boldsymbol{\Xi}^{\mathrm{I}}_{k} = \mathbb{E}\left[\left(\boldsymbol{\chi}^{\mathrm{I1}}_{k} + \boldsymbol{\chi}^{\mathrm{I2}}_{k}\right)\left(\boldsymbol{\chi}^{\mathrm{I1}}_{k} + \boldsymbol{\chi}^{\mathrm{I2}}_{k}\right)^{\mathrm{H}}\right]. \tag{3.18}$$

The overall noise signal at node k is given by

$$\boldsymbol{\chi}^{\mathrm{N}}_{k} = \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k}, \tag{3.19}$$

therefore the noise covariance matrix at node k is given by

$$\boldsymbol{\Xi}^{\mathrm{N}}_{k} = \mathbb{E}\left[\left(\boldsymbol{\chi}^{\mathrm{N}}_{k}\right)\left(\boldsymbol{\chi}^{\mathrm{N}}_{k}\right)^{\mathrm{H}}\right]. \tag{3.20}$$

Two-way relaying requires two time slots for a transmission from transmitter j to receiver k [RW07], hence the factor $\frac{1}{2}$ needs to be multiplied with the MIMO data rate expression. Therefore, the achievable data rate at receiver k transmitted by node j is given by

$$R_{jk} = \frac{1}{2}\log_2\left(\left|\mathbf{I} + \frac{\boldsymbol{\Xi}^{\mathrm{U}}_{jk}}{\boldsymbol{\Xi}^{\mathrm{I}}_{k} + \boldsymbol{\Xi}^{\mathrm{N}}_{k}}\right|\right). \tag{3.21}$$

Thus, the achievable sum rate of the system is given by

$$R_{\mathrm{sum}} = \sum_{\substack{j\in\mathcal{K},\\ k=\Pi(j)}} R_{jk}. \tag{3.22}$$

3.4 Degrees of Freedom

In this section, the degrees of freedom (DoF) metric is briefly described. The DoF of a wireless interference network are the interference free signal dimensions, i.e., the number of data streams that can be transmitted simultaneously without interference [CJ08b]. The DoF-metric is a valuable first order approximation of the capacity at the high SNR region [Jaf13; CJ08b], i.e, the high SNR performance will be characterized by the DoF-metric. In general, the investigation of the DoF is much simpler than the investigation of the capacity of a wireless interference channel [WGJ11]. If the limit $\frac{R_{\text{sum}}}{\log_2(SNR)}$ exists as $SNR \to \infty$, this limiting value is said to be the achievable DoF, given by

$$\text{DoF} = \lim_{SNR \to \infty} \frac{R_{\text{sum}}}{\log_2(SNR)} \tag{3.23}$$

[Jaf13]. Hence, the DoF characterizes the slope of the sum rate curve at high SNR values. The DoF-metric is an important measurement to evaluate the performance of interference limited systems.

3.5 Communication Pairs Connected to the Same Multiple Relays

3.5.1 Introduction

In this section, one of most challenging parts of partially connected networks which is the handling of communication pairs connected to the same multiple relays is investigated. A communication pair which is connected to multiple relays will be served by these multiple relays exploiting that all connected relays can receive and retransmit the useful signal of this communication pair. Hence, $\mathcal{R}(k) = \mathcal{R}(j)$ for all communication pairs (j, k) in the entire network. It is assumed that all communication pairs in the entire network are served by at least one relay. In Section 3.5.3, a new three step IA scheme is proposed and in Section 3.5.4, the corresponding closed form solution is presented. Finally, in Section 3.5.7, the performance of the new algorithms is investigated and evaluated.

The content of this section has been published by the author of this thesis in [PGL+14] and [PLWK15].

3.5.2 Interference Alignment Conditions

In this section, the IA conditions of a partially connected relay aided pair-wise communication network are introduced. In this thesis, the alignment of signal spaces based on linear processing (beamforming) is considered. In order to achieve IA at the receivers, the whole receive space is divided into two disjoint subspaces, the useful signal subspace (USS) and the interference signal subspace (ISS). At each receiving node, all interference signals should be aligned in the ISS so that the USS contains only the useful signal. After performing IA, the aligned interference inside the ISS is nullified by a projection of the receive signal onto a subspace orthogonal to the ISS subspace. The useful signal within the USS is certainly not canceled by this so-called ZF receiver due to the disjoint subspaces. This can be achieved by designing the transmit, relay and receive filter properly. The self-interference can in general be in the USS or the ISS. This problem is equivalent to the existence of a transmit filter $\mathbf{V}_j$, $\text{rank}(\mathbf{V}_j) = d$, a relay processing matrix $\mathbf{G}_q$, $\text{rank}(\mathbf{G}_q) = R_q$ and a receive filter $\mathbf{U}_k^{\text{H}}$, $\text{rank}(\mathbf{U}_k^{\text{H}}) = d$, such that the following IA conditions are fulfilled.

These IA conditions are given by

$$\mathbf{U}_k^{\text{H}} \sum_{q \in \mathcal{R}(k)} \mathbf{H}_{q,k}^{\text{rd}} \mathbf{G}_q \mathbf{H}_{i,q}^{\text{sr}} \mathbf{V}_i = \mathbf{0}, \quad \forall i \in \{i \in \mathcal{K}(q) : i \neq k, j\}, \tag{3.24}$$

$$\text{rank}\left(\mathbf{U}_k^{\text{H}} \sum_{q \in \mathcal{R}(k)} \mathbf{H}_{q,k}^{\text{rd}} \mathbf{G}_q \mathbf{H}_{j,q}^{\text{sr}} \mathbf{V}_j \right) = d. \tag{3.25}$$

(3.24) may be interpreted as a condition for the existence of an interference free space of dimension d at receiver k of the communication pair (j, k). In other words, by (3.24) it is guaranteed that the interference signals are within an $(N_k - d)$-dimensional subspace at receiver k that can be zero-forced by the receive filter $\mathbf{U}_k^{\text{H}}$. (3.25) is a condition which grantees that the desired signal is resolvable within the interference free subspace by spanning a d-dimensional subspace at receiver k after the receive filter. (3.24) and (3.25) are necessary as well as sufficient for the feasibility of IA. The receive filter nullifies the signals within the ISS, but not the useful signals within the USS. Therefore, both subspaces, the USS and the ISS, have to be disjoint from each other.

3.5.3 Interference Alignment Scheme

3.5.3.1 MAC phase: Simultaneous Signal Alignment

In this section, the MAC phase transmission strategy, if both nodes of a pair are connected to the same multiple relays, is described. The underlying system model has been introduced in Section 3.2. In the following, a new technique called simultaneous signal alignment (SSA) is developed, which offers the possibility for communication pairs connected to the same multiple relays to perform signal alignment (SA) simultaneously at multiple relays. It is considered that the relays have the minimum required number of antennas to achieve IA. SA was proposed in [GWK11] in order to perform IA in a small fully connected single relay pair-wise communication network. SSA is a generalization of SA in order to handle larger partially connected more realistic network structures, in which communication pairs can be served by multiple relays.

In the MAC phase, all nodes $k \in \mathcal{K}$ of the entire network simultaneously transmit their data to all connected relays. Hence, during the MAC phase, each relay $q \in \mathcal{Q}$ receives signals from all nodes in its subnetwork. In total $\sum_{i\in\mathcal{K}(q)} d$ data streams are transmitted in the MAC phase. It is assumed that $\sum_{i\in\mathcal{K}(q)} d$ data streams cannot be spatially separated in an R_q-dimensional relay space of relay q. It is shown that SSA is necessary in order to achieve IA at the receivers if the number of relay antennas is limited. The objective is to develop a relay aided IA algorithm for large partially connected networks. To achieve an IA solution and to reliably decode the useful signal, it is necessary that the unknown interference signal and the useful signal are in linearly independent subspaces at the receivers, i.e., the USS and the ISS have to be linearly independent. The self-interference can be in the USS or the ISS. It is assumed that the self-interference can be perfectly canceled at the receiver. The signal space of any transmitting node is of dimension N_k and spanned by the columns of the corresponding channel matrix $\mathbf{H}^{\text{sr}}_{k,q}$. Hence, signals transmitted by node k will be in an N_k-dimensional subspace at relay $q \in \mathcal{R}(k)$. The USS needs to have at least the dimension of the data vector. Therefore, a USS of size d has to be reserved for the useful signal in order to satisfy the IA conditions introduced in Section 3.5.2, i.e., (3.24) and (3.25). The interference signals have to be within the remaining $(N_k - d)$-dimensional ISS. The columns of $\mathbf{H}^{\text{sr}}_{k,q}\mathbf{V}_k$ span a d-dimensional subspace within the N_k-dimensional subspace at relay $q \in \mathcal{R}(k)$.

To avoid inter-pair interference, the signals of all node pairs which are connected to relay $q \in \mathcal{Q}$ should be pair-wise aligned and linearly independent of each other pair's signals at relay q. This means that each node of the communication pair (j, k), $\forall j \neq k,\ j, k \in \mathcal{K}(q)$ designs its transmit filter in such a way that the two d-dimensional subspaces spanned by node j and node k are pairwise aligned in a subspace of the entire signal space at relay q.

The first SSA condition to align the signals from communication pair (j,k) at relay q is given by

$$\operatorname{span}\left(\mathbf{H}^{\mathrm{sr}}_{j,q}\mathbf{V}_j\right) = \operatorname{span}\left(\mathbf{H}^{\mathrm{sr}}_{k,q}\mathbf{V}_k\right) \tag{3.26}$$

where span($\mathbf{A}$), $\mathbf{A} \in \mathbb{C}^{n\times m}$ denotes the subspace spanned by the columns of the matrix $\mathbf{A}$, i.e., $\operatorname{span}(\mathbf{A}) = \{\mathbf{Ax} : \mathbf{x} \in \mathbb{C}^m\}$. This condition is equal to the SA condition proposed in [GWK11] and is sufficient if a communication pair is only connected to a single relay. After performing SA or SSA, there are $\frac{1}{2}\sum_{i\in\mathcal{K}(q)} d$ effective data streams at relay q.

If a communication pair (j,k) inside the intersection area shall additionally perform SA at a second relay $\tilde{q} \in \mathcal{Q} \setminus \{q\}$, this pair has to fulfill a second SSA condition additionally to (3.26). This second condition is given by

$$\operatorname{span}\left(\mathbf{H}^{\mathrm{sr}}_{j,\tilde{q}}\mathbf{V}_j\right) = \operatorname{span}\left(\mathbf{H}^{\mathrm{sr}}_{k,\tilde{q}}\mathbf{V}_k\right). \tag{3.27}$$

It is possible to add further conditions in the same way, but for simplicity of the notation, in this thesis we only consider intersections of at most two subnetworks.

3.5.3.2 BC phase: Simultaneous Channel Alignment and Transceive Zero-Forcing

In this section, the BC phase transmission strategy, if both nodes of a pair are connected to the same multiple relays, is described. A new filter design concept called simultaneous channel alignment (SCA) is proposed. SCA is a generalization of channel alignment (CA) which was proposed in [GWK11], in order to perform IA in small fully connected single relay aided pairwise communication networks.

After SSA, $\frac{1}{2}\sum_{i\in\mathcal{K}_q} d$ effective data streams are available at relay q. In the BC phase, the coefficients of the relay processing matrix are chosen such that IA is achieved at the receiving nodes. After achieving an IA solution at each node, the receive filter can be designed in order to nullify the interference and to maintain the useful signal at the nodes. The signal space of any receiving node is of dimension N_k and spanned by the columns of the corresponding channel matrix $\mathbf{H}^{\mathrm{rdH}}_{q,k}$. The relay and the receive filter have to be chosen such that (3.24) and (3.25) are satisfied. There are $\frac{1}{2}\sum_{i\in\mathcal{K}_q} d$ effective channels at which the relay has to perform transmit ZF in order to nullify the interference at the receivers.

Each node served by relay q designs its receive filter such that the effective channels of the communication pair (j,k) span the same subspace at relay q. This condition is extended to SCA in this section. Like SA and CA are dual problems [GWK11], SSA and SCA are also dual problems. The first SCA condition for the communication pair (j,k) is given by

$$\operatorname{span}\left(\mathbf{H}_{q,j}^{\mathrm{rdH}}\mathbf{U}_j\right) = \operatorname{span}\left(\mathbf{H}_{q,k}^{\mathrm{rdH}}\mathbf{U}_k\right). \tag{3.28}$$

This first SCA condition is equal to the CA condition proposed in [GWK11]. Hence, the first SCA condition is valid for the nodes which are in the set $\mathcal{K}^{\wedge}(q)$, i.e., for nodes for which CA is sufficient. If a communication pair is located inside the set $\mathcal{K}^{\cap}(q,\tilde{q})$, a second condition, given by

$$\operatorname{span}\left(\mathbf{H}_{\tilde{q},j}^{\mathrm{rdH}}\mathbf{U}_j\right) = \operatorname{span}\left(\mathbf{H}_{\tilde{q},k}^{\mathrm{rdH}}\mathbf{U}_k\right) \tag{3.29}$$

has to be fulfilled to perform SCA. Since SSA and SCA are dual problems, determining the solution space for SCA is dual to determining the SSA solution space.

After SSA and SCA, there are $\frac{1}{2}\sum_{i\in\mathcal{K}_q} d$ effective data streams and $\frac{1}{2}\sum_{i\in\mathcal{K}_q} d$ effective channels.

In this paragraph, transceive zero-forcing (TxZF) performed by the relay is described. At the input of each receiver, the interference signals are inside the ISS and the useful signal is in the USS, both subspaces are disjoint of each other. It is not necessary that the interference is already zero at the input of the receive filter $\mathbf{U}_k^{\mathrm{H}}$. However, the interference at the output of the receive filter will be zero for sure. This is achieved by the relay filters $\mathbf{G}_q$, which are based on the effective channels.

3.5.3.3 Properness Conditions for SSA and SCA

In this section, the properness conditions which has to be fulfilled to perform SSA and SCA is derived. A system is defined to be proper if and only if the number of independent variables in the system is larger than or equal to the number of independent equations in the system [YGJK09; YGJK10; TGR09]. Hence, whether a system is proper or improper is based on the number of antennas at the nodes and at the relay. The properness condition is neither a necessary nor a sufficient condition, but proper systems are likely to be feasible, i.e., it is likely that an IA solution for the system exists [YGJK09]. In other words, proper systems are almost surely feasible, but improper systems are for sure not. Because of the limited signal space dimensions not all systems configurations are feasible. Therefore, the determination of the properness conditions is an impotent step in order to find a feasible system configuration. The properness of a system depends on finding matrices which satisfy the set of equations given by the IA conditions. The properness condition is a necessary as well as sufficient condition if the IA problem is linear [Str09].

In general, the feasibility of an IA problem is associated with the solvability of a set of polynomial equations [RLW13].

Communication pairs which are connected to the same multiple relays, i.e., communication pairs inside the intersection area, are performing SSA and SCA. If a communication pair is only connected to a single relay, the alignment problem can be reduced to SA and CA [GWK11]. SSA and SCA as well as SA and CA are performed at the node in the different groups. The number of effective data streams $\frac{1}{2}\sum_{i\in\mathcal{K}(q)} d$, derived in Section 3.5.3.1, at each relay results in the condition

$$R_q = \frac{1}{2} \sum_{i\in\{\mathcal{K}_q:(i,\Pi(i))\in\mathcal{K}_q\}} d \tag{3.30}$$

for the number of required antennas at each relay. Hence, the relay is always able to perform TxZF. Due to the decoupling of the trilinear IA problem into three linear subproblems named SSA/SA, SCA/CA and TxZF, the properness conditions derived in this section becomes necessary as well as sufficient. The properness of a system is defined by the number N_k of antennas at the nodes as well as on the number of equations given by the SSA and SCA conditions. From (3.26), (3.27) and (3.28), (3.29) it is known that SSA and SCA are dual problems. The involved number of equations and signal dimensions is the same for both problems. Hence, it is sufficient to define the properness based on SSA.

The signal space at a node has to be large enough such that the communication pair (j,k) can select a common subspace in the desired relay signal spaces. If a communication pair can only be served by a single relay q, this pair has to select a d-dimensional signal space at relay q. If a communication pair is served by the relays q and $\tilde{q}$, this pair has to select a d-dimensional signal space at relay q and $\tilde{q}$. This selection of a common subspace is necessary to perform SA and CA for nodes which are served by a single relay, i.e., $|\mathcal{R}^{\cap}(j,k)| = 1$, and to perform SSA and SCA for the nodes served by multiple relays, i.e., $|\mathcal{R}^{\cap}(j,k)| > 1$, which mean that nodes are inside the intersection area.

The signal space of transmitter k is of dimension N_k and the signal space at relay q is of dimension R_q. The two N_k-dimensional signal spaces of the communication pair (j,k) have to intersect in an at least d-dimensional subspace in the R_q-dimensional relay space. Applying the subspace dimension theorem [Str09], one gets the condition

$$d \leq 2N_k - R_q, \tag{3.31}$$

which has to be fulfilled in order to perform SA and CA at relay q. This condition is equivalent to the condition mentioned in [GWK11], which considers only a single relay in a fully connected network.

Hence, the total required number of antennas for each communication pair which is only connected to a single relay is given by

$$N_k \geq \frac{R_q + d}{2}, \quad \forall k \in \left\{\mathcal{K}(q) : (j,k) \notin \mathcal{K}^{\cap}(q,\tilde{q}),\ \forall \tilde{q} \in \mathcal{Q} \setminus q\right\}. \tag{3.32}$$

The condition if the two N_k-dimensional signal spaces of the communication pair (j,k) have to intersect in an at least d-dimensional subspace in the common relay space of relay q and $\tilde{q}$ is given by

$$d \leq 2N_k - R_q - R_{\tilde{q}}. \tag{3.33}$$

Hence, the total required number of antennas at each node inside an intersection area is given by

$$N_k \geq \frac{R_q + R_{\tilde{q}} + d}{2}, \quad \forall k \in \left\{\mathcal{K}^{\cap}(q,\tilde{q}) : (j,k) \in \mathcal{K}^{\cap}(q,\tilde{q})\right\}. \tag{3.34}$$

3.5.4 Interference Alignment Algorithm

3.5.4.1 Simultaneous Signal Alignment

In this section, an algorithm to achieve an SSA solution at all connected relays is described. For simplicity of the notation, as mentioned in Section 3.5.3, it is assumed that an intersection area consists of at most an intersection of two subnetworks, i.e, the nodes inside the intersection area will be served by two relays. However, the presented algorithm can easily be extended to the general case. The closed form solution for the two transmit filters $\mathbf{V}_j$ and $\mathbf{V}_k$ of the considered communication pair (j,k) is derived.

The first SSA condition (3.26) requires that both nodes of a communication pair are equipped with the same number of antennas, i.e., $N_k = N_j$, and that they transmit the same number of data streams d. Therefore, the columns of $\mathbf{H}_{jq}^{\mathrm{sr}}\mathbf{V}_j$ span a d-dimensional subspace in the N_k-dimensional subspace spanned by the channel $\mathbf{H}_{jq}^{\mathrm{sr}}$. Similarly, the columns of $\mathbf{H}_{kq}^{\mathrm{sr}}\mathbf{V}_k$ span a d-dimensional subspace in the N_k-dimensional subspace spanned by the channel $\mathbf{H}_{kq}^{\mathrm{sr}}$. In order to satisfy the first SSA condition (3.26), these two d-dimensional subspaces spanned by the columns of $\mathbf{H}_{jq}^{\mathrm{sr}}\mathbf{V}_j$ and $\mathbf{H}_{kq}^{\mathrm{sr}}\mathbf{V}_k$ have to overlap. In other words, the two N_k-dimensional subspace spanned by the channels $\mathbf{H}_{jq}^{\mathrm{sr}}$ and $\mathbf{H}_{kq}^{\mathrm{sr}}$ have to intersect in an at least d-dimensional subspace at relay q. If the dimension of the intersection subspace is larger than d, then there exists a solution space instead of a unique solution.

Let the columns of $\mathbf{A}_q = \begin{bmatrix} \mathbf{A}_{j,q} \\ \mathbf{A}_{k,q} \end{bmatrix}$ span the entire SA solution space. Hence, the transmit filters $\mathbf{V}_k$ and $\mathbf{V}_j$ of the communication pair (j,k) are a subspace of the space spanned by the columns of $\mathbf{A}_q$, given by

$$\operatorname{span}\left(\begin{bmatrix} \mathbf{V}_j \\ \mathbf{V}_k \end{bmatrix}\right) \subseteq \operatorname{span}\left(\begin{bmatrix} \mathbf{A}_{j,q} \\ \mathbf{A}_{k,q} \end{bmatrix}\right). \tag{3.35}$$

To determine the entire solution space, (3.26) can be rewritten, without loss of generality, as a homogeneous linear system of equations given by

$$\underbrace{\begin{bmatrix} \mathbf{H}^{\mathrm{sr}}_{j,q} & -\mathbf{H}^{\mathrm{sr}}_{k,q} \end{bmatrix}}_{\mathbf{H}^{\mathrm{ss}}_{j,k,q}} \cdot \underbrace{\begin{bmatrix} \mathbf{A}_{j,q} \\ \mathbf{A}_{k,q} \end{bmatrix}}_{\mathbf{A}_q} = \mathbf{0}. \tag{3.36}$$

The entire solution space $\mathbf{A}_q = \begin{bmatrix} \mathbf{A}^{\mathrm{T}}_{j,q} & \mathbf{A}^{\mathrm{T}}_{k,q} \end{bmatrix}^{\mathrm{T}}$ is determined by taking the null space of $\mathbf{H}^{\mathrm{ss}}_{j,k,q} = \begin{bmatrix} \mathbf{H}^{\mathrm{sr}}_{j,q} & -\mathbf{H}^{\mathrm{sr}}_{k,q} \end{bmatrix} \in \mathbb{C}^{R_q \times 2N_k}$, given by

$$\underbrace{\begin{bmatrix} \mathbf{A}_{j,q} \\ \mathbf{A}_{k,q} \end{bmatrix}}_{\mathbf{A}_q} = \operatorname{null}\left(\mathbf{H}^{\mathrm{ss}}_{j,k,q}\right), \tag{3.37}$$

where $\mathbf{A}_q$ denotes a basis of the nullspace. Hence, the columns of the matrix $\mathbf{A}_q$ span the entire nullspace of $\mathbf{H}^{\mathrm{ss}}_{j,k,q}$.

In order to satisfy the second SSA condition (3.27), the two d-dimensional subspaces spanned by the columns of $\mathbf{H}^{\mathrm{sr}}_{j\tilde{q}}$ and $\mathbf{H}^{\mathrm{sr}}_{k\tilde{q}}$ have to intersect in an at least d-dimensional subspace at relay $\tilde{q}$.

The corresponding homogeneous linear system of equations is given by

$$\underbrace{\begin{bmatrix} \mathbf{H}^{\mathrm{sr}}_{j,\tilde{q}} & -\mathbf{H}^{\mathrm{sr}}_{k,\tilde{q}} \end{bmatrix}}_{\mathbf{H}^{\mathrm{ss}}_{j,k,\tilde{q}}} \cdot \underbrace{\begin{bmatrix} \mathbf{A}_{j,\tilde{q}} \\ \mathbf{A}_{k,\tilde{q}} \end{bmatrix}}_{\mathbf{A}_{\tilde{q}}} = \mathbf{0}. \tag{3.38}$$

If the two solution spaces $\mathbf{A}_q$ and $\mathbf{A}_{\tilde{q}}$ have an intersection, i.e, $\mathbf{A}_q \cap \mathbf{A}_{\tilde{q}} \neq \emptyset$, it is possible to achieve signal alignment simultaneously at two different relays q and $\tilde{q}$. The condition under which a common solution space exists has been introduced in Section 3.5.3.3. The entire SSA solution space $\mathbf{A}_q \cap \mathbf{A}_{\tilde{q}} = \mathbf{A}$ is given by

$$\underbrace{\begin{bmatrix} \mathbf{A}_j \\ \mathbf{A}_k \end{bmatrix}}_{\mathbf{A}} = \operatorname{null}\left(\mathbf{H}^{\mathrm{ss}}_{j,k,q}\right) \cap \operatorname{null}\left(\mathbf{H}^{\mathrm{ss}}_{j,k,\tilde{q}}\right). \tag{3.39}$$

Equation (3.39) can be rewritten as

$$\underbrace{\begin{bmatrix} \mathbf{A}_j \\ \mathbf{A}_k \end{bmatrix}}_{\mathbf{A}} = \mathrm{null}\left(\begin{bmatrix} \mathbf{H}^{\mathrm{ss}}_{j,k,q} \\ \mathbf{H}^{\mathrm{ss}}_{j,k,\tilde{q}} \end{bmatrix} \right), \tag{3.40}$$

taking into account the properties mentioned in [Str09].

The columns of $\mathbf{A}$ span a $\left(2N_k - R_q - R_{\tilde{q}}\right)$-dimensional solution space which fulfills (3.26) and (3.27). If a node is only connected to relay q, the spanned solution space is of dimension $\left(2N_k - R_q\right)$. The precoding filters $\mathbf{V}_j$ and $\mathbf{V}_k$ are chosen from the solution space as

$$\begin{bmatrix} \mathbf{V}_j \\ \mathbf{V}_k \end{bmatrix} = \begin{bmatrix} \mathbf{A}_j \\ \mathbf{A}_k \end{bmatrix} \cdot \mathbf{\Phi}_{\mathrm{MAC}}, \tag{3.41}$$

where $\mathbf{\Phi}_{\mathrm{MAC}}$ is a matrix with d columns and rank d selecting one possible solution of the whole solution space.

The matrices $\mathbf{\Phi}_{\mathrm{MAC}}$ of (3.41) can be optimized to maximize a given objective, e.g., the sum rate. Any selection of the matrices $\mathbf{\Phi}_{\mathrm{MAC}}$ with d columns and rank d will lead to an IA solution.

3.5.4.2 Simultaneous Channel Alignment

From conditions (3.26), (3.27), (3.28) and (3.29) it can be seen that SSA and SCA are dual problems. Hence, the algorithm described in Section 3.5.4.1 can be adapted to the SCA problem, by replacing the MAC channels and the transmit filters with the corresponding BC channels and the receive filters.

Let the columns of $\mathbf{B} = \begin{bmatrix} \mathbf{B}_j \\ \mathbf{B}_k \end{bmatrix}$ span the entire SCA solution space. Similar to the SSA solution space, the SCA space is given by

$$\underbrace{\begin{bmatrix} \mathbf{B}_j \\ \mathbf{B}_k \end{bmatrix}}_{\mathbf{B}} = \mathrm{null}\left(\begin{bmatrix} \mathbf{H}^{\mathrm{ss}'}_{j,k,q} \\ \mathbf{H}^{\mathrm{ss}'}_{j,k,\tilde{q}} \end{bmatrix} \right), \tag{3.42}$$

where $\mathbf{H}^{\mathrm{ss}'}_{j,k,q} = \begin{bmatrix} \mathbf{H}^{\mathrm{rdH}}_{q,j} & -\mathbf{H}^{\mathrm{rdH}}_{q,k} \end{bmatrix} \in \mathbb{C}^{R_q \times 2N_k}$ and $\mathbf{H}^{\mathrm{ss}'}_{j,k,\tilde{q}} = \begin{bmatrix} \mathbf{H}^{\mathrm{rdH}}_{\tilde{q},j} & -\mathbf{H}^{\mathrm{rdH}}_{\tilde{q},k} \end{bmatrix} \in \mathbb{C}^{R_{\tilde{q}} \times 2N_k}$. The columns of $\mathbf{B}$ span a $\left(2N_k - R_q - R_{\tilde{q}}\right)$-dimensional solution space in the common relay space of relay q and $\tilde{q}$. Hence, (3.28) and (3.29) are fulfilled. If a communication pair is only served via a single relay q, the spanned solution space is of dimension $2N_k - R_q$. The two receive filters $\mathbf{U}_j^{\mathrm{H}}$ and $\mathbf{U}_k^{\mathrm{H}}$ are chosen out of this solution space as

$$\begin{bmatrix} \mathbf{U}_j \\ \mathbf{U}_k \end{bmatrix} = \begin{bmatrix} \mathbf{B}_j \\ \mathbf{B}_k \end{bmatrix} \cdot \mathbf{\Phi}_{\mathrm{BC}}, \tag{3.43}$$

where $\mathbf{\Phi}_{\mathrm{BC}}$ is a matrix with d columns and rank d selecting one possible solution of the whole solution space.

The matrix $\mathbf{\Phi}_{\mathrm{BC}}$ of (3.43) can be optimized in order to maximize a given objective, e.g., the sum rate. Any arbitrary selection of the matrix $\mathbf{\Phi}_{\mathrm{BC}}$ with d columns and rank d will lead to an IA solution.

3.5.4.3 Transceive Zero-Forcing

In this section, the relay transceive zero forcing filter is designed. Transceive zero forcing is a combination of receive and transmit zero forcing [TV05]. The relay performs receive ZF to spatially separate the $\frac{1}{2}\sum_{i\in\mathcal{K}_q} d$ effective data streams. Afterwards, the relay performs transmit ZF to spatially orthogonalize the signals transmitted through the $\frac{1}{2}\sum_{i\in\mathcal{K}_q} d$ effective channels. To satisfy the transceive zero-forcing condition, a system of linear equations given by

$$\mathbf{I}_R = \mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}} \cdot \mathbf{G}_q \cdot \mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}}, \tag{3.44}$$

where $\mathbf{I}_R$ is an $R\times R$ identity matrix, must be fulfilled. Let $\mathbf{G}_q^{\mathrm{RX\,H}}$ and $\mathbf{G}_q^{\mathrm{TX}}$ denote the receive and transmit zero-forcing matrices, respectively. The effective channels in the MAC and BC phase are given by

$$\begin{aligned} \mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}} &= \begin{bmatrix} \mathbf{H}_{x,q}^{\mathrm{sr}}\mathbf{V}_x & \cdots & \mathbf{H}_{y,q}^{\mathrm{sr}}\mathbf{V}_y \end{bmatrix}, \\ & \qquad x,y \in \left\{x,y : x,y \in \mathcal{K}(q); x \neq y; \operatorname{span}\left(\mathbf{H}_{x,q}^{\mathrm{sr}}\mathbf{V}_x\right) \neq \operatorname{span}\left(\mathbf{H}_{y,q}^{\mathrm{sr}}\mathbf{V}_y\right)\right\} \qquad (3.45) \\ \mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}} &= \begin{bmatrix} \mathbf{U}_x^{\mathrm{H}}\mathbf{H}_{q,x}^{\mathrm{rd}} \\ \vdots \\ \mathbf{U}_y^{\mathrm{H}}\mathbf{H}_{q,y}^{\mathrm{rd}} \end{bmatrix}, \\ & \qquad x,y \in \left\{x,y : x,y \in \mathcal{K}(q); x \neq y; \operatorname{span}\left(\mathbf{U}_x^{\mathrm{H}}\mathbf{H}_{q,x}^{\mathrm{rd}}\right) \neq \operatorname{span}\left(\mathbf{U}_y^{\mathrm{H}}\mathbf{H}_{q,y}^{\mathrm{rd}}\right)\right\} \qquad (3.46) \end{aligned}$$

respectively. $\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}}$ is of dimension $R_q \times \frac{1}{2}\sum_{i\in\mathcal{K}(q)} d$ and $\mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}$ is of dimension $\frac{1}{2}\sum_{i\in\mathcal{K}(q)} d \times R_q$. The matrices in (3.45) and (3.46) are square matrices of size $\frac{1}{2}\sum_{i\in\mathcal{K}(q)} d \times \frac{1}{2}\sum_{i\in\mathcal{K}(q)} d$ if (3.30) is taken into account. The subspace spanned by the effective data stream of a given communication pair is disjoint from the subspaces of all other communication pairs. Hence, the matrices $\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}}$ and $\mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}$ are non-singular with probability one and therefore invertible.

Then $\mathbf{G}_q^{\mathrm{RX\,H}}$ and $\mathbf{G}_q^{\mathrm{TX}}$ are uniquely determined by

$$\mathbf{G}_q^{\mathrm{RX\,H}} = \left(\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}}\right)^{-1}, \tag{3.47}$$

$$\mathbf{G}_q^{\mathrm{TX}} = \left(\mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}\right)^{-1}. \tag{3.48}$$

The entire relay processing matrix is given by

$$\mathbf{G}_q = \beta_q \cdot \mathbf{G}_q^{\mathrm{TX}} \cdot \mathbf{G}_q^{\mathrm{RX\,H}} = \beta_q \cdot \left(\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}} \cdot \mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}\right)^{-1}, \tag{3.49}$$

where β_q is the normalization factor determined in (3.10) such that the relay transmit power constraint is fulfilled.

3.5.5 Required CSI

In this section, the required CSI at the nodes connected to a single relay, at the nodes connected to multiple relays and at the relay itself is described.

For the algorithm proposed Section 3.5.4 the nodes of a communication pair which are just connected to a single relay q require only pair-wise CSI to determine the transmit filter $\mathbf{V}_k$ and the receive filter $\mathbf{U}_k^{\mathrm{H}}$, see (3.26) and (3.28). Pair-wise CSI means that each node of a communication pair has to know its own channel to the connected relay, in the MAC and BC phase and the channel of its communication partner to the connected relay, in the MAC and BC phase. This is the same amount of required CSI, at a communication pair, to perform IA in fully connected relay aided networks.

The nodes of a communication pair inside the intersection area which perform SSA and SCA require multiple pair-wise CSI to determine the transmit filter $\mathbf{V}_k$ and the receive filter $\mathbf{U}_k^{\mathrm{H}}$, see (3.26), (3.27), (3.28) and (3.29). Multiple pair-wise CSI means that each node of a communication pair has to know its own channels to all connected relays, in the MAC and BC phase and the channels of its communication partner to all connected relays, in the MAC and BC phase.

Typically, in fully connected networks, the relay in relay aided IA algorithms requires global CSI to perform transceive ZF [GWK11], where global CSI means that all channels in the MAC and BC phase are known at the relay. In partially connected networks, this condition can be relaxed by exploiting the fact that some interference links among the subnetworks are missing or negligibly weak.

Relay q has to know the channels of all nodes inside its own subnetwork, referred to as subnetwork CSI, as well as the transmit and receive filter of all nodes in is own subnetwork. These filters depends on multiple pair-wise CSI, if a communication pair is inside the intersection area. Hence, relay q requires more than just subnetwork CSI to determine the relay processing matrix $\mathbf{G}_q$, but less than global CSI. In summary, it can be said that the required CSI to perform interference alignment in a partially connected network is less than global CSI. Therefore, the required CSI in partially connected networks is termed local CSI, in this thesis.

3.5.6 Algorithm Zero-Forcing Inter-Subnetwork Interference

Instead of performing SSA at multiple relays, there are also other conceivable possibilities to establish a reliable communication in relay aided partially connected pair-wise communication networks. One possible alternative algorithm is that a communication pair inside an intersection area is only served by a single relay, whereas the other connected relays treat these signals as interference and zero-force them. This algorithm was proposed by the author of this thesis in [PGL+14].

Since not all communication pairs connected to a relay are served by that relay, it is useful to introduce the set $\mathcal{K}^{\text{ser}}(q)$, which denotes the set of nodes served by relay q. Hence, $\mathcal{K}^{\text{ser}}(q)$ is a proper subset of $\mathcal{K}(q)$, i.e., $\mathcal{K}^{\text{ser}}(q) \subset \mathcal{K}(q)$.

The communication pairs in the set $\mathcal{K}^{\text{ser}}(q)$ design their filters according to the IA algorithm of Section 3.5.4, to avoid inter-pair interference. Hence, the signals of the communication pairs served by relay q are pair-wise aligned in the R_q-dimensional relay space of relay q. All pair-wise aligned signals span a subspace of dimension

$$R_q^{\text{q1}} = |\mathcal{K}^{\text{ser}}(q)|\, d/2 \tag{3.50}$$

at relay q. The unaligned signals, i.e., signals from communication pairs served by an other relay, span a subspace of dimension

$$R_q^{\text{q2}} = |\mathcal{K}(q) \setminus \mathcal{K}^{\text{ser}}(q)|\, d \tag{3.51}$$

at relay q. From these subspaces, one can derive a condition for the required number of antennas at relay q, given by

$$R_q = \left(\frac{1}{2}\, |\mathcal{K}^{\text{ser}}(q)| + |\mathcal{K}(q) \setminus \mathcal{K}^{\text{ser}}(q)|\right) d. \tag{3.52}$$

In order to cancel the interference, the relay performs transmit and receive zero forcing, which is known as transceive zero forcing [TV05]. Since the relay filters are designed according to the filter design in Section 3.5.4.3, these are not derived in this section.

The two N_k-dimensional signal spaces of the communication pair (j,k) have to intersect in an at least d-dimensional subspace in the R_q-dimensional relay space. Applying the subspace dimension theorem [Str09], one gets the condition

$$d \leq 2N_k - R_q, \tag{3.53}$$

which has to be fulfilled. Hence, the required number of antennas at each node of a communication pair is given by

$$N_k \geq \frac{R_q + d}{2}, \quad \forall k \in \mathcal{K}^{\text{ser}}(q). \tag{3.54}$$

3.5.7 Performance Analysis

3.5.7.1 Introduction

In this section, the performance of the proposed IA algorithm presented in Section 3.5 and the performance of the proposed algorithm zero-forcing inter-subnetwork interference as noise presented in Section 3.5.6 are investigated through numerical simulations. Both algorithms are designed for partially connected relay aided pairwise communication networks. Two different reference algorithms are considered for performance comparison. These reference algorithms will be briefly introduced in this section. To evaluate the performance of the algorithms, the sum rate introduced in Section 3.3 and the DoF introduced in Section 3.4 are considered. As introduced in Section 3.4, the DoF is defined as the total number of interference free data streams that can be transmitted in one channel use. The sum rate is simulated over a large SNR range, because the DoF analysis is only valid for an asymptotically high SNR. The sum rates achieved by these algorithms are obtained through numerical MATLAB simulations.

In the following the assumptions regarding the simulation are briefly described. The algorithms themselves are valid for the assumptions mentioned in Sections 2.3, 3.5.4 and 3.5.6.

