Joint Relaying and Spatial Sharing Multicast Scheduling for mmWave Networks

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Abstract—Millimeter-wave (mmWave) communication plays a vital role in disseminating large volumes of data in beyond-5G networks efficiently. Unfortunately, the directionality of mmWave communication significantly complicates efficient data dissemination, particularly in multicasting, which is gaining more and more importance in emerging applications (e.g., V2X, public safety, massive IoT). While multicasting for systems operating at lower frequencies (i.e., sub-6GHz) has been extensively studied, they are sub-optimal for mmWave systems as mmWave has significantly different propagation characteristics, i.e., using the directional transmission to compensate for the high path loss and thus promoting spectrum sharing. In this paper, we propose novel multicast scheduling algorithms by jointly exploiting relaying and spatial sharing gains while aiming to minimize the multicast completion time. We first characterize the problem with a comprehensive model and formulate it with an integer linear program (ILP). We further design a practical and scalable semi-distributed algorithm named mmDiMu, based on gradually maximizing the transmission throughput over time. Finally, we carry out validation through extensive simulations in different scales, and the results show that mmDiMu significantly outperforms conventional algorithms with around 95% reduction on multicast completion time.

Index Terms—Millimeter-wave (mmWave) networks, multicasting, relay, spatial sharing, scheduling.

I. INTRODUCTION

Millimeter-wave (mmWave) communication fulfills the demand for multi-gigabit-per-second (Gbps) throughput and low-latency communication even for extremely dense networks [1], which are usually not easy to sustain with traditional communications operating at sub-6GHz frequencies. Despite its benefits, mmWave communications suffer from very high attenuation, resulting in dramatic penetration loss. To compensate for this loss, directional transmissions are typically employed where the coverage of communication is constrained to a rather small area, e.g., to the line of sight in the extreme case. This limitation poses new challenges to guarantee efficient content dissemination for various delay-sensitive multicast applications (e.g., data broadcasting in vehicular networks to support autonomous driving, high-definition video broadcasting in a concert hall, and public-safety use cases).

Although multicast scheduling has been widely explored for networks operating at sub-6GHz frequencies, the specific benefits and challenges of mmWave multicast scheduling remain understudied [2]. In particular, multicast scheduler designs for sub-6GHz communications assume the availability of omnidirectional transmission, and thus a transmitter can schedule transmissions to any arbitrary subset of nodes within a specific range simultaneously. However, directional mmWave communications undermine this assumption and render these designs inapplicable.

One trivial design for mmWave multicast scheduling can simply employ multiple directional unicast and/or multicast transmissions to sequentially serve all multicast nodes. The behavior of such a scheduler is illustrated in Fig. 1a, where the source node (labeled as $(\overline{0})$) transmits sequentially in sectors 1 to 5 to serve multicast nodes (1) to (6), respectively. However, this design is extremely inefficient, and a straightforward improvement can be applied if we consider beam grouping based on adaptive beamforming [2]-[4]. This design is illustrated in Fig. 1b where nodes that are closer to the source nodes (i.e., (1), (2), and (3)) are served with a wider beam, while the father nodes (i.e., (4), and (5)) and the nodes that are not in proximity with the other nodes (i.e., 6) with narrower beams. Although the adaptive method provides higher flexibility in grouping the receiving nodes, it comes at the expense of more complex beamforming and costly antenna architecture.

As seen, the designs mentioned above rely only on singlehop transmissions, which can be problematic in many practical scenarios. More specifically, there might exist nodes that are not directly reachable by the source, or the transmission rates are low due to large distance (i.e., node (5) in sector 4) or the presence of blockages [5], [6]. In such cases, a relay-aided transmission is inevitable to ensure reachability and guarantee high-performance multicasting (in terms of throughput and delay). Enabling relay allows a node to be a transmitting node as soon as it receives the data from another node. As shown in Fig. 1c, upon receiving data from node (0) in the first time slot, nodes (3), and (4) act as the relay node for node (1)(2), and (5), respectively. With this flexibility, we can break down a low-rate multicast transmission into a combination of multiple high-rate unicast and/or multicast transmissions that can be scheduled separately. Interestingly, we can then leverage the limited coverage of directional transmissions in mmWave to increase spatial gain which is brought by reduced interference among concurrent (unicast/multicast) transmissions; in Fig. 1c, the transmission in links $(0 \rightarrow 6)$, $(3 \rightarrow 1)$ (2), and $(4 \rightarrow 5)$ can occur concurrently without mutually interfering each other.