- It is assumed that the channel between each node and all relays connected to this node is an i.i.d. frequency-flat Rayleigh fading MIMO channel [LS03]. Hence, the channel matrices are of full rank, almost surely.
- The channel matrices are normalized such that the average received power is the same as the average transmit signal power.
- Channel reciprocity is assumed, i.e., the MAC and BC phase channel matrices are the complex conjugate transpose of each other.
- It is assumed that all nodes are served by at least one relay.
- Due to the considered statistical channel model, the channel amplitude may vary for different realizations. Hence, all simulation results are averaged over 10^4 independent channel realizations. For each channel realization, all filters are designed according to the considered algorithm and the corresponding sum rate is calculated. The average sum rate which is plotted in this section is therefore an average over all 10^4 independent channel realizations. 10^4 independent channel realizations are large enough to get a sufficiently small confidence interval for the sum rate.
- For simplicity, it is assumed that the noise variance is equal at all nodes and all relays, i.e., $\sigma^2 = \sigma^2_{\mathrm{n},k} = \sigma^2_{\mathrm{r},q}$, $\forall k \in \mathcal{K}$, $\forall q \in \mathcal{Q}$.

As the first reference algorithm, an iterative IA algorithm, termed ITR, is considered. The algorithm presented in this paragraph minimizes the leakage interference [GCJ08] to approach an IA solution in a fully connected relay aided network. This reference algorithm is an extension of the algorithm proposed in [GAWK13], in which the transmit filters $\mathbf{V}_i\ \forall i \in \mathcal{K}$ are fixed a priori. Since this algorithm is designed for fully connected networks, all channel coefficients are non-zero, i.e., all nodes have a connection to all relays in the network. Let $\mathbf{r} = \begin{bmatrix}\mathbf{r}_1^{\mathrm{T}} & \mathbf{r}_2^{\mathrm{T}} & \cdots & \mathbf{r}_q^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$ be the concatenated received signal of all Q relays in the network. Since the relays are not sharing their received signals, one can define an overall block diagonal relay processing matrix $\mathbf{G}$. This concatenated block diagonal matrix $\mathbf{G}$ contains the matrices $\mathbf{G}_q$, $\forall q \in \mathcal{Q}$, as defined in Section 3.2, as sub-matrices. Instead of a individual power constraint of each relay q, as introduced in Section 3.2, the overall relay of this ITR algorithm has a global power constraint. It is assumed that this overall relay has a maximum transmit power, denoted by $P_{\mathrm{r,max}}^{\mathrm{block}}$.

The overall relay processing matrix is given by

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \cdots & \mathbf{0} \\ \vdots & & & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{G}_q \end{bmatrix} \tag{3.55}$$

and is normalized such that the maximum transmit power constraint

$$\mathbb{E}\left\{\left\|\beta\tilde{\mathbf{G}}\mathbf{r}\right\|_F^2\right\} \le P_{\mathrm{r,max}}^{\mathrm{block}} \tag{3.56}$$

with

$$\mathbf{G} = \beta \cdot \tilde{\mathbf{G}} \tag{3.57}$$

is fulfilled, where $\tilde{\mathbf{G}}$ denotes the unnormalized precoder and β the normalization factor.

As defined in Section 3.2, each node has a maximum transmit power of $P_{\mathrm{n,max}}$. The leakage interference minimization can be formulated as the following optimization problem, given by

$$\begin{aligned} \text{minimize} \quad & \|\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rd}}\mathbf{G}\mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i\|_{\mathrm{F}}^2,\ \forall i \neq j, k \\ \text{subject to} \quad & \|\mathbf{V}_i\|_F^2 \le P_{\mathrm{n,max}}, \\ & \mathbb{E}\left\{\left\|\beta\tilde{\mathbf{G}}\mathbf{r}\right\|_F^2\right\} \le P_{\mathrm{r,max}}^{\mathrm{block}}. \end{aligned} \tag{3.58}$$

where $\mathbf{H}_k^{\mathrm{rd}} = \begin{bmatrix}\mathbf{H}_{1,k}^{\mathrm{rd}} & \mathbf{H}_{2,k}^{\mathrm{rd}} & \cdots & \mathbf{H}_{\mathrm{Q},k}^{\mathrm{rd}}\end{bmatrix}$ and $\mathbf{H}_i^{\mathrm{sr}} = \begin{bmatrix}\mathbf{H}_{i,1}^{\mathrm{srH}} & \mathbf{H}_{i,2}^{\mathrm{srH}} & \cdots & \mathbf{H}_{i,\mathrm{Q}}^{\mathrm{srH}}\end{bmatrix}^{\mathrm{H}}$.

The entire algorithm is illustrated in Figure 3.3. First, the transmit filter $\mathbf{V}_j$ and the overall relay processing matrix $\mathbf{G}$ are arbitrarily initialized. In the next three steps, two variables are held temporarily fixed to find a solution for the remaining unfixed variable. Each of these steps is a separate minimization problem, over only one of the three matrices $\mathbf{V}_j, \mathbf{G}, \mathbf{U}_k^{\mathrm{H}}$. Such an optimization is known as alternating minimization [PH09; BH02]. These three steps are repeated until a stop criterion is reached. This can be, for example, the leakage interference or the number of iterations. In this thesis, 1000 iterations are

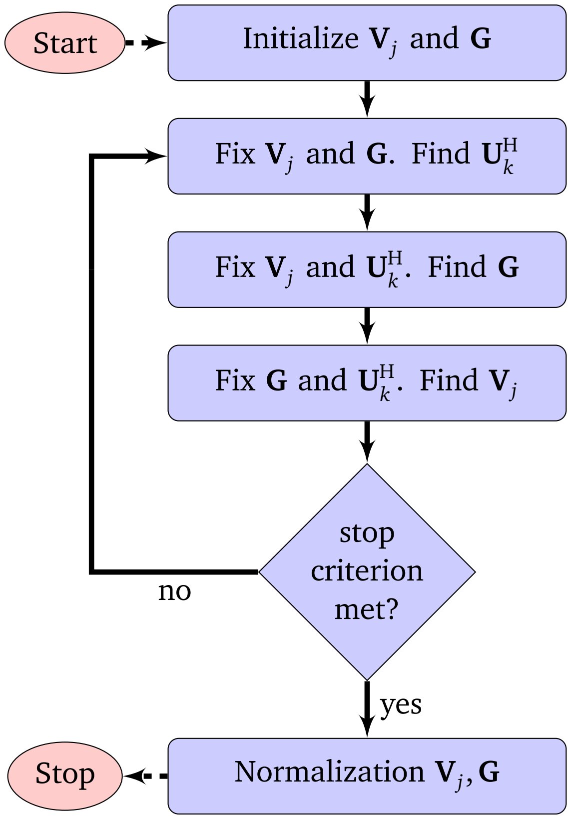


Figure 3.3. Iterative IA algorithm to approach an IA solution in a fully connected pair-wise communication network, considering multiple relays.

considered as stop criterion for the simulation. The matrices $\mathbf{V}_j$ and $\mathbf{G}$ are normalized to fulfill the power constraints. The final result of this algorithm strongly depends on the initialization values because the optimization problem is non-convex. Hence, it cannot be guaranteed that the algorithm converges to the global minimum. It has to be noted, that this algorithm requires perfect global CSI at all nodes and relays.

The second reference algorithm, termed orthogonal resources (OR), is an IA algorithm based on [GWK11] designed for fully connected two-way relaying networks. In this OR algorithm, the subnetworks are treated as individual fully connected networks. The relay processing matrix and transmit and receive filters of all nodes inside such a fully connected subnetwork are designed according to the IA algorithm proposed in [GWK11]. To omit interference between the subnetworks, caused by communication pairs belonging to several subnetworks, the individual fully connected subnetworks communicate one after the other. In other words, orthogonal resources, time slots in this algorithm, are utilized to establish an interference-free communication.

It should be noted that there are no algorithms in the state of the art literature for partially connected relay-aided pairwise communication networks that would be suitable as reference algorithms.

3.5.7.2 Degrees of Freedom Analysis

In this section, the DoF achieved by the proposed IA algorithm and the DoF achieved by the proposed algorithm zero-forcing inter-subnetwork interference as noise are investigated and compared with the two chosen reference algorithms. As introduced in Section 3.4, the DoF is defined as the total number of interference free data streams that can transmitted in one channel use. Hence, the achievable DoF are depending on the number of communication pairs that can simultaneously communicate without interference. The closed-form IA algorithm, proposed in Section 3.5.4, which utilizes SSA and SCA to perform IA, is termed simultaneous signal and channel alignment (SSCA) algorithm in this section. The algorithm introduced in Section 3.5.6, which zero-forces the inter-subnetwork interference and utilizes only SA and CA is termed zero-forcing inter-subnetwork interference (ZFISI) algorithm in this section. The main difference between these two algorithms is that communication pairs that are connected to multiple relays can be served by multiple relays considering the SSCA algorithm, but only by one relay considering the ZFISI algorithm. If the ZFISI algorithm is utilized, a communication pair that is connected to multiple relays is only served by a single relay, whereas the other relays treat signals from these nodes as interference and suppress them.

Two different partially connected networks are considered for the DoF analysis and the investigation of the sum rate. In the first considered network N1, $K = 16$ communication pairs are equally distributed over 4 subnetworks. Since each of these subnetworks contains a single relay, in total $Q = 4$ relays are in the network. Each relay is connected to 5 communication pairs, as shown in Figure 3.4. The second considered network N2 contains $K = 10$ communication pairs distributed over 3 subnetworks, as shown in Figure 3.5. Let $K_q = |\mathcal{K}(q)|$, where $|\cdot|$ denote the cardinality. In network N2, relay 1 is connected to $K_1 = 4$ communication pairs and relay 2 and relay 3 are connected to $K_2 = K_3 = 5$ communication pairs. The network N2 was selected to demonstrate that the proposed algorithms work for different numbers of nodes per subnetwork and for multiple nodes within the intersection of two subnetworks.

It has to be noted that the properness conditions of the different algorithms considered in this section do not allow to consider always the same number of antennas at the relays and nodes. Hence, six different scenarios are considered for each of the two networks. The scenarios considered for the first network N1 are shown in Table 3.1 and the scenarios considered for the second network N2 are shown in Table 3.2. It should be noted that the nodes of a communication pair are each equipped with the same number of antennas, i.e., $N_j = N_k \forall (j,k)$.

In scenario A1.1, each of the $Q = 4$ relays is equipped with $R_q = 5$, $\forall q \in \{1,2,3,4\}$ antennas according to (3.30), the number of antennas at the nodes are determined according to (3.32) and (3.34). Hence, the properness conditions of the SSCA algorithm are fulfilled and IA can be performed. The proposed SSCA algorithm serves $K = 16$ communication pairs simultaneously and achieves, therefore, 16 DoF.

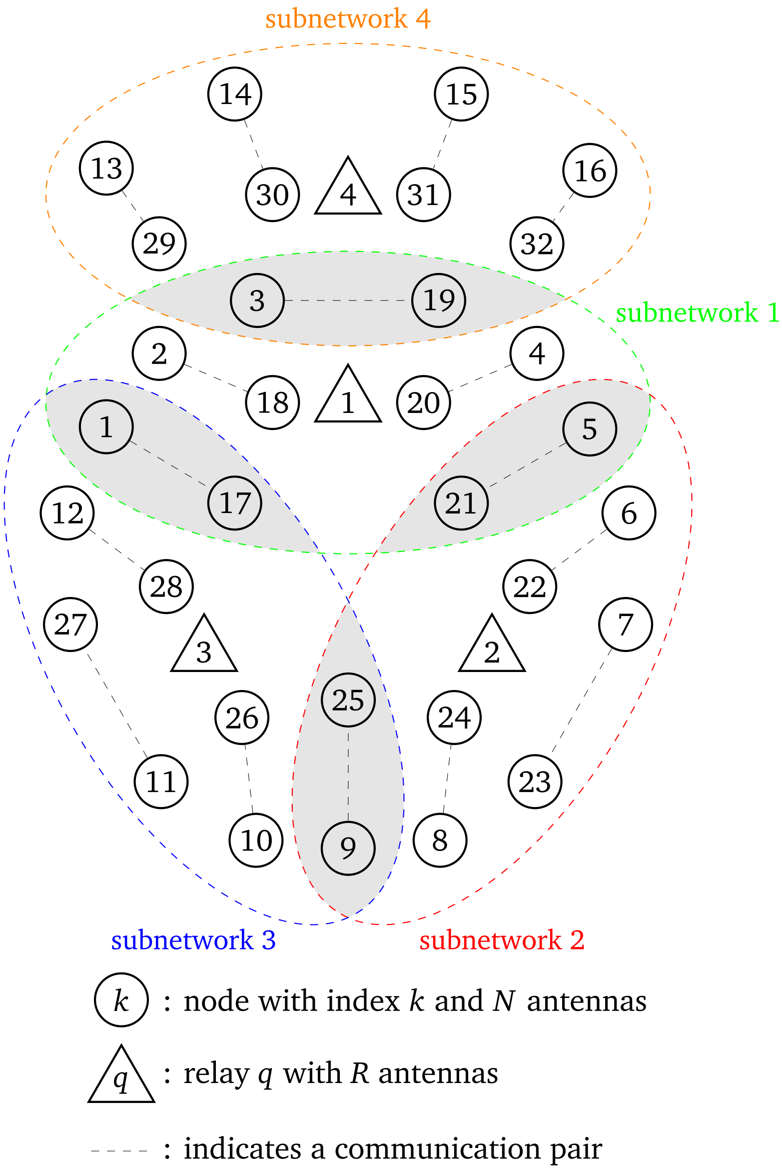


Figure 3.4. Partially connected network N1 with $K = 16$ communication pairs in 4 subnetworks.

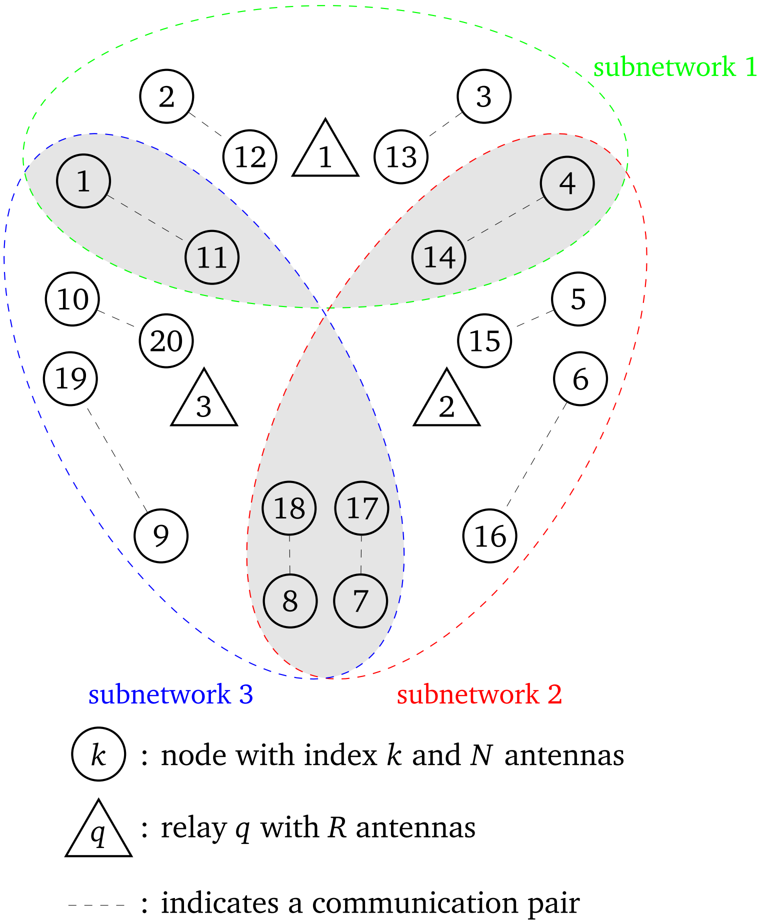


Figure 3.5. Partially connected network N2 with $K = 10$ communication pairs in 3 subnetworks.

Table 3.1. Considered scenarios for the first network N1 and the achievable DoF

Scenario	Algorithm	K	d	Q	Antennas Relays $\left[R_1 \quad R_2 \quad \cdots \quad R_Q\right]$	Antennas Nodes $N_j = N_k \forall (j,k)$ $\left[N_1 \quad N_2 \quad \cdots \quad N_K\right]$	DoF
A1.1	SSCA	16	1	4	[5 5 5 5]	[6 3 6 3 6 3 3 3 6 3 3 3 3 3 3 3]	16
A1.2	ZFISI	16	1	4	[6 6 6 6]	[4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4]	16
A1.3	ZFISI	12	1	4	[5 5 5 5]	[3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3]	12
A1.4	OR	16	1	4	[5 5 5 5]	[3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3]	4
A1.5	ITR	16	1	4	[5 5 5 5]	[6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6]	-
A1.6	ITR	8	1	4	[5 5 5 5]	[6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6]	8

Table 3.2. Considered scenarios for the second network N2 and the achievable DoF

Scenario	Algorithm	K	d	Q	Antennas Relays $\left[R_1 \;\; R_2 \;\; \cdots \;\; R_Q\right]$	Antennas Nodes $N_j = N_k \forall (j,k)$ $\left[N_1 \;\; N_2 \;\; \cdots \;\; N_K\right]$	DoF
A2.1	SSCA	10	1	3	[4 5 5]	[5 3 3 5 4 4 6 6 3 3]	10
A2.2	ZFISI	10	1	3	[5 6 7]	[3 3 3 4 4 4 4 4 4 4]	10
A2.3	ZFISI	8	1	3	[4 5 5]	[4 3 3 4 4 4 4 4 4 4]	8
A2.4	OR	10	1	3	[4 5 5]	[3 3 3 3 3 3 3 3 3 3]	4
A2.5	ITR	10	1	3	[5 5 5]	[6 6 6 6 6 6 6 6 6 6]	-
A2.6	ITR	7	1	3	[5 5 5]	[6 6 6 6 6 6 6 6 6 6]	7

In order to serve the same number $K = 16$ of communication pairs using the ZFISI algorithm, each of the $Q = 4$ relays requires $R_q = 6$, $\forall q \in \{1, 2, 3, 4\}$ antennas according to condition (3.52), see scenario A1.2. The number of antennas at the nodes has to fulfill (3.54), to achieve an IA solution. Therefore, each of the $2K$ nodes is equipped with $N_k = 4$, $\forall k \in \mathcal{K}$ antennas, as shown in Table 3.1. The ZFISI algorithm can serve $K = 16$ communication pairs simultaneously and achieves therefore 16 DoF. However, the ZFISI algorithm requires one more antenna at the relays than the SSCA algorithm. For the nodes inside the intersection area, the SSCA algorithm has a drawback, these nodes require two more antennas in comparison to the ZFISI algorithm to achieve the same DoF.

If the ZFISI algorithm shall be performed considering the same number of antennas at the relay as in scenario N1SS1, only $K = 12$ communication pairs can be served simultaneously, see scenario A1.3. Hence, the communication pairs $(4, 20)$, $(7, 23)$, $(11, 27)$ and $(16, 27)$ are not served in scenario A1.3. These nodes can be served by using TDMA in addition to the SCP algorithm, for example. In this scenario $N_k = 3$, $\forall k \in \mathcal{K}$ antennas at each node are sufficient. However, the number of antennas at the nodes does not influence the achievable DoF. Since only $K = 12$ communication pairs each transmitting $d = 1$ data stream are served simultaneously, only 12 DoF are achieved in scenario A1.3.

In scenario A1.4, the OR reference algorithm is considered. Each of the subnetworks consists of $K = 5$ communication pairs and a single relay. Due to the intersection of the subnetworks, the OR reference algorithm requires 8 time-slots to ensure that each communication pair communicates at least once free of inter-subnetwork interference. In contrast to the SSCA algorithm and the ZFISI algorithm, the OR algorithm achieves only 4 DoF. This is because the OR algorithm requires four times the number of time slots compared to the SSCA algorithm and the ZFISI algorithm to enable a bidirectional communication between the communication pairs. This will be verified in the sum-rate performance section.

In the scenarios A1.5 and A1.6 the ITR reference algorithm is taken into account, the relays are equipped with $R_q = 5$, $\forall q \in \{1, 2, 3, 4\}$ antennas and the nodes are equipped with $N_k = 6$, $\forall k \in \mathcal{K}$ antennas. $N_k = 6$ was selected because this is the maximum number of antennas required at the node, regardless of which algorithm is applied, see Table 3.1. In scenario A1.5, the ITR algorithm is not able to find an IA solution for the $K = 16$ communication pairs. However, for $K = 8$ communication pairs, the ITR algorithm finds an IA solution and achieves $K = 8$ DoF.

In all scenarios considered for network N1, the proposed algorithms SSCA and ZFISI achieve more DoF than both reference algorithms. The OR reference algorithm requires four times the number of time-slots than the proposed algorithm, this results in less DoF. Since the ITR reference algorithm considers a fully connected network, i.e., all channel coefficients are non-zero, more interference links have to be aligned in this fully connected network than in a partially connected network. Hence, the ITR reference algorithm is only able to find an IA solution if $K = 8$ and is interference limited for $K = 16$. These results for network N1 are shown in Table 3.1.

In this paragraph, network N2 is considered and investigated. In the scenarios A2.1, A2.2 and A2.3 the properness conditions belonging to the algorithms SSCA and ZFISI are fulfilled. Hence, in these three scenarios, the DoF are equal to the number K of communication pairs. Since, the OR reference algorithm requires three times the number of time-slots compared to the proposed algorithms SSCA, and ZFISI in scenario A2.4 only 4 DoF can be achieved. The ITR algorithm considered in the scenarios A2.5 and A2.6 works only if $R_1 = R_2 = R_3$ as well as $N_1 = N_2 = \cdots = N_{10}$. In scenario A2.5, the ITR algorithm is not able to find an IA solution, therefore the DoF is not determined. In scenario A2.6, the ITR algorithm finds an IA solution and achieves 7 DoF. A fully connected network has, in general, more interference links that have to be aligned at the receivers than a partially connected network. Therefore, the ITR algorithms achieve less DoF than the proposed algorithms SSCA and ZFISI. In all scenarios, considered for the network N2, the proposed algorithms SSCA and ZFISI achieve more DoF than both reference algorithms. These results are shown in Table 3.2.

It is worth mentioning that an increase in the number of antennas at the nodes does not affect the DoF of the SSCA, ZFISI, and OR algorithms, as the number of antennas at the nodes does not affect the number of nodes that can be served simultaneously.

3.5.7.3 Sum Rate Analysis

In this section, the sum rate performance of the proposed algorithms SSCA and ZFISI is compared with the two reference algorithms ITR and OR. The algorithms SSCA, ZFISI, and OR choose an arbitrary signal and channel alignment solution out of the corresponding entire solution space, i.e., the matrix $\boldsymbol{\Phi}_{\mathrm{MAC}}$ in (3.41) and the matrix $\boldsymbol{\Phi}_{\mathrm{BC}}$ in (3.43) are arbitrarily selected from the complex space $\mathbb{C}$. Hence, an increase in the number of antennas at the node does not influence the performance of these three algorithms. If one increases the number of antennas at the nodes, the corresponding solution space will increase as well which leads to further optimization possibilities. However, such further optimization is not part of this section. The ITR algorithm, in contrast, can utilize additional antennas at the nodes to minimize the leakage interference in the network, which is in favor of the ITR algorithm. Each node $k \in \mathcal{K}$ has a transmit power $P_{\mathrm{n,max}} = P$. The ITR algorithm has a global power constraint, i.e., $P_{\mathrm{r,max}}^{\mathrm{block}} = KP$. The other algorithms have an individual power constraint, i.e., $P_{\mathrm{r,max}} = \frac{1}{Q}KP$, $\forall q \in \mathcal{Q}$. However, the sum of relay transmit powers is the same. The global power constraint favors the ITR algorithm since the ITR algorithm can distribute the total transmit power arbitrarily among the relays.

Figure 3.6 shows the average sum rate in bit per channel use as a function of P/σ^2, considering network N1, shown in Figure 3.4. The solid blue curve corresponds to the average sum rate of the proposed SSCA algorithm, taking into account scenario A1.1. The loosely dashed red and yellow curves correspond to the average sum rate of the proposed ZFISI algorithm, taking into account scenario A1.2 and A1.3, respectively. The dashed violet curve, the dotted blue and green curves correspond to the average sum rate of the reference algorithms, taking into account the scenarios A1.4, A1.5, and A1.6, respectively.

The slope of the curves represents the achievable DoF of the different algorithms. It can be seen, that the SSCA algorithm achieves the highest sum rate in the high P/σ^2 region and 16 DoF. The ZFISI algorithm in scenario A1.2 achieves the same DoF, but requires an additional antenna at each relay. However, the SSCA algorithm is still better than the ZFISI algorithm in terms of sum rate. One reason for this is that in the ZFISI algorithm, the nodes inside the intersection area will be only served by one relay, while the other relay treats these signals as interference and suppresses them, which results in a power loss. For a given number of antennas at the relay, the SSCP algorithm achieves more DoF than the ZFISI algorithm. This is because communication pairs within the intersection area are served by multiple relays when the SSCA algorithm is applied. With SSA and SCA, used by the SSCA algorithm, more communication pairs can be served simultaneously, resulting in more DoF. The ITR algorithm for $K = 16$ communication pairs considered in A1.5 converges to a finite average sum rate in the high P/σ^2 region, due to interference leakage and is, therefore, interference limited. That implies that the ITR algorithm in scenario A1.5 cannot find an IA solution, i.e., IA is infeasible for the ITR algorithm in scenario A1.5. In scenario A1.6 the ITR algorithm finds an IA solution but can only serve $K = 8$ communication pairs simultaneously. Even though the favorable assumptions of the ITR algorithm, the proposed algorithms achieve better performance. The OR algorithm achieves an interference free communication, but the lowest sum rate of all considered algorithms. The reason for this is that this algorithm requires four times the number of time slots than all other considered algorithms. The sum rate of this algorithm is given by

$$R_{\mathrm{T}} = \frac{\sum_{q=1}^{Q} |\mathcal{K}_q|}{|\mathcal{K}|} \frac{1}{4} \sum_{Q}^{q=1} R_{\mathrm{sum},q}^{\mathrm{fully}}, \tag{3.59}$$

where $R_{\mathrm{sum},q}^{\mathrm{fully}}$ denotes the achievable sum rate of subnetwork q. This results in $\frac{16}{20}\frac{1}{4}\sum_{Q}^{q=1} R_{\mathrm{sum},q}^{\mathrm{fully}} = \frac{1}{5}\sum_{Q}^{q=1} R_{\mathrm{sum},q}^{\mathrm{fully}}$ for network N1. The fraction $\frac{16}{20}$ is required because the OR algorithm serves a total of 20 nodes compared to the SSCA algorithm, which serves 16 nodes.

Figure 3.7 shows the average sum rate in bit per channel use as a function of P/σ^2, considering network N2, shown in Figure 3.4. It can seen that the SSCA algorithm achieves the highest sum rate in the high P/σ^2 region and achieves more DoF than ITR and OR. The ZFISI algorithm in scenario A2.2 achieves the same DoF, but requires an additional antenna at each relay. However, the SSCA algorithm performs still better than the ZFISI algorithm in terms of sum rate. In contrast to the fact that the SSCA and the ZFISI algorithms achieve more DoF in the scenarios A2.1, A2.2, and A2.3 than the ITR algorithm in scenario A2.6, for $P/\sigma^2 < 25$dB, the absolute value of the sum rate achieved by the ITR algorithm in scenario A2.6 is higher than that of the SSCA and ZFISI algorithm. This is mainly due to the reduced number K of communication pairs and the large number N_k of antennas at the nodes.

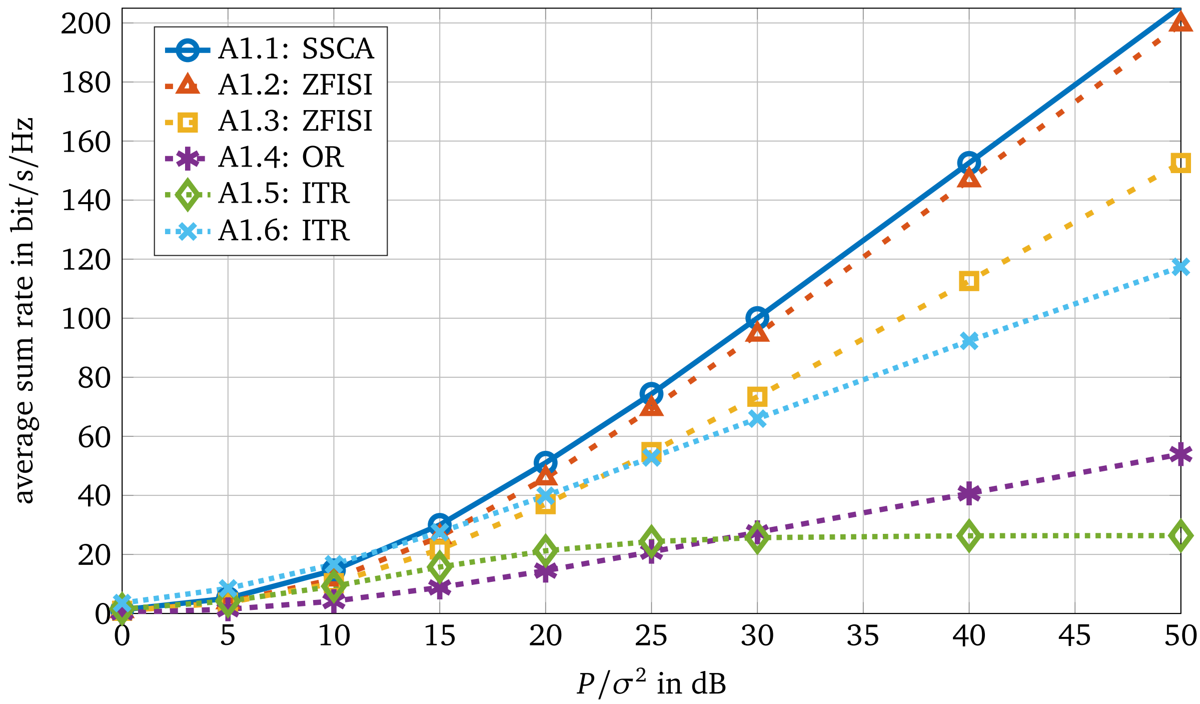


Figure 3.6. Average sum rate versus P/σ^2 for the scenarios that consider network N1.

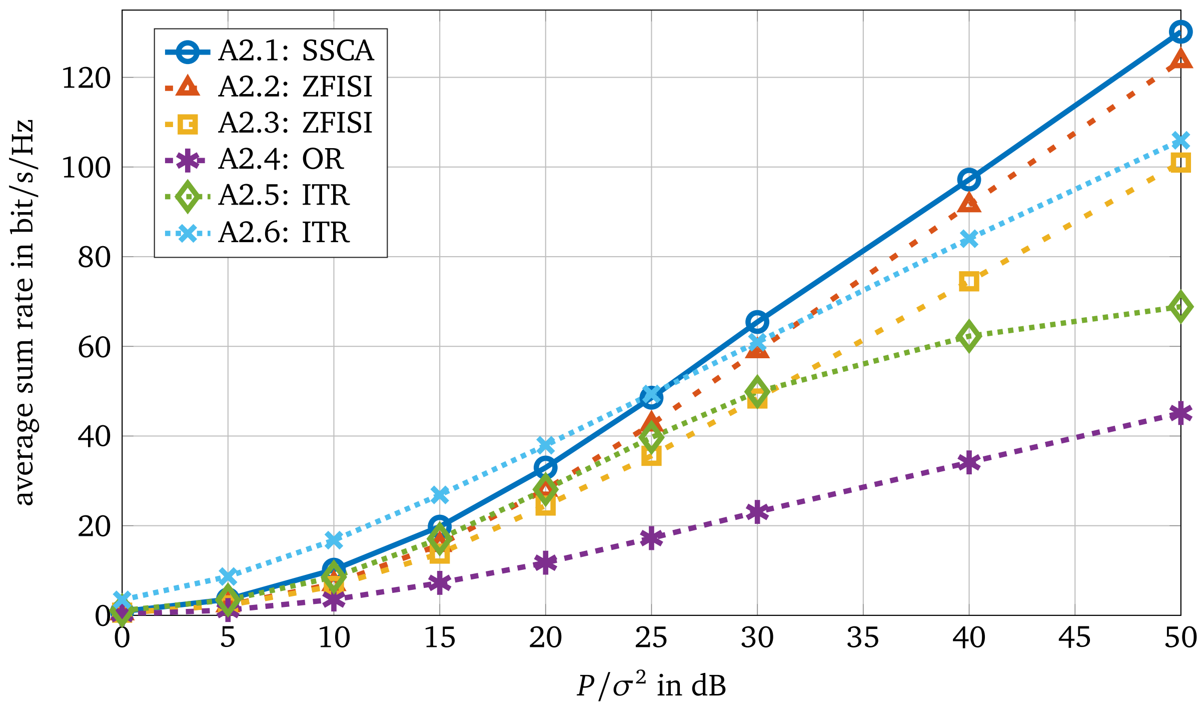


Figure 3.7. Average sum rate versus P/σ^2 for the scenarios that consider network N2.

3.6 Nodes of Communication Pairs Connected to at Least One Common Relay with an Additional Connection to Another Relay

3.6.1 Introduction

In this section, the focus is on communication pairs were out of this pair a single node has an additional connection to another relay. It is assumed that all nodes in the entire network are served by at least one relay. Due to the considered two-way relaying protocol, only relays which have a connection to both nodes of a communication pair can assist the pair-wise communication of a pair. Beside the handling of communication pairs investigated in Section 3.5 which are connected to the same multiple relays, this is a new challenging problem, especially in large partially connected networks. In large networks, it may happen that not both nodes of a communication pair are connected to the same relays. Rather, in partially connected networks it is even likely that not all nodes of a communication pair are connected to the same relays. The additional connection of a single node to a relay which cannot assist the communication of this communication pair leads to inter-subnetwork interference in the network. Nodes which are in addition connected to a relay that cannot assist the communication of these nodes receive only inter-subnetwork interference via this additional communication link and no useful signal. Therefore, this additional link is called inter-subnetwork interference link in the following. This inter-subnetwork interference can either be minimized or even canceled by the relay or the node which is connected via the inter-subnetwork interference link. One trivial solution to cancel this type of interference via the relay, is to assume that the relay has enough antennas to spatially suppress or align the interference received via the inter-subnetwork interference link.

In this section, it is assumed that the relay does not have the capability to minimize or align the inter-subnetwork interference. Instead, the nodes which suffer from inter-subnetwork interference have to design their transmit filters in order to minimize the inter-subnetwork interference. One advantage of the algorithm proposed in this section is that the relay which cannot assist the communication can design its filter without any CSI about the node which cannot be served by this relay. If the relay should suppress the interference, the relay would require CSI about the inter-subnetwork interference link. In Section 3.6.2, a new interference power minimization algorithm which can be solved in closed form and minimizes the inter-subnetwork interference in the whole network is proposed. In Section 3.6.3, the performance of the new closed form algorithm is investigated and evaluated.

The content of this section has been published by the author in [PLWK16].

3.6.2 Interference Power Minimization Algorithm

3.6.2.1 Interference Power Minimization

In this section, an inter-subnetwork interference power minimization algorithm for large partially connected networks is presented. In the following, it is assumed that all nodes are served by at least one relay and that single nodes of a communication pair can have an additional connection to at most one relay, which cannot assist the communication. Communication pairs which are only connected to a single relay will be served by this relay. These nodes can perform SA and CA at this single relay to achieve an IA solution, as mentioned in Section 3.5 and proposed in [GWK11] originally for fully connected networks. If both nodes of a communication pair are connected to the same multiple relays, this pair can perform SSA and SCA at all connected relays as proposed in Section 3.5, in order to achieve an IA solution. The nodes shall perform SA and CA at all relays which can serve these nodes, because this transmission technique is able to maximize the DoF in a two-way relaying network. If the nodes perform SA at the relays and these aligned signals are linearly independent of the signals of each other node pair, inter-pair interference is avoided. Nodes which are connected via an inter-subnetwork interference link to a relay suffer from inter-subnetwork interference. In this section, it is assumed, that the relay does not have the capability to minimize or align the inter-subnetwork interference signal. Instead, the nodes will design their transmit-filters in order to minimize inter-subnetwork interference.

In the following, node j of the communication pair (j,k) shall be the node with an additional connection to a relay which cannot assist the communication of node j, i.e., node j suffers from inter-subnetwork interference. For simplicity of the notation, an intersection of two subnetworks is assumed. Let $\check{q}$ denote the relay which has a connection to both nodes of the communication pair (j,k), i.e., which can serve this communication pair (j,k). The relay which has an additional connection to node j of the communication pair (j,k) is denoted by $\bar{q}$. Hence, $\bar{q}$ cannot assist the communication of the communication pair (j,k).

The power which relay $q \in \mathcal{Q}$ receives from node $i \in \mathcal{K}(q)$ in the MAC phase, is given by

$$\left\|\mathbf{H}_{i,q}^{\mathrm{sr}}\mathbf{V}_i\mathbf{d}_i\right\|_{\mathrm{F}}^2 = \mathrm{Tr}\left(\mathbf{d}_i^{\mathrm{H}}\mathbf{V}_i^{\mathrm{H}}\mathbf{H}_{i,q}^{\mathrm{srH}}\mathbf{H}_{i,q}^{\mathrm{sr}}\mathbf{V}_i\mathbf{d}_i\right). \tag{3.60}$$

Due to the assumption that the transmit symbols are independent and identically distributed (i.i.d.), so that $\mathbb{E}[\mathbf{d}_j\mathbf{d}_j^{\mathrm{H}}] = \mathbf{I}_d$, $\forall j \in \mathcal{K}$ holds, (3.60) can by simplified to

$$\left\|\mathbf{H}_{i,q}^{\mathrm{sr}}\mathbf{V}_i\right\|_{\mathrm{F}}^2 = \mathrm{Tr}\left(\mathbf{V}_i^{\mathrm{H}}\mathbf{H}_{i,q}^{\mathrm{srH}}\mathbf{H}_{i,q}^{\mathrm{sr}}\mathbf{V}_i\right). \tag{3.61}$$

Hence, the minimization of the inter-subnetwork interference can be formulated as the following optimization problem given by

$$\begin{aligned} \underset{\mathbf{V}_j\in\mathbb{C}^{N_j\times d}}{\text{minimize}} \quad & \mathrm{Tr}\left(\mathbf{V}_j^{\mathrm{H}}\mathbf{H}_{j,\bar{q}}^{\mathrm{srH}}\mathbf{H}_{j,\bar{q}}^{\mathrm{sr}}\mathbf{V}_j\right) \\ \text{subject to} \quad & \|\mathbf{V}_j\|_F^2 \le P_{\mathrm{n,max}}. \end{aligned} \tag{3.62}$$

The optimum $\mathbf{V}_j$ can be mathematically derived by applying the Lagrangian method [BV04] and the property that the eigenvalues of the Hermitian matrix $\mathbf{H}^{\mathrm{srH}}_{j,\bar{q}}\mathbf{H}^{\mathrm{sr}}_{j,\bar{q}}$ are real [HJ13]. Hence, $\mathbf{V}_j$ which minimizes the interference power of the d data streams is given by

$$\mathbf{V}_{j,\min} = \chi_{\min,d}\left(\mathbf{H}^{\mathrm{srH}}_{j,\bar{q}}\mathbf{H}^{\mathrm{sr}}_{j,\bar{q}}\right), \tag{3.63}$$

where $\chi_{\min,x}(.)$ denotes a matrix containing the eigenvectors corresponding to the x smallest eigenvalues of the matrix within the brackets as its columns. To avoid inter-pair interference, the communication pair (j,k) performs SA at the relay which serve this communication pair, i.e. the nodes design their transmit filters in such a way that the signals of the communication pair (j,k) are pairwise aligned in a subspace of the entire signal space at relay q. Hence, the first SSA condition (3.26) which is equivalent to the SA condition has to be fulfilled. Without loss of generality, the first SSA condition (3.26) can be rewritten as a system of linear equations, given by

$$\mathbf{H}^{\mathrm{sr}}_{k,\breve{q}}\mathbf{v}^{(l)}_k - \mathbf{H}^{\mathrm{sr}}_{j,\breve{q}}\mathbf{v}^{(l)}_j = \mathbf{0}, \tag{3.64}$$

where $\mathbf{v}^{(l)}_j$ and $\mathbf{v}^{(l)}_k$ denote the l^{th} column of $\mathbf{V}_j$ and $\mathbf{V}_k$, respectively. If one fixes $\mathbf{V}_j$ in (3.64) by $\mathbf{V}_{j,\min}$ from (3.63), (3.64) will end up in an inhomogeneous system of linear equations, given by

$$\mathbf{H}^{\mathrm{sr}}_{k,\breve{q}}\mathbf{v}^{(l)}_k = \mathbf{H}^{\mathrm{sr}}_{j,\breve{q}}\mathbf{v}^{(l)}_{j,\min}. \tag{3.65}$$

Because of the fixed $\mathbf{V}_j$, one loses $N_j d$ variables to solve (3.65). Such a system of equations can have a single unique solution, no solution, which means an empty solution set, or infinitely many solutions [Beu14; Str09]. The set of solution vectors of (3.65) spans the solution space. If $\mathrm{Rank}\left(\mathbf{H}^{\mathrm{sr}}_{k,\breve{q}}\right) = R_{\breve{q}}$, (3.65) has one particular solution and $N_k - R_{\breve{q}}$ special solutions in the null space of $\mathbf{H}^{\mathrm{sr}}_{k,\breve{q}}$, if $\mathrm{Rank}\left(\mathbf{H}^{\mathrm{sr}}_{k,\breve{q}}\right) < N_k$. If $R_{\breve{q}} > N_k$ the inhomogeneous system in (3.65) is an overdetermined system of equations, i.e., a system of equations which has more equations than unknown variables. Such an overdetermined system has usually no solution [Str09]. Hence, $\mathbf{v}^{(l)}_{k,\min}$ is in this case determined by the least squares solution

$$\mathbf{v}^{(l)}_{k,\min} = \mathbf{H}^{\mathrm{sr}\dagger}_{k,\breve{q}}\mathbf{H}^{\mathrm{sr}}_{j,\breve{q}}\mathbf{v}^{(l)}_{j,\min}. \tag{3.66}$$

The transmit filters $\mathbf{V}_{k,\min}$ and $\mathbf{V}_{j,\min}$ have to fulfill the first SSA condition or in general the SA condition (3.26) in order to avoid inter-pair interference. To guarantee that the transmit filters $\mathbf{V}_{k,\min}$ and $\mathbf{V}_{j,\min}$ are in the SA solution space $\mathbf{A} = \mathrm{null}\left(\left[\mathbf{H}^{\mathrm{sr}}_{j,\breve{q}} \quad -\mathbf{H}^{\mathrm{sr}}_{k,\breve{q}}\right]\right)$ (3.37), the transmit filters $\mathbf{V}_{k,\min}$ and $\mathbf{V}_{j,\min}$ are orthogonal projected onto $\mathrm{span}(\mathbf{A})$. An orthogonal projection is a projection onto $\mathrm{span}(\mathbf{A})$ along $\mathrm{span}(\mathbf{A})\perp$, where $\mathrm{span}(\mathbf{A})\perp$ denotes the orthogonal complement of $\mathrm{span}(\mathbf{A})$. The projection matrix $\mathbf{P}$ and is given by

$$\mathbf{P} = \mathbf{A}\left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}}, \tag{3.67}$$

where $\left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right) = \mathbf{I}_{(2N_k - R_{\breve{q}})}$. The transmit filters of the communication pair (j,k) which minimize the inter-subnetwork interference power and which still fulfill the SA condition (3.26) are given by

$$\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix} = \alpha \cdot \mathbf{P}\begin{bmatrix}\mathbf{V}_{j,\min} \\ \mathbf{V}_{k,\min}\end{bmatrix}, \tag{3.68}$$

where α is determined such that the node transmit power constraint is fulfilled. It is worth to mention that the relays which have in addition connections to single nodes of communication pairs can determine their filters without any CSI of these single nodes.

3.6.2.2 Properness Condition

In this section, the properness condition to perform IA and to minimize inter-subnetwork interference is introduced. The number of required antennas at each relay is given by (3.30) in order to guarantee that the relay is always able to perform Transceive ZF. However, the relay has not enough degree of freedom to handle the inter-subnetwork interference in the network. The minimum required number of antennas at each node to perform SA is given by the equal sign in (3.32). Additional antennas at the nodes can be utilized to minimize or even cancel the inter-subnetwork interference in the network, i.e., the greater than sign in (3.32). Four different ranges for the number of antennas N_j at node j, which suffers from inter-subnet interference, are defined below. An overview of these ranges is shown in Table 3.3. In the first range I., which is not shown in the table,

Table 3.3. Different ranges for the number of antennas on nodes suffering from inter-subnet interference.

II. No optimization	III. Optimization	IV. Orthogonal
$N_j = \frac{R_{\check{q}} + d}{2}$	$\frac{R_{\check{q}} + d}{2} < N_j < R_{\bar{q}} + d$	$N_j \geq R_{\bar{q}} + d$

(3.32) is not fulfilled and SA is therefore not possible. In the second range II., the number N_j of antennas corresponds to the minimum required number to perform SA, i.e., a minimization of the interference power at $\bar{q}$ is not possible. The third range III., represents a range in which minimization of the interference power at $\bar{q}$ is possible, but the interference cannot be canceled completely. In the forth range IV., node j is equipped with enough antennas in order to transmit orthogonal to the channel from node j to relay $\bar{q}$, i.e., relay $\bar{q}$ will receive no interference from node j anymore. Hence, a further increase of the number N_j of antennas has no further influence on the interference. Communication pairs which are connected to only one relay design their filters according to (3.37) in order to perform SA at this relay and have therefore to fulfill (3.32). Communication pairs which are connected to at least two relays design there filters according to (3.40) in order to perform SSA at all connected relays and have therefore to fulfill (3.34).

3.6.3 Performance Analysis

3.6.3.1 Introduction

In this section, the performance of the interference power minimization algorithm presented in Section 3.6.2.1 is investigated. This algorithm is referred to as interference power minimization (IPM) in the following. The achievable DoF, as well as the sum rate, are considered to evaluate the performance of the algorithm. As introduced in Section 3.4, the DoF is defined as the total number of interference free data streams that can be transmitted in one channel use. In the following, three different scenarios are considered for the evaluation. Since a DoF analysis is only valid for an asymptotically high SNR, the sum rate over a larger SNR range is simulated in order to assess the performance. The sum rates achieved by these algorithms are obtained through numerical MATLAB simulations.

In the following the assumptions regarding the simulation are briefly described. The algorithm itself is valid for the assumptions mentioned in Section 2.3 and Section 3.6.2.1.

- It is assumed that the channel between each node and all relays connected to this node is an i.i.d frequency-flat Rayleigh fading MIMO channel [LS03]. Hence, the channel matrices are of full rank, almost surely.
- The channel matrices are normalized such that the average received power is the same as the average transmit signal power.
- Channel reciprocity is assumed, i.e., the MAC and BC phase channel matrices are the complex conjugate transposes of each other.
- It is assumed that all nodes are served by at least one relay.
- Due to the considered statistical channel model, the channel amplitude may vary for different realizations. Hence, all simulation results are averaged over 10^4 independent channel realizations. For each channel realization, all filters are designed according to the considered algorithm and the corresponding sum rate is calculated. The average sum rate which is plotted in this section is therefore an average over all 10^4 independent channel realizations. 10^4 independent channel realizations are large enough to get a sufficiently small confidence interval for plotting the sum rate.

In this paragraph, the chosen reference algorithm is briefly described. The reference algorithm designs its filters according to the algorithm introduced in Section 3.5.4. This algorithm can achieve an IA solution in a partially connected network if both nodes of a communication pair are connected to the same relays. However, in this algorithm, neither the relays nor the node suffering from inter-subnetwork interference can handle or even suppress inter-subnetwork interference. This algorithm is termed SSCA in this section.

3.6.3.2 DoF Analysis

In this section, the DoF achieved by the proposed interference power minimization scheme is investigated. Three different scenarios are considered for investigation, termed O1, O2, O3. All three scenarios consist of two subnetworks, each containing a single relay. The subnetworks intersect partly so that some nodes belong to both subnetworks. Nodes which are inside of the intersection area of these two subnetworks are surly connected to both relays. Table 3.4 gives an overview of the different considered scenarios, the number of nodes, the given number of antennas at the relays and at the nodes and the number of transmitted data streams are shown. It is important to know that $N_j = N_k$ for all communication pairs (j,k). Hence, only N_j is listed in Table 3.4. The number of antennas marked with x in Table 3.4 is not predefined fixed, it is a simulation parameter. This parameter is selected, according to the optimization ranges mentioned in Table 3.3. All other nodes are equipped with the minimum required number of antennas in order to perform SA or SSA at the relays, according to (3.32) and (3.34). The number of antennas at relay 1 and relay 2 are determined according to (3.30). The size of a network is defined by the number of communication pairs within a subnetwork. The three different scenarios are depicted in Figure 3.8, Figure 3.9 and Figure 3.10, to visualize the connections.

Table 3.4. Considered scenarios and the number of antennas

			Antennas Relays		Antennas Nodes $N_j = N_k \forall (j,k)$							
Scenario	K	d	R_1	R_2	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
O1	8	1	4	5	3	3	x	5	3	3	3	3
O2	5	1	3	3	2	x	4	2	2	-	-	-
O3	8	1	5	4	3	3	x	3	5	3	3	3

These scenarios are selected because they cover the following main cases:

- O1: Both subnetworks are of different sizes and the node suffering from inter-subnetwork interference is served by the smaller subnetwork. The size of subnetwork 1 is 4 and the size of subnetwork 2 is 5.
- O2: Both subnetworks are of the same size. The size of subnetwork 1 is 3 and the size of subnetwork 2 is 3.
- O3: Both subnetworks are of different sizes and the node suffering from inter-subnetwork interference is served by the larger subnetwork. The size of subnetwork 1 is 5 and the size of subnetwork 2 is 4.

Scenarios O1 is shown in Figure 3.8. The connection sets of Scenario O1 are given by $\mathcal{K}(1) = \{1,2,3,4,9,10,11,12\}$ and $\mathcal{K}(2) = \{4,5,6,7,8,11,12,13,14,15,16\}$. The nodes of the set $\mathcal{K}(1) \cap \mathcal{K}(2) = \{4, 11, 12\}$ are belonging to both subnetworks, i.e., are connected to both relays Q1 and Q2. Node 11 is the node which suffers from inter-subnetwork interference. Hence, this node has to design its transmit filter in order to minimize the inter-subnetwork interference. In order to perform SA at relay 1, each node of the communication pair $(3, 11)$ needs $N_3 = \lceil \frac{R_1+1}{2} \rceil = 3$ antennas, according to (3.32). To totally cancel out the inter-subnetwork interference, the communication pair $(3, 11)$ would require $N_3 = \lceil R_2 + 1 \rceil = 6$ antennas, according to Table 3.3.

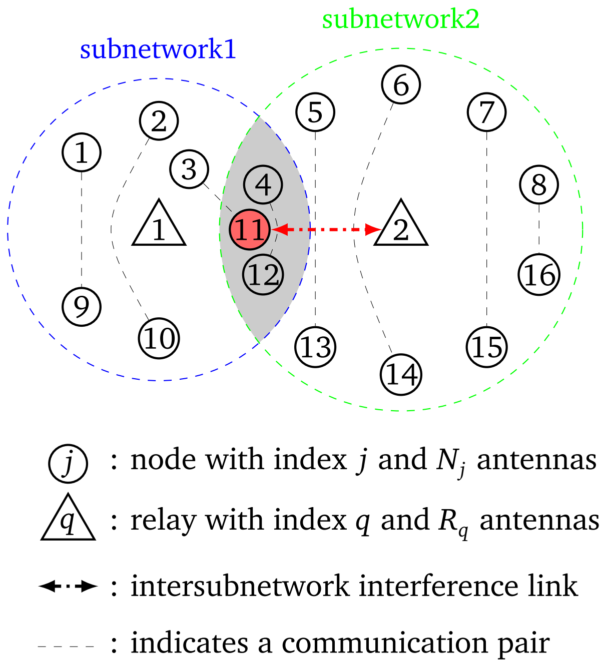


Figure 3.8. Partially connected network, Scenario O1

The second scenarios O2 is shown in Figure 3.9. The connection sets of Scenario O2 are given by $\mathcal{K}(1) = \{1, 2, 3, 6, 7, 8\}$ and $\mathcal{K}(2) = \{3, 4, 5, 7, 8, 9, 10\}$. The nodes of the set $\mathcal{K}(1) \cap \mathcal{K}(2) = \{3, 7, 8\}$ are belonging to both subnetworks, i.e., are connected to both relays Q1 and Q2. Node 7 is the node which suffers from inter-subnetwork interference. In order to perform SA at relay 1, each node of the communication pair $(2, 7)$ needs $N_2 = \lceil \frac{R_1+1}{2} \rceil = 2$ antennas, according to (3.32). To totally cancel out the inter-subnetwork interference, the communication pair $(2, 7)$ would require $N_2 = \lceil R_2 + 1 \rceil = 5$ antennas, according to Table 3.3.

Finally, The tried considered scenario O3 is shown in Figure 3.10. The connection sets of Scenario O1 are given by $\mathcal{K}(1) = \{1, 2, 3, 4, 5, 9, 10, 11, 12, 13\}$ and $\mathcal{K}(2) = \{5, 6, 7, 8, 11, 13, 14, 15, 16\}$. The nodes of the set $\mathcal{K}(1) \cap \mathcal{K}(2) = \{5, 11, 13\}$ are belonging to both subnetworks, i.e., are connected to both relays Q1 and Q2. Node 11 is the node which suffers from inter-subnetwork interference. In order to perform SA at relay 1, each node of the communication pair $(3, 11)$ needs $N_3 = \lceil \frac{R_1+1}{2} \rceil = 3$ antennas, according to (3.32). To totally cancel out the inter-subnetwork interference, the communication pair $(3, 11)$ would require $N_3 = \lceil R_2 + 1 \rceil = 5$ antennas, according to Table 3.3.

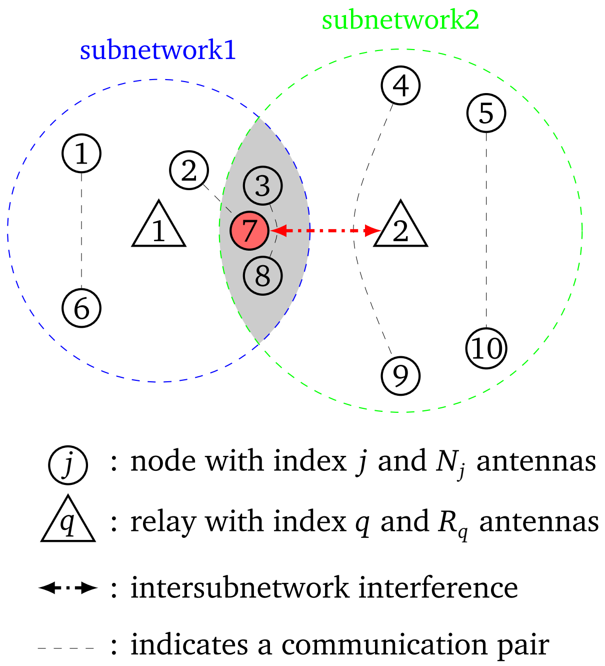


Figure 3.9. Partially connected network, Scenario O2

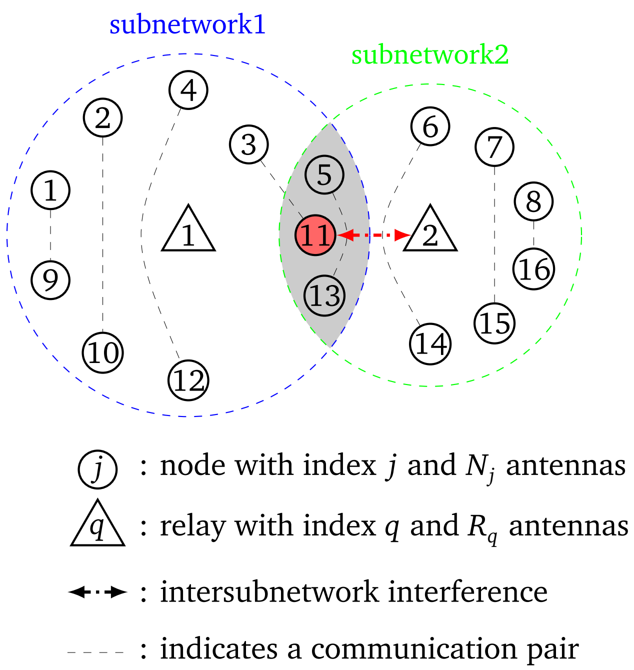


Figure 3.10. Partially connected network, Scenario O3

In a two-way relaying network, two time slots are necessary in order to perform a bidirectional communication. Hence, the $2K$ nodes of the whole network, transmitting d data streams each, can achieve Kd DoF. These DoF can only be achieved in an interference-free communication, i.e., only if the node suffering from inter-subnetwork interference designs it transmit-filter orthogonal to the inter-subnetwork interference link. Hence, the node which suffers from inter-subnetwork interference has to fulfill the third column (Orthogonal) of Table 3.3. The behavior will be verified in the sum rate analysis of the algorithm. Table 3.5 shows the DoF achievable by the proposed algorithm if all inter-subnetwork interference is canceled. The reference algorithm cannot achieve an interference-free communication and is therefore not shown in Table 3.5. The reason for this is that the reference algorithm cannot suppress the inter-subnetwork interference.

Table 3.5. Considered scenarios and the achievable DoF

			Antennas Relays		Antennas Nodes $N_j = N_k \forall (j,k)$								DoF
Scenario	K	d	R_1	R_2	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	IPM
O1	8	1	4	5	3	3	6	5	3	3	3	3	8
O2	5	1	3	3	2	4	4	2	2	-	-	-	5
O3	8	1	5	4	3	3	6	3	5	3	3	3	8

3.6.3.3 Sum Rate Analysis

In this section, the sum rate performance of the proposed interference power minimization scheme is analyzed and compared with the reference algorithm. Let $P_{n,\max} = P$ denote the power of each of the $2K$ nodes in the network. Further, let $P_{q,\max} = \frac{1}{Q}KP$ denote the transmit power at each of the Q relays. The noise power at each node and at each relay is assumed to be the same for the simulation and is denoted by $\sigma^2 = \sigma_k^2 = \sigma_q^2$, $\forall k \in \mathcal{K}, \forall q \in \mathcal{Q}$.

In the following, the sum rate performance of the IPM algorithm is compared with the SSCA reference algorithm. Both algorithms will be applied to all three considered scenarios O1, O2, and, O3. Each of the $2K$ nodes wants to transmit $d = 1$ data stream to its communication partner. The ratio between the maximum transmit power P and the noise power σ^2 is termed SNR.

Firstly, the sum rate performance of the IPM algorithm is compared with the SSCA reference algorithm, considering scenario O1. Figure 3.11 shows the sum rate performance of the proposed IPM algorithm for $N_{11} = 6$ antennas and the IPM algorithm for $N_{11} = 5$ antennas, in comparison to the SSCA reference algorithm for $N_{11} = 6$, as a function of $\frac{P}{\sigma^2}$. The solid lines and the dashed line represent the IPM algorithm and the SSCA algorithm, respectively. It can be seen that the proposed IPM algorithm performs better than the SSCA reference algorithm in the high SNR region. Up to an SNR of 15 dB, the performance of the algorithms is almost the same. The reason for this behavior is that in

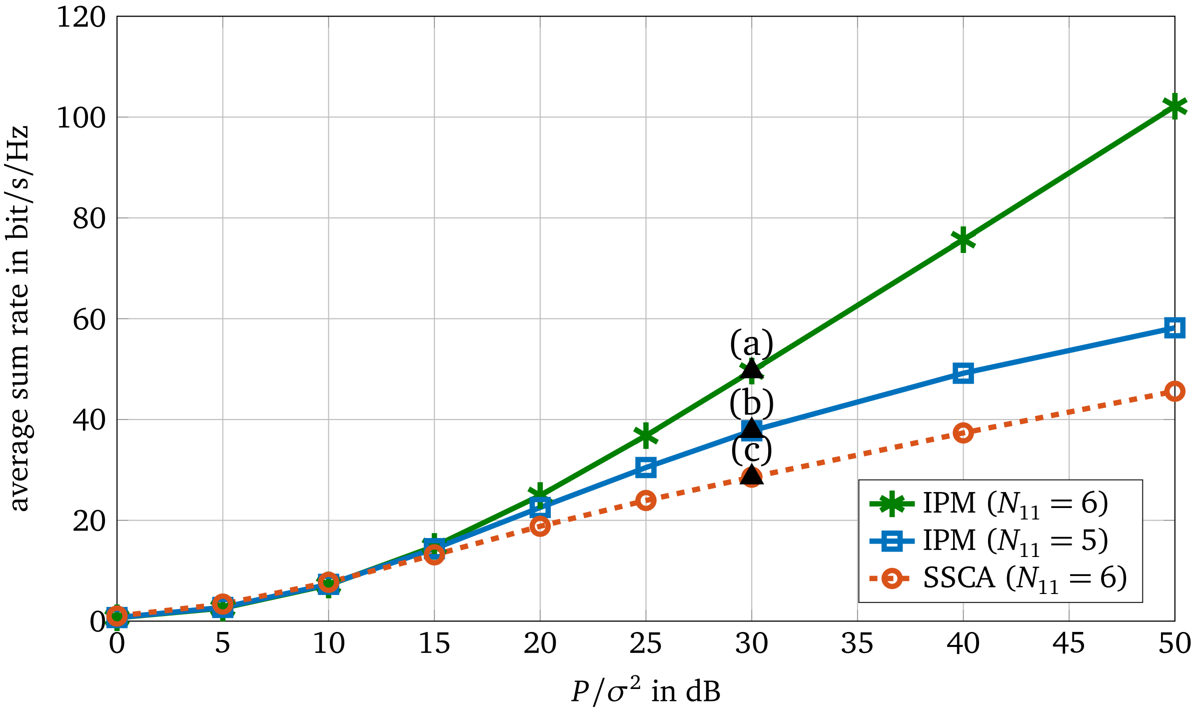


Figure 3.11. Sum rate performance of the IPM algorithm in comparison to the SSCA algorithm, for the Scenario O1

the low SNR region, noise is the dominating factor that is not taken into account by the algorithms. The IPM algorithm outperforms the SSCA algorithm in the high SNR region, even if the number of antennas at node 11 is reduced from $N_{11} = 6$ to $N_{11} = 5$. The IPM algorithm utilizes additional antennas at the nodes, i.e. the larger than symbol in (3.32), to minimize inter-subnetwork interference. Whereas, the SSCA algorithm cannot utilize additional antennas at the node to further improve the sum rate. The proposed IPM algorithm establishes an interference free communication of all $2K$ nodes in the whole network, if $N_{11} > 6$. This means that relay 2 which is in addition connected to the single node 11 of the communication pair $(3, 11)$ receives no interference from node 11. This is achieved by designing the transmit filter of node 11 in such a way that the transmit filter is in the null-space of the channel from node 11 to relay 2. From the DoF definition, it becomes clear that for $N_{11} > 6$ the DoF is maximized, because there is no more interference in the network if all nodes are transmitting simultaneously. In the case of $N_{11} = 5$ and taking the IPM algorithm into account, interference-free communication is not possible. However, the IPM algorithm sill outperforms the SSCA algorithm. Because the dimension of the solution space (3.40) is still larger than 1 and therefore can be utilized for inter-subnetwork power minimization.

Figure 3.12 shows the sum rate performance of the IPM algorithm, in comparison to the SSCA reference algorithm, as a function of N_{11} for a fixed ratio of $\frac{P}{\sigma^2} = 30\text{dB}$. This ration is chosen because it is a good tradeoff between noise power and interference power in the network. In general, any value of $\frac{P}{\sigma^2}$ can be chosen. The four ranges introduced in

Table 3.3 are also shown in Figure 3.12. It is immediately obvious that the number of

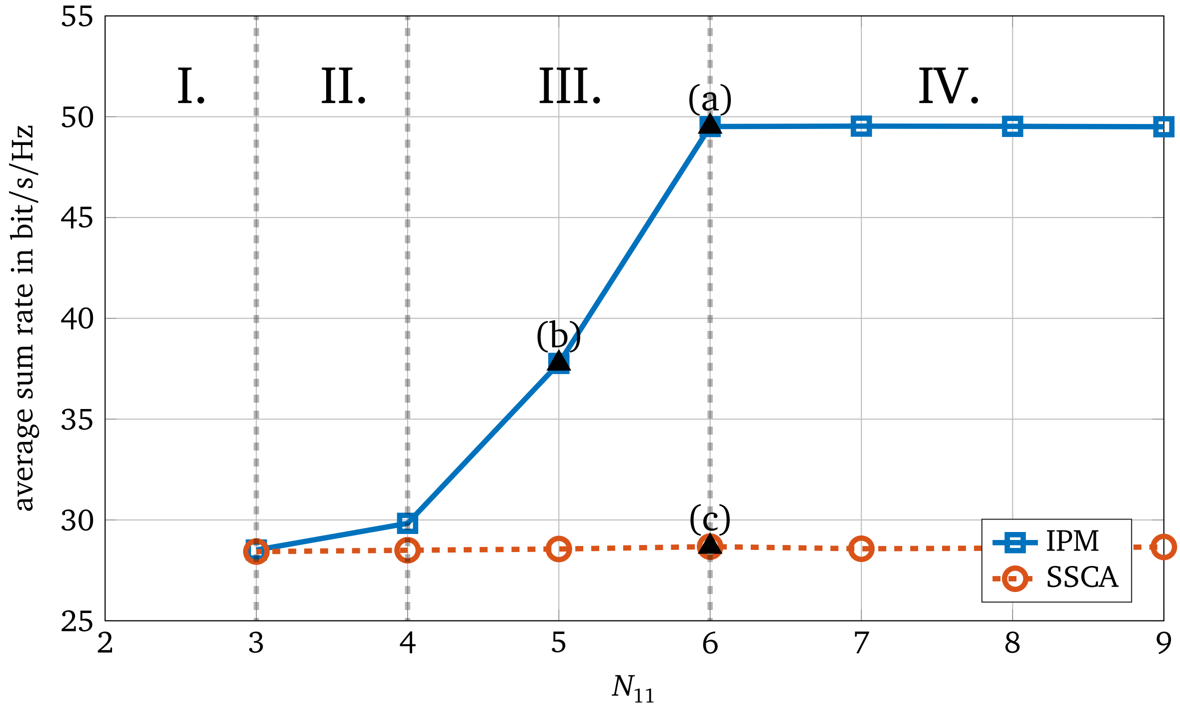


Figure 3.12. Average sum rate versus the number of antennas N_{11} at node 11, which suffers from inter-subnetwork interference, for the Scenario O1 at $\frac{P}{\sigma^2} = 30$dB. For I. - IV. see Table 3.3

antennas N_{11} at node 11 does not affect the sum rate performance of the SSCA reference algorithm. The proposed IPM, on the other hand, can achieve approximately a 75% higher sum rate, if N_{11} is greater than or equal to 6. In contrast to the IPM algorithm, the SSCA algorithm chooses an arbitrary solution out of the entire solution space (3.40). An increase in the number of antennas N_{11}, therefore, has on average no influence on the sum rate performance of the SSCA algorithm. As long as the number of antennas N_{11} is larger than the minimum required number of antennas to perform SA at relay 1, which serves the communication pair $(3, 11)$, the proposed IPM algorithm outperforms the SSCA reference algorithm . Figure 3.12 shows, that a further increase of the number of antennas N_{11} at node 11 beyond 6 has almost no influence on the sum rate at an SNR of 30dB. This is due to the fact that all inter-subnetwork has already been eliminated. The marker (a) in Figure 3.11 and Figure 3.12 represents a point with the same sum rate, number of antennas at N_{11} and $\frac{P}{\sigma^2}$, this also applies for the markers (b) an (c), respectively.

In this following, the sum rate performance of the IPM algorithm is compared with the reference algorithm SSCA considering scenario O2. Figure 3.13 shows the sum rate performance of the proposed IPM algorithm for $N_7 = 4$ antennas and the IPM algorithm for $N_7 = 3$ antennas, in comparison to the SSCA reference algorithm for $N_7 = 4$, as a function of $\frac{P}{\sigma^2}$. With regard to the sum rate, the same behavior can be observed as in scenario O1.

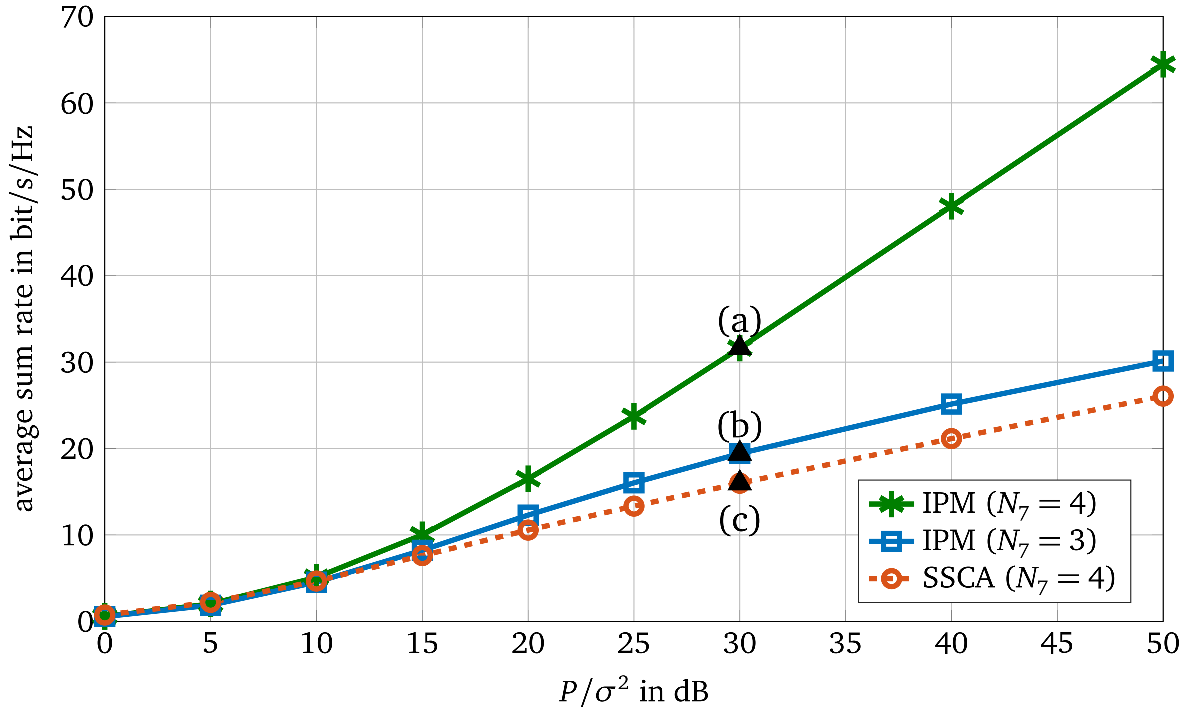


Figure 3.13. Sum rate performance of the IPM algorithm in comparison to the SSCA algorithm, for the Scenario O2

This means that the proposed IPM algorithm will also be gainful if both relays serve the same number of communication pairs.

Figure 3.14 shows the sum rate performance of the IPM algorithm, in comparison to the SSCA reference algorithm for, as a function of N_7 for a fixed ratio of $\frac{P}{\sigma^2} = 30$dB. It can be seen that the sum rate performance of the SSCA reference algorithm is not affected by the number of antennas at node 7. The proposed IPM algorithm, on the other hand, can achieve approximately a 100% higher sum rate, if $N_7 \geq 4$. The marker (a) in Figure 3.13 and Figure 3.14 represents a point with the same sum rate, number of antennas at N_7 and $\frac{P}{\sigma^2}$, this also applies for the markers (b) an (c), respectively.

Finally, the sum rate performance of the IPM algorithm is compared with the SSCA reference algorithm considering scenario O3. Figure 3.15 shows the sum rate performance of the proposed IPM algorithm for $N_{11} = 6$ antennas and the IPM algorithm for $N_{11} = 5$ antennas, in comparison to the SSCA reference algorithm for $N_{11} = 6$, as a function of $\frac{P}{\sigma^2}$. In contrast to scenario O1, in scenario O3 the relay which has an inter-subnetwork interference link connection is connected to more communication pairs than the other relay, i.e., the sizes of the subnetworks are interchanged, see Figure 3.8 and Figure 3.10. It can be seen that the proposed IPM algorithm performs better than the SSCA reference algorithm in the high SNR region. This is even the case if the number of antennas at node 11 is reduced from $N_{11} = 6$ to $N_{11} = 5$. The reason for this is the multidimensional SA solution space (3.32) used by the IPM algorithm to minimize the inter-subnetwork

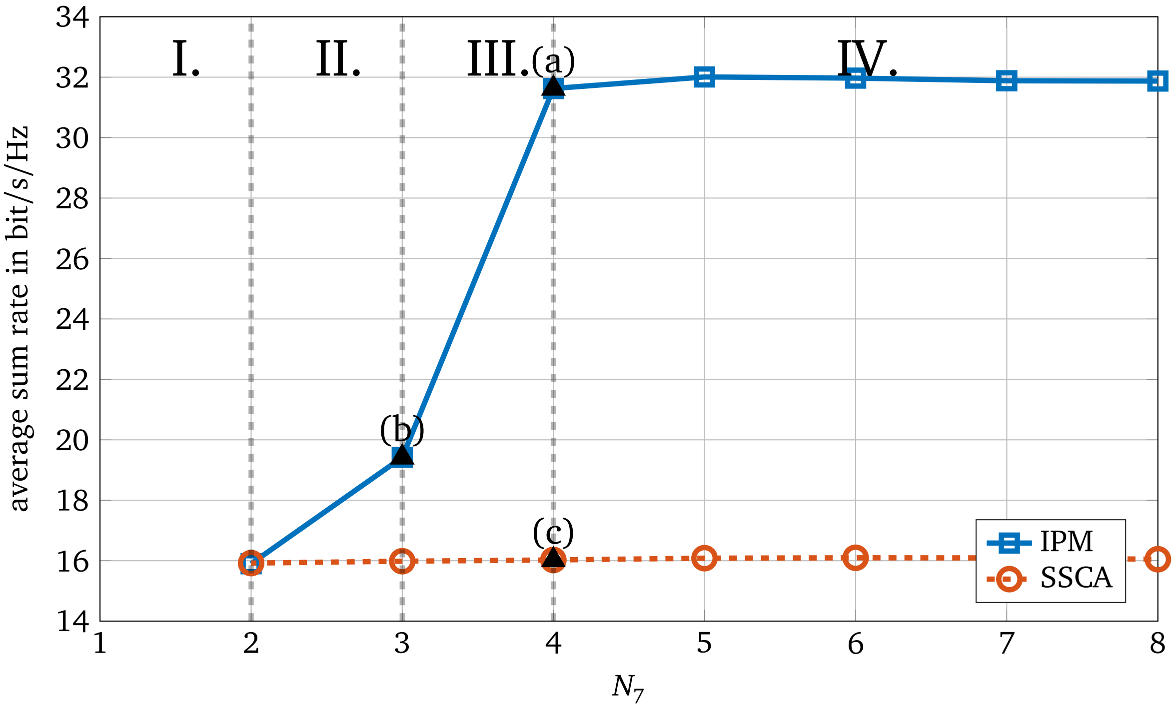


Figure 3.14. Average sum rate versus the number of antennas N_7 at node 7, which suffers from inter-subnetwork interference, for the Scenario O2 at $\frac{P}{\sigma^2} = 30$dB. For I. - IV. see Table 3.3

interference power. It is worth to mention, that in Figure 3.15 IPM for $N_{11} = 5$ and IPM for $N_{11} = 6$ achieve the same DoF. This can be observed by the slope of the curves, which remains the same. The IPM algorithm achieves gain even if the smaller of the two subnetworks suffers from inter-subnetuwerk interference. The proposed IPM algorithm establishes an interference-free communication of all $2K$ nodes in the whole network, if $N_{11} > 5$.

Figure 3.16 shows the sum rate performance of the IPM algorithm, in comparison to the SSCA reference algorithm for, as a function of N_{11} for a fixed ratio of $\frac{P}{\sigma^2} = 30$dB. It can be seen that a further increase of the number of antennas N_{11} beyond $N_{11} = 5$ leads in contrast to Scenario 1 to a further performance gain. The reason for this gain in terms of sum rate is the smaller SA solution space in Scenario O3 in comparison to the SA solution space in Scenario O1. The SA solution space in Scenario O1 has 8-dimensions, whereas the SA solution space in Scenario O3 has only 5-dimensions. From Figure 3.16 and Figure 3.12 it can be observed that for $N_{11} \rightarrow \infty$ in both scenarios the same sum rate can be achieved. In both scenarios, the total number of data streams, number of communication pairs and total transmit power are the same.