We believe the optimal performance of mmWave systems should jointly exploit the *relaying* and *spatial sharing* aspects



(a) Sequential multicast with fixed (b) Sequential multicast with adaptive (c) Joint relay and spatial sharing with fixed beamwidth (S = 5). (c) Joint relay and spatial sharing with fixed beamwidth (S = 2).

Fig. 1: Different multicast mechanisms with different gray shades represent the transmission/multicast sessions in different time slots, and S indicates the total slots. The number at each sector's arc indicate the index of each sector.

of mmWave communications. *Thus far, the existing works have considered either of the aspects but never jointly.* This motivates us to design new mmWave multicast scheduling algorithms integrating both aspects. Unsurprisingly, the joint optimization is complicated, and the specific challenge resides in designing efficient communication group composition and spatial sharing scheduling. With both spatial and temporal factors involved, the relay nodes have to be determined gradually, and the source and the relay nodes have to select their target nodes carefully depending on how the communication will affect the total completion time.

To address these challenges, we provide a comprehensive model and an integer linear program (ILP) to characterize the problem, with the objective of minimizing the multicast completion time (i.e., the time required for all nodes to receive the intended data). The ILP aims to find the optimal scheduling policy that determines the transmitting nodes and their corresponding receivers at each time slot. Specifically, it jointly minimizes the duration of each time slot accounting for all *concurrent transmissions*¹ while selecting the optimal relay node. Exploiting spatial sharing in the relay-aided multicast transmission requires careful scheduling, both spatially and temporally, which is usually not of concern in the conventional multicast. Hence, the problem formulation for directional multicasting is significantly different and inherently more complicated than that of the conventional multicast scheduling in the literature. Ultimately, solving the ILP provides a tight lower bound for the multicast completion time in a mmWave network leveraging both relaying and spatial sharing gains.

To account for the deployment in real-world scenarios where equipment has constrained computational power and to ensure scalability, we further design a lightweight semidistributed algorithm, namely mmDiMu. The high-level idea is to exploit concurrency by allowing each transmitting node to autonomously decide and transmit to its target node(s). The set of target nodes for each transmitting node is determined based on the signal-to-noise ratio (SNR) of nodes and is updated after every transmission time slot.

The following summarizes the contributions of this paper:

 We identify the challenges and opportunities in mmWave multicast scheduling and provide an ILP formulation that finds the optimal scheduling policy by jointly leveraging relaying and spatial sharing gains.

- Due to the exponential complexity of the ILP-based solution (namely ILP), we propose a lightweight and scalable heuristic, namely mmDiMu.
- We validate the performance of the proposed algorithms through extensive simulations where ILP demonstrates a substantial gain in completion time as compared to all other algorithms. While there is a slight gap between mmDiMu and ILP solutions, we can observe a remarkable improvement over the existing algorithms, i.e., FHMOB in [7], and OMS in [8] for sub-6GHz and the adaptive beamwidth algorithm (i.e., Adapt) in [2] for mmWave.
- We also evaluate the impact of interference of mmDiMu in scenarios with different beamwidths and show that its impact is marginal even for extreme-density scenarios.
- We provide valuable insights into the design of mmWave multicast systems and design guidelines depending on the network's density and system configurations.

II. RELATED WORK

As a key technology for beyond-5G networks, mmWave has been considered for many emerging applications (e.g., autonomous driving, public safety, and mobile video streaming) that typically require the distribution of data in large volumes with low latency. Unfortunately, directional mmWave links suffer from limited coverage which complicates multicasting. Most existing works on mmWave mainly focus on unicast transmissions. That said, the challenges and benefits of mmWave multicast remain understudied. Here, we present the state of the art of multicast techniques for both sub-6GHz and mmWave networks, while differentiating them with our proposed approach.

A. Sub-6GHz multicasting

The most basic type of multicasting is broadcast, in which *all nodes are served simultaneously*. In this case, the transmit rate is limited by the node with the worst channel quality. Improving over this basic technique, many opportunistic multicast techniques are proposed in [9]–[11] and the references therein. These techniques exploit multiuser diversity by opportunistically transmitting to an arbitrary subset of the nodes with better instantaneous channel quality. As a result, they outperform the broadcast scheme and achieve higher throughput. However, this technique still suffers from poor performance when the network has nodes located at its edge. In the extreme case (i.e., when many nodes are located at the edge), it performs similarly to a broadcast scheme.

¹ In mmWave communication systems, the terminology of spatial sharing is also commonly referred to as concurrent transmission. In this paper, these terms are used interchangeably.

Overcoming the above issue, the research community has explored multicast beamforming which uses beamforming techniques that focuses the transmit power at the directions of interest by adjusting the antenna gains. As a result, it improves the SNR of the nodes in these directions. This technique is first used in [12], where the authors first use omnidirectional multicast to transmit to nodes with better channel quality and then use directional multicast to transmit sequentially to the remaining nodes. To further improve the system performance, a better method applies beamforming weights at the antenna leading to the maximization of the worst SNR at the expense of degrading the SNR of other nodes. Many research works demonstrate this technique yields a high system throughput [8], [13], [14] and minimizes completion time [7], [15], [16].

The aforementioned works mainly focus on scheduling the subset of nodes in a system to achieve the intended goal, where neither coverage nor blockage is an issue. Specifically, a source node can simultaneously transmit to any arbitrary subset or even all nodes if desired. Nevertheless, operating at high frequency, mmWave communications are prone to extremely high attenuation and penetration loss. The scheduling problem is further exacerbated with the use of directional beamforming, which makes it impossible to serve any arbitrary nodes in the system simultaneously. As a result, the multicasting techniques designed for sub-6GHz communication yield suboptimal performance for mmWave communications. To shed light on this aspect, we specifically benchmarked the performance of our proposed algorithms to two seminal multicast schedulers used in sub-6GHz systems (i.e., in [7], [8]) in Section V.

B. mmWave multicasting

An initial work addressing the need to redesign mmWave multicast scheduler is in [3], where the authors emphasize on the use of adaptive beamwidth to improve the grouping of the multicast nodes to achieve higher throughput. A similar work in [2] investigates the trade-off between transmission beamwidth and achievable SNR. These schedulers may require a high level of beamwidth adaptation and complex precoders to form arbitrary beams to provide coverage to the multicast nodes. Therefore, this design increases the complexity and the cost of the antenna design. In contrast, with a highly reduced complexity, the authors in [17] present a practical IEEE 802.11ad compliance approach where a codebook-based scheduler with one radio frequency (RF) chain is applied.

All the works mentioned above consider only single-hop multicasting in which the multicast transmission rate remains limited to the nodes located farthest from the source node without leveraging spatial sharing. Later, the benefits of relay and spatial sharing are *separately considered* in [18] and [19] to improve the multicast rate and spectral efficiency, respectively. In [18], the authors exploit relaying only to overcome non-line-of-sight paths, but not for performance optimization. In [19], the authors leverage spatial sharing in which they enable the simultaneous transmission of single-hop unicast and multicast sessions to increase network efficiency.



Fig. 2: Data dissemination via multicast scheduling for L = 8. The number on the edges indicates the number of required transmission slots.

To sum up, all the works mentioned above either consider *multi-hop relay* or *spatial sharing*, but not jointly. To the best of our knowledge, we are the first to jointly consider both to minimize the data delivery time (i.e., completion time) for mmWave multicast communications.

III. SYSTEM MODEL AND ASSUMPTIONS

We consider a mmWave network composed of N + 1randomly distributed nodes denoted by set $\mathcal{N} = \{0, 1, ..., N\}$, where node 0 represents the source and the other nodes n = 1, ..., N are interested in receiving data of size B from the source. We assume relaying is enabled in the network, where, once a node receives the data, it can multicast the data to other nodes. We consider a time-slotted system where the number of time slots is denoted by variable S, and the set of time slots is given by $\mathcal{S} = \{1, ..., S\}$. The length of each time slot can be unequal, but only one-hop transmission is allowed per time slot. To exploit spatial sharing, multiple concurrent transmissions can coexist at each time slot.

We call a node that has data to transmit to other node(s) a parent node (PN)², and $\mathcal{P}^s \subset \mathcal{N}$ denotes the set of PNs at time slot s. A node that will receive data at time slot s is called a child node (CN), where $\mathcal{C}_m^s \subset \mathcal{N}$ denotes the set of CNs of PN m. A node can serve as PN in multiple time slots, and the data must be completely delivered to all its CNs in each time slot. Therefore, $\mathcal{C}_m^{s_1} \neq \mathcal{C}_m^{s_2}$ for any $s_1, s_2 \in \mathcal{S}$ and $s_1 \neq s_2$. Each node in the network has a *fixed* transmit power and L equal-width orthogonal lobes numbered counterclockwise starting from 0°, denoted by $\mathcal{L} = \{1, ..., L\}$. For each node $m \in \mathcal{N}$, we denote by \mathcal{N}_m^l the set of nodes that are within the coverage of lobe $l \in \mathcal{L}$ of the node. For example, we have $\mathcal{N}_0^1 = \{1, 2\}$ and $\mathcal{N}_3^4 = \{5\}$ in Fig. 2. Note that, as the lobes are orthogonal, a node can activate more than one lobe simultaneously.