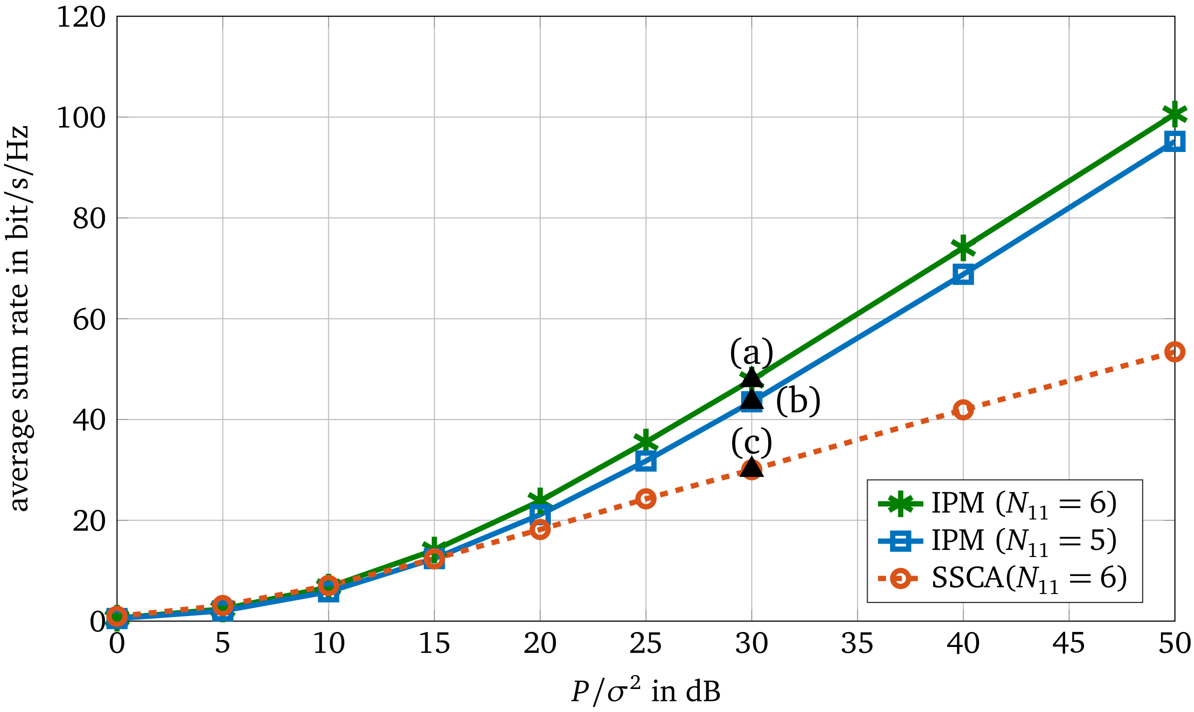


Figure 3.15. Sum rate performance of the IPM_closed algorithm in comparison to the SSCA algorithm, for the Scenario O3

3.7 Summary

In this chapter, a large partially connected relay aided pair-wise communication network has been considered. At the beginning of this chapter, an appropriated system model has been introduced. It has been shown that a large, partially connected relay aided pair-wise communication network can be portioned into several subnetworks which are fully connected. Only relays which are connected to both nodes of a communication pair can serve this pair, i.e., they can assist the communication of this pair. Two different cases have been considered in this chapter. In the first case, communication pairs may be connected to multiple relays. In the second case, a single node of a communication pair may be in addition connected to a relay that cannot assist the communication of this pair.

- For the first case, new techniques called SSA and SCA has been introduced to perform SA and CA at multiple relays simultaneously. The properness conditions for SSA and SCA are derived and it has been shown that local CSI is sufficient to perform IA in partially connected networks. Further, has been shown that this proposed closed-form solution achieves more DoF than the reference algorithms and has better sum-rate performance, especially in the high SNR-region.
- For the second case, a new closed form algorithm which minimizes the inter-subnetwork interference power in the whole partially connected network has been

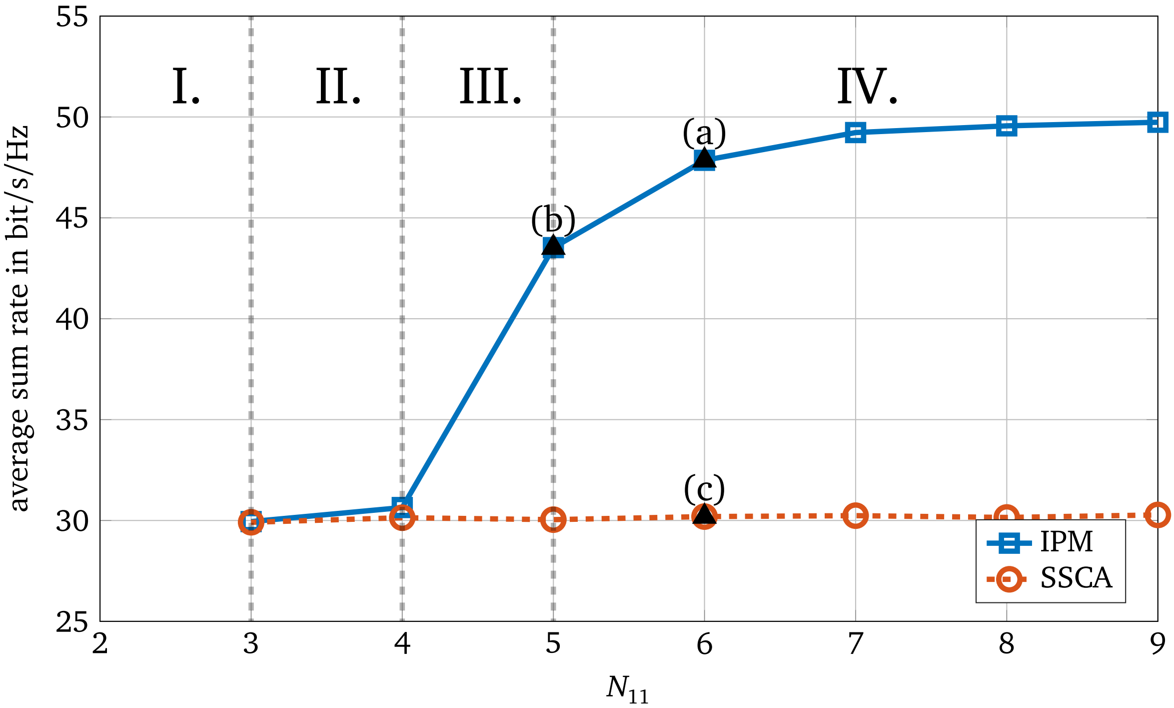


Figure 3.16. Average sum rate versus the number of antennas N_{11} at node 11, which suffers from inter-subnetwork interference, for the Scenario O3 at $\frac{P}{\sigma^2} = 30$dB. For I. - IV. see Table 3.3

proposed. The dependency of the performance on the number of antennas at the nodes has been investigated. It has been shown that the proposed inter-subnetwork interference power minimization algorithm can achieve an interference-free communication under certain conditions. The simulation results show that the proposed inter-subnetwork interference power minimization algorithm achieves a higher sum rate in comparison to the considered reference algorithm.

Chapter 4

Interference Alignment in Multi-Group Multi-Way Fully Connected Relaying Networks

4.1 Introduction

In this chapter, IA in a fully connected multi-group multi-way relaying network is considered. In such a network, multiple nodes form a group and each of these nodes wants to share its information with all other nodes in its group via an intermediate relay. In its general form, such networks consists of multiple groups containing an arbitrary number of nodes each. The group-wise exchange of data between the nodes inside a group is performed via the multi-way relaying protocol. Fully connected means that all nodes in all groups are connected to the relay. An algorithm which achieves IA at each receiving node in a fully connected multi-group multi-way relaying network which maximizes the DoF was proposed in [GAL+14]. The main drawback of the algorithm proposed in [GAL+14] is that the number of required antennas at the relay scales linearly with the number of nodes per group and the number of groups itself. For typical multi-group applications mentioned in Section 2.2.2, the algorithm proposed in [GAL+14] are not really practical due to the mentioned drawback. The algorithm proposed in this chapter is a more scalable solution to deal with a large number of nodes per group and therefore to overcome the main drawback mentioned above. In Section 4.2, the system model for a fully connected multi-group multi-way relaying network considering an amplify-and-forward relay is introduced. In Section 4.3, the system model for a fully connected multi-group multi-way relaying network considering an decode-and-forward relay is introduced. In Section 4.4, the sum rate expression considering an amplify-and-forward relay is defined. In Section 4.5, the sum rate expression considering an amplify-and-forward relay is defined. In Section 4.6, a multicast IA algorithm is proposed. Finally, in Section 4.6.4, the performance of the proposed multicast interference alignment algorithm considering an amplify-and-forward as well as a decode-and-forward relay is investigated.

A part of the content of this section has been published by the author of this thesis in [PSK17].

4.2 System Model

In this section, the system model of the considered multi-group multi-way relaying network is introduced. An example scenario of a fully connected multi-group multi-way relaying network consisting of $L = 3$ groups and $S_1 = S_2 = S_3 = 3$ nodes per group is shown in Figure 4.1. This type of network topology has been explained in Section 2.2.2. Each of the $L \geq 1$ groups contains $S_l > 2$ multi-antenna nodes. For $S_l = 2$, the problem would be reduced to the pairwise communication network introduced in Chapter 3 and [GAK+13].

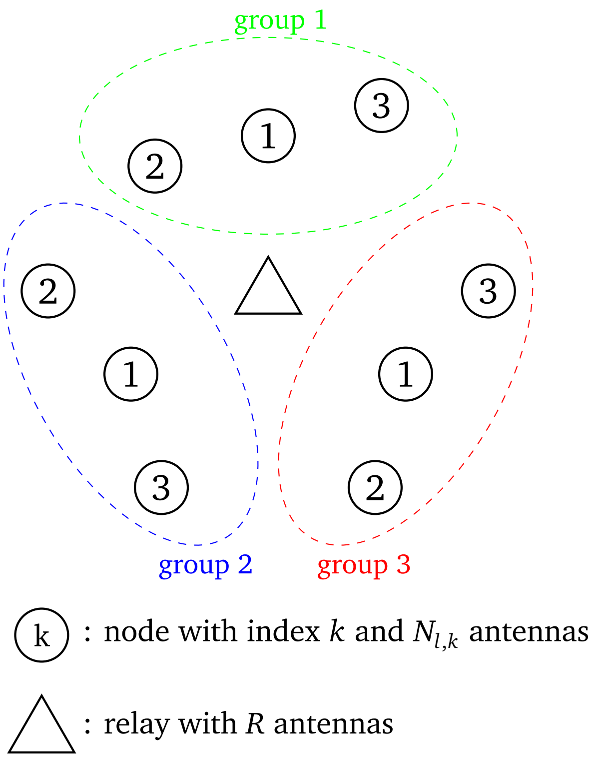


Figure 4.1. Multi-group multi-way relay network with $L = 3$ groups and $S_l = 3$, $\forall l \in \mathcal{L} = \{1, 2, 3\}$ nodes per group

Let $l \in \mathcal{L} = \{1, ..., L\}$ denote the group index and $k \in \mathcal{G}_l = \{1, ..., S_l\}$ the node index of group l, respectively. The k-th node in the l-th group is denoted by $[l, k]$ and is equipped with $N_{l,k}$ antennas. Each of the S_l nodes in the l-th group wants to share $d \leq N_{l,k}$ data streams with the $S_l - 1$ other nodes in its group. Nodes in neighboring groups are not interested in these data streams, i.e., these data streams would be interference. Nodes cannot overhear the data streams transmitted by other nodes in the entire network, i.e., there are no direct links between the nodes themselves. Hence, the communication between the nodes takes place via an intermediate half-duplex relay equipped with R antennas. Hence, all nodes in the set

$$\mathcal{G} = \bigcup_{l \in \mathcal{L}} \mathcal{G}_l \tag{4.1}$$

are connected to this intermediate relay.

To exchange information between the S_l nodes in group $l \in \mathcal{L}$ groupwise via the intermediate relay, a transmission scheme with M MAC phases and B MC phases is considered. In each of the B MC phases, $S_l - 1$ nodes per group shall receive the data stream previously transmitted from the node which is not served in this MC phase. Hence, the relay needs estimates of all data streams after the M MAC phases, to multicast a specific data stream only to nodes which want to receive this data stream. The nodes within a group can transmit either d data streams or nothing in each MAC phase. However, it is assumed that at least one node of each group is transmitting d data streams in each MAC phase. Since the relay has to separate all received data streams, the minimum required number of antennas at the relay is given by

$$R \geq Ld. \tag{4.2}$$

Let $\mathcal{G}_{l,m}$ denote the set of nodes selected for the transmission in group l in MAC phase $m = 1, ..., M$. In each of the $b = 1, ..., B$ MC phases a MIMO interference multicast channel is created, by separating the R relay antennas into L clusters, each serving a specific group of nodes and aligning the interference generated to the other groups. For example, the allocation of in total 6 relay antennas to 2 clusters, each cluster serving one of in total 2 groups is illustrated in Figure 4.2. The relay multicasts a linearly processed version of the signals received in the MAC phases to $S_l - 1$ nodes per group which want to receive a signal, in each MC phase. In each MC phase, $S_l - 1$ nodes within a group receive the same signal via a specific cluster belonging to that group. Because the signals are only transmitted to nodes that want to receive, no node receives self-interference. Hence, a self-interference cancelation at the nodes is not required.

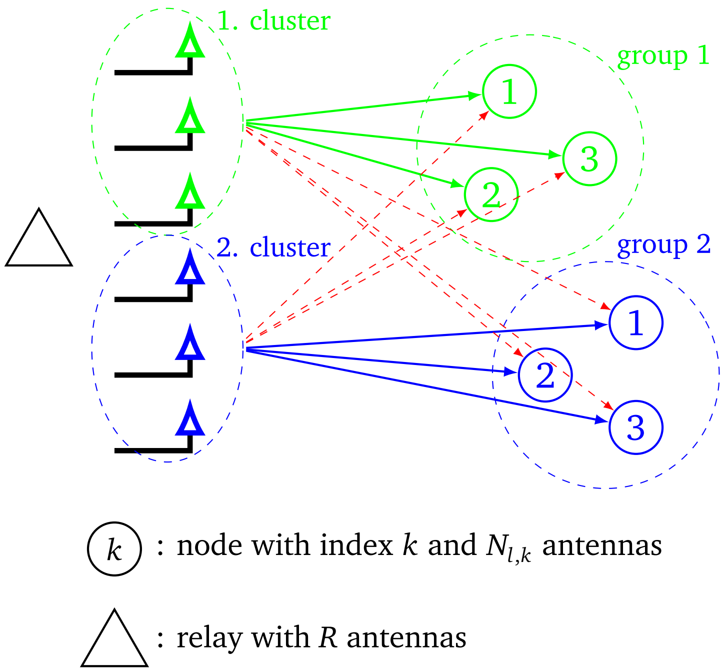


Figure 4.2. Relay antenna allocation to two clusters

Figure 4.3 shows the equivalent low pass signal model of a fully connected multi-group multi-way relaying network. Let $\mathbf{d}_{l,k} \in \mathbb{C}^{d\times 1}$ denote the data vector originating from node $[l,k]$. The covariance matrix of $\mathbf{d}_{l,k}$ is given by

$$\mathbf{R}_{d_{l,k}} = \mathbb{E}[\mathbf{d}_{l,k}\mathbf{d}_{l,k}^{\mathrm{H}}], \quad \forall k \in \mathcal{G} \text{ and } \forall l \in \mathcal{L}. \tag{4.3}$$

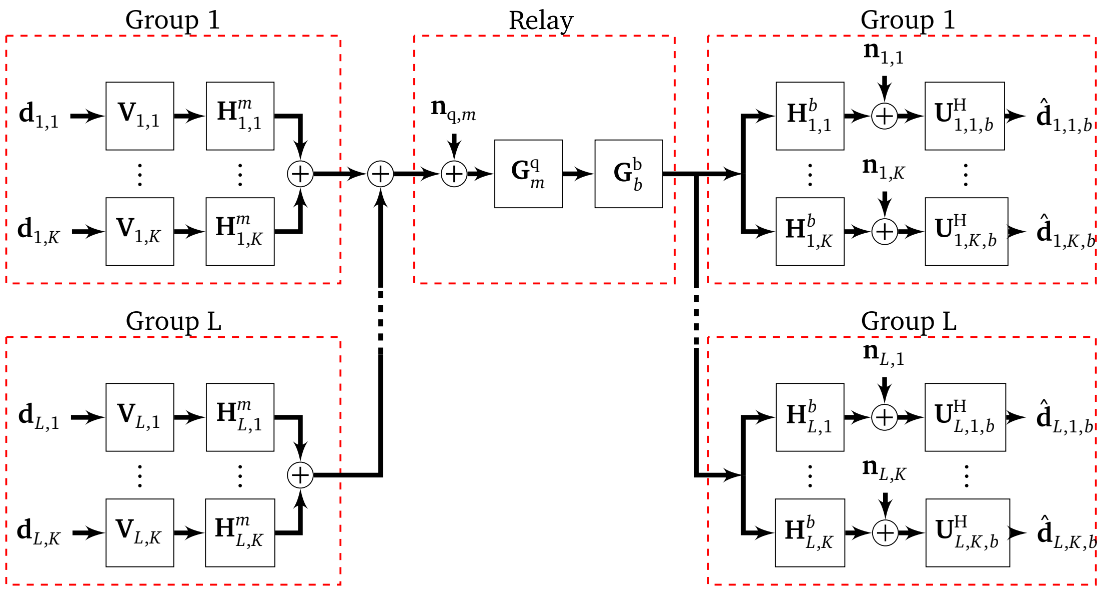


Figure 4.3. System model of the fully connected multi-group multi-way relaying network

It is assumed that the transmit symbols are independent and identically distributed (i.i.d.), so that $\mathbb{E}[\mathbf{d}_{l,k}\mathbf{d}_{l,k}^{\mathrm{H}}] = \mathbf{I}_d$, $\forall k \in \mathcal{G}$ and $\forall l \in \mathcal{L}$ and $\mathbb{E}[\mathbf{d}_{l,k}\mathbf{d}_{l,j}^{\mathrm{H}}] = \mathbf{0}_d$, $\forall k \neq j$ holds. The matrix $\mathbf{V}_{l,k} \in \mathbb{C}^{N\times d}$ denote the precoding matrix of node $[l,k]$. For simplicity of the notation, it is assumed that each of the nodes has a maximum transmit power denoted by $P_{\mathrm{n,max}}$. The precoder normalization to satisfy the maximum transmit power constraint is given by

$$||\mathbf{V}_{l,k}||_F^2 \leq P_{\mathrm{n,max}}, \quad \forall k \in \mathcal{G} \text{ and } \forall l \in \mathcal{L}. \tag{4.4}$$

Let $\mathbf{H}_{l,k}^{\mathrm{m}} \in \mathbb{C}^{R\times N_{l,k}}$ and $\mathbf{H}_{l,k}^{\mathrm{b}} \in \mathbb{C}^{N_{l,k}\times R}$ denote the frequency-flat, quasi-static MIMO channel matrix between node $[l,k]$ and the relay during the MAC phases and between the relay and node $[l,k]$ in the MC phases, respectively. The channels are assumed to be constant over the M MAC phases and over the B MC phases. It is assumed that the channel matrices are mutually independent and of full rank. Further, let $\mathbf{n}_{\mathrm{q},m} = \mathcal{CN}(0, \sigma_{\mathrm{q},m}^2 \mathbf{I}_R) \in \mathbb{C}^{R\times 1}$ denote the noise at the relay in MAC phase m and $\mathbf{n}_{l,k,b} = \mathcal{CN}(0, \sigma_{l,k}^2 \mathbf{I}_N) \in \mathbb{C}^{N\times 1}$ denote the noise at node $[l,k]$ in MC phase b. The components of the noise vectors $\mathbf{n}_{\mathrm{q},m}$ and $\mathbf{n}_{l,k,b}$ are i.i.d. zero-mean complex Gaussian random variables with variance $\sigma_{\mathrm{q},m}^2$ and $\sigma_{l,k}^2$, respectively. The signal received at the relay in MAC phase m is given by

$$\mathbf{r}_m^{\mathrm{q}} = \mathbf{G}_m^{\mathrm{q}} \left(\sum_{l=1}^{L} \sum_{k\in\mathcal{G}_{l,m}} \mathbf{H}_{l,k}^{\mathrm{m}} \mathbf{V}_{l,k} \mathbf{d}_{l,k} + \mathbf{n}_{\mathrm{q},m} \right) \in \mathbb{C}^{R\times 1}, \tag{4.5}$$

where $\mathbf{G}_m^{\mathrm{q}} \in \mathbb{C}^{R\times R}$ denotes the receive processing matrix of the relay.

The relay has enough antennas to spatially separate the data vectors received in each MAC phase. Therefore, after all MAC phases are completed, the relay will have separated all data vectors. These separated data vectors will be estimated by the relay. The estimated data vector at the relay, related to a given node, can be expressed as

$$\mathbf{r}_{l,k} = \mathbf{G}_{l,k}\left(\sum_{l'=1}^{L}\sum_{k'\in\mathcal{G}_{l',\check{m}}} \mathbf{H}^{\mathrm{m}}_{l',k'}\mathbf{V}_{l',k'}\mathbf{d}_{l',k'} + \mathbf{n}_{\mathrm{q},\check{m}}\right) \in \mathbb{C}^{d\times 1}, \tag{4.6}$$

where $\mathbf{G}_{l,k} \in \mathbb{C}^{d\times R}$ is a submatrix of $\mathbf{G}^{\mathrm{q}}_{m}$ obtained by extracting the d rows related to user $[l,k]$ from $\mathbf{G}^{\mathrm{q}}_{m} = \left[\mathbf{G}^{\mathrm{H}}_{1,k'_1}\cdots\mathbf{G}^{\mathrm{H}}_{L,k'_2}\right]^{\mathrm{H}}, \forall k'_1, k'_2 \in \mathcal{G}_{l,m}$, taking into account the MAC phase $\check{m}$ in which the node's signal was transmitted. Equation (4.6) can be rewritten by a few mathematical steps as:

$$\begin{aligned}\mathbf{r}_{l,k} &= \mathbf{G}_{l,k}\mathbf{H}^{\mathrm{m}}_{l,k}\mathbf{V}_{l,k}\mathbf{d}_{l,k} + \mathbf{G}_{l,k}\sum_{\substack{k'\in\mathcal{G}_{l,\check{m}},\\ k'\neq k}} \mathbf{H}^{\mathrm{m}}_{l,k'}\mathbf{V}_{l,k'}\mathbf{d}_{l,k'} \\ &\quad + \mathbf{G}_{l,k}\sum_{\substack{l'=1,\\ l'\neq l}}^{L}\sum_{k'\in\mathcal{G}_{l',\check{m}}} \mathbf{H}^{\mathrm{m}}_{l',k'}\mathbf{V}_{l',k'}\mathbf{d}_{l',k'} + \mathbf{G}_{l,k}\mathbf{n}_{\mathrm{q},\check{m}}.\end{aligned} \tag{4.7}$$

The first term and the second term in (4.7) are the signals from node k in group l and the signals from all other nodes in group l, respectively. The third term in (4.7) represents the interference signal which node k in group l receives from all other groups in the network. The fourth term represents the effective noise of node k in group l. In the following, we assume that the receive processing matrix $\mathbf{G}^{\mathrm{q}}_{m}$ performs receive zero-forcing. Hence, (4.7) can be further simplified to

$$\mathbf{r}_{l,k} = \mathbf{d}_{l,k} + \mathbf{G}_{l,k}\mathbf{n}_{\mathrm{q},\check{m}}. \tag{4.8}$$

Let $\mathbf{H}^{\mathrm{b}}_{l,k,j}$ denote a channel submatrix of $\mathbf{H}^{\mathrm{b}}_{l,k} = \left[\mathbf{H}^{\mathrm{b}}_{l,k,1}\cdots\mathbf{H}^{\mathrm{b}}_{l,k,L}\right] \in \mathbb{C}^{N_{l,k}\times R}$, representing a channel between the jth-cluster of antennas at the relay to node k in group l, for all $j \in \{1,\ldots,L\}$. By this splitting of the R relay antennas into L clusters, a MIMO interference multicast channel is created. In general, each cluster can have a different cluster size R^{C}_{l}. For simplicity of the notation and without loss of generality, an equal splitting among the relay antennas is assumed, i.e., each antenna cluster contains $R^{\mathrm{C}}_{l} = R/L$ antennas. Let $\mathbf{G}^{\mathrm{b}}_{j,b} \in \mathbb{C}^{R/L\times d}$ denote the relay precoding matrix of antenna cluster j in phase b. The relay has a maximum transmit power, denoted by $P_{r,\max}$. Hence, the relay precoding matrix $\mathbf{G}^{\mathrm{b}}_{j,b}$ has to normalized be such that the maximum total power constraint

$$\sum_{j=1}^{L}\mathrm{E}\left\{\left\|\beta^{\mathrm{b}}_{b}\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b}\mathbf{r}_{j,b}\right\|^{2}_{\mathrm{F}}\right\} \leq P_{r,\max} \tag{4.9}$$

with

$$\mathbf{G}^{\mathrm{b}}_{j,b} = \beta^{\mathrm{b}}_{b}\cdot\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b} \tag{4.10}$$

is fulfilled in each MC phase b, where $\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b} \in \mathbb{C}^{R/L\times d}$ denotes the unnormalized precoders and β^{b}_{b} the normalization factor related to MC phase b. After a few mathematical steps and plugging (4.8) into (4.9), one will end up with the normalization factor

$$\beta^{\mathrm{b}}_{b} = \sqrt{\frac{P_{r,\max}}{\sum_{j=1}^{L}\mathrm{Tr}\left[\tilde{\mathbf{G}}^{\mathrm{b,H}}_{j,b}\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b}\left(\mathbf{I}+\sigma^{2}_{\mathrm{q},m}\mathbf{G}_{j,b}\mathbf{G}^{\mathrm{H}}_{j,b}\right)\right]}}. \tag{4.11}$$

In [PSK17], published by the author of this thesis, the splitting of the R relay antennas into L clusters was predefined, i.e., the first R/L antennas are chosen for the first group, the second R/L antennas are chosen for the second group etc. In order to generalize this selection, let $\mathbf{O}_j \in \mathbb{B}^{R\times R/L}$, $\mathbb{B}=\{0,1\}$, $\forall j \in \mathcal{L}$ denote the antenna selection matrix. $\mathbf{O}_j$ has to be a matrix with at most one 1 in each row and at least one 1 in each column. Hence, $\mathbf{O}_j$ has to fulfill

$$\operatorname{rank}\left(\mathbf{O}_j\right)=\frac{R}{L}. \tag{4.12}$$

As an example, if $R=6$ and $L=3$ the antenna selection matrices

$$\mathbf{O}_1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\\0&0&0\end{bmatrix},\quad \mathbf{O}_2=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}, \tag{4.13}$$

will select the first R/L antennas for the first group and the second R/L for the second group. Let $\mathbf{U}^{\mathrm{H}}_{l,k,b} \in \mathbb{C}^{d\times N}$ denote the receive zero-forcing filter at node $[l,k]$ which nullifying the interference signals. The estimated data vector at node $[l,k]$ in MC phase b (with $b \neq k$) is given by

$$\hat{\mathbf{d}}_{l,k,b}=\mathbf{U}^{\mathrm{H}}_{l,k,b}\left(\sum_{j=1}^{L}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_j\mathbf{G}^{\mathrm{b}}_{j,b}\mathbf{r}_{j,b}+\mathbf{n}_{l,k,b}\right). \tag{4.14}$$

Taking (4.8) into account, (4.14) can be written as

$$\begin{aligned}\hat{\mathbf{d}}_{l,k,b} \quad = \quad & \mathbf{U}^{\mathrm{H}}_{l,k,b}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_l\mathbf{G}^{\mathrm{b}}_{l,b}\mathbf{d}_{l,b}+\mathbf{U}^{\mathrm{H}}_{l,k,b}\sum_{\substack{j=1,\\ j\neq l}}^{L}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_j\mathbf{G}^{\mathrm{b}}_{j,b}\mathbf{d}_{j,b}\\ & +\mathbf{U}^{\mathrm{H}}_{l,k,b}\left(\sum_{j=1}^{L}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_j\mathbf{G}^{\mathrm{b}}_{j,b}\mathbf{G}_{j,b}\mathbf{n}_{\mathrm{q},\check{b}}+\mathbf{n}_{l,k,b}\right).\end{aligned} \tag{4.15}$$

The first and the second term in (4.15) are the useful signal and the interference from other groups, respectively. The last term represents the effective noise at node k in group l during MC phase b.

4.3 System Model: Decode-and-Forward Relay

In this section, the system model of the considered multi-group multi-way relaying network, taking into account a decode-and-forward, is introduced. A multicast transmission requires a separation of the received data streams at the relay. Due to this separation, it can be assumed that the received data streams can also be decoded. As already mentioned, in general, a decode-and-forward relay is not necessary to perform IA. In comparison to an amplify-and-forward relay, considered in Section 4.2, a decode-and-forward relay has the advantage that the noise at the relay does not propagate to the nodes. Hence, some equations of the system model introduced in Section 4.2 have to be rewritten. Decode-and-forward relays are in general more complex than amplify-and-forward relays. If a decode-and-forward relay is considered, the relay has to decode the data streams transmitted by the nodes. It is assumed that the relay decodes all received data streams correctly. The equations of the system model that must be rewritten when considering a decode-and-forward relay are derived below. A multicast transmission requires a separation of the received data streams at the relay.

The estimated data vector at the relay, related to a given node, can be expressed as

$$\mathbf{r}_{l,k}^{\mathrm{DF}} = \mathbf{G}_{l,k}\left(\sum_{l'=1}^{L}\sum_{k'\in\mathcal{G}_{l',\check{m}}} \mathbf{H}_{l',k'}^{\mathrm{m}}\mathbf{V}_{l',k'}\mathbf{d}_{l',k'}\right) \in \mathbb{C}^{d\times 1}, \tag{4.16}$$

in comparison to (4.6) for the amplify-and-forward relay.

(4.16) can be rewritten by a few mathematical steps as follows:

$$\begin{aligned}\mathbf{r}_{l,k}^{\mathrm{DF}} &= \mathbf{G}_{l,k}\mathbf{H}_{l,k}^{\mathrm{m}}\mathbf{V}_{l,k}\mathbf{d}_{l,k} + \mathbf{G}_{l,k}\sum_{\substack{k'\in\mathcal{G}_{l,\check{m}},\\ k'\neq k}} \mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k'}\mathbf{d}_{l,k'}\\ &\quad + \mathbf{G}_{l,k}\sum_{\substack{l'=1,\\ l'\neq l}}^{L}\sum_{k'\in\mathcal{G}_{l',\check{m}}} \mathbf{H}_{l',k'}^{\mathrm{m}}\mathbf{V}_{l',k'}\mathbf{d}_{l',k'}.\end{aligned} \tag{4.17}$$

The first term and the second term in (4.17) are the signals from node k in group l and the signal from all other nodes in group l, respectively. The third term in (4.17) represents the interference signal which node k in group l receives from all other groups in the network. In comparison to (4.7) there is no effective noise term in (4.17).

Since zero-forcing is assumed as receive processing matrix, (4.17) can be further simplified to

$$\mathbf{r}_{l,k}^{\mathrm{DF}} = \mathbf{d}_{l,k}. \tag{4.18}$$

The relay precoding matrix $\mathbf{G}_{j,b}^{\mathrm{b}}$ has to be normalized such that the maximum total power constraint

$$\sum_{j=1}^{L} \mathrm{E}\left\{\left\|\beta_b^{\mathrm{DF}}\tilde{\mathbf{G}}_{j,b}^{\mathrm{b}}\mathbf{d}_{j,b}\right\|_{\mathrm{F}}^{2}\right\} \le P_{r,\max} \tag{4.19}$$

with

$$\mathbf{G}_{j,b}^{\mathrm{b,DF}} = \beta_b^{\mathrm{DF}} \cdot \tilde{\mathbf{G}}_{j,b}^{\mathrm{b}} \tag{4.20}$$

is fulfilled in each MC phase b, where $\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b} \in \mathbb{C}^{R/L\times d}$ denotes the unnormalized precoders and β^{DF}_{b} the normalization factor related to MC phase b.

Plugging (4.18) into (4.19) will end up with the normalization factor

$$\beta^{\mathrm{DF}}_{b} = \sqrt{\frac{P_{r,\max}}{\sum_{j=1}^{L} \mathrm{Tr}\left[\tilde{\mathbf{G}}^{b,\mathrm{H}}_{j,b}\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b}\right]}}. \tag{4.21}$$

The estimated data vector at node $[l,k]$ in MC phase b (with $b \neq k$) taking (4.18) into account is given by

$$\begin{aligned}\hat{\mathbf{d}}^{\mathrm{DF}}_{l,k,b} \quad = \quad & \mathbf{U}^{\mathrm{H}}_{l,k,b}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_{l}\mathbf{G}^{\mathrm{b,DF}}_{l,b}\mathbf{d}_{l,b} + \mathbf{U}^{\mathrm{H}}_{l,k,b}\sum_{\substack{j=1,\\ j\neq l}}^{L}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_{j}\mathbf{G}^{\mathrm{b,DF}}_{j,b}\mathbf{d}_{j,b}\\ & + \mathbf{U}^{\mathrm{H}}_{l,k,b}\mathbf{n}_{l,k,b}.\end{aligned} \tag{4.22}$$

The first and the second term in (4.22) are the useful signal and the interference from other groups, respectively. The last term represents the effective noise at node k in group l during MC phase b.

4.4 Achievable Sum Rate AF

In this section, the achievable sum data rate of an amplify-and-forward MIMO multi-group multi-way relaying network is derived. The achievable sum rate of the entire network is the sum of data rates at each receiving node. The data vector $\mathbf{d}_{l,k}$, $\forall k \in \mathcal{G}, \forall l \in \mathcal{L}$ is a circular symmetric Gaussian random vector [Gal08; NM93]. As described in Section 3.3, the achievable data rate at a MIMO receiver is given by

$$R_{\mathrm{MIMO}} = \log_2\left(|\mathbf{I} + \mathbf{SINR}|\right). \tag{4.23}$$

The useful signal at node k in group l in MC phase b is given by

$$\boldsymbol{\chi}^{\mathrm{U,AF}}_{l,k,b} \quad = \quad \mathbf{U}^{\mathrm{H}}_{l,k,b}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_{l}\mathbf{G}^{\mathrm{b}}_{l,b}\mathbf{d}_{l,b}, \tag{4.24}$$

and the useful signal covariance matrix is given by

$$\mathbf{\Xi}^{\mathrm{U,AF}}_{l,k,b} \quad = \quad \mathbb{E}\left[\left(\boldsymbol{\chi}^{\mathrm{U}}_{l,k,b}\right)\left(\boldsymbol{\chi}^{\mathrm{U}}_{l,k,b}\right)^{\mathrm{H}}\right]. \tag{4.25}$$

The inter-group interference signal at node $[l,k]$ in MC phase b is given by

$$\boldsymbol{\chi}^{\mathrm{I,AF}}_{l,k,b} \quad = \quad \mathbf{U}^{\mathrm{H}}_{l,k,b}\sum_{\substack{j=1,\\ j\neq l}}^{L}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_{j}\mathbf{G}^{\mathrm{b}}_{j,b}\mathbf{d}_{j,b}, \tag{4.26}$$

and the inter-group interference covariance matrix is given by

$$\boldsymbol{\Xi}^{\mathrm{I,AF}}_{l,k,b} \quad = \quad \mathbb{E}\left[\left(\boldsymbol{\chi}^{\mathrm{I}}_{l,k,b}\right)\left(\boldsymbol{\chi}^{\mathrm{I}}_{l,k,b}\right)^{\mathrm{H}}\right]. \tag{4.27}$$

The overall noise signal at node $[l,k]$ in MC phase b is given by

$$\boldsymbol{\chi}^{\mathrm{N,AF}}_{l,k,b} \quad = \quad \mathbf{U}^{\mathrm{H}}_{l,k,b}\left(\sum_{j=1}^{L}\mathbf{H}^{\mathrm{b}}_{l,k}\mathbf{O}_j\mathbf{G}^{\mathrm{b}}_{j,b}\mathbf{G}_{j,b}\mathbf{n}_{\mathrm{q},\check{b}} + \mathbf{n}_{l,k,b}\right), \tag{4.28}$$

and the noise covariance matrix is given by

$$\boldsymbol{\Xi}^{\mathrm{I,AF}}_{l,k,b} \quad = \quad \mathbb{E}\left[\left(\boldsymbol{\chi}^{\mathrm{N}}_{l,k,b}\right)\left(\boldsymbol{\chi}^{\mathrm{N}}_{l,k,b}\right)^{\mathrm{H}}\right]. \tag{4.29}$$

The data rate at node $[l,k]$ in phase b is given by

$$R_{l,k,b} = \log_2\left(\left|\mathbf{I}_d + \frac{\boldsymbol{\Xi}^{\mathrm{U,AF}}_{l,k,b}}{\boldsymbol{\Xi}^{\mathrm{I,AF}}_{l,k,b} + \boldsymbol{\Xi}^{\mathrm{N,AF}}_{l,k,b}}\right|\right). \tag{4.30}$$

In amplify-and-forward multi-group multi-way relaying each node k in group l has to ensure that its data stream can be decoded correctly by all $\mathcal{G}_l \setminus \{k\}$ intended nodes in group l. Hence, the maximum transmitting rate of each node in group l has to be smaller than or equal to the smallest achievable receiving rate of any node in group l.

The data rate of node k in group l is, therefore, given by

$$R_{l,k} = \min\left(R_{l,k,b}\right). \tag{4.31}$$

The sum rate of group l is given by

$$R_l = \frac{1}{M+B}\sum_{k\in\mathcal{G}_l}(S_l - 1)\min\left(R_{l,k,b}\right), \tag{4.32}$$

where the factor $(S_l - 1)$ is needed because only $(S_l - 1)$ nodes inside a group want to receive the signal transmitted by a single node of a group. The multiplication with the fraction $\frac{1}{M+B}$ is necessary because $M + B$ time slots are required for one communication cycle.