We adopt a path-loss model used in [20] (will be detailed in Section V-A), and the received rate is computed using the Shannon capacity model from [21]. We denote by $\gamma_{m,n}$ the SNR of the received signal at CN *n* from PN *m*. A node is a target node (TN) of a PN if it has the lowest SNR among all the CNs within the same lobe of the PN. Note that a PN has a TN for every lobe it steers its beam toward. We denote by \mathcal{G}_m^s the set of all TNs of PN *m* at time slot *s*. Given \mathcal{C}_m^s , the set \mathcal{G}_m^s can be formally defined as

$$\mathcal{G}_m^s = \{n \mid \gamma_{m,n} = \min\{\gamma_{m,u}\}, u \in \mathcal{N}_m^l \cap \mathcal{C}_m^s, \forall l \in \mathcal{L}\}.$$
 (1)

 $^{^2}$ A PN node is either node 0, or a node that has received the multicast data from another node at the previous time slot.

 $|\mathcal{G}_m^s| = 0$ indicates node m does not transmit at time slot sand $|\mathcal{G}_m^s| = L$ indicates node n steers its beam towards all directions, where $|\cdot|$ gives the cardinality of a set. Since a TN has the lowest SNR among all CNs within the same lobe, the maximum rate which determines the transmission time of a PN depends on the SNR of the set of its TNs $\mathcal{G}_m^s \subseteq \mathcal{C}_m^s$. Given the TN set (\mathcal{G}_m^s) of a PN m, the optimal transmitting rate for the PN is as presented in [7]. Our focus is on obtaining the optimal \mathcal{C}_m^s for each node m at each time slot s. As our transmit power is fixed, activating more lobes may simultaneously results in a lower transmission rate. Let $r^*(\mathcal{G}_m^s)$ be the optimal transmit rate. The time required for PN m to complete the data transmissions to all its TNs (including its CNs) at time slot s is given by

$$t_m(\mathcal{G}_m^s) = \frac{B}{r^*(\mathcal{G}_m^s)}.$$
(2)

At each time slot, multiple PN can transmit simultaneously, exploiting spatial sharing. As a result, the duration of a time slot is the longest transmission at the time slot, that is,

$$t^{s}(\mathcal{N}) = \max_{m \in \mathcal{N}} \{t_{m}(\mathcal{G}_{m}^{s})\}.$$
(3)

Our objective in this work is to minimize the total duration of all the time slots in S, namely multicast *completion time*, by jointly minimizing t^s and S, and to determine the set of PNs and their corresponding CNs in each time slot. The completion time T can be expressed by

$$T(\mathcal{N}) = \sum_{s \in \mathcal{S}} t^s(\mathcal{N}). \tag{4}$$

The following constraints should be considered. First, all nodes have to receive the data within *S* time slots, i.e.,

$$\bigcup_{m \in \mathcal{N}, s \in \mathcal{S}} \mathcal{C}_m^s = \mathcal{N} \setminus \{0\}.$$
 (5)

Then, a node can only transmit data to other nodes if it has already received the data, i.e.,

$$\forall s \ge 2, m \in \mathcal{P}^s \implies m \in \bigcup_{\substack{x \in \mathcal{P}^{s'} \\ 1 \le s' \le s - 1}} \mathcal{C}_x^{s'} \cup \{0\}.$$
(6)

IV. PROPOSED APPROACHES

In this section, we describe our solutions to the min-time mmWave multicast scheduling problem. We first provide an ILP formulation that gives a precomputed optimal schedule, and then we propose a more scalable online distributed algorithm.

A. Optimum Solution by ILP

We first define terms and variables using a toy example in Fig. 2. We define K as the number of elements in the power set of $\mathcal{N} \setminus \{0\}$, excluding the empty set, i.e., $K = 2^N - 1$. In Fig. 2, we have N = 5, K = 31, and L = 8.

• \mathbf{g}_m^s (target vector of PN m in time slot s): a binary vector $\mathbf{g}_m^s = [g_{m,1}^s \dots g_{m,N}^s]^{\mathsf{T}} \in \{0,1\}^N$ in which $(.)^{\mathsf{T}}$ is the transpose operator and $g_{m,n}^s = 1$ if node n is a TN of PN m in time slot s. For example, in Fig. 2, nodes (2) and (3) are the TNs of the source in the first time slot, and hence the target vector is $\mathbf{g}_0^1 = [01100]^{\mathsf{T}}$. There are K possible combinations for a target vector for each PN.

- U (target matrix): a binary matrix of size N × K. Each of the columns of U represents a possible choice for a target vector, where g^s_m is a column of U. In fact, U is independent of the nodes, and it shows the state-space of the target vector g^s_m, m ∈ N. Precisely, U = [u^T₁,..., u^T_K] where u_k is a 1 × N binary vector. We form U by filling u_k, 1 ≤ k ≤ K, via the reverse (rev) of the N-bit decimal-to-binary (dec2bin) conversion of the index k. For instance, u₆ = rev([dec2bin(6)]) = rev([00110]) = [01100] and in Fig. 2, based on the definition of TN in (7), we have g¹₀ = u^T₆.
- p^s_m (PN vector of PN m in time slot s): a binary vector p^s_m = [p^s_{m,1},..., p^s_{m,K}][†] ∈ {0,1}^K, ∀m ∈ N and ∑^K_{k=1} p^s_{m,k} ≤ 1. If node m is a PN at time slot s, then ∑^K_{k=1} p^s_{m,k} = 1, otherwise, ∑^K_{k=1} p^s_{m,k} = 0. Precisely, p^s_{m,k} = 1 if PN m chooses the k-th column of U as its target vector. Given p^s_m, the TNs of PN m is obtained by g^s_m = Up^s_m. (7)
- \mathbf{N}_m (observation matrix): $\mathbf{N}_m = [\mathbf{n}_m^1, ..., \mathbf{n}_m^L]$ is a binary matrix of size $N \times L$, defined for every $m \in \mathcal{N}$. For each node m, \mathbf{N}_m indicates with which lobe can node m cover the other nodes using a single-hop transmission. Precisely, $\mathbf{N}_m(n, l) = 1$ if node n is within lobe l of node m. For the example in Fig. 2, we have

$$\mathbf{N}_{0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$
 (8)

• \mathbf{C}_m (CNs matrix): a binary matrix of size $N \times K$ that shows if a PN m transmits to its TNs, which of the other nodes fall within the coverage area of the PN. While \mathbf{g}_m^s represents the set \mathcal{G}_m^s of TNs of PN *m*, defined in (7), \mathbf{C}_m corresponds to the set \mathcal{C}_m^s of CNs of PN m, which can also be served given the TNs in \mathbf{g}_m^s . Let \mathbf{g}_m^s be the target vector of PN m corresponding to the k-th column of U, then the elements of the k-th column of C_m , which are equal to 1, represent all the nodes that can be served by such a target vector. To clarify, let node $n \in \mathcal{N}_m^l$ be a target node of PN m given \mathbf{g}_m^s corresponding to the k-th column of U. Based on the definition, since node n as the TN of PN m is always in C_m^s , then, $C_m(n,k) = 1$. Further, we have $\mathbf{C}_m(u,k) = 1$ if $\gamma_{m,u} \ge \gamma_{m,n}, \forall u \in \mathcal{N}_m^l$. Based on item (ii), for the source node in Fig. 2, we have $\mathbf{C}_0(:,6) = [11100]^{\mathsf{T}}$ which corresponds to $\mathbf{u}_6^{\mathsf{T}}$. Given the PN vector \mathbf{p}_m^s , we denote all the CNs, covered by PN m, by a binary vector $\hat{\mathbf{c}}_m^s = [\hat{c}_{m,1}^s, \dots, \hat{c}_{m,N}^s]^{\mathsf{T}}$ where $\hat{c}_{m,n}^s = 1$ if node n is covered by PN m at time slot s. $\hat{\mathbf{c}}_m^s$ is thus obtained by

$$\hat{\mathbf{c}}_m^s = \mathbf{C}_m \mathbf{p}_m^s. \tag{9}$$

• \mathbf{t}_m (transmission duration): $\mathbf{t}_m = [t_{m,1}, ..., t_{m,K}] \in \mathbb{R}^K, m \in \mathcal{N}$, a real-valued vector. If a PN *m* chooses the *k*-th column of U as its target vector \mathbf{g}_m^s , then, $t_{m,k}$ shows the duration of transmission defined in (2).