Thus, the achievable sum rate in the entire multi-group multi-way relaying network in given by

$$R^{\mathrm{AF}}_{\mathrm{sum}} = \sum_{l=1}^{L}R_l = \frac{1}{M+B}\sum_{l=1}^{L}\sum_{k\in\mathcal{G}_l}R_{l,k}. \tag{4.33}$$

4.5 Achievable Sum Rate DF

4.5.1 Introduction

In this section, the achievable sum rate of a decode-and-forward MIMO multi-group multi-way relaying network is derived. The achievable sum rate of the entire network is the sum of data rates at each receiving node. In decode-and-forward multi-group multi-way relaying, the relay decodes the received data streams, i.e., after M MAC-phases the relay has received and decoded all data streams of all $L\sum_{l=1}^{L} S_l$ nodes in the entire network. Hence, the calculation of the sum rate can be divided into two independent steps. In the first step, the MAC-phase rate which is achievable at the relay is determined. In the second step, the MC-phase rate which is achieved at the nodes is determined. The relay has to re-encode the data streams received in the MAC phases and transmit them to all the nodes in the different groups. The relay has to adapt the transmitting rate to the channels so that the data streams can be correctly decoded by each intended receiving node.

4.5.2 MAC phase data rate

The estimated data vector at the relay is shown in (4.17).

The useful signal covariance matrix at the relay of node $[l,k]$ transmitted in phase m is given by

$$\boldsymbol{\Xi}_{l,k}^{\mathrm{U,DF}} = \mathbb{E}\left[\left(\mathbf{G}_{l,k}\mathbf{H}_{l,k}^{\mathrm{m}}\mathbf{V}_{l,k}\mathbf{d}_{l,k}\right)\left(\mathbf{G}_{l,k}\mathbf{H}_{l,k}^{\mathrm{m}}\mathbf{V}_{l,k}\mathbf{d}_{l,k}\right)^{\mathrm{H}}\right] = \mathbf{G}_{l,k}\mathbf{H}_{l,k}^{\mathrm{m}}\mathbf{V}_{l,k}\mathbf{V}_{l,k}^{\mathrm{H}}\mathbf{H}_{l,k}^{\mathrm{mH}}\mathbf{G}_{l,k}^{\mathrm{H}}. \quad (4.34)$$

The intra group interference covariance matrix at the relay caused by $\mathcal{G}_{l,\check{m}} \setminus \{k\}$ transmitted in phase $\check{m}$ is given by

$$\begin{aligned}\boldsymbol{\Xi}_{l,k}^{\mathrm{I1,DF}} &= \mathbb{E}\left[\left(\mathbf{G}_{l,k}\sum_{k'\in\mathcal{G}_{l,\check{m}}\setminus k}\mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k'}\mathbf{d}_{l,k'}\right)\left(\mathbf{G}_{l,k}\sum_{k'\in\mathcal{G}_{l,\check{m}}\setminus k}\mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k'}\mathbf{d}_{l,k'}\right)^{\mathrm{H}}\right] \\ &= \sum_{k'\in\mathcal{G}_{l,\check{m}}\setminus k}\;\sum_{k''\in\mathcal{G}_{l,\check{m}}\setminus k}\mathbf{G}_{l,k}\mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k''}\mathbf{V}_{l,k''}^{\mathrm{H}}\mathbf{H}_{l,k''}^{\mathrm{mH}}\mathbf{G}_{l,k}^{\mathrm{H}}. \end{aligned} \quad (4.35)$$

The inter group interference covariance matrix at the relay caused by $\mathcal{G}_{l',\check{m}}$, $\forall l' \in \mathcal{L} \setminus \{l\}$

transmitted in phase $\breve{m}$ is given by

$$\begin{aligned}\boldsymbol{\Xi}_{l,k}^{\mathrm{I2,DF}} &= \mathbb{E}\left[\left(\mathbf{G}_{l,k}\sum_{\substack{l'=1,\\ l'\neq l}}^{L}\sum_{k'\in\mathcal{G}_{l,\breve{m}}}\mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k'}\mathbf{d}_{l,k'}\right)\left(\mathbf{G}_{l,k}\sum_{\substack{l'=1,\\ l'\neq l}}^{L}\sum_{k'\in\mathcal{G}_{l,\breve{m}}}\mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k'}\mathbf{d}_{l,k'}\right)^{\mathrm{H}}\right] \\ &= \left(\mathbf{G}_{l,k}\sum_{\substack{l'=1,\\ l'\neq l}}^{L}\sum_{k'\in\mathcal{G}_{l,\breve{m}}}\mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k'}\right)\left(\mathbf{G}_{l,k}\sum_{\substack{l'=1,\\ l'\neq l}}^{L}\sum_{k'\in\mathcal{G}_{l,\breve{m}}}\mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k'}\right)^{\mathrm{H}}. \end{aligned} \tag{4.36}$$

The noise covariance matrix at the relay corresponding to $[l,k]$ is given by

$$\boldsymbol{\Xi}_{l,k}^{\mathrm{N,DF}} = \mathbb{E}\left[\left(\mathbf{G}_{l,k}\mathbf{n}_{\mathrm{q},\breve{m}}\right)\left(\mathbf{G}_{l,k}\mathbf{n}_{\mathrm{q},\breve{m}}\right)^{\mathrm{H}}\right] = \mathbb{E}\left[\left(\mathbf{G}_{l,k}\mathbf{n}_{\mathrm{q},\breve{m}}\mathbf{n}_{\mathrm{q},\breve{m}}^{\mathrm{H}}\mathbf{G}_{l,k}^{\mathrm{H}}\right)\right] = \mathbf{G}_{l,k}\mathbf{G}_{l,k}^{\mathrm{H}}\sigma_{\mathrm{q},\breve{m}}^{2}. \tag{4.37}$$

The achievable rate of node $[l,k]$ at the relay is given by

$$R_{l,k}^{\mathrm{MAC}} = \log_2\left(\left|\mathbf{I}_d + \frac{\boldsymbol{\Xi}_{l,k}^{\mathrm{U,DF}}}{\boldsymbol{\Xi}_{l,k}^{\mathrm{I1,DF}} + \boldsymbol{\Xi}_{l,k}^{\mathrm{I2,DF}} + \boldsymbol{\Xi}_{l,k}^{\mathrm{N,DF}}}\right|\right). \tag{4.38}$$

4.5.3 BC/MC phase data rate

The estimated data vector at node $[l,k]$ in phase b is given in (4.22).

The useful signal covariance matrix at node $[l,k]$ in phase b is given by

$$\begin{aligned}\boldsymbol{\Xi}_{l,k,b}^{\mathrm{U,MC}} &= \mathbb{E}\left[\left(\mathbf{U}_{l,k,b}^{\mathrm{H}}\mathbf{H}_{l,k}^{\mathrm{b}}\mathbf{O}_{l}\mathbf{G}_{l,b}^{\mathrm{b}}\mathbf{d}_{l,b}\right)\left(\mathbf{U}_{l,k,b}^{\mathrm{H}}\mathbf{H}_{l,k}^{\mathrm{b}}\mathbf{O}_{l}\mathbf{G}_{l,b}^{\mathrm{b}}\mathbf{d}_{l,b}\right)^{\mathrm{H}}\right] \\ &= \mathbf{U}_{l,k,b}^{\mathrm{H}}\mathbf{H}_{l,k}^{\mathrm{b}}\mathbf{O}_{l}\mathbf{G}_{l,b}^{\mathrm{b}}\mathbf{G}_{l,b}^{\mathrm{bH}}\mathbf{O}_{l}^{\mathrm{H}}\mathbf{H}_{l,k}^{\mathrm{bH}}\mathbf{U}_{l,k,b}. \end{aligned} \tag{4.39}$$

The inter group interference covariance matrix at node $[l,k]$ in phase b is given by

$$\begin{aligned}\boldsymbol{\Xi}_{l,k,b}^{\mathrm{I,MC}} &= \mathbb{E}\left[\left(\mathbf{U}_{l,k,b}^{\mathrm{H}}\sum_{\substack{j=1,\\ j\neq l}}^{L}\mathbf{H}_{l,k}^{\mathrm{b}}\mathbf{O}_{j}\mathbf{G}_{j,b}^{\mathrm{b}}\mathbf{d}_{j,b}\right)\left(\mathbf{U}_{l,k,b}^{\mathrm{H}}\sum_{\substack{j=1,\\ j\neq l}}^{L}\mathbf{H}_{l,k}^{\mathrm{b}}\mathbf{O}_{j}\mathbf{G}_{j,b}^{\mathrm{b}}\mathbf{d}_{j,b}\right)^{\mathrm{H}}\right] \\ &= \left(\mathbf{U}_{l,k,b}^{\mathrm{H}}\sum_{\substack{j=1,\\ j\neq l}}^{L}\mathbf{H}_{l,k}^{\mathrm{b}}\mathbf{O}_{j}\mathbf{G}_{j,b}^{\mathrm{b}}\right)\left(\mathbf{U}_{l,k,b}^{\mathrm{H}}\sum_{\substack{j=1,\\ j\neq l}}^{L}\mathbf{H}_{l,k}^{\mathrm{b}}\mathbf{O}_{j}\mathbf{G}_{j,b}^{\mathrm{b}}\right)^{\mathrm{H}}. \end{aligned} \tag{4.40}$$

The noise covariance matrix at node $[l,k]$ is given by

$$\begin{aligned}\boldsymbol{\Xi}^{\text{N,MC}}_{l,k,b} &= \mathbb{E}\left[\left(\mathbf{U}^{\text{H}}_{l,k,b}\mathbf{n}_{l,k,b}\right)\left(\mathbf{U}^{\text{H}}_{l,k,b}\mathbf{n}_{l,k,b}\right)^{\text{H}}\right] = \mathbb{E}\left[\left(\mathbf{U}^{\text{H}}_{l,k,b}\mathbf{n}_{l,k,b}\mathbf{n}^{\text{H}}_{l,k,b}\mathbf{U}_{l,k,b}\right)\right] \\ &= \mathbf{U}^{\text{H}}_{l,k,b}\mathbf{U}_{l,k,b}\sigma^2_{l,k,b}. \end{aligned} \tag{4.41}$$

The data rate at node $[l,k]$ in phase b is given by

$$R^{MC}_{l,k,b} = \log_2\left(\left|\mathbf{I}_d + \frac{\boldsymbol{\Xi}^{\text{U,MC}}_{l,k,b}}{\boldsymbol{\Xi}^{\text{I,MC}}_{l,k,b} + \boldsymbol{\Xi}^{\text{N,MC}}_{l,k,b}}\right|\right). \tag{4.42}$$

4.5.4 Achivable sum rate

In a decode-and-forward two-way relay network the achievable data rate at node k is given by $R_k = \min\left(R^{\text{MAC}}_l, R^{\text{BC}}_k\right)$, if the nodes k and l are a communication pair. The rate R^{MAC}_l denotes the rate that is achieved at the relay from node l in the MAC phase and R^{BC}_k denotes the rate achieved at node k from the relay in the BC phase [RW07].

The relay has to make sure, that all $S_l - 1$ nodes in group l in MC phase b can decode the data stream transmitted by node k in the MAC phases. Therefore, the relay cannot transmit with rates higher than the receiving data rate.

The data stream of node k in group l is transmitted to all other nodes in group l in MC phase $b = k$.

The data rate of transmitting node k at receiving node r is given by

$$R_{l,k,r} = \min\left(R^{\text{MAC}}_{l,k}, \min_{r' \in \mathcal{G}\setminus\{k\}}\left(R^{\text{MC}}_{l,r',k}\right)\right), \tag{4.43}$$

i.e., $R_{l,k,r} = R_{l,k,1}$, $\forall r \in \mathcal{G} \setminus k$.

The data rate at all $\mathcal{G} \setminus \{k\}$ receiving node in group l is given by

$$R_{l,k} = \sum_{r \in \mathcal{G}\setminus k} R_{l,k,r} = (S_l - 1) R_{l,k,1}. \tag{4.44}$$

The sum rate in group l is given by

$$R_l = \frac{1}{M + S_l}\sum_{k \in \mathcal{G}} R_{l,k}. \tag{4.45}$$

The achievable sum rate in the entire network in given by

$$R_{\text{sum}}^{\text{DF}} = \sum_{l=1}^{L} R_l = \sum_{l=1}^{L} \frac{1}{M+S_l} \sum_{k\in\mathcal{G}} R_{l,k}. \tag{4.46}$$

If all nodes inside group l should communicate with the same data rate, the achievable sum rate in the entire network is given by

$$R_{\text{sym}}^{\text{DF}} = \sum_{l=1}^{L} \frac{1}{M+S_l} S_l \left(S_l - 1\right) \min_{\{k,r\}\in\mathcal{G}|k\neq r} R_{l,k,r}. \tag{4.47}$$

4.6 Multicast Interference Alignment Scheme

4.6.1 Introduction

In this section, a multicast IA algorithm for fully connected multi-group multi-way relaying networks is proposed. In the proposed algorithm, the number of antennas at the relay is independent of the number of nodes per group, which is an important property in wireless communication networks, since the number of antennas is limited in general. The idea behind this algorithm is, that in each of the MC phases, a MIMO interference multicast channel is created by dividing the number of antennas at the relay into as many clusters as groups in the network. Each of these clusters serves a specific group of nodes and transmits its signal in such a way that the signals transmitted from different clusters are aligned at the non-intended multicast groups. The proposed algorithm supports different numbers of antennas at the relay for each given system configuration. Hence, one can choose between performance and saving the required hardware resources. The underlying system model was introduced in Section 4.2 and Section 4.3. Hence, the variables and assumptions mentioned in these sections are still valid.

A part of the content of this section has been published by the author and his co-authors in [PSK17].

4.6.2 Required number of MAC and MC phases

For simplicity of the notation, in the following it is assumed that $S_l = S$, $\forall l \in \mathcal{L}$, i.e., each group contains the same number S of nodes. The other variables are as defined in Section 4.2. In each MAC-phase $m = 1, ..., M$,

$$S_{\text{MAC}} = \frac{R}{Ld} \tag{4.48}$$

nodes per group can be active and transmitting data to the relay, so that the relay is able to separate the data streams from the nodes of the different groups. Consequently, the relay receives $S_{\text{MAC}}L$ data streams in each MAC-phase. This leads to a total number of $M = \frac{SLd}{R}$ required MAC phases. For the special case where $R = Ld$, $M = S$ MAC phases are required. In this special case, only one node per group is active and transmits in each of the MAC phases to the intermediate relay. Since in total Sd data streams have to be exchanged in each group, $B = S$ MC phases are required, omitting self-interference.

4.6.3 Filter Design

4.6.3.1 Transmit Filter Design of the Nodes

Since the relay has enough antennas to spatially separate all received data streams during each MAC phase, the relay is able to cancel the whole interference. Hence, the node's precoding matrix is designed in order to maximize the SNR of the received signal at the relay. The SNR of node $[l,k]$ can be maximized by assigning the d strongest singular values of the channel matrix $\mathbf{H}^{\text{m}}_{l,k}$ to the precoding matrix $\mathbf{V}_{l,k}$, belonging to node $[l,k]$. Therefore, the transmit filter at each node is based on the channel between that node and the relay. Thus, local CSI is sufficient for the transmit filter design. The singular-value decomposition (SVD) of $\mathbf{H}^{\text{m}}_{l,k}$ is given by

$$\text{SVD}\left(\mathbf{H}^{\text{m}}_{l,k}\right) = \mathbf{\Gamma}_{l,k}\mathbf{\Sigma}_{l,k}\mathbf{\Lambda}^{\text{H}}_{l,k}, \tag{4.49}$$

where $\mathbf{\Gamma}_{l,k} \in \mathbb{C}^{R\times R}$ and $\mathbf{\Lambda}_{l,k} \in \mathbb{C}^{N\times N}$ are orthogonal matrices containing the singular vectors of $\mathbf{H}^{\text{m}}_{l,k}$. Matrix $\mathbf{\Sigma}_{l,k} \in \mathbb{C}^{R\times N}$ contains the singular values. The transmit filter of node $[l,k]$ is given by

$$\mathbf{V}_{l,k} = \sqrt{\frac{P_{\text{n,max}}}{d}}\mathbf{\Lambda}_{l,k,1\ldots d}, \tag{4.50}$$

where $\mathbf{\Lambda}_{l,k,1\ldots d}$ represents a matrix containing as columns the singular vectors corresponding to the d largest singular values of $\mathbf{\Sigma}_{l,k}$.

4.6.3.2 Relay Receive Processing Matrix

The relay receive processing matrix uses a receive zero-forcing approach to cancel out the entire interference signals at the relay. Let us define the following equivalent channel for each MAC phase m:

$$\begin{aligned}\mathbf{H}^{\text{m}}_{m} \quad &= \quad \left[\mathbf{H}^{\text{m}}_{1,k1}\mathbf{V}_{1,k1}\cdots\mathbf{H}^{\text{m}}_{1,k2}\mathbf{V}_{1,k2}\cdots\mathbf{H}^{\text{m}}_{L,k1}\mathbf{V}_{L,k1}\cdots\mathbf{H}^{\text{m}}_{L,k2}\mathbf{V}_{L,k2}\right] \in \mathbb{C}^{R\times R},\\ &\qquad k_1,\, k_2 \in \mathcal{G}_{l,m},\, k_1 \neq k_2.\end{aligned} \tag{4.51}$$

The equivalent channel matrix $\mathbf{H}_m^{\mathrm{m}}$ has the dimension $R \times R$ under consideration of (4.48). The matrix $\mathbf{H}_m^{\mathrm{m}}$ is a square matrix, which is non-singular with probability one. Hence, the Zero-Forcing matrix $\mathbf{G}_m^{\mathrm{q}}$ can be uniquely determined by taking the inverse of the equivalent channel matrix $\mathbf{H}_m$, given by

$$\mathbf{G}_m^{\mathrm{q}} = \left(\mathbf{H}_m^{\mathrm{m}}\right)^{-1}. \tag{4.52}$$

4.6.3.3 Relay Precoding Matrix

Let us define $\mathbf{\Phi}_{l,k,b} \in \mathbb{C}^{N \times d}$ as an orthonormal basis of the receive subspace at node k in group l in MC phase b. The interference power remaining in the received signal at each node k in all groups l after the left multiplication with $\mathbf{\Phi}_{l,k,b}^{\mathrm{H}}$ is a measure of the quality of the alignment and is termed leakage interference [GCJ08]. The objective is to minimize the leakage interference at the receivers and is given by

$$\begin{aligned} &\text{minimize} \quad \mathrm{Tr}\left(\sum_{\substack{j=1,\\ j\neq l}}^{L}\sum_{\substack{k=1,\\ k\neq b}}^{S} \mathbf{\Phi}_{l,k,b}^{\mathrm{H}}\mathbf{H}_{l,k}^{\mathrm{b}}\mathbf{O}_j\mathbf{G}_{j,b}^{\mathrm{b}}\mathbf{G}_{j,b}^{\mathrm{b,H}}\mathbf{O}_j^{\mathrm{H}}\mathbf{H}_{l,k}^{\mathrm{b,H}}\mathbf{\Phi}_{l,k,b}\right) \\ &\text{subject to} \quad \mathbf{\Phi}_{l,k,b}^{\mathrm{H}} \cdot \mathbf{\Phi}_{l,k,b} = \mathbf{I},\ \forall l \in \mathcal{L}, \forall k \in \mathcal{G}_l, \forall b \\ &\qquad\qquad\quad ||\mathbf{G}_{j,b}^{\mathrm{b}}||_F^2 \leq P_{\mathrm{r,max}}. \end{aligned} \tag{4.53}$$

The concept of alternating optimization, in this case alternating minimization [PH09; Byr11], is employed to determine the relay precoding matrix $\mathbf{G}_{j,b}^{\mathrm{b}}$. An alternating minimization algorithm requires several steps to solve the entire problem. In each step, all variables except one hold temporarily fixed to solve the remaining unfixed variable. In other words, in each step only one variable is optimized [BH02]. This process is repeated until a certain stop criterion is reached. An alternating minimization can only be applied if there exists a closed solution for each of the variables, assuming all other variables are fixed. Let $\nu_{\min,d}(.)$ denote an operation delivering a matrix containing the eigenvectors corresponding to the d smallest eigenvalues of the matrix within the brackets as its columns. The algorithm applied in each MC phase b in order to determine the relay precoding matrix $\mathbf{G}_{j,b}^{\mathrm{b}}$ and the receive space $\mathbf{\Phi}_{l,k,b}$ is described in Algorithm 1. The antenna selection matrix $\mathbf{O}_j$ is a priori fixed in this thesis and therefore not optimized. The subspace $\mathbf{\Phi}_{l,k,b}^{\mathrm{H}}$ is reserved for node $[l,k]$ in MC phase b. Hence, the interference at receiver $[l,k]$ has to be orthogonal to $\mathbf{\Phi}_{l,k,b}^{\mathrm{H}}$ in MC phase b to nullify leakage interference. The algorithm finds a precoding matrix $\mathbf{G}_{l,b}^{\mathrm{b}}$ such that the interference caused at each node in group l in phase b has maximum squared Euclidean distance between it and the subspace spanned by the columns of each $\mathbf{\Phi}_{l,k,b}^{\mathrm{H}}$.

Algorithm 1 Alternating optimization algorithm

1. Randomly initialize precoders $\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b}$ for $j = 1, \ldots, L$.
2. Find the basis of the interference subspace
$$\mathbf{\Phi}_{l,k,b} = \nu_{\min,d}\left(\sum_{\substack{j=1,\\ j\neq l}}^{L} \mathbf{H}^{\mathrm{b}}_{l,k,j}\mathbf{O}_j\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b}\tilde{\mathbf{G}}^{\mathrm{b,H}}_{j,b}\mathbf{O}^{\mathrm{H}}_j\mathbf{H}^{\mathrm{b,H}}_{l,k,j}\right)$$
for $l = 1, \ldots, L$ and $k = 1, \ldots, S$, with $k \neq b$.
3. Find the unnormalized precoders
$$\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b} = \nu_{\min,d}\left(\sum_{\substack{l=1,\\ l\neq j}}^{L}\sum_{\substack{k=1,\\ k\neq b}}^{S} \mathbf{O}^{\mathrm{H}}_j\mathbf{H}^{\mathrm{b,H}}_{l,k,j}\mathbf{\Phi}_{l,k,b}\mathbf{\Phi}^{\mathrm{H}}_{l,k,b}\mathbf{H}^{\mathrm{b}}_{l,k,j}\mathbf{O}_j\right)$$
for $j = 1, \ldots, L$.
4. Repeat steps 2 and 3 until convergence.
5. Calculate β^{b}_b according to (4.11) and then normalize precoders as $\mathbf{G}^{\mathrm{b}}_{j,b} = \beta^{\mathrm{b}}_b\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b}$ for $j = 1, \ldots, L$.

4.6.3.4 Receive Filter Matrix

The interference signals among the groups are aligned at the receivers, after applying Algorithm 1. This interference can be canceled by the left multiplication of $\mathbf{\Phi}^{\mathrm{H}}_{l,k,b}$ by the received signal vector. In order to separate the desired spatial streams at each node $[l,k]$ a zero-forcing approach is employed. Hence, the receive processing matrix in each phase b is given by

$$\begin{aligned}\mathbf{U}^{\mathrm{H}}_{l,k,b} &= \left(\mathbf{\Phi}^{\mathrm{H}}_{l,k,b}\mathbf{H}^{\mathrm{b}}_{l,k,l}\mathbf{G}^{\mathrm{b}}_{l,b}\right)^{-1}\mathbf{\Phi}^{\mathrm{H}}_{l,k,b}, \\ &\quad \forall l \in \mathcal{L}; \quad \forall k \in \mathcal{G},\ k \neq b.\end{aligned} \tag{4.54}$$

4.6.3.5 Properness Conditions: Multicast Interference Alignment

In this section, the properness condition for the multicast IA algorithm introduced in Section 4.6 is derived. A system is defined to be proper if and only if the number of independent variables in the system is larger than or equal to the number of independent constraints in the system [YGJK09; YGJK10; TGR09].

As introduced in Section 4.2, each node is equipped with $N_{l,k}$ antennas and the transmit filter matrix is of dimension $N_{l,k} \times d$. Hence, $N_{l,k}d$ variables are available at node k in group l. To spatially separate the d data streams at the receivers, the columns of the filter matrix have to be linearly independent. Therefore, d^2 variables at node $[l,k]$ have to be fixed so that the columns of the transmit filter become linearly independent [YGJK09]. Hence, $\left(N_{l,k}d - d^2\right)$ free variables are available at node k in group l. The entire relay space is R-dimensional, as introduced in Section 4.2. This relay space is divided into L

clusters of dimension R_l^{C}, $\forall l \in \mathcal{L}$, i.e., R_l^{C} denotes the number of antennas at the antenna cluster belonging to group l. The counting of the free variables at the relay takes place in the same way as for the nodes. Therefore, the total number of free variables is given by

$$X_{\mathrm{v}} = \sum_{l=1}^{L} \left(R_l^{\mathrm{C}} - d\right) d + \sum_{l=1}^{L} \sum_{k \in \mathcal{G}_l} \left(N_{l,k} - d\right) d. \tag{4.55}$$

In each of the L groups the signals form $(L-1)$ groups have to be aligned in a subspace linearly independent of the useful signal subspace of this group. Hence, the total number of zero forcing constrains in the system is given by

$$X_{\mathrm{c}} = \sum_{l=1}^{L} \sum_{k \in \mathcal{G} \setminus \mathcal{G}_l} d^2. \tag{4.56}$$

For a system to be proper the number of variables X_{v} has to be larger than or equal to the number of constraints X_{c}. Hence, the properness condition is given by

$$\begin{gathered} \sum_{l=1}^{L} \left(R_l^{\mathrm{C}} - d\right) d + \sum_{l=1}^{L} \sum_{k \in \mathcal{G}_l} \left(N_{l,k} - d\right) d \geq \sum_{l=1}^{L} \sum_{k \in \mathcal{G} \setminus \mathcal{G}_l} d^2 \\ \mathcal{G}_l = \{1, \ldots, S_l\}\,;\ \mathcal{G} = \bigcup_l \mathcal{G}_l \\ N_{l,k} \geq d\ \forall l, k;\ R_l^{\mathrm{C}} \geq d\ \forall l \end{gathered} \tag{4.57}$$

For the special case $R_l^{\mathrm{C}} = \frac{R}{L}$ and $N_{l,k} = N$, considered by the author of this thesis in [PSK17], the condition in (4.57) can be simplified as

$$L \cdot \left(\frac{R}{L} - d\right) d + LS \cdot (N-d)\, d \geq (L-1)\, LSd^2, \tag{4.58}$$

$$\frac{R}{L} - d + SN \geq LSd. \tag{4.59}$$

This leads to

$$\frac{R}{L} - d \geq S \cdot (Ld - N), \tag{4.60}$$

$$\min\left(\frac{R}{L}, N\right) \geq d, \tag{4.61}$$

4.6.4 Performance Analysis

4.6.4.1 Introduction

In this section, the performances of the proposed multicast IA algorithm is analyzed and compared with a reference IA algorithm proposed in [GAL+14]. The achivable DoF, as well as the sum rate, are considered to evaluate the performance of the algorithms. In Section 4.6.4.2, the different simulation scenarios are introduced and the DoF of the proposed algorithm is compared with the reference algorithm. In Section 4.6.4.3, the sum rates achieved by the proposed algorithm are compared with the selected reference algorithm. The sum rates achieved by these algorithms are obtained through numerical MATLAB simulations.

In the following the assumptions regarding the simulation are briefly described. The algorithm themselves is valid for the assumption mentioned in Section 2.3 and Section 4.6.

- It is assumed that the channel between each node and the relay is an i.i.d frequency-flat Rayleigh fading MIMO channel [LS03]. Hence, the channel matrices are of full rank, almost surely.
- The channel matrices are normalized such that the average received power is the same as the average transmit signal power.
- Due to the considered statistical channel model, the channel amplitude may vary for different realizations. Hence, all simulation results are averaged over 10^4 independent channel realizations. For each channel realization, all filters are designed according to the considered algorithm and the corresponding sum rate is calculated. The average sum rate which is plotted in this section is therefore an average over all 10^4 independent channel realizations. 10^4 independent channel realizations are large enough to get a sufficiently small confidence interval for plotting the sum rate.

4.6.4.2 Degrees of Freedom Analysis

In this section, the DoF of the proposed multicast IA algorithm for fully connected multi-group multi-way relaying networks are investigated and compared with the selected reference algorithm proposed in [GAL+14]. The properness condition of the proposed algorithm has been derived in Section 4.6.3.5. For the case in which $N = Ld$ and $R/L \geq d$ hold, the system is always feasible, independent of the number S of nodes per group. From (4.60) and (4.61) it becomes obvious that when satisfying $N = Ld$ and $R/L \geq d$, the dimensioning of the number of relay and node antennas does not depend on S anymore. This leads to

$$\begin{aligned} \frac{R}{L} &\geq d, && (4.62)\\ \min\left(\frac{R}{L}, N\right) &\geq d. && (4.63) \end{aligned}$$

In the following, two different networks are considered for performance evaluation. The first network GF1 consists of $L = 3$ groups, each containing $S = 3$ nodes of these each want to transmit $d = 1$ data stream to all other group members, see Figure 4.1. The second network GF2 consists of $L = 3$ groups, each containing $S = 6$ nodes of these each want to transmit $d = 1$ data stream to all other group members. The different values for the number S of nodes in each group allow verifying the influence of the group size on the dimensioning of the system parameters, for example, the number of antennas required at the relay. The properness conditions of the proposed algorithm and the reference algorithm do not allow us to consider the same number of antennas at the relay. In total, ten different scenarios for simulation and performance analysis are considered.

Table 4.1 gives an overview of the different scenarios considered in this section, the number of nodes S, the number of groups in the network L, the number of data streams transmitted by each node d, the number of antennas at the relay R and the numbers of antennas at each of the S_l nodes $N_{l,k}$ is shown. In the scenarios F1.1 and F1.2, network GF1 and an amplify-and-forward relay are considered. In the scenarios F1.4 and F1.5, network GF1 and an decode-and-forward relay are considered. Scenario F1.3 considers network GF1 applying the reference algorithm. In the scenarios F2.1, F2.2, F2.3 and F2.4, network GF2 and an amplify-and-forward relay applying the proposed algorithm are considered. Scenario F2.5 considers network GF2 applying the reference algorithm. It is worth to mention, that it is not possible to apply the proposed algorithm as well as the reference algorithm on exactly the same scenarios. This can be seen from the properness conditions of both algorithms.

In this paragraph the reference algorithm is briefly explained. The reference algorithm from the literature was proposed in [GAL+14]. In [GAL+14] the relay does not have enough antennas to spatially separate the data streams transmitted by the nodes. The relay assist the process of performing IA at the receiving nodes in the different group. IA is performed in three linearly independent steps, which are group signal alignment (GSA), group channel alignment (GCA) and transceive zero forcing. The reference algorithm requires a single MAC phase, i.e., all nodes of all groups transmitting simultaneously to the relay. $S-1$ BC phases are required in order to grantee that each node can decode the data stream transmitted by all other nodes in its own group, assuming self-interference can perfectly be canceled. The reference algorithm considers that the nodes may have different numbers of antennas.

The DoF of the proposed algorithm can be determined by calculating the ratio between the total number of data streams

$$X_{\mathrm{d}} = (LS(S-1)d) \tag{4.64}$$

and the total number of phases

$$X_{\mathrm{s}} = \left(\frac{SLd}{R+S}\right). \tag{4.65}$$

The DoF of the reference algorithm was derived in [GAL+14] and is given by $L(S-1)d$.

A comparison between the proposed multicast algorithm and the reference algorithm, with regard to their dimensioning parameters and DoF expression is shown in Table 4.2.

Table 4.1. Considered scenarios for multicast IA

					Antennas Nodes $N_{l,k} \forall l \in \mathcal{L}$						Phases		
Scenario	S	L	d	R	$N_{l,1}$	$N_{l,2}$	$N_{l,3}$	$N_{l,4}$	$N_{l,5}$	$N_{l,6}$	MAC	MC/BC	DoF
F1.1	3	3	1	3	3	3	3	-	-	-	3	3	3
F1.2	3	3	1	9	3	3	3	-	-	-	1	3	4.5
F1.3	3	3	1	6	2	2	3	-	-	-	1	2	6
F2.1	6	3	1	3	3	3	3	3	3	3	6	6	7.5
F2.2	6	3	1	6	3	3	3	3	3	3	3	6	10
F2.3	6	3	1	9	3	3	3	3	3	3	2	6	11.25
F2.4	6	3	1	18	3	3	3	3	3	3	1	6	12.86
F2.5	6	3	1	15	2	2	3	3	3	3	1	5	15
F1.4	3	3	1	3	3	3	3	-	-	-	3	3	3
F1.5	3	3	1	9	3	3	3	-	-	-	1	3	4.5

Table 4.2. System dimensioning and DoF expression

	Approach	
	Reference algorithm	Proposed MC algorithm
Relay antennas	$R = Ld(S-1)$	$R \geq Ld$
Node antennas	$\sum_k N_{l,k} \geq R + d,\ \forall l \in \mathcal{L}$	$N = Ld$
MAC phases	1	$\frac{SLd}{R}$
BC/MC phases	$S-1$	S
DoF	$L(S-1)d$	$\frac{L(S-1)d}{(Ld/R)+1}$

It can be seen from Table 4.2 that the required number of relay antennas of the proposed algorithm is independent of S. The dependency on S is shifted to the number of required MAC phases, which might be a favorable property, as we are trading physical antenna resources for additional time phases. The reference algorithm, in contrast, has a single MAC phase, but at the cost of a potentially high number of relay antennas as S increases. In terms of DoF, the proposed multicast approach suffers a penalty when using the smallest number of relay antennas, but it has the flexibility to use more relay antennas. The number of relay antennas should be an integer multiple of Ld, but such that the number SLd/R of MAC phases is also an integer. As the number of relay antennas increases, the DoF performance of the proposed algorithm approaches that of the reference algorithm, proved by

$$\lim_{R\to\infty} \frac{L(S-1)d}{(Ld/R)+1} = L(S-1)d. \tag{4.66}$$

In the first considered network GF1, the proposed multicast algorithm can employ half the number of relay antennas of the reference algorithm, achieving half the number of DoF. Increasing the number of relay antennas is not beneficial in the first considered network, since a larger number of antennas at the relay than for the reference algorithm is required and still a lower number of DoF is achieved, see Table 4.1. This behavior will be verified in the sum rate analysis of this section.

In the second network GF2, the proposed multicast algorithm has higher flexibility than in the first network. The multicast algorithm can work with a fifth of the number of antennas than the reference algorithm at the relay, achieving still half of the DoF of the reference. If one increases the number of antennas at the relay the DoF will also increase, see Table 4.1. This behavior will be verified in the sum rate analysis of this section.

Note that the restriction that the number SLd/R of MAC phases should be an integer does not allow, for the considered scenarios, a comparison between the reference and the proposed algorithms with the same number of relay antennas.

4.6.4.3 Sum Rate Analysis

In this section the sum rate performance of the proposed multicast interference alignment algorithm for fully connected multi-group multi-way relaying networks presented in Section 4.6 is analyzed and compared with the reference algorithm. Let $P_{n,\max} = P$ denote the transmit power of each of the LS nodes in the network. Further, Let $P_{\mathrm{r},\max} = LSP$ denote the transmit power of the relay. The noise power at each node and at the relay is assumed to be the same for the simulation and is denoted by $\sigma^2 = \sigma^2_{l,k} = \sigma^2_{\mathrm{q},m}$, $\forall k \in \mathcal{G}$. In the following, the ratio of P/σ^2 is termed SNR.

Since a DoF analysis is only valid for an asymptotically high SNR, a large range of SNR values of both algorithms is simulated and shown in this section. This makes the assessment of the performance of the algorithms easier. The alternating minimization algorithm, Algorithm 1, of the proposed multicast algorithm considers 10 iterations, which were verified to achieve convergence.

Figure 4.4 shows the sum rate performance as a function of P/σ^2 for network GF1 with $L = 3$ groups and $S = 3$ nodes per group, considering an amplify-and-forward relay, i.e., the scenarios F1.1, F1.2, and F1.3 are shown in Figure 4.4. The dashed line and the full lines represent the sum-rate achieved by the reference algorithm and the proposed multicast algorithm, respectively. It can be observed from Figure 4.4 that the proposed multicast algorithm with $R = 3$, scenario F1.1, outperforms the reference algorithm till an SNR of roughly 20 dB. The performance of the proposed multicast algorithm improves for $R = 9$, scenario F1.2, but at the cost of using more antennas at the relay than for the reference algorithm, scenario F1.3. The proposed multicast algorithm with $R = 9$, scenario F1.2, outperforms the reference algorithm, scenario F1.3, till an SNR of roughly 40 dB. In the proposed multicast algorithm, the nodes design their transmit filters to maximize the SNR of the received signal at the relay. Hence, the proposed multicast algorithm outperforms the reference algorithm in the low SNR region. The reference algorithm does not care about the useful signal power and has, therefore, a bad performance in the low SNR region. It can also be seen that the slope of the three curves in Figure 4.4 is different, the slope of the sum-rate curve is a measure for the DoF. These slopes correspond to the DoF given in Table 4.1.

Figure 4.5 shows the sum rate performance as a function of P/σ^2 for network GF2 with $L = 3$ groups and $S = 6$ nodes per group, considering an amplify-and-forward relay, i.e., the scenarios F2.1, F2.2, F2.3, F2.4 and F2.5 are shown in Figure 4.4. If the number of nodes per group increases from 3 to 6, rather significant performance gains are achieved by the proposed multicast algorithm. For $R = 3$, $R = 6$ and $R = 9$ antennas at the relay, the proposed multicast algorithm outperforms the reference algorithm, with $R = 15$ antennas at the relay, in terms of sum rate up to the SNR values of roughly 30 dB, 42 dB, and 52 dB, respectively. The scenarios F2.2 with $R = 6$ antennas at the relay and F2.3 with $R = 9$ antennas at the relay are a good trade-off between the number of relay antennas and sum-rate performance. In both of these scenarios, the reference algorithm with $R = 15$ antennas at the relay is outperformed by the proposed multicast algorithm in terms of

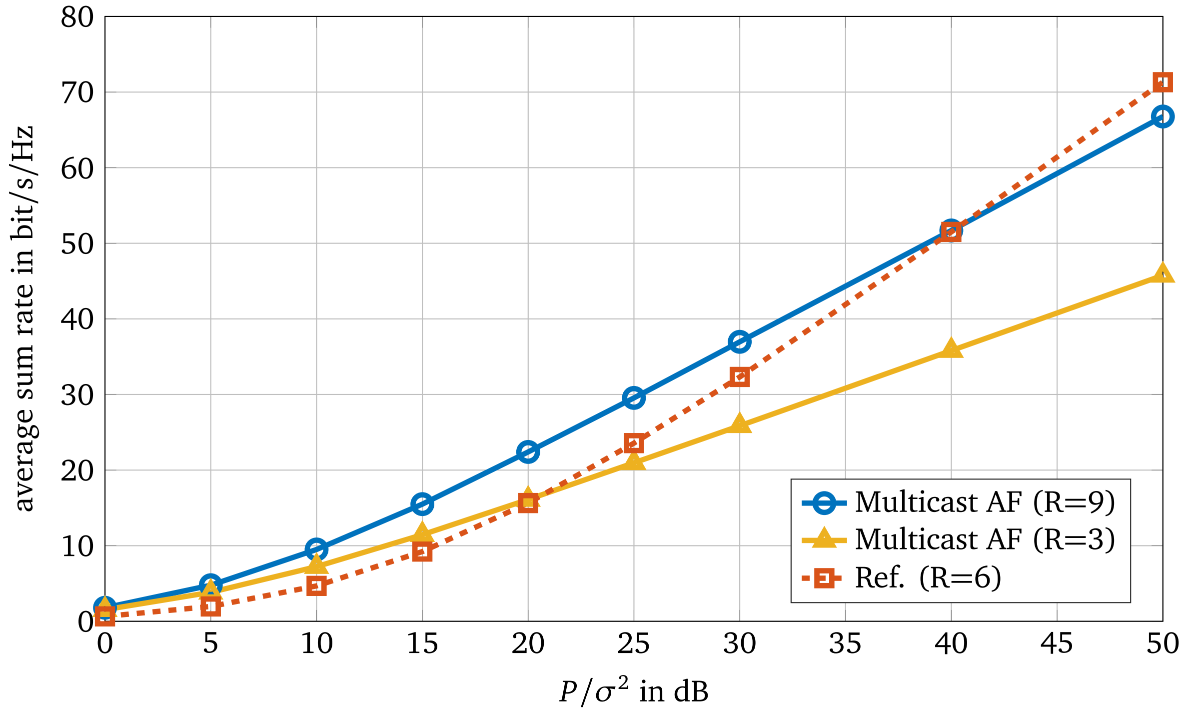


Figure 4.4. Sum rate performance of the proposed multicast algorithm and the reference algorithm versus P/σ^2 for network GF1 with $L = 3$, $d = 1$ and $S = 3$

sum-rate over the entire simulated SNR range. Whereby, the proposed algorithm even requires fewer antennas at the relay then the reference algorithm.

In Figure 4.6 the number R of relay antennas is plotted versus the number S of nodes per group, for a fixed SNR value of 20 dB. All nodes are equipped with the minimum required number of antennas, specified in the system dimension equations in Table 4.2. The triangle markers denote the proposed multicast algorithm and the circle markers denote the reference algorithm, the marker size scales with the average sum rate. The proposed multicast algorithm is much more flexible than the reference algorithm, e.g., for $R = 9$ antennas the reference algorithm can only handle $S = 4$ nodes per group, whereas the proposed algorithm can handle $S = 3$ with 1 MAC phase, $S = 6$ with 2 MAC phase, $S = 9$ with 3 MAC phase and so on. From the color of the markers, it can also be seen that the achievable sum rate of the reference algorithm does not increase as much as the sum rate of the proposed algorithm. If only 1 MAC phase is considered, the proposed algorithm can serve one node less than the reference algorithm for all relay antenna configurations. The reason for this is the additional required MC phase in comparison to the reference algorithm.

In Figure 4.7, the average sum rate of the proposed algorithm is plotted versus the number R of relay antennas for different ratios $\frac{S}{R}$, for an SNR of 20 dB. The colors, as well as the marker shapes, indicate different constant ratios of $\frac{S}{R}$. The dotted lines are lines on which there are markers corresponding to the same number S of nodes. This graph

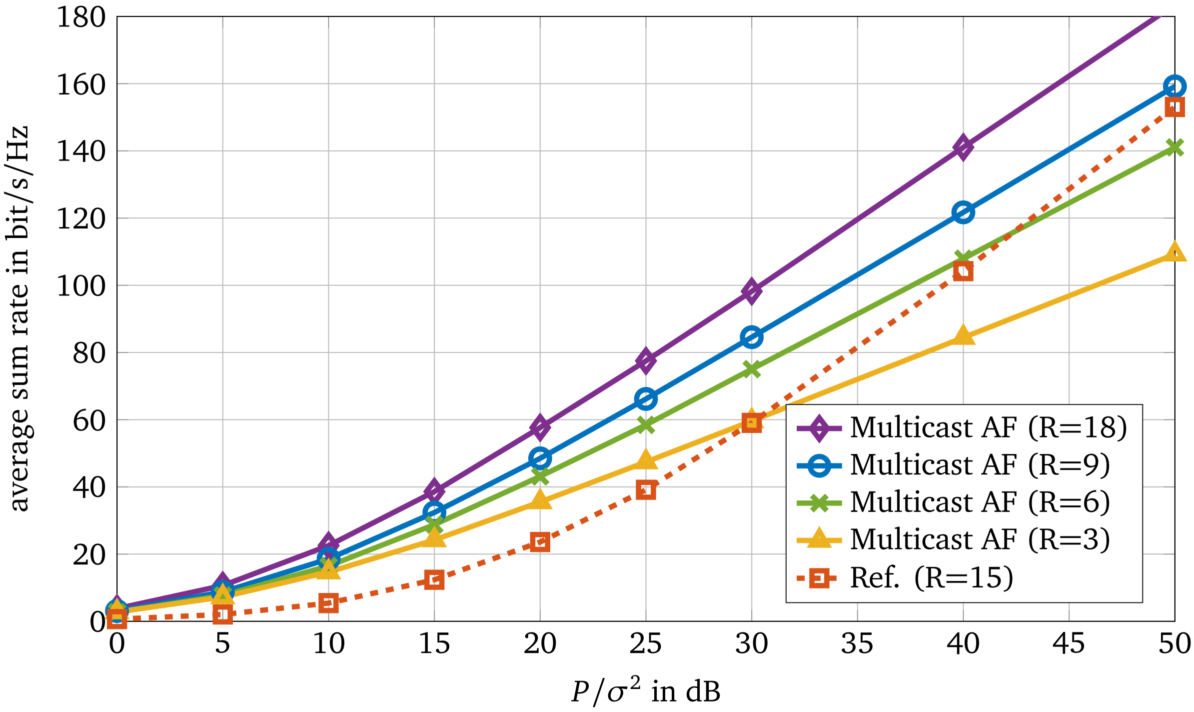
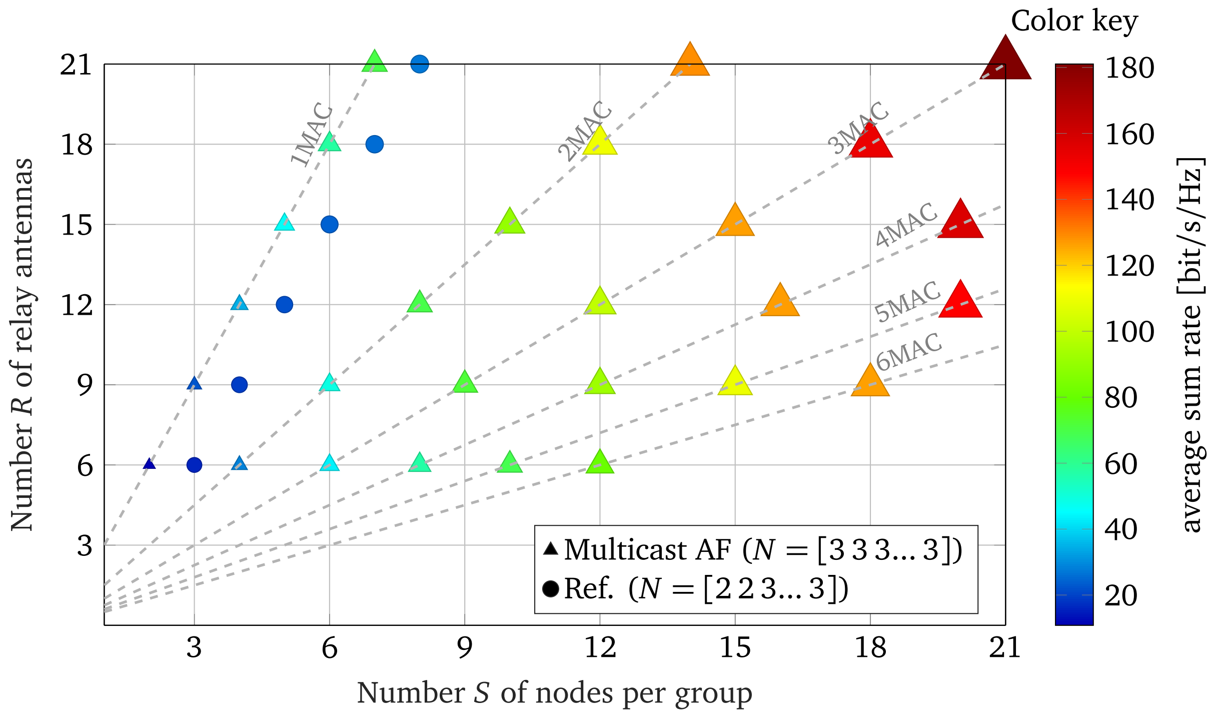


Figure 4.5. Sum rate performance of the proposed multicast algorithm and the reference algorithm versus P/σ^2 for network GF2 with $L = 3$, $d = 1$, $S = 6$.

is a good way to visualize the possible dimensioning parameters of a system with $L = 3$ groups in which each node is equipped with $N = 3$ antennas and wants to transmit $d = 1$ data stream, e.g., if $S = 6$, R can be 3, 6, 9, or 18. If $R = 12$, $S = 4$, $S = 8$, $S = 12$, $S = 16$, $S = 20$ or, $S = 24$ nodes per group can be served.

In this paragraph, a decode-and-forward (DF) relay is considered. In comparison to an amplify-and-forward relay, a decode-and-forward relay has the advantage that the noise at the relay does not propagate to the nodes. The average sum-rate as a function of P/σ^2 for network GF1, considering an amplify-and-forward relay and a decode-and-forward is shown in Figure 4.8. This means the performance of the scenarios F1.1, F1.2, F1.4, F1.5, and F1.3 are compared in Figure 4.8. A decode-and-forward relay does not change the DoF, see Figure 4.8. The slope of the sum-rate curves remains the same but is shifted upwards, because the noise power at the receiving nodes is less than in the amplify-and-forward relay case. Decode-and-forward relays are in general more complex than amplify-and-forward relays. Therefore, one has to decide if one wants to spend this effort to achieve a slightly higher sum rate.

It can be summarized that the proposed multicast algorithm presents a better sum rate performance up to a certain SNR value than the reference algorithm. As the number of relay antennas is increased the slope of the sum rate curve increases as well, pushing the point at which the reference algorithm outperforms the proposed one to a higher SNR value.



The marker size scales with the average sum rate.

Figure 4.6. Number R relay antennas versus number S of nodes in each group for an SNR $= 20$ dB, considering network a network with $L = 3$ groups were each node wants to transmit $d = 1$ data stream.

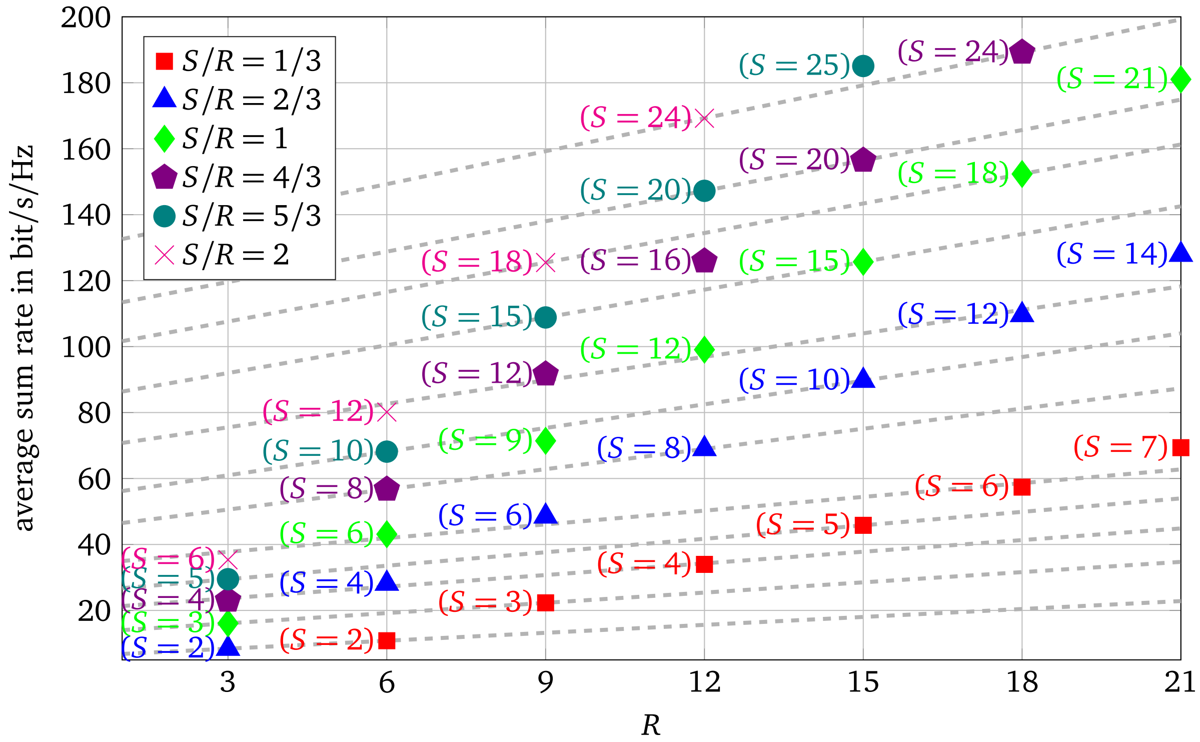


Figure 4.7. Average sum rate versus number R of antennas at the relay for different ratios $\frac{S}{R}$ at SNR =20 dB, for a network with $L = 3$, $d = 1$, $N = 3$

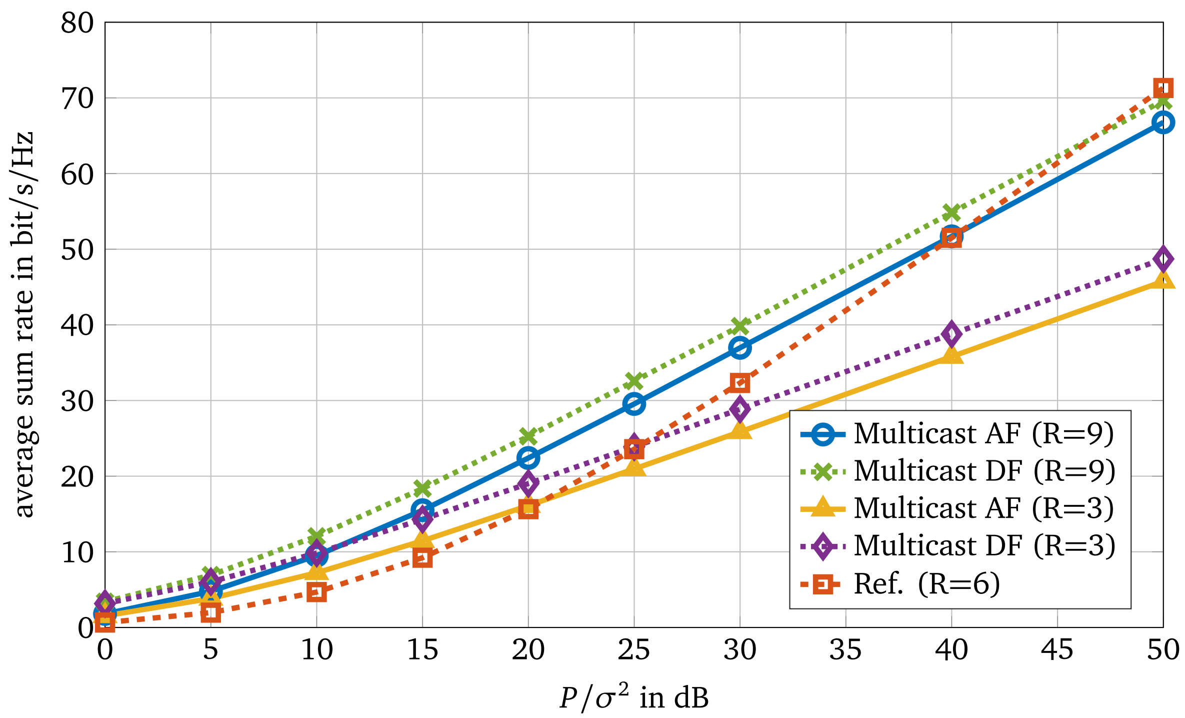


Figure 4.8. Sum rate performance of the proposed multicast algorithm versus P/σ^2 for network GF1, considering an AF and a DF relay, with $L = 3$, $d = 1$ and $S = 3$

4.7 Summary

In this chapter, IA in a fully connected multi-group multi-way relaying network has been considered. At the beginning of this chapter, an appropriated system model has been introduced. A multicast IA algorithm has been proposed in order to perform IA in a multi-group multi-way relaying network. Furthermore, the properness condition for the proposed multicast IA algorithm has been derived. The main advantage of the proposed multicast IA algorithm is that the minimum required number of antennas at the relay is independent of the number of users per group, which is an important property since physical antenna resources are limited in general. The algorithm proposed in this chapter is flexible in the sense that it supports different numbers of antennas at the relay for each given system configuration, which allows to achieve different trade-offs between performance and required hardware resources. An amplify-and-forward as well as a decode-and-forward relay has been considered. It has been shown that the proposed multicast IA algorithm requires fewer antennas at the relay than the reference algorithm from the literature. The simulation results show that the proposed multicast IA algorithm outperforms the reference algorithm for a broad range of SNR values. It has also been shown that the upper limit of this SNR range, up to which the proposed algorithm achieves a higher sum rate than the reference algorithm, increases as the number of relay antennas is increased.

Chapter 5

Interference Alignment in Multi-Group Multi-Way Partially Connected Relaying Networks

5.1 Introduction

In this chapter, IA in a partially connected multi-group multi-way relaying network is considered. In such a partially connected multi-group multi-way relaying network, not all groups of nodes are connected to all relays in the entire network. However, any group is connected to at least one relay that serves this group of nodes. This leads to a network that may contain two or more fully connected subnetworks that intersect. Hence, this network topology can be represented as multiple subnetworks, where each of these contains a single relay and all groups of nodes connected to this relay. The relays are used to manipulate the effective channel to achieve IA at each receiver. Each node inside a group wants to share its information with all other nodes in its group, but not with nodes in other groups of the network. Groups belonging to multiple subnetworks are located inside the intersection area for at least two subnetworks. The most challenging part of this network topology is the handling of the groups that are connected to multiple relays, i.e., of groups belonging to multiple subnetworks. Therefore, this chapter proposes a new algorithm for the treatment of these types of groups.

In Section 5.2, the system model for a partially connected multi-group multi-way relaying network is introduced. In Section 5.3, the sum rate expression of the considered partially connected mult-group multi-way relaying network is derived. In Section 5.4 the closed-form algorithm to achieve IA at each receiver is proposed. For this purpose, the concept of SSA and SCA, proposed in Section 3.5.3, is generalized in order to handle groups with more than two members. Finally, in Section 5.5, the performance of the proposed IA algorithm is investigated.

The content of this section has been published by the author of this thesis in [PVK19].

5.2 System Model

In this section, the system model of the considered partially connected multi-group multi-way relaying network is introduced. This type of network topology has been explained in Section 2.2.3. Such a partially connected multi-group multi-way relaying network consists of $L \geq 1$ groups which are served by $Q \geq 1$ amplify-and-forward half-duplex multi antenna relays. The l^{th} group, $l \in \mathcal{L} = \{1, ..., L\}$, contains S_l multi-antenna half-duplex nodes which want to communicate groupwise. The communication between the nodes inside a group takes place via at least one intermediate relay. The direct links between the nodes are irrelevant because a multi-way transmission scheme is utilized. An example scenario consisting of $L = 5$ groups, $Q = 2$ subnetworks and $S_1 = S_2 = S_3 = S_4 = S_5 = 3$ nodes per group is shown in Figure 5.1. It is assumed that the relays

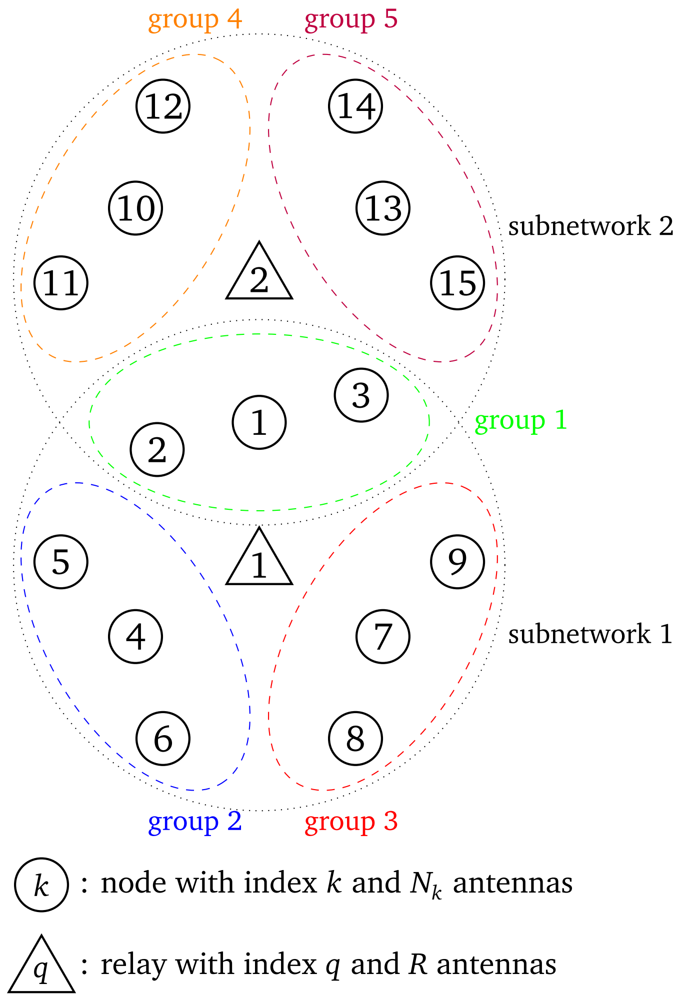


Figure 5.1. Partially connected multi-group multi-way relaying network with $L = 5$ groups, $S = 3$ nodes in each group and $Q = 2$ relays.

know which nodes belong to which group. Nodes in the first group are indicated with the set $\mathcal{G}_1 = \{1, ..., S_1\}$, nodes in the second group are indicated with the set $\mathcal{G}_2 = \{S_1 + 1, ..., S_1 + S_2\}$, and so on. In general, the l^{th} group contains nodes with the indices in the set $\mathcal{G}_l = \{a_l, ..., b_l\}$, where $a_l = \sum_{j=1}^{l-1} S_j + 1$ and $b_l = \sum_{j=1}^{l} S_j$. Each node only wants to exchange data with nodes in its own group and belongs only to a single group, i.e., $\mathcal{G}_l \cap \mathcal{G}_k = \emptyset$, $\forall l \neq k$. The set of all nodes in the entire network is given by $\mathcal{G} = \cup_{l=1}^{L} \mathcal{G}_l$. Let $\mathcal{G}(q)$ denote the set of nodes which are connected to relay $q \in \mathcal{Q} = \{1, ..., L\}$ and $\mathcal{R}(k)$ the set of relays which are connected to node k. Let L_q denote the number of groups connected to relay q. Node k in group l is equipped with N_k antennas and wants to share $d \leq N_k$ data streams with the other nodes in group l. For simplicity, it is assumed that all groups have the same number of nodes, i.e, $S_l = S$ and all relays are equipped with R_q antennas. In order to exchange information between the S nodes in group l groupwise, a transmission scheme with one MAC phase and b BC phases is considered. Let $\mathbf{d}_k \in \mathbb{C}^{d\times 1}$ denote the data vector originating from node k. The covariance matrix of $\mathbf{d}_k$ is given by

$$\mathbf{R}_{\mathbf{d}_k} = \mathbb{E}[\mathbf{d}_k \mathbf{d}_k^{\mathrm{H}}], \quad \forall k \in \mathcal{G}. \tag{5.1}$$

It is assumed that the transmit symbols are i.i.d., so that $\mathbb{E}[\mathbf{d}_k \mathbf{d}_k^{\mathrm{H}}] = \mathbf{I}_d$, $\forall k \in \mathcal{G}$ and $\mathbb{E}[\mathbf{d}_k \mathbf{d}_j^{\mathrm{H}}] = \mathbf{0}_d$, $\forall k \neq j$ holds. The matrix $\mathbf{V}_k \in \mathbb{C}^{N_k \times d}$ denote the precoding matrix of node k. For simplicity of the notation, it is assumed that each of the nodes has a maximum transmit power denoted by $P_{\mathrm{n,max}}$. To satisfy the maximum transmit power constraint of the nodes, the precoders are normalized given by

$$||\mathbf{V}_k||_F^2 \leq P_{\mathrm{n,max}}, \quad \forall k \in \mathcal{G}. \tag{5.2}$$

Let $\mathbf{H}_{k,q}^{\mathrm{nr,m}} \in \mathbb{C}^{R_q \times N_k}$ and $\mathbf{H}_{k,q}^{\mathrm{rn,b}} \in \mathbb{C}^{N_k \times R_q}$ denote the frequency-flat, quasi-static MIMO channel matrix between node k and relay q during the MAC phase and between relay q and node k in the BC phases, respectively. The channels are assumed to be constant over the BC phases. It is assumed that the channel matrices are mutually independent and of full rank. The components of the noise vectors $\mathbf{n}_{\mathrm{r},q} = \mathcal{CN}(0, \sigma_{\mathrm{r},q}^2 \mathbf{I}_{R_q}) \in \mathbb{C}^{R_q \times 1}$ at the relay and $\mathbf{n}_{\mathrm{n},k} = \mathcal{CN}(0, \sigma_{\mathrm{n},k}^2 \mathbf{I}_{N_k}) \in \mathbb{C}^{d \times N_k}$ at the nodes are i.i.d. complex Gaussian random variables.

The signal received at relay q in the MAC phase is given by

$$\mathbf{r}_q = \sum_{k \in \mathcal{G}(q)} \mathbf{H}_{k,q}^{\mathrm{nr,m}} \mathbf{V}_k \mathbf{d}_k + \mathbf{n}_{\mathrm{r},q}. \tag{5.3}$$

The relay retransmits this received signal to all connected nodes after performing linear signal processing. For simplicity of the notation, it is assumed that each of the relays has a maximum transmit power denoted by $P_{r,\max}$. The relay processing matrix of relay q in BC Phase p is given by $\mathbf{G}_q^p$. This relay processing matrix is normalized so that the transmit power constraint of relay q given by

$$\mathrm{E}\left\{ \left\| \beta_{q,p} \tilde{\mathbf{G}}_q^p \mathbf{r}_q \right\|_{\mathrm{F}}^2 \right\} \leq P_{r,\max} \tag{5.4}$$

with

$$\mathbf{G}_q^p = \beta_{q,p} \cdot \tilde{\mathbf{G}}_q^p \tag{5.5}$$

is fulfilled in each BC phase p, where $\tilde{\mathbf{G}}_q^p \in \mathbb{C}^{R_q \times R_q}$ denotes the unnormalized precoders and $\beta_{q,p}$ the normalization factor related to BC phase p.

The received signal at node k in BC phase p is given by

$$\begin{aligned}
\mathbf{y}_k^p \quad = \quad & \sum_{q\in\mathcal{R}(k)} \sum_{\substack{j\in\mathcal{G}_l,\\ j\neq k}} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{j,q}^{\mathrm{nr,m}} \mathbf{V}_j \mathbf{d}_j \\
& + \sum_{q\in\mathcal{R}(k)} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{k,q}^{\mathrm{nr,m}} \mathbf{V}_k \mathbf{d}_k \\
& + \sum_{q\in\mathcal{R}(k)} \sum_{i\in\mathcal{G}(q)\setminus\mathcal{G}_l} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{i,q}^{\mathrm{nr,m}} \mathbf{V}_i \mathbf{d}_i \\
& + \sum_{q\in\mathcal{R}(k)} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k}, \quad \forall k \in \mathcal{G}_l.
\end{aligned} \tag{5.6}$$

The first and second term in (5.6) are the useful and self-interference signal, respectively. The third term represents the interference from other groups and the last two terms represent the effective noise at node k. It is assumed that the self-interference can be perfectly canceled at each receiving node.

To achieve an IA solution, the useful and interference signals have to be in linearly independent subspaces at the receivers. Hence, the filters at the nodes and relays are designed in such a way that all interference signals are aligned within an $(N_k - d)$-dimensional interference subspace at receiving node k and the useful signals are within a d-dimensional useful signal subspace disjoint from the interference subspace during each of the b BC phases. At each receiving node, a two stage receive filter is considered. Let $\mathbf{U}_k^{\mathrm{H}} \in \mathbb{C}^{d\times N_k}$ denote the first stage receive filter at node k which nullifies the interference signals in each BC phase. Hence, before the first stage receive filter, the useful signals have to be in a subspace disjoint from the interference subspace. The output of the first stage receive filter after BC phase p at node k is given by

$$\begin{aligned}
\mathbf{s}_k^p \quad = \quad & \mathbf{U}_k^{\mathrm{H}} \sum_{q\in\mathcal{R}(k)} \sum_{\substack{j\in\mathcal{G}_l,\\ j\neq k}} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{j,q}^{\mathrm{nr,m}} \mathbf{V}_j \mathbf{d}_j \\
& + \mathbf{U}_k^{\mathrm{H}} \sum_{q\in\mathcal{R}(k)} \sum_{i\in\mathcal{G}(q)\setminus\mathcal{G}_l} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{i,q}^{\mathrm{nr,m}} \mathbf{V}_i \mathbf{d}_i \\
& + \mathbf{U}_k^{\mathrm{H}} \left(\sum_{q\in\mathcal{R}(k)} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k} \right), \quad \forall k \in \mathcal{G}_l.
\end{aligned} \tag{5.7}$$

The first stage receive filter nullifies the interference signal at the receivers by a projection of the received signal onto a subspace orthogonal to the aligned interference signals. This means that the first stage receive filter is a ZF filter. The useful signal located in the d-dimensional subspace at the receivers shall not be nullified by the first stage receive filter. Hence, the conditions for node $k \in \mathcal{G}_l$ to perform IA are given by

$$\mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{i,q}^{\mathrm{nr,m}} \mathbf{V}_i \quad = \quad \mathbf{0}, \quad \forall i \in \mathcal{G}(q) \setminus \mathcal{G}_l, \tag{5.8}$$

$$\mathrm{rank}\left(\mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{j,q}^{\mathrm{nr,m}} \mathbf{V}_j \right) \quad = \quad d, \quad \forall j \in \mathcal{G}_l,\ j \neq k. \tag{5.9}$$

In the d-dimensional useful subspace, there are $(S-1)d$ useful signals. These $(S-1)d$ useful signals cannot be spatially separated using a single BC phase. Hence, the relays have to transmit d linearly independent linear combinations of the $(S-1)d$ useful signals in the p BC phases in order to spatially separate the useful signals via joint possessing over all BC phase. Hence, $S-1$ BC phases are required in total to achieve IA at each receiver. The second stage receive filter $\mathbf{Q}_{k,q}^{\mathrm{H}}$ is applied to the output signal concatenated over $S-1$ phases of the first stage receive filter. Then, $\hat{\mathbf{d}}_k$ contains all $(S-1)d$ estimated symbols from the different nodes within the same group and is given by

$$\hat{\mathbf{d}}_k = \mathbf{Q}_{k,q}^{\mathrm{H}} \left[\mathbf{s}_k^{1\mathrm{T}} \quad \ldots \quad \mathbf{s}_k^{(S-1)\mathrm{T}} \right]^{\mathrm{T}}. \tag{5.10}$$

5.3 Achievable Sum Rate

In this section, the achievable sum rate of a partially connected multi-group multi-way relaying network is derived. The achievable sum rate of the entire network is the sum of data rates achieved at each receiving node. The data vector $\mathbf{d}_k$, $\forall k \in \mathcal{G}$ is a circular symmetric Gaussian random vector [Gal08; NM93]. As described in Section 3.3, the achievable data rate at a MIMO receiver is given by

$$R_{\mathrm{MIMO}} = \log_2 \left(|\mathbf{I} + \mathbf{SINR}| \right). \tag{5.11}$$

The useful signal at node k in BC phase p is given by

$$\boldsymbol{\chi}_{k,p}^{\mathrm{GP,U}} = \mathbf{U}_k^{\mathrm{H}} \sum_{q \in \mathcal{R}(k)} \sum_{\substack{j \in \mathcal{G}_l, \\ j \neq k}} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{j,q}^{\mathrm{nr,m}} \mathbf{V}_j \mathbf{d}_j, \tag{5.12}$$

and the useful signal covariance matrix is given by

$$\boldsymbol{\Xi}_{k,p}^{\mathrm{GP,U}} = \mathbb{E}\left[\left(\boldsymbol{\chi}_{k,p}^{\mathrm{GP,U}} \right) \left(\boldsymbol{\chi}_{k,p}^{\mathrm{GP,U}} \right)^{\mathrm{H}} \right]. \tag{5.13}$$

The interference signal at node k in BC phase p is given by

$$\boldsymbol{\chi}_{k,p}^{\mathrm{GP,I}} = \mathbf{U}_k^{\mathrm{H}} \sum_{q \in \mathcal{R}(k)} \sum_{i \in \mathcal{G}(q) \setminus \mathcal{G}_l} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{H}_{i,q}^{\mathrm{nr,m}} \mathbf{V}_i \mathbf{d}_i, \tag{5.14}$$

and the interference covariance matrix is given by

$$\boldsymbol{\Xi}_{k}^{\mathrm{GP,I}} = \mathbb{E}\left[\left(\boldsymbol{\chi}_{k,p}^{\mathrm{GP,I}} \right) \left(\boldsymbol{\chi}_{k,p}^{\mathrm{GP,I}} \right)^{\mathrm{H}} \right]. \tag{5.15}$$

The overall noise signal at node k in BC phase p is given by

$$\boldsymbol{\chi}_{k,p}^{\mathrm{GP,N}} = \mathbf{U}_k^{\mathrm{H}} \left(\sum_{q \in \mathcal{R}(k)} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^p \mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k} \right), \tag{5.16}$$

and the noise covariance matrix is given by

$$\boldsymbol{\Xi}_{k,b}^{\mathrm{GP,N}} = \mathbb{E}\left[\left(\boldsymbol{\chi}_{k,p}^{\mathrm{GP,N}}\right)\left(\boldsymbol{\chi}_{k,p}^{\mathrm{GP,N}}\right)^{\mathrm{H}}\right]. \quad (5.17)$$

The data rate at node k in phase p is, therefore, given by

$$R_{k,p}^{\mathrm{GP}} = \log_2\left(\left|\mathbf{I}_d + \frac{\boldsymbol{\Xi}_{k,p}^{\mathrm{GP,U}}}{\boldsymbol{\Xi}_{k,p}^{\mathrm{GP,I}} + \boldsymbol{\Xi}_{k,p}^{\mathrm{GP,N}}}\right|\right). \quad (5.18)$$

In amplify-and-forward multi-group multi-way relaying, each node $k \in \mathcal{G}_l$ has to ensure that its data stream can be decoded correctly by all $\mathcal{G}_l \setminus \{k\}$ intended nodes in group l. Hence, the maximum transmitting rate of each node in group l has to be smaller than or equal to the smallest achievable receiving rate of any node in group l.

The maximum achievable data rate of the data stream transmitted by node $k \in \mathcal{G}_l$ is, therefore, given by

$$R_k^{\mathrm{GP}} = (S-1)\min\left(R_{k,p}^{\mathrm{GP}}\right), \quad (5.19)$$

where the factor $S-1$ is needed because only $S-1$ nodes inside a group want to receive the signal transmitted by a single node inside a group.

Thus, the achievable sum rate in the entire multi-group multi-way relaying network is given by

$$R_{\mathrm{sum}}^{\mathrm{GP}} = \frac{1}{S}\sum_{l=1}^{L}\sum_{k\in\mathcal{G}_l} R_k^{\mathrm{GP}}. \quad (5.20)$$

The multiplication with the fraction $\frac{1}{S}$ is necessary because S time slots are required for one communication cycle, i.e., 1 MAC phase and $S-1$ BC phases.

5.4 Interference Alignment Algorithm

5.4.1 Introduction

In large partially connected multi-group multi-way relaying networks, it is challenging to handle groups of nodes that are connected to multiple relays. Groups of nodes which are connected to a single relay can perform group signal alignment (GSA) and group channel alignment (GCA) at this relay, as proposed for fully connected multi-group multi-way relaying networks in [GAL+14]. Groups of nodes which are connected to multiple relays shall be served by these multiple relays. Hence, the SSA and SCA techniques introduced in Section 3.5.3 are generalized for multi-group multi-way relaying networks. The pairwise communication, considered in Section 3.5.3, can be considered as a special case of multi-group multi-way relaying communication. In this special case, each group has only two members. The new technique is called simultaneous group signal alignment (SGSA) and simultaneous group channel alignment (SGCA). This new technique decouples the process of IA into linearly independent problems: SGSA, SGCA and transceive zero-forcing. The underlying system model has been introduced in Section 5.2. It is assumed that groups may be connected to multiple relays or a single relay. For simplicity, an intersection of only two subnetworks is considered in the following.

5.4.2 MAC phase: Simultaneous Group Signal Alignment

In this section, the MAC phase transmission strategy and SGSA, if a group of nodes is connected to the same multiple relays, is described. Since only one MAC phase is assumed, all $L_q S$ nodes connected to relay q have to transmit simultaneously. Hence, $L_q Sd$ data streams are transmitted to relay q in this single MAC phase. This means, that each node belonging to the set $\mathcal{G}(q)$ transmits d data streams to relay q. It is assumed that $L_q Sd$ data streams cannot be spatially separated in the R_q-dimensional relay space of relay q. However, the precoding filters of the nodes are designed in such a way that all Sd data stream from all node in group l are within an $(S-1)d$-dimensional subspace at relay q. Hence, $R_q = L_q(S-1)d$ antennas are required at relay q. It is to note, that in this $L_q(S-1)d$-dimensional relay space, $L_q Sd$ data streams cannot be spatially separated. After performing GSA or SGSA, there are $L_q(S-1)d$ effective data streams at relay q.

Let the columns of

$$\mathbf{\Delta}_{l,q} = \begin{bmatrix} \mathbf{A}_{a_l,q} \\ \mathbf{A}_{a_l+1,q} \\ \vdots \\ \mathbf{A}_{b_l,q} \end{bmatrix} \tag{5.21}$$

span the solution space corresponding to the first SGSA condition. The first SGSA condition is given by the following system of linear equations:

$$\underbrace{\begin{bmatrix} \mathbf{H}^{\text{nr,m}}_{a_l,q} & \mathbf{H}^{\text{nr,m}}_{a_l+1,q} & \cdots & \mathbf{H}^{\text{nr,m}}_{b_l,q} \end{bmatrix}}_{\mathbf{H}^{\text{eff,m}}_{l,q}} \cdot \begin{bmatrix} \mathbf{A}_{a_l,q} \\ \mathbf{A}_{a_l+1,q} \\ \vdots \\ \mathbf{A}_{b_l,q} \end{bmatrix} = \mathbf{0}. \tag{5.22}$$

The entire solution space $\mathbf{\Delta}_{l,q}$ corresponding to the first SGSA condition is determined by taking the null space of $\mathbf{H}^{\text{eff,m}}_{l,q}$, given by

$$\mathbf{\Delta}_{l,q} = \begin{bmatrix} \mathbf{A}_{a_l,q} \\ \mathbf{A}_{a_l+1,q} \\ \vdots \\ \mathbf{A}_{b_l,q} \end{bmatrix} = \text{null}\left(\mathbf{H}^{\text{eff,m}}_{l,q}\right). \tag{5.23}$$

For groups which are only connected to a single relay, this first condition is sufficient. Hence, the first SGSA condition is the same as the GSA condition in [GAL+14], which was developed for fully connected multi-group multi-way relaying networks.

If a group is besides relay q also connected to relay $\tilde{q}$, in addition, a second SGSA condition has to be fulfilled. This second SGSA condition is given by

$$\underbrace{\begin{bmatrix} \mathbf{H}^{\text{nr,m}}_{a_l,\tilde{q}} & \mathbf{H}^{\text{nr,m}}_{a_l+1,\tilde{q}} & \cdots & \mathbf{H}^{\text{nr,m}}_{b_l,\tilde{q}} \end{bmatrix}}_{\mathbf{H}^{\text{eff,m}}_{l,\tilde{q}}} \cdot \underbrace{\begin{bmatrix} \mathbf{A}_{a_l,\tilde{q}} \\ \mathbf{A}_{a_l+1,\tilde{q}} \\ \vdots \\ \mathbf{A}_{b_l,\tilde{q}} \end{bmatrix}}_{\mathbf{\Delta}_{l,\tilde{q}}} = \mathbf{0}, \tag{5.24}$$

where $\mathbf{\Delta}_{l,\tilde{q}}$ denotes the entire solution space corresponding to the second SGSA condition.

If the two solution spaces $\mathbf{\Delta}_{l,q}$ and $\mathbf{\Delta}_{l,\tilde{q}}$ have an intersection, i.e., $\mathbf{\Delta}_{l,q} \cap \mathbf{\Delta}_{l,\tilde{q}} \neq 0$, it is possible to achieve SGSA at the two relays q and $\tilde{q}$, i.e. GSA at the relays q and $\tilde{q}$ simultaneously. This common solution space is given by

$$\operatorname{span}\left(\begin{bmatrix} \mathbf{V}_{a_l} \\ \mathbf{V}_{a_l+1} \\ \vdots \\ \mathbf{V}_{b_l} \end{bmatrix}\right) \subseteq \operatorname{span}(\mathbf{\Delta}_l) = \operatorname{span}\left(\begin{bmatrix} \mathbf{A}_{a_l} \\ \mathbf{A}_{a_l+1} \\ \vdots \\ \mathbf{A}_{b_l} \end{bmatrix}\right) = \operatorname{null}\left(\begin{bmatrix} \mathbf{H}^{\text{eff,m}}_{l,q} \\ \mathbf{H}^{\text{eff,m}}_{l,\tilde{q}} \end{bmatrix}\right). \tag{5.25}$$

Hence, the transmit filters $\mathbf{V}_{a_l}$, $\mathbf{V}_{a_l+1}$ to $\mathbf{V}_{b_l}$ of the nodes in group l are in a subspace of the space spanned by the columns of $\mathbf{\Delta}_l$. In other words, the transmit filters $\mathbf{V}_{a_l}$, $\mathbf{V}_{a_l+1}$ to $\mathbf{V}_{b_l}$ are a subset of the intersection of the null spaces of $\mathbf{H}^{\text{eff,m}}_{l,q}$ and $\mathbf{H}^{\text{eff,m}}_{l,\tilde{q}}$. The transmit filters $\mathbf{V}_k$, $\forall k \in \mathcal{G}(q)$ are chosen out of the solution space $\mathbf{\Delta}_l$ and need to be of full rank d so that the d data streams transmitted by node k in $\mathcal{G}(q)$ span a d-dimensional subspace at relay q.

5.4.3 BC phase: Simultaneous Group Channel Alignment

In this section, the BC phase strategy and SGCA, if a group of nodes is connected to the same multiple relays, is described. In order to achieve IA in each of the BC phases, SGCA is performed at the receiving nodes and transceive zero forcing at the relays. All nodes design their first stage receive filter such that the effective channels of all S nodes in group l span a $(S-1)d$-dimensional subspace in the corresponding $L_q(S-1)d$-dimensional relay space of relay q. Therefore, the interference signals will be in an (N_k-d)-dimensional interference subspace orthogonal to the d-dimensional subspace spanned by the columns of $\mathbf{U}_k^{\text{H}}$. Like SSA and SCA introduced in Section 3.5.3, SGSA and SGCA are dual problems. Hence, the two SGCA conditions can be derived analog to SGSA introduced in

Section 5.4.2. The first SGCA condition is, therefore, given by

$$\underbrace{\begin{bmatrix}\mathbf{B}_{a_l,q} & \mathbf{B}_{a_l+1,q} & \cdots & \mathbf{B}_{b_l,q}\end{bmatrix}}_{\boldsymbol{\Delta}_{l,q}^{\mathrm{rn,b}}} \cdot \underbrace{\begin{bmatrix}\mathbf{H}_{a_l,q}^{\mathrm{rn,b}} \\ \mathbf{H}_{a_l+1,q}^{\mathrm{rn,b}} \\ \vdots \\ \mathbf{H}_{b_l,q}^{\mathrm{rn,b}}\end{bmatrix}}_{\mathbf{H}_{l,q}^{\mathrm{eff,b}}} = \mathbf{0}, \tag{5.26}$$

and the second SGCA condition is given by

$$\underbrace{\begin{bmatrix}\mathbf{B}_{a_l,\tilde{q}} & \mathbf{B}_{a_l+1,\tilde{q}} & \cdots & \mathbf{B}_{b_l,\tilde{q}}\end{bmatrix}}_{\boldsymbol{\Delta}_{l,\tilde{q}}^{\mathrm{rn,b}}} \cdot \underbrace{\begin{bmatrix}\mathbf{H}_{a_l,\tilde{q}}^{\mathrm{rn,b}} \\ \mathbf{H}_{a_l+1,\tilde{q}}^{\mathrm{rn,b}} \\ \vdots \\ \mathbf{H}_{b_l,\tilde{q}}^{\mathrm{rn,b}}\end{bmatrix}}_{\mathbf{H}_{l,\tilde{q}}^{\mathrm{eff,b}}} = \mathbf{0}. \tag{5.27}$$

Determining the solution space of SGCA is dual to determining the SGSA solution space. Hence, the entire SGCA solution space is given by

$$\mathrm{span}\left(\begin{bmatrix}\mathbf{U}_{a_l}^{\mathrm{H}}\mathbf{U}_{a_l+1}^{\mathrm{H}}\cdots\mathbf{U}_{b_l}^{\mathrm{H}}\end{bmatrix}\right) \subseteq \mathrm{span}\left(\boldsymbol{\Delta}_l^{\mathrm{rn,b}}\right) = \mathrm{null}\left(\begin{bmatrix}\mathbf{H}_{l,q}^{\mathrm{eff,b}} \\ \mathbf{H}_{l,\tilde{q}}^{\mathrm{eff,b}}\end{bmatrix}\right). \tag{5.28}$$

The receive filters $\mathbf{U}_{a_l}^{\mathrm{H}}$, $\mathbf{U}_{a_l+1}^{\mathrm{H}}$ to $\mathbf{U}_{b_l}^{\mathrm{H}}$ are chosen from the solution space $\boldsymbol{\Delta}_l^{\mathrm{rn,b}}$. Similar to SGSA, after performing GCA or SGCA, there are $L_q(S-1)d$ effective channels at which the relay has to perform transmit ZF in order to nullify the interference at the receivers.

5.4.4 Properness Conditions

In this section, the properness condition which has to be fulfilled to perform SGSA and SGCA is derived. The number of antennas at relay q is $R_q = L_q(K-1)$, as already derived in Section 5.4.2. The signal space of a node has to be large enough such that all nodes in a group can select a common subspace in the desired signal spaces at relay q, if the group is inside the set $\mathcal{G}(q)$, or at the relays q and $\tilde{q}$ if the group is inside the set $\mathcal{G}(q) \cap \mathcal{G}(\tilde{q})$.

If a group is inside the set $\mathcal{G}(q)$, the columns of matrix (5.23) span a $\left(\sum_{k=1}^{S} N_k - R_q\right)$-dimensional solution space. Hence, the first SGSA condition is fulfilled if and only if

$$\sum_{k=1}^{S} N_k \geq R_q + d, \quad \forall l \in \{1, \cdots, L\}. \tag{5.29}$$

If a group is inside the set $\mathcal{G}(q) \cap \mathcal{G}(\tilde{q})$, the columns of matrix (5.25) span a $\left(\sum_{k=1}^{S} N_k - \sum_{j\in\mathcal{R}(l)} R_j\right)$-dimensional solution space. Hence, the first and the second SGSA condition are simultaneously fulfilled if and only if

$$\sum_{k=1}^{S} N_k \geq R_q + R_{\tilde{q}} + d, \quad \forall l \in \{1, \cdots, L\}. \tag{5.30}$$

5.4.5 Transceive Zero Forcing

In this section, the design of the relay processing matrix is described. As derived in Section 5.4.2 and Section 5.4.3, there are $L_q(S-1)d$ effective data streams and $L_q(S-1)d$ effective channels at relay q after applying SGSA and SGCA. The relays perform transceive zero forcing in order to transmit these $L_q(S-1)d$ effective data streams to all nodes in the set $\mathcal{G}(q)$. The effective channels of the MAC phase and BC phases are given by

$$\mathbf{H}^{\mathrm{MAC}}_{\mathrm{eff},l,q} = \left[\mathbf{H}^{\mathrm{nr,m}}_{a_l,q}\mathbf{V}_{a_l} \;\; \cdots \;\; \mathbf{H}^{\mathrm{nr,m}}_{b_l,q}\mathbf{V}_{b_l}\right], \tag{5.31}$$

$$\mathbf{H}^{\mathrm{BC}}_{\mathrm{eff},l,q} = \left[\left(\mathbf{U}_{a_l}\mathbf{H}^{\mathrm{rn,b}}_{a_l,q}\right)^T \;\; \cdots \;\; \left(\mathbf{U}_{b_l}\mathbf{H}^{\mathrm{rn,b}}_{b_l,q}\right)^T\right]^T, \tag{5.32}$$

$\forall q;\ l \in \mathcal{G}(q)$. Let $\mathbf{G}^{\mathrm{rxH}}_q$ and $\mathbf{G}^{\mathrm{tx}}_q$ denote the receive and transmit zero forcing matrices of relay q, respectively. These matrices are given by

$$\mathbf{G}^{\mathrm{rxH}}_q = \left[\mathbf{H}^{\mathrm{MAC}}_{\mathrm{eff},1,q} \;\; \mathbf{H}^{\mathrm{MAC}}_{\mathrm{eff},2,q} \;\; \cdots \;\; \mathbf{H}^{\mathrm{MAC}}_{\mathrm{eff},L_q,q}\right]^{-1}, \tag{5.33}$$

$$\mathbf{G}^{\mathrm{tx}}_q = \left[(\mathbf{H}^{\mathrm{BC}}_{\mathrm{eff},1,q})^{\mathrm{T}} \;\; (\mathbf{H}^{\mathrm{BC}}_{\mathrm{eff},2,q})^{\mathrm{T}} \;\; \cdots \;\; (\mathbf{H}^{\mathrm{BC}}_{\mathrm{eff},L_q,q})^{\mathrm{T}}\right]^{(-1)\mathrm{T}}. \tag{5.34}$$

The matrices of the right-hand side of (5.33) and (5.34) are square matrices, which are non-singular with probability one and therefore invertible.

Let $\mathbf{P}_p$ denotes a block diagonal precoding matrix corresponding to BC phase p. The matrix $\mathbf{P}_p$ has to be chosen such that in the $(S-1)$ BC phases, $(S-1)d$ linearly independent linear combinations are received at the receivers. The whole relay processing matrix of relay q in BC phase p is, therefore, given by

$$\mathbf{G}^{p}_q = \beta_{q,p} \cdot \mathbf{G}^{\mathrm{rxH}}_q \mathbf{P}_p \mathbf{G}^{\mathrm{tx}}_q, \tag{5.35}$$

where $\beta_{q,p}$ is the normalization factor in (5.4) such that the relay transmit power constraint is fulfilled.

It is worth to mention that any block-diagonal matrix $\mathbf{P}_p$ arbitrarily chosen will almost surely be a valid solution.

5.4.6 Group Signal Separation

In this section, the second stage receive filter is designed. After applying the first stage receive filter, the interference will be zero and the useful signals will be in a d-dimensional subspace at each node. Let $\mathbf{H}_{k,j,q}^{\mathrm{p(eff)}} = \mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{k,q}^{\mathrm{rn,b}} \mathbf{G}_q^{p} \mathbf{H}_{j,q}^{\mathrm{nr,m}} \mathbf{V}_j$ denote the effective channel between nodes $j \in \mathcal{G}(q)$ and $k \in \mathcal{G}(q)$ of group l connected to relay q in BC phase p. Hence, the effective channel from all $(S-1)$ nodes in group l to node k is given by

$$\mathbf{H}_{k,q}^{\mathrm{(p)eff}} = \left[\mathbf{H}_{k,1,q}^{\mathrm{(p)eff}} \quad \cdots \quad \mathbf{H}_{k,i,q}^{\mathrm{(p)eff}} \quad \cdots \quad \mathbf{H}_{k,K,q}^{\mathrm{(p)eff}} \right]_{i \neq k}, \ \forall q \in \{1, \cdots, Q\}. \tag{5.36}$$

The effective channel from all the $S-1$ nodes to node k in group l over all $(S-1)$ BC phases is given by

$$\mathbf{H}_{k,q}^{\mathrm{eff}} = \left[\left(\mathbf{H}_{k,q}^{\mathrm{(1)eff}} \right)^{\mathrm{T}} \quad \left(\mathbf{H}_{k,q}^{\mathrm{(2)eff}} \right)^{\mathrm{T}} \quad \cdots \quad \left(\mathbf{H}_{k,q}^{\mathrm{(K\text{-}1)eff}} \right)^{\mathrm{T}} \right]^{\mathrm{T}}. \tag{5.37}$$

The second stage receive filter separates the useful signals received from the nodes within a group and is designed as a zero forcing filter in order to spatially separate the $(S-1)d$ data streams. Hence, the second stage receive filter is determined by taking the inverse of $\mathbf{H}_{k,q}^{\mathrm{eff}}$ and therefore given by

$$\mathbf{Q}_{k,q}^{\mathrm{H}} = \left(\mathbf{H}_{k,q}^{\mathrm{eff}} \right)^{-1}. \tag{5.38}$$

The matrix on the right-hand side of (5.38) is a square matrix of full rank and therefore invertible. Nodes which are connected to multiple relays have to apply all q second stage filters in order to separate the useful signals.

5.5 Performance Analysis

This section examines the performance of the IA algorithm in Section 5.4, which is designed for partially connected multi-group multi-way relaying networks. This algorithm is referred to as simultaneous group signal and channel alignment (SGSCA) in the following. The proposed SGSCA algorithm is compared with two different reference algorithms. To evaluate the performance of the algorithms, the sum rate and the DoF are considered. Since a DoF analysis is only valid for an asymptotically high SNR, the sum rate over a large SNR range is simulated to assess the performance. The sum rates achieved by these algorithms are obtained through numerical MATLAB simulation.

In the following the assumptions regarding the simulation are briefly described. The algorithms themselves are valid for the assumptions mentioned in the sections 2.3 and 5.4.

- It is assumed that the channel between each node and all relays connected to this node is an i.i.d frequency-flat Rayleigh fading MIMO channel [LS03]. Hence, the channel matrices are of full rank, almost surely.

- It is assumed that all groups of nodes are served by at least one relay
- Due to the considered statistical channel model, the channel amplitude may vary for different realizations. Hence, all simulation results are averaged over 10^4 independent channel realizations. For each channel realization, all filters are designed according to the considered algorithm and the corresponding sum rate is calculated. The average sum rate which is plotted in this section is therefore an average over all 10^4 independent channel realizations. 10^4 independent channel realizations are large enough to get a sufficiently small confidence interval for plotting the sum rate.
- For simplicity, it is assumed that the noise variance is equal at all nodes and all relays without loss of generality, i.e., $\sigma^2 = \sigma^2_{\mathrm{n},k} = \sigma^2_{\mathrm{r},q}$, $\forall k \in \mathcal{G}$, $\forall q \in \{1, \cdots, Q\}$.

In this paragraph, the two chosen reference algorithms are introduced and briefly described. Both reference algorithms are based on the IA algorithm proposed in [GAL+14], i.e., the nodes and the relays design the filters accordingly. The algorithm in [GAL+14] achieves an IA solution in a multi-group multi-way relaying network considering a single relay, i.e., all groups of nodes are connected to this single relay. Hence, the algorithm in [GAL+14] can only achieve an IA solution in a fully connected network. The first reference algorithm is termed treat as interference (TaI). In this TaI reference algorithm the group located inside the intersection area of two subnetworks is only served by one connected relay, the other relay cannot assist the communication and treats the signals from nodes of this group as interference. The second reference algorithm is termed group signal alignment utilizing orthogonal resources (GSAOR). In this GSAOR reference algorithm, the communication of the groups belonging to a subnetwork takes place one after the other, i.e., the groups inside subnetworks utilize orthogonal resources for communication. This will avoid interference between the subnetworks.

It should be noted that there exists no other algorithm in the literature than the proposed algorithm that takes into account a partially connected relay aided multi-group communication.

5.5.1 Degrees of Freedom Analysis

In this section, the DoF achieved by the proposed SGSCA IA algorithm is investigated and compared with the two chosen reference algorithms. The main difference between the proposed algorithm and the two reference algorithms is that the proposed SGSCA algorithm is designed in order to handle groups of nodes that are connected to multiple relays.

A multi-group multi-way relaying network with $L = 5$ groups, $S = 4$ nodes in each group and $Q = 2$ relays is considered for the DoF analysis and the investigation of the sum rate.

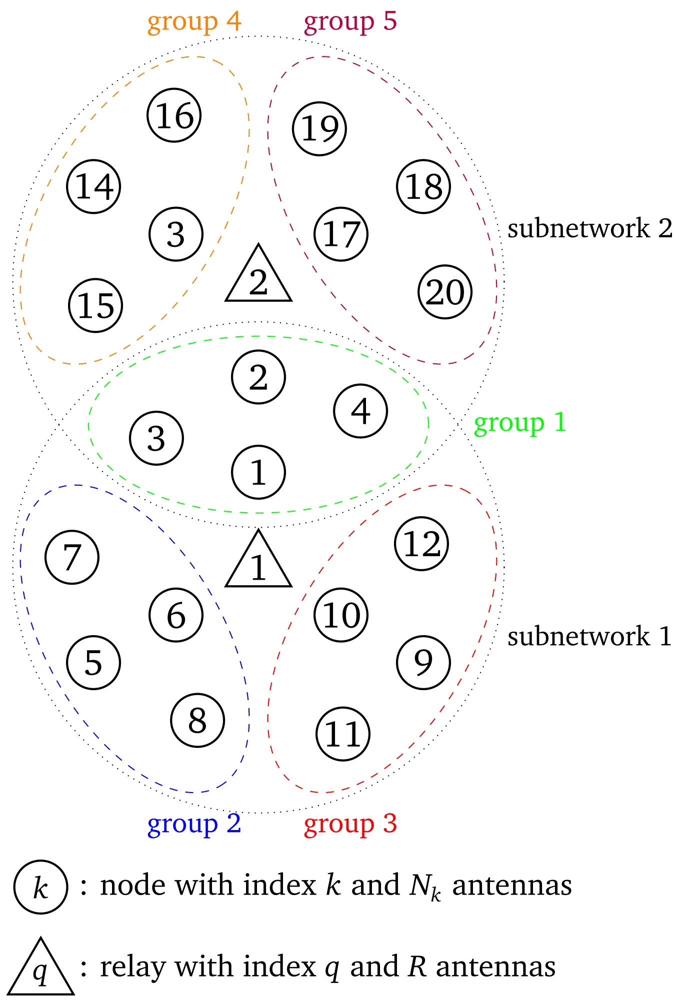


Figure 5.2. Partially connected multi-group multi-way relay network with $S = 4$ nodes in each of the $L = 5$ groups aided by $Q = 2$ relays.

A single group of the entire network is connected to both relays. The other groups are only connected to a single relay and equally distributed to them, see Figure 5.2.

The scenarios in Table 5.1 are considered for simulation. The difference between these two scenarios is the number of antennas at the nodes inside the group connected to both relays. The proposed SGSCA algorithm considers the C1 scenario and the reference algorithms GSAOR and TaI consider the C2 scenario. The properness condition (5.30) shows that the nodes of a group within the intersection area require more antennas at each node than the nodes of a group connected to only one relay if the nodes inside the intersection area are served by multiple relays. However, the two reference algorithms can also be applied to scenario C1. Since the two reference algorithms cannot use the additional antennas to increase the performance, the average performance will be the same.

The proposed SGSCA algorithm as well as the GSAOR reference algorithm achieve an in-

Table 5.1. Considered multi-group multi-way relaying scenarios

					Relays		Nodes					DoF		
Scenario	S	L	d	Q	R_1	R_2	N_1 - N_4	N_5 - N_8	N_9 - N_{12}	N_{13} - N_{16}	N_{17} - N_{20}	SGSCA	GSAOR	TaI
C1	4	5	1	2	9	9	[5 5 5 5]	[3 3 2 2]	[3 3 2 2]	[3 3 2 2]	[3 3 2 2]	15	7	6
C2	4	5	1	2	9	9	[3 3 2 2]	[3 3 2 2]	[3 3 2 2]	[3 3 2 2]	[3 3 2 2]	-	7	6

terference free communication. However, the GSAOR reference algorithm cannot serve the same number of nodes simultaneously as the proposed SGSCA algorithm. Therefore, the GSAOR reference algorithm is not able to achieve the same DoF. The GSAOR reference algorithm requires twice the number of transmission phases as the proposed SGSCA algorithm. The TaI reference algorithm requires the same number of transmission phases as the proposed algorithm, but cannot establish an interference-free communication. The relay that does not assist the communication suffers therefore from inter-subnetwork interference. Hence, the TaI reference algorithm is not able to achieve the same DoF as the proposed algorithm. An overview of the DoF achievable by the different algorithms is shown in Table 5.1 and will be verified in the sum rate analysis of this section.

5.5.2 Sum Rate Analysis

In this section, the sum rate performance of the proposed IA algorithm for partially connected multi-group multi-way relaying networks, presented in Section 5.4, is analyzed and compared with the reference algorithms. Let $P_{n,\max} = P$ denote the transmit power of each of the LS nodes in the entire network. Further, Let $P_{\mathrm{r},\max} = \frac{1}{Q}LSP$ denote the transmit power of each relay. In the following, the ratio P/σ^2 is termed SNR.

Since a DoF analysis is only valid for an asymptotically high SNR, a large range of SNR values of all three algorithms is simulated and shown in this section. This makes the assessment of the performance of the algorithms easier.

Figure 5.3 shows the sum rate performance as a function of P/σ^2 for the network shown in Figure 5.2. The solid line and the dashed lines represent the sum-rate achieved by the proposed SGSCA algorithm and the two reference algorithms, respectively. It can be observed from Figure 5.3 that the proposed algorithm outperforms both reference algorithm in terms of sum rate. The reason for this is on the one hand the interference-free communication of the proposed algorithm in comparison to the TaI reference algorithm and on the other hand the lower number of required time slots in comparison to the GSAOR reference algorithm. It can also be seen that the slope of the three curves in Figure 5.3 is different. The slope of a sum rate curve indicates the DoF. These slopes correspond to the DoF given in Table 5.1 and confirm, therefore, the stated values in Section 5.5.1.

Applying the algorithm proposed in [GAL+14] would require more antennas at the nodes in each group and considers only a single relay compared to the proposed algorithm for partially connected networks. Hence, a direct comparison with the algorithm proposed in [GAL+14] for fully connected networks is not appropriate. The other way around the proposed SGSCA algorithm can be applied to fully connected networks.

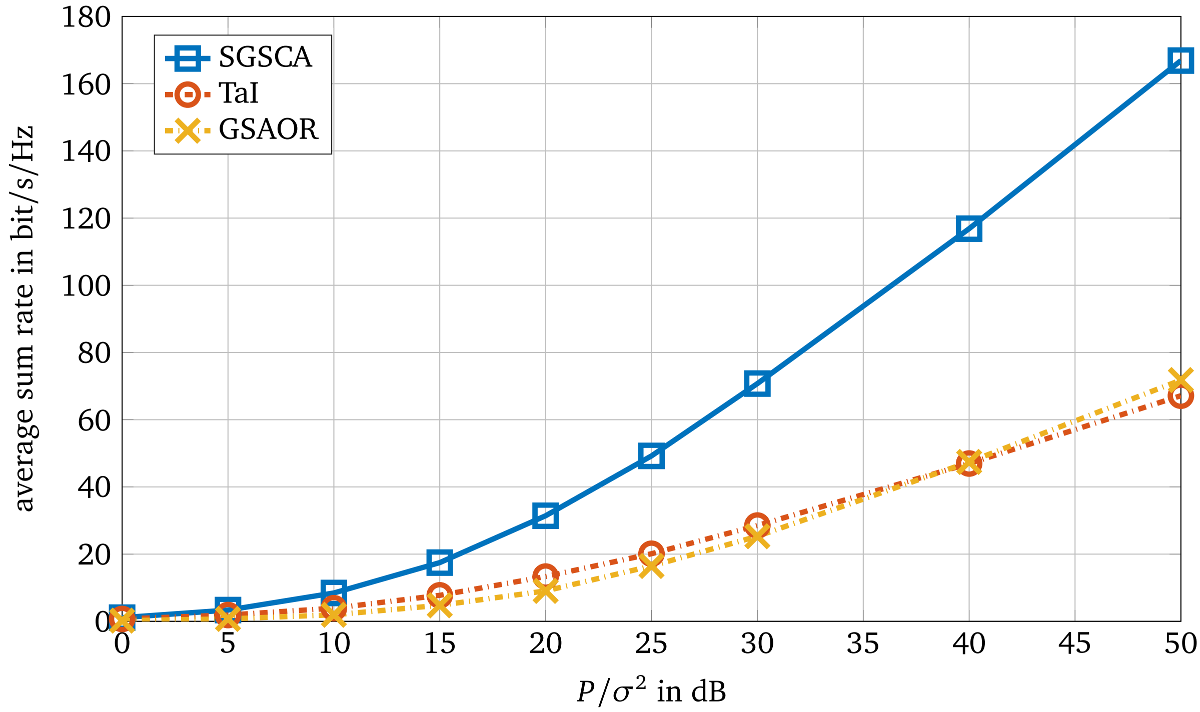


Figure 5.3. Sum rate performance of a partially connected multi-group multi-way relaying network with $L = 5$ groups, $K = 4$ nodes in each group and $Q = 2$ relays.

5.6 Summary

In this chapter, IA in a partially connected multi-group multi-way relaying network has been considered. At the beginning of this chapter, an appropriated system model is introduced. It has been shown that a large, partially connected multi-group multi-way relay network can be portioned into several subnetworks which are fully connected. The most challenging part of such a partially multi-group multi-way relaying connected network is the handling of the groups of nodes which are connected to multiple relays. A new technology called SGSA and SGCA was introduced to enable such groups to be served simultaneously via multiple relays. The properness conditions for SGSA and SGCA are derived by counting the number of free variables in a system of equations. Through simulation, it has been shown that the proposed IA algorithm outperforms the reference algorithms in terms of sum rate and DoF.

Chapter 6
Conclusion and Outlook

6.1 Conclusion

In this thesis, three important interference-limited relay aided wireless network topologies have been investigated, the partially connected relay aided multi-pair pair-wise communication network, the fully connected multi-group multi-way relaying network and the partially connected multi-group multi-way relay network. In the fully connected multi-group multi-way relay network topology, only a single relay has been considered in this thesis, i.e., the entire communication takes place via this intermediate relay. In the two partially connected network topologies, multiple relays have been considered in this thesis, i.e., the entire communication may take place via several relays. In contrast to the conventional use of relays for coverage extension, the relays in this thesis are used to help the process of IA. Partial connectivity plays an important role in large wireless networks where the assumption that all nodes are connected to all relays does not hold due to high path losses or shadowing. In such large networks, the distances between different nodes may differ a lot, leading to links of considerably different signal strengths, where sufficiently weak links may be neglected. Hence, large networks are in general partially connected.

In Chapter 1, the importance of interference management in wireless communication networks is discussed and IA is introduced as a promising technique to handle interference in wireless networks. Furthermore, an overview of the state of the art, related to the topics covered by this thesis, is presented. The challenges involved in performing IA in the three investigated network topologies are listed. Based on the discussed state of the art, open issues addressed in this thesis are formulated. Following this, the contributions of this thesis are summarized.

In Chapter 2, the considered network topologies are briefly described and the assumptions which are valid throughout the entire thesis are introduced. Furthermore, typical applications of the considered topologies are mentioned.

In Chapter 3, the focus is on large partially connected relay aided pair-wise communication networks. To investigate this network topology, an appropriate system model is introduced. The concept of an appropriate partitioning of a partially connected network into subnetworks which are fully connected has been introduced. Hence, a partially connected network consists of multiple subnetworks, where each of these subnetworks contains a single amplify-and-forward relay and all nodes being connected to this relay. Some nodes or even communication pairs may be connected to multiple relays. It is assumed that all nodes and relays in the entire network are half-duplex devices. The bidirectional pair-wise communication between the multi-antenna nodes takes place via the intermediate multi-antenna relays, using the two-way relaying protocol. The two-way relaying protocol enables a bidirectional communication in two time-slots. Only relays which are connected to both nodes of a communication pair can serve this pair, i.e., they can assist the communication of this pair. Hence, it is assumed that all communication pairs in the entire network are served by at least one relay. The most challenging part of such a partially connected network is the handling of nodes which are connected to multiple relays. Hence, a new technique called SSA and SCA has been introduced to perform SA and CA at multiple relays simultaneously. This new technique enables the decomposition of IA into three independent steps termed SSA, SCA and transceive zero forcing. The properness conditions for SSA and SCA are derived by counting the number of free variables in a system of equations. It has been shown that local CSI is sufficient to perform IA in partially connected networks, whereas in fully connected relay aided networks global CSI is required in general. A closed-form solution has been obtained, for the case that communication pairs may be connected to multiple relays. It is shown through simulation, that this proposed closed-form solution achieves more DoF than the reference algorithms and has better sum-rate performance, especially in the high SNR-region. Especially in large wireless networks, it may happen that not both nodes of a communication pair are connected to the same relays. If a single node of a communication pair is in addition connected to a relay which, therefore, cannot assist the communication, this node receives only interference and no useful signal from this relay. Such a node suffers from inter-subnetwork interference, due to the connection by an inter-subnetwork link to the additional relay. Hence, in this thesis, a new closed form algorithm which minimizes the inter-subnetwork interference power in the whole partially connected network has been proposed. Furthermore, the dependency of the performance on the number of antennas at the nodes is investigated. It has been shown that the proposed inter-subnetwork interference power minimization algorithm can achieve an interference free communication under certain conditions. The simulation results show that the proposed inter-subnetwork interference power minimization algorithm achieves a higher sum rate in comparison to the considered reference algorithm.

In Chapter 4, the focus is on fully connected multi-group multi-way relaying networks. To investigate this network topology, an appropriate system model is introduced. In such a network, multiple nodes form a group and each node wants to share its message with all other nodes in its group via an intermediate relay. It is assumed that all nodes and the relay are half-duplex devices. The group-wise communication between the multi-antenna nodes inside a group takes place via the intermediate multi-antenna relay, using a transmission strategy considering multiple MAC phases and multiple MC phases. In this thesis, a multicast IA algorithm for multi-group multi-way relaying networks has been proposed. In each of the multicast phases, a MIMO interference multicast channel is created by separating the antennas of the relay into clusters. Each of these clusters serves a specific group of nodes and transmits in such a way that the signals transmitted from different clusters are aligned at the nodes of the non-intended multicast groups. The advantage of this proposed algorithm is that the minimum required number of antennas at the relay is independent of the number of nodes per group, which is an important property since the number of antennas available at the relay is limited in general. The algorithm proposed in this thesis is flexible in the sense that it supports different numbers of antennas at the relay for each given system configuration, which allows to achieve different trade-offs between performance and required hardware resources. To multicast a specific data stream the relay needs estimates of all data streams transmitted by the nodes. Hence, an amplify-and-forward or a decode-and-forward relay can be utilized without changing the requirements on the number of antennas at the relay. Therefore, an amplify-and-forward as well as a decode-and-forward relay have been considered in this thesis. The properness condition for the proposed multicast IA is derived by counting the number of free variables in a system of equations. It has been shown that the proposed multicast IA algorithm outperforms the reference algorithm from literature for a broad range of SNR values. To achieve this, the proposed algorithm even requires fewer antennas at the relay than the reference algorithm.

In Chapter 5, the focus is on partially connected multi-group multi-way relay networks. In contrast to the multi-group multi-way relaying network investigated in Chapter 4 partial connectivity and multiple relays are considered. To investigate this network topology, an appropriate system model is introduced. Such a partially connected network consists of multiple subnetworks, where each of these contains a single amplify-and-forward relay and all groups connected to this relay. This means that not all groups of nodes are connected to all relays in the network. However, any group is connected to at least one relay which serves this group of nodes. In such a network, each node wants to share its data with all other nodes in its group, but not with nodes in other groups. It is assumed that all nodes in the entire network are half-duplex devices. In order to exchange data between the nodes inside a group groupwise, a transmission scheme with one MAC phase and multiple BC phases is considered. The most challenging part of such a partially connected network is the handling of the groups which are connected to multiple relays. Hence, a new technique called SGSA and SGCA has been introduced to perform SA and CA in partially connected multi-group multi-way relaying networks. The concept of SSA and SCA presented in this thesis has been generalized to cover groups with more than two members. The properness conditions for SGSA and SGCA are derived by counting the number of free variables in a system of equations. A closed-form solution has been obtained. It has been shown that the proposed IA algorithm outperforms the reference algorithm in terms of sum rate and DoF.

6.2 Outlook

In this thesis, three important interference-limited network topologies have been investigated, the partially connected relay aided pair-wise communication, the fully connected relay aided group communication and the partially connected relay aided group communication. However, there are additional interference-limited networks that have not been considered in this thesis, e.g., a cellular network or a mixture of different topologies, i.e., a heterogeneous network structure. In the following, some additional research topics related to the content of this thesis offering new challenges, are briefly discussed.

In this thesis, the perfect global CSI assumption for IA algorithms, in general, assumed in fully connected relay aided pair-wise communication networks, has already been relaxed in the case of a partially connected network. However, even for the IA algorithms proposed in this thesis, perfect CSI is assumed to be available inside the subnetworks. In realistic networks, the channels have to be estimated, which leads in general to no perfect CSI due to estimation errors caused by quantization inaccuracy or outdated CSI. Furthermore, parts of this thesis assume perfect self-interference cancellation, which is also based on perfect CSI. Hence, the investigation of the topologies considered in this thesis under the assumption of outdated CSI or imperfect CSI offers new research challenges. Furthermore, different pilot transmission-based CSI estimation schemes could be investigated and developed.

In this thesis, it is assumed that all available communication links are of similar strength. However, considering links of different strengths is more realistic and it is, therefore, important to investigate such networks under consideration of real-world scenarios.

For the partially connected pair-wise communication networks investigated in this thesis, only the minimum required number of antennas at the relays is considered. Additional antennas at the relays are not supported by the algorithms proposed for pair-wise communication. However, additional antennas at the relays could be utilized to reduce the number of antennas at the nodes, for a fixed number of communication pairs or data streams in the network. This would result in IA algorithms that can be applied more flexibly to given networks. Further, the optimization possibilities of the node transmit filters have not been investigated. If the nodes, in a pair-wise communication network, are equipped with more than the minimum required number of antennas to perform IA, the solution space will be larger than one. Hence, this is an obvious optimization possibility.

For the multi-group multi-way relaying scenario investigated in this thesis, it is assumed that the number of nodes in each group is equal. However, the case considering different numbers of nodes per group is still an open topic, in fully connected networks as well as in partially connected networks. It is not appropriate to assume that in realistic networks, the number of nodes in each group is equal. For instance, different numbers of nodes in each group could be served in a fully connected network by generalizing the multicast IA algorithm proposed in this thesis, utilizing antenna clusters of different sizes. The properness condition for this generalization has already been derived in this thesis. Future work could also extend the multicast algorithm proposed in this thesis for partially connected multi-group multi-way relaying networks. This would result in a more scalable and flexible algorithm for partially connected multi-group multi-way relaying networks. Furthermore, if not all nodes inside a group are transmitting simultaneously, this offers the possibility to utilize direct links between nodes.

The cellular network topology has not been considered in this thesis. However, this topology is an important network topology of our daily life and needs, therefore, to be investigated. In such a network, multiple base stations want to serve multiple terminals, in general. In [LAG+15] we considered a fully connected relay-aided cellular network and proposed an IA scheme. In [LAG+15] a two-hop transmission scheme that exploits the direct links between the base stations and mobile stations is applied. However, considering two-way relaying in cellular networks is still an open problem. In real-world cellular networks, the assumption that all terminals are connected to all base stations does not hold due to path losses or shadowing. Hence, especially the investigation of large partially connected cellular networks with multiple base stations, relays and terminals is a new research challenge. The algorithms proposed in this thesis may be generalized to a cellular network for this purpose. To perform relay-aided IA in such a cellular network, the properness and feasibility conditions have to be derived. Besides the generalization of the algorithms proposed in this thesis, it is also important to consider a different amount of available CSI at the base stations and mobile stations.

List of Acronyms

AWGN	Additive White Gaussian Noise
BC	Broadcast
CA	Channel Alignment
CFS	Closed Form Solution
CSI	Channel State Information
DoF	Degrees of Freedom
FDMA	Frequency Division Multiple Access
GCA	group channel alignment
GSA	group signal alignment
IA	Interference Alignment
IoT	Internet of Things
ISS	Interference Signal Subspace
M2M	Machine to Machine
MAC	Multiple Access
MC	Multicast
MIMO	Multiple Input Multiple Output
MMSE	Minimum Mean Square Error
MSE	Mean Squared Error
SA	Signal Alignment
SCA	Simultaneous Channel Alignment
SGCA	Simultaneous Group Channel Alignment
SGSA	Simultaneous Group Signal Alignment
SINR	Signal-to-Interference plus Noise Ratio
SNR	Signal to Noise Ratio
SSA	simultaneous signal alignment
SVD	Sigular-Value-Decomposition

TDMA Time Division Multiple Access

USS Useful Signal Subspace

ZF Zero Forcing

List of Symbols

$[l,k]$	k-th node in the l-th group
(j,k)	Communication pair consisting of node j and node k
$\mathbf{A}$	SSA solution space at for relay
$\mathbf{A}_q$	SA solution space at for relay q
$\mathbf{A}_{k,q}$	Solution space of node k connected to relay q
b	phase index
B	Number of MC phases
$\mathbf{B}_q$	CA solution space at for relay q
$\mathbf{B}_{k,q}$	Solution space of node k connected to relay q
$\mathbf{B}$	SCA solution space at for relay
β	Normalization factor
β_q	Normalization factor related to relay q
β_b^{b}	Normalization factor related to MC phase b
$\beta_{q,p}$	Normalization factor related to BC phase p and relay q
d	Number of transmitted data streams
$\mathbf{d}_k$	Data vector originating from node k
$\mathbf{d}_{l,k}$	Data vector originating from node $[l,k]$
$\hat{\mathbf{d}}_k$	Estimated data vector at node k
$\mathbf{d}_{l,k}$	Data vector originating from node $[l,k]$
$\hat{\mathbf{d}}_{l,k,b}^{\mathrm{DF}}$	Estimated data vector at node $[l,k]$ in MC phase b
$\hat{\mathbf{d}}_{l,k,b}$	Estimated data vector at node $[l,k]$ in MC phase b
$\mathbf{\Delta}_{l,q}$	Solution space at relay q
$\mathbf{\Delta}_l$	SGSA solution space
$\mathbf{\Delta}_{l,q}^{\mathrm{b}}$	Solution space at relay q for BC
$\mathbf{\Delta}_l^{\mathrm{b}}$	SGCA solution space
$\mathbf{G}_q$	Linear signal processing matrix of relay q
$\tilde{\mathbf{G}}_q$	Unnormalized processing matrix of relay q
$\mathbf{G}_q^{\mathrm{RX\,H}}$	Relay receive zero forcing matrix at relay q
$\mathbf{G}_q^{\mathrm{TX}}$	Relay transmit zero forcing matrix at relay q
$\mathbf{G}$	Overall relay processing matrix
$\tilde{\mathbf{G}}$	Unnormalized overall relay processing matrix
$\mathbf{G}_{l,k}$	Submatrix of $\mathbf{G}_m^{\mathrm{q}}$
$\mathbf{G}_m^{\mathrm{q}}$	Receive processing matrix of the relay in MAC phase m

$\mathbf{G}^{\mathrm{b}}_{j,b}$	Relay precoding matrix of antenna cluster j in phase b
$\tilde{\mathbf{G}}^{\mathrm{b}}_{j,b}$	Unnormalized relay precoding matrix of antenna cluster j in phase b
$\mathbf{G}^{p}_{q}$	Relay processing matrix of relay q in phase p
$\tilde{\mathbf{G}}^{p}_{q}$	Unnormalized Relay processing matrix of relay q in phase p
$\mathbf{G}^{\mathrm{rxH}}_{q}$	Receive zero forcing matrices of relay q
$\mathbf{G}^{\mathrm{tx}}_{q}$	Transmit zero forcing matrices of relay q
$\mathcal{G}_{l,m}$	Nodes selected for the transmission in group l in MAC phase m
$\mathcal{G}_{l}$	Set of node indices that are in group l
$\mathcal{G}$	Set of node indices in the entire network
$\mathcal{G}(q)$	Set of nodes which are connected to relay q
$\mathbf{H}^{\mathrm{sr}}_{k,q}$	MIMO channel matrix between node k and relay q in the MAC phase
$\mathbf{H}^{\mathrm{rd}}_{q,k}$	MIMO channel matrices between relay q and node k in the BC phase
$\mathbf{H}^{\mathrm{m}}_{l,k}$	MIMO channel matrix between node $[l,k]$ and the relay during the MAC phases
$\mathbf{H}^{\mathrm{b}}_{l,k}$	MIMO channel matrix between node $[l,k]$ and the relay during the MC phases
$\mathbf{H}^{\mathrm{b}}_{l,k,j}$	Channel submatrix of $\mathbf{H}^{\mathrm{b}}_{l,k} = \left[\mathbf{H}^{\mathrm{b}}_{l,k,1} \cdots \mathbf{H}^{\mathrm{b}}_{l,k,L}\right] \in \mathbb{C}^{N_{l,k} \times R}$
$\mathbf{H}^{\mathrm{nr,m}}_{k,q}$	MIMO channel matrix between node k and relay q in the MAC phase(s)
$\mathbf{H}^{\mathrm{rn,b}}_{k,q}$	MIMO channel matrix between relay q and node k in the BC phase(s)
$\mathbf{H}^{\mathrm{eff,m}}_{l,q}$	Effective channel of all nodes in group l connected to relay q
$\mathbf{H}^{\mathrm{eff,b}}_{l,q}$	Effective channel of relay q to all nodes in group l connected
$\mathbf{H}^{\mathrm{MAC}}_{\mathrm{eff},l,q}$	Effective channel of the MAC phase
$\mathbf{H}^{\mathrm{BC}}_{\mathrm{eff},l,q}$	Effective channel of the BC phases
$\mathbf{H}^{(\mathrm{p})\mathrm{eff}}_{k,q}$	Effective channel
$\mathbf{H}^{\mathrm{eff}}_{k,q}$	Effective channel
j	node index
k	node index
K	Number of communication pairs
$\mathcal{K}(q)$	Set of nodes which are connected to relay q
$\mathcal{K}$	Set of nodes in the whole network
$\mathcal{K}^{\wedge}(q)$	Set of nodes which are only connected to relay q
$\mathcal{K}^{\cap}(q1,q2)$	Set of nodes inside the intersection area between q_1 and q_2
$\mathcal{K}^{\mathrm{ser}}(q)$	Set of nodes served by relay q
L	Number of groups
l	Group index
L_q	Number of nodes connected to relay q

$\mathcal{L}$	Set of groups in the entire network
M	Number of MAC phases
m	MAC phase index
M_{v}	Number of variables
M_{e}	Number of equations
M_{c}	Number of constraints
$N_{l,k}$	Number of antennas at node k in group l
N_k	Number of antennas at node k
$\mathbf{n}_{\mathrm{q},m}$	Noise at the relay in MAC phase m
$\mathbf{n}_{l,k,b}$	Noise at node $[l,k]$ in MC phase b
$\mathbf{n}_{\mathrm{r},q}$	Noise vector at relay q
$\mathbf{n}_{\mathrm{n},k}$	Noise vector at node k
$\boldsymbol{\Xi}_k^{\mathrm{I}}$	Interference covariance matrix
$\boldsymbol{\Xi}_k^{\mathrm{N}}$	Noise covariance matrix at node k
$\boldsymbol{\Xi}_{jk}^{\mathrm{U}}$	Useful signal covariance matrix
$\mathbf{O}_j$	Antenna selection matrix
p	BC phase index
$P_{\mathrm{r,max}}^{\mathrm{block}}$	Maximum transmit power of the relay
$P_{\mathrm{n,max}}$	Maximum transmit power of the nodes
$P_{r,\mathrm{max}}$	Maximum transmit power of each relay
$\tilde{q}$	Relay index if a node is connected to two relays
Q	Number of subnetworks or number of relays in a network
q	Relay index
$\mathcal{Q}$	Set of relay indices of the entire network
$\mathbf{Q}_{k,q}^{\mathrm{H}}$	Second stage receive filter at node k connected to relay q
$\mathbf{r}$	Overall received signal at the relay
R	Number of relay antennas
$\mathcal{R}(k)$	Set of relays which are connected to node k
$\mathcal{R}^{\cap}(j,k)$	Set of relays which are connected to the communication pair (j,k)
R_{MIMO}	Achievable data rate at a MIMO receiver
R_{jk}	Achievable data rate at receiver k transmitted by node j
R_{sum}	Achievable sum rate
$R_{\mathrm{sum}}^{\mathrm{GP}}$	Achievable sum data rate of a partially connected multi-group multi-way relaying network
$\mathbf{R}_{\mathbf{d}_{l,k}}$	Covariance matrix of $\mathbf{d}_{l,k}$
R_l^{C}	Antenna cluster size

$R_{\text{sum}}^{\text{DF}}$	Achievable sum rate in the entire network, considering DF
$R_{\text{sym}}^{\text{DF}}$	Achievable sum rate in the entire network assuming symmetric transmission, considering DF
$R_{\text{sum}}^{\text{AF}}$	Achievable sum rate
$\mathbf{r}_m^{\text{q}}$	Signal received at the relay in MAC phase m
$\mathbf{r}_{l,k}$	Estimated data vector at the relay
$\mathbf{r}_{l,k}^{\text{DF}}$	Estimated data vector at the relay, considering DF
R_q	Number of relay antennas at relay q
$\mathbf{R}_{\mathbf{d}_k}$	Covariance matrix of $\mathbf{d}_k$
$\mathbf{r}_q$	Received signal at relay q
$\sigma_{\text{n},k}^2$	Variance of the noise at node k
$\sigma_{\text{r},q}^2$	Variance of the noise at relay q
$\sigma_{\text{q},m}^2$	Noise variance at the relay in MAC phase m
$\sigma_{l,k}^2$	Noise variance at node $[l,k]$ in MC phase b
S_l	Number of nodes in the l^{th} group
S	Number of nodes in each group
S_{MAC}	Number of nodes per group that can be active and transmitting data to the relay
$\mathbf{s}_k^p$	Output of the first stage receive filter after BC phase p at node k
$\mathbf{v}_k^{(l)}$	The l^{th} column of $\mathbf{V}_k$
$\mathbf{V}_{l,k}$	Precoding matrix of node $[l,k]$
$\mathbf{V}_k$	Precoding matrix of node k
$\mathbf{U}_{l,k,b}^{\text{H}}$	Receive zero-forcing filter at node $[l,k]$
$\mathbf{U}_k^{\text{H}}$	First stage receive filter at node k
$\mathbf{\Phi}_{\text{MAC}}$	Selection matrix
$\mathbf{\Phi}_{\text{BC}}$	Selection matrix
$\mathbf{\Phi}_{l,k,b}$	Orthonormal basis of the receive subspace at node k in group l in MC phase b
X_{v}	Number of free variables
X_{c}	Number of constrains
$\boldsymbol{\chi}_k^{\text{I1}}$	Intra-subnetwork interference at node k
$\boldsymbol{\chi}_k^{\text{I2}}$	Inter-subnetwork interference at node k
$\boldsymbol{\chi}_{jk}^{\text{U}}$	Useful signal at node k transmitted by node j
$\boldsymbol{\chi}_k^{\text{N}}$	Noise signal at node k
$\mathbf{y}_k$	Received signal at node k
$\mathbf{y}_k^p$	Received signal at node k in BC phase p

$k = \Pi(j)$	Communication partner index function
$\mathbb{C}$	Set of complex numbers
$\mathbb{R}$	Set of real numbers
$(.)^{\mathrm{T}}$	Transpose of a vector or a matrix
$(.)^{*}$	Complex conjugate of a scalar, vector or matrix
$(.)^{\mathrm{H}}$	Complex conjugate transpose of a vector or matrix
$(.)^{-1}$	Inverse of a square matrix
$\mathbb{E}[.]$	Expectation of the element inside the brackets
$\lvert\mathcal{K}\rvert$	Cardinality of $\mathcal{K}$
$\lvert\mathbf{A}\rvert$	Determinant of the matrix $\mathbf{A}$
$\mathbf{I}_N$	$N \times N$ identity matrix
tr(.)	Trace, i.e., sum of the main diagonal elements of the matrix inside the brackets
$\lVert\mathbf{A}\rVert_{\mathrm{F}}$	Frobenious norm of $\mathbf{A}$, given by $\lVert\mathbf{A}\rVert_{\mathrm{F}} = \sqrt{\mathrm{tr}\,(\mathbf{A}^{\mathrm{H}}\mathbf{A})}$
$\log_2$	Logarithm with base 2
$\nu_{\mathrm{min},d}(.)$	Delivers a matrix containing the eigenvectors corresponding to the d smallest eigenvalues of the matrix within the brackets, as its columns
null($\mathbf{A}$)	Null space of the matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$, given by $\{\mathbf{x} \in \mathbb{C}^m : \mathbf{A}\mathbf{x} = \mathbf{0}\}$
span($\mathbf{A}$)	Span of a matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$, given by span$(\mathbf{A}) = \{\mathbf{A}\mathbf{x} : \mathbf{x} \in \mathbb{C}^m\}$
$SVD(\mathbf{A})$	Singular-value decomposition of $\mathbf{A}$
$\chi_{\mathrm{min},x}(.)$	Matrix containing the eigenvectors corresponding to the x smallest eigenvalues of the matrix within the brackets as its columns

Bibliography

[AK10a] A. U. T. Amah and A. Klein, "Beamforming-Based Physical Layer Network Coding for Non-Regenerative Multi-Way Relaying", *EURASIP Journal on Wireless Communications and Networking*, 2010.

[AK10b] A. U. T. Amah and A. Klein, "Pair-aware transceive beamforming for non-regenerative multi-user two-way relaying", in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, 2010. DOI: `10.1109/icassp.2010.5494901`.

[AK11a] A. U. T. Amah and A. Klein, "Regenerative Multi-Group Multi-Way Relaying", *IEEE Transactions on Vehicular Technology*, vol. 60, no. 7, pp. 3017–3029, Sep. 2011.

[AK11b] A. Amah and A. Klein, "Non-regenerative multi-antenna multi-group multi-way relaying", *EURASIP Journal on Wireless Communications and Networking*, 2011.

[ANP13] M. Alam, R. H. Nielsen, and N. R. Prasad, "The evolution of M2M into IoT", in *Proc. 2013 First International Black Sea Conference on Communications and Networking (BlackSeaCom)*, Jul. 2013, pp. 112–115.

[AV09] V. Annapureddy and V. Veeravalli, "Gaussian Interference Networks: Sum Capacity in the Low-Interference Regime and New Outer Bounds on the Capacity Region", *IEEE Transactions on Information Theory*, vol. 55, no. 7, pp. 3032–3050, 2009.

[BCC+07] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communications*. Cambridge university press, 2007.

[BCT11] G. Bresler, D. Cartwright, and D. Tse, "Feasibility of interference alignment for the MIMO interference channel: The symmetric square case", *Computing Research Repository - CORR*, Apr. 2011.

[Beu14] A. Beutelspacher, *Lineare Algebra, Eine Einführung in die Wissenschaft der Vektoren, Abbildungen und Matrizen*. Springer Spektrum, 2014.

[BH02] J. C. Bezdek and R. J. Hathaway, *Some Notes on Alternating Optimization*. Springer, 2002.

[BKW+09] S. Berger, M. Kuhn, A. Wittneben, T. Unger, and A. Klein, "Recent advances in amplify-and-forward two-hop relaying", *IEEE Communications Magazine*, vol. 47, no. 7, pp. 50–56, Jul. 2009.

[Bro16] E. Brown. (2016). Who Needs the Internet of Things?, [Online]. Available: `https://www.linux.com/news/who-needs-internet-things`.

[BV04] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[Byr11] C. L. Byrne, "Alternating Minimization and Alternating Projection Algorithms: A Tutorial", *Department of Mathematical Sciences University of Massachusetts Lowell*, pp. 1–41, 2011.

[Cis17] Cisco, "Cisco Visual Netwiking Index: Global Mobile Data Traffic Forcast Update, 2016-2021", Cisco, 2017.

[CJ07] V. R. Cadambe and S. A. Jafar, "Interference Alignment and the Degrees of Freedom for the K-User Interference Channel", *CoRR*, 2007.

[CJ08a] V. Cadambe and S. Jafar, "Can feedback, cooperation, relays and full duplex operation increase the degrees of freedom of wireless networks?", in *Proc. IEEE International Symposium on Information Theory (ISIT08)*, 2008, pp. 1263–1267.

[CJ08b] ——, "Interference Alignment and Degrees of Freedom of the K-User Interference Channel", *IEEE Transactions on Information Theory*, pp. 3425–3441, Aug. 2008.

[CJ09] V. R. Cadambe and S. A. Jafar, "Interference Alignment and the Degrees of Freedom of Wireless X Networks", *IEEE Transactions on Information Theory*, pp. 3893–3908, Sep. 2009.

[DK12a] H. Degenhardt and A. Klein, "Filter design with optimized numbers of data streams for multi-pair two-way relaying under asymmetric rate requirements", in *Proc. International ITG Workshop on Smart Antennas (WSA)*, Mar. 2012, pp. 294–300.

[DK12b] ——, "Self-interference aware MIMO filter design for non-regenerative multi-pair two-way relaying", in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, Apr. 2012, pp. 272–276.

[DK13] ——, "A Network Coding Approach to Non-Regenerative Multi-Antenna Multi-Group Multi-Way Relaying", in *Proc. IEEE Vehicular Technology Conference (VTC Spring)*, Jun. 2013, pp. 1–5.

[Eri18] Ericsson, "Ericsson Mobility Report", Ericsson, 2018.

[GAK+13] R. S. Ganesan, H. Al-Shatri, A. Kuehne, T. Weber, and A. Klein, "Pair-Aware Interference Alignment in Multi-User Two-Way Relay Networks", *IEEE Transactions on Wireless Communications*, vol. 12, no. 8, pp. 3662–3671, Aug. 2013, ISSN: 1536-1276.

[GAL+14] R. Ganesan, H. Al-Shatri, X. Li, T. Weber, and A. Klein, "Multi-group multi-way relaying with reduced number of relay antennas", in *Proc. 2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2014, pp. 2714–2718.

[Gal08] R. G. Gallager, *Principles of Digital Communication*. Cambridge University Press, 2008.

[Gan15] R. Ganesan, "Relay-aided interference alignment for bi-directional communications in wireless networks", PhD thesis, TU-Darmstadt, Mar. 2015.

[GAWK13] R. Ganesan, H. Al-Shatri, T. Weber, and A. Klein, "Relay-aided interference alignment for bidirectional communications in multi-pair multi-relay networks", in *Proc. 2013 IEEE on Signal Processing Advances in Wireless Communications (SPAWC)*, 2013, pp. 66–70.

[GCJ08] K. Gomadam, V. Cadambe, and S. Jafar, "Approaching the Capacity of Wireless Networks through Distributed Interference Alignment", in *Proc. IEEE Global Telecommunications Conferenc (GLOBECOM)*, Dec. 2008, pp. 1–6.

[GCJ11] ——, "A Distributed Numerical Approach to Interference Alignment and Applications to Wireless Interference Networks", *IEEE Transactions on Information Theory*, pp. 3309–3322, 2011.

[GG11] M. Guillaud and D. Gesbert, "INTERFERENCE ALIGNMENT IN THE PARTIALLY CONNECTED K-USER MIMO INTERFERENCE CHANNEL", *in Proc. Euro. Signal Proc. Conf.*, pp. 1–5, 2011.

[GJ10] T. Gou and S. Jafar, "Degrees of Freedom of the K User M × N MIMO Interference Channel", *IEEE Transactions on Information Theory*, pp. 6040–6057, 2010.

[GJJV03] A. Goldsmith, S. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels", *IEEE Journal on Selected Areas in Communications*, pp. 684–702, Jun. 2003.

[GJW11] T. Gou, S. A. Jafar, and C. Wang, "On the Degrees of Freedom of Finite State Compound Wireless Networks", *IEEE Transactions on Information Theory*, pp. 3286–3308, Jun. 2011.

[GLA+13] R. S. Ganesan, H. Q. Le, H. Al-Shatri, T. Weber, and A. Klein, "Closed form solution and useful signal power maximization for interference alignment in multi-pair two-way relay networks", in *Proc. IEEE, in Wireless Communications and Networking Conference*, 2013, pp. 3414–3419.

[GMK10] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "Interference alignment for the K user MIMO interference channel", in *Proc. IEEE International Symposium on Information Theory*, Jun. 2010.

[GS05] A. B. Gershman and N. D. Sidiropoulos, *Space-Time Processing for MIMO communications*. Wiley, 2005.

[GWK11] R. Ganesan, T. Weber, and A. Klein, "Interference Alignment in Multi-User Two Way Relay Networks", in *Proc. 2011 IEEE Vehicular Technology Conference (VTC Spring11)*, May 2011.

[GWKA13] R. Ganesan, T. Weber, A. Klein, and H. Al-Shatri, "Iterative MMSE filter design for multi-pair two-way multi-relay networks", 2013.

[GYGP09] D. Gunduz, A. Yener, A. Goldsmith, and H. V. Poor, "The multi-way relay channel", in *IEEE International Symposium on Information Theory (ISIT)*, Jun. 2009, pp. 339–343.

[GYGP13] ——, "The Multiway Relay Channel", *IEEE Transactions on Information Theory*, pp. 51–63, Jan. 2013.

[HBG+13] Y. Hua, D. W. Bliss, S. Gazor, Y. Rong, and Y. Sung, "Guest Editorial: Theories and Methods for Advanced Wireless Relays — Issue II", *IEEE Journal on Selected Areas in Communications*, pp. 1361–1367, Aug. 2013.

[HJ13] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge, 2013.

[Jaf11] S. A. Jafar, *Interference Alignment — A New Look at Signal Dimensions in a Communication Network*. now, 2011.

[Jaf13] S. A. Jafar, "Topological Interference Management through Index Coding", *ArXiv e-prints*, pp. 1–79, Jan. 2013. [Online]. Available: `http://arxiv.org/abs/1301.3106`.

[Jaf14] S. Jafar, "Topological Interference Management Through Index Coding", *IEEE Transactions on Information Theory*, pp. 529–568, Jan. 2014.

[JF07] S. Jafar and M. Fakhereddin, "Degrees of Freedom for the MIMO Interference Channel", *IEEE Transactions on Information Theory*, vol. 53, no. 7, pp. 2637–2642, 2007.

[JMLZ18] X. Jing, L. Mo, H. Liu, and C. Zhang, "Linear Space-Time Interference Alignment for K -User MIMO Interference Channels", *IEEE Access*, vol. 6, pp. 3085–3095, 2018.

[JS06] S. A. Jafar and S. Shamai, "Degrees of Freedom Region for the MIMO X Channel", *CoRR*, 2006.

[JS08] S. Jafar and S. Shamai, "Degrees of Freedom Region of the MIMO X Channel", *IEEE Transactions on Information Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.

[JS10] J. Joung and A. Sayed, "Multiuser Two-Way Amplify-and-Forward Relay Processing and Power Control Methods for Beamforming Systems", *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1833–1846, Mar. 2010.

[JV10] S. Jafar and S. Vishwanath, "Generalized Degrees of Freedom of the Symmetric Gaussian K User Interference Channel", *IEEE Transactions on Information Theory*, vol. 56, no. 7, pp. 3297–3303, 2010.

[JZD17] X. Jiang, B. Zheng, and Y. Du, "Interference alignment in two-way relay networks via rank constraints rank minimization", *China Communications*, vol. 14, no. 7, pp. 1–9, Jul. 2017.

[Kay93] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall, 1993.

[KRWY11] L. Ke, A. Ramamoorthy, Z. Wang, and H. Yin, "Degrees of freedom region for an interference network with general message demands", in *IEEE International Symposium on Information Theory Proceedings (ISIT)*, 2011, pp. 36–40.

[LAG+14] X. Li, H. Al-Shatri, R. Ganesan, D. Papsdorf, A. Klein, and T. Weber, "Relay-aided interference alignment for multiple partially connected subnetworks", in *Proc. IEEE Eleventh International Symposium on Wireless Communication Systems (ISWCS14)*, Aug. 2014, pp. 121–125.

[LAG+15] X. Li, H. Al-Shatri, R. S. Ganesan, D. Papsdorf, A. Klein, and T. Weber, "Uplink-Downlink Duality of Interference Alignment in Cellular Relay Networks", in *Proc. 10th International ITG Conference on Systems, Communications and Coding (SCC 2015)*, Feb. 2015, pp. 1–6.

[LD18] W. Li and M. Dong, "Joint Relay Beamforming and Receiver Processing for Multi-Way Multi-Antenna Relay Networks", *IEEE Transactions on Communications*, vol. 66, no. 2, pp. 576–588, Feb. 2018.

[LDLG11] C. Leow, Z. Ding, K. Leung, and D. Goeckel, "On the Study of Analogue Network Coding for Multi-Pair, Bidirectional Relay Channels", *IEEE Transactions on Wireless Communications*, vol. 10, no. 2, pp. 670–681, Feb. 2011.

[LJ16] N. Lee and R. W. H. Jr, "Advanced interference management technique: potentials and limitations", *IEEE Wireless Communications*, vol. 23, no. 3, pp. 30–38, Jun. 2016.

[LLC10] N. Lee, J.-B. Lim, and J. Chun, "Degrees of Freedom of the MIMO Y Channel: Signal Space Alignment for Network Coding", *IEEE Transactions on Information Theory*, vol. 56, no. 7, pp. 3332–3342, Jul. 2010.

[LPKW15] X. Li, D. Papsdorf, A. Klein, and T. Weber, "An Interference Alignment Scheme using Partial CSI for Large Partially Connected Relay Networks", in *Proc. IEEE Twelfth International Symposium on Wireless Communication Systems (ISWCS15)*, Aug. 2015.

[LS03] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. Cambridge University Press, 2003.

[LT15] K. Liu and M. Tao, "Generalized Signal Alignment: On the Achievable DoF for Multi-User MIMO Two-Way Relay Channels", *IEEE Transactions on Information Theory*, vol. 61, no. 6, pp. 3365–3386, Jun. 2015.

[LTXL14] K. Liu, M. Tao, Z. Xiang, and X. Long, "Generalized Signal Alignment For MIMO Two-Way X Relay Channels", *ArXiv e-prints*, vol. 6, pp. 1–6, Feb. 2014. arXiv: `1402.1607 [cs.IT]`.

[MMK08] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, "Communication Over MIMO X Channels: Interference Alignment, Decomposition, and Performance Analysis", *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3457–3470, Aug. 2008.

[NAH11] B. Nosrat-Makouei, J. Andrews, and R. Heath, "MIMO Interference Alignment Over Correlated Channels With Imperfect CSI", *IEEE Transactions on Signal Processing*, vol. 59, no. 6, pp. 2783–2794, Jun. 2011.

[NLL10] H. Ning, C. Ling, and K. Leung, "Relay-aided interference alignment: Feasibility conditions and algorithm", in *IEEE International Symposium on Information Theory Proceedings (ISIT)*, 2010, pp. 390–394.

[NM93] F. D. Neeser and J. L. Massey, "Proper complex random processes with applications to information theory", *IEEE Transactions on Information Theory*, vol. 39, no. 4, pp. 1293–1302, Jul. 1993.

[NMK10] B. Nourani, S. Motahari, and A. Khandani, “Relay-aided Interference Alignment for the quasi-static interference channel”, in *Proc. IEEE International Symposium on Information Theory Proceedings (ISIT)*, 2010, pp. 405–409.

[NSSG09] F. Negro, S. P. Shenoy, D. T. M. Slock, and I. Ghauri, “Interference alignment limits for K-user frequency-flat MIMO”, in *Proc. 17th European Signal Processing Conference, Eusipco*, Glasgow, UNITED KINGDOM, Aug. 2009.

[PGL+14] D. Papsdorf, R. S. Ganesan, X. Li, T. Weber, and A. Klein, “Combining Interference Alignment and Two-Way Relaying in Partially Connected Networks with only Local CSI”, in *Proc. IEEE Eleventh International Symposium on Wireless Communication Systems (ISWCS14)*, Barcelona, Spain, Aug. 2014.

[PH09] S. Peters and R. Heath, “Interference alignment via alternating minimization”, in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Apr. 2009, pp. 2445–2448.

[PLWK15] D. Papsdorf, X. Li, T. Weber, and A. Klein, “Interference Alignment in Partially Connected Multi-User Two-Way Relay Networks”, in *Proc. IEEE 26th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC15)*, Hong Kong, China, Aug. 2015.

[PLWK16] ——, “Inter-Subnetwork Interference Minimization in Partially Connected Two-Way Relaying Networks”, in *Proc. IEEE 84th Vehicular Technology Conference (VTC-Fall)*, Sep. 2016, pp. 1–6.

[POJ+17] F. H. Panahi, T. Ohtsuki, W. Jiang, Y. Takatori, and T. Nakagawa, “Joint Interference Alignment and Power Allocation for Multi-User MIMO Interference Channels under Perfect and Imperfect CSI”, *IEEE Transactions on Green Communications and Networking*, 2017.

[Pro01] J. G. Proakis, *Digital Communications*. McGRAW-HILL International Edition, 2001.

[PSK17] D. Papsdorf, Y. C. B. Silva, and A. Klein, “Multicast interference alignment in a multi-group multi-way relaying network”, in *Proc. 2017 IEEE International Conference on Communications (ICC)*, May 2017, pp. 1–6.

[PVK19] D. Papsdorf, S. Velumani, and A. Klein, “MIMO Multi-Group Multi-Way Relaying: Interference Alignment in a Partially Connected Network”, in *Proc. IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2019, pp. 1–6.

[RLW13] L. Ruan, V. Lau, and M. Win, “The Feasibility Conditions for Interference Alignment in MIMO Networks”, *IEEE Transactions on Signal Processing*, vol. 61, no. 8, pp. 2066–2077, Apr. 2013.

[RW07] B. Rankov and A. Wittneben, “Spectral efficient protocols for half-duplex fading relay channels”, *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 379–389, Feb. 2007.

[SHV08] M. Shen, A. Host-Madsen, and J. Vidal, “An improved interference alignment scheme for frequency selective channels”, in *Proc. IEEE International Symposium on Information Theory*, Jul. 2008, pp. 559–563.

[SLTL10] H. Shen, B. Li, M. Tao, and Y. Luo, "The New Interference Alignment Scheme for the MIMO Interference Channel", in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, Apr. 2010, pp. 1–6.

[Str09] G. Strang, *Introduction to Linear Algebra*. Wellesley - Cambridge Press, 2009.

[SVJS08] S. Sridharan, S. Vishwanath, S. Jafar, and S. Shamai, "On the capacity of cognitive relay assisted Gaussian interference channel", in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Jul. 2008, pp. 549–553.

[TGR09] R. Tresch, M. Guillaud, and E. Riegler, "On the achievability of interference alignment in the K-user constant MIMO interference channel", in *Statistical Signal Processing, 2009. SSP '09. IEEE/SP 15th Workshop on*, 2009, pp. 277–280. DOI: 10.1109/SSP.2009.5278586.

[TV05] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, 2005.

[UK08] T. Unger and A. Klein, "Duplex Schemes in Multiple Antenna Two-Hop Relaying", *Hindawi Publishing Corporation*, 2008.

[WGJ11] C. Wang, T. Gou, and S. A. Jafar, "Subspace Alignment Chains and the Degrees of Freedom of the Three-User MIMO Interference Channel", *CoRR*, 2011.

[WY13] F. Wang and X. Yuan, "Zero-forcing multi-way relaying with sum rate maximisation", *TRANSACTIONS ON EMERGING TELECOMMUNICATIONS TECHNOLOGIES*, 2013.

[YGJK09] C. Yetis, T. Gou, S. Jafar, and A. Kayran, "Feasibility Conditions for Interference Alignment", in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, 30 2009-dec. 4 2009, pp. 1–6.

[YGJK10] ——, "On Feasibility of Interference Alignment in MIMO Interference Networks", *IEEE Transactions on Signal Processing*, vol. 58, no. 9, pp. 4771–4782, Sep. 2010.

[ZWQW18] S. Zeng, C. Wang, C. Qin, and W. Wang, "Interference Alignment Assisted by D2D Communication for the Downlink of MIMO Heterogeneous Networks", *IEEE Access*, vol. 6, pp. 24 757–24 766, 2018.

Own Publications

[LAG+14] X. Li, H. Al-Shatri, R. Ganesan, D. Papsdorf, A. Klein, and T. Weber, "Relay-aided interference alignment for multiple partially connected subnetworks", in *Proc. IEEE Eleventh International Symposium on Wireless Communication Systems (ISWCS14)*, Aug. 2014, pp. 121–125.

[LAG+15] X. Li, H. Al-Shatri, R. S. Ganesan, D. Papsdorf, A. Klein, and T. Weber, "Uplink-Downlink Duality of Interference Alignment in Cellular Relay Networks", in *Proc. 10th International ITG Conference on Systems, Communications and Coding (SCC 2015)*, Feb. 2015, pp. 1–6.

[LPKW15] X. Li, D. Papsdorf, A. Klein, and T. Weber, "An Interference Alignment Scheme using Partial CSI for Large Partially Connected Relay Networks", in *Proc. IEEE Twelfth International Symposium on Wireless Communication Systems (ISWCS15)*, Aug. 2015.

[PGL+14] D. Papsdorf, R. S. Ganesan, X. Li, T. Weber, and A. Klein, "Combining Interference Alignment and Two-Way Relaying in Partially Connected Networks with only Local CSI", in *Proc. IEEE Eleventh International Symposium on Wireless Communication Systems (ISWCS14)*, Barcelona, Spain, Aug. 2014.

[PLWK15] D. Papsdorf, X. Li, T. Weber, and A. Klein, "Interference Alignment in Partially Connected Multi-User Two-Way Relay Networks", in *Proc. IEEE 26th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC15)*, Hong Kong, China, Aug. 2015.

[PLWK16] ——, "Inter-Subnetwork Interference Minimization in Partially Connected Two-Way Relaying Networks", in *Proc. IEEE 84th Vehicular Technology Conference (VTC-Fall)*, Sep. 2016, pp. 1–6.

[PSK17] D. Papsdorf, Y. C. B. Silva, and A. Klein, "Multicast interference alignment in a multi-group multi-way relaying network", in *Proc. 2017 IEEE International Conference on Communications (ICC)*, May 2017, pp. 1–6.

[PVK19] D. Papsdorf, S. Velumani, and A. Klein, "MIMO Multi-Group Multi-Way Relaying: Interference Alignment in a Partially Connected Network", in *Proc. IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2019, pp. 1–6.

Supervised Student Theses

Name	Title of the thesis	Thesis type	Date
Schmidt, Florian	Investigation of the necessity to align weak intersubnetwork Interferences in partially connected networks	Bachelor thesis	05/2015
Sun, Yuanchao	Weighted Factor Based Iterative MMSE Filter-Design in Partially Connected Two-Way Relaying Networks	Master Thesis	07/2016
Luan, Jun	Investigation of Multi-Group Multi-Way Relaying Networks	Master Thesis	09/2016

Lebenslauf

Name:	Daniel Papsdorf
Anschrift:	Goldschmittstr. 11; 55270 Ober-Olm
Geburtsdatum:	14.12.1983
Geburtsort:	Wiesbaden
Familienstand:	Verheiratet

Schulausbildung

09/2003 – 07/2004	Fachoberschule BBS1 in Mainz

Studium

09/2004 – 02/2009	Studium der Allgemeinen Elektrotechnik an der Fachhochschule Bingen, Studienabschluß: Diplom(FH)
10/2010 – 06/2013	Studium der Elektrotechnik an der Technischen Universität Darmstadt, Studienabschluß: Master of Science

Berufstätigkeit

09/2000 – 07/2003	Ausbildung zum Prozessleitelektroniker bei der, Nestle AG, Mainz
07/2003 – 09/2003	Prozessleitelektroniker bei der Nestle AG, Mainz
03/2009 – 09/2010	Entwicklungsingenieur R&D bei EAE electronics, Mainz
10/2010 – 09/2011	Entwicklungsingenieur R&D (Studienbegleitend / Teilzeit) bei EAE electronics, Mainz
10/2011 – 05/2013	Entwicklungsingenieur R&D Drehmomentmesstechnik (Studienbegleitend) bei HBM Messtechnik, Darmstadt

06/2013 – 07/2013	Entwicklungsingenieur R&D Drehmomentmesstechnik bei HBM Messtechnik, Darmstadt
08/2013 – 07/2018	Wissenschaftlicher Mitarbeiter am Fachgebiet Kommunikationstechnik, Institut für Nachrichtentechnik, Technische Universität Darmstadt
seit 11/2018	Wissenschaftlicher Mitarbeiter am Institut für angewandte Ökologie, Hochschule Geisenheim University

Darmstadt, 02. Dezember 2019

Daniel Papsdorf

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