Matrices $\mathbf{U}, \mathbf{N}_m, \mathbf{C}_m, \mathbf{t}_m$ can be calculated given the distribution of nodes in the network, while $\mathbf{p}_m^s, \forall m \in \mathcal{N}, s \in \mathcal{S}$

are to be found by the ILP. Using these terms, the ILP formulation is provided as follows.

$$\min_{p_{m,k}^s} \quad T(\mathcal{N}) = \sum_{s \in \mathcal{S}} \max_{m \in \mathcal{N}} \{ \mathbf{t}_m \mathbf{p}_m^s \}$$
(10a)

s.t.
$$\sum_{k=1}^{K} p_{m,k}^{s} = \begin{cases} 1 & m = 0, s = 1\\ 0 & m = [1, ..., N], s = 1\\ < 1 & s > 2 \end{cases}$$
(10b)

$$\sum_{k=1}^{K} p_{m,k}^{s} \leq \sum_{s'=1}^{s-1} \sum_{x \in \mathcal{N} \setminus \{m\}} \hat{c}_{x,m}^{s'}, \forall m \in \mathcal{N} \setminus \{0\}, s \ge 2 \quad (10c)$$

$$\begin{aligned} (\mathbf{g}_m^s)^{\mathsf{T}} \mathbf{n}_m^l &\leq 1, & \forall m \in \mathcal{N}, \forall s, \forall l & (10d) \\ p_{m,k}^s &\in \{0,1\}, & \forall m \in \mathcal{N}, \forall s, \forall k & (10e) \end{aligned}$$

As mentioned, $p_{m,k}^s \in \{0, 1\}$ in (10) is the decision variable, which determines the TNs of PN m as in (7). (10b) expresses that only the source node can transmit at s = 1 since all the other nodes has not receive the multicast data. In the following time slots, any of the nodes in \mathcal{N} could be a PN given that it has received the data in any previous time slots $1 \le s' \le s - 1$ the constraint in (10c) indicates this. Finally, (10d) guarantees that the number of TN in a lobe is at most one.

The problem is NP-hard [22]. Although NP-hard, its running time depends on the number of integer variables. Our proposed ILP has $N \times (2^N + 1)$ variables, and thus a complexity of $O(2^N)$ which exponentially increases with N. Clearly, the ILP-based solution has an exponential time complexity, and it can only be solved for very small problem instances (i.e., small N). For this reason, in the next section, we design a practical and lower complexity heuristic.

B. Semi-distributed Multicast Scheduling

Our semi-distributed multicast scheduling heuristic, namely mmDiMu, accounts for both relay and spatial sharing. By having each PN deciding, with minimal information, its CNs to transmit to, mmDiMu is scalable and semi-distributed in nature as opposed to the centralized ILP solution. The pseudocode of the algorithm is as shown in Algorithm 1. In what follows, we elaborate on the detail of the algorithm.

We use W to denote the set of waiting nodes that have not received the intended data. Initially, i.e., at the first time slot, node 0 is the only PN in set \mathcal{P}^1 , and we have $\mathcal{W} = \{1, ..., N\}$ We use D to denote the channel's SNR matrix, where D(m, n). represents the SNR between nodes m and n. At each of the following time slots $s \ge 2$, we select for each node $n \in W$ the PN m in \mathcal{P}^s with the maximum SNR $\mathbf{D}(m, n)$. In the case where a node has the same SNR for two or more PNs, it will randomly select one of the PNs. After this process, for each node $m \in \mathcal{P}^s$ we obtain its CN set \mathcal{C}_m^s at this time slot, and we apply the opportunistic multicast scheduling that maximizes the sum throughput to select the set of nodes from \mathcal{C}_m^s for PN m to transmit to. The intuition lies in maximizing the achievable rate for each transmission session to promote minimum session transmission time and thus resulting in minimum completion time. Once receiving the data, a node will be removed from the set W and added to the PN set \mathcal{P}^{s+1} The above process is repeated until all nodes receive the data. Algorithm 1 Pseudocode of mmDiMu algorithm

1: Input: N, D Initialize counters: time slot $s \leftarrow 1$ 2. 3: Initialize waiting node set: $\mathcal{W} \leftarrow \{1, ..., N\}$ Initialize PN sets: $\mathcal{P}^s \leftarrow \{0\}, \forall s$ 4: Initialize CN sets: $\mathcal{C}_m^s \leftarrow \emptyset, \forall m, \forall s$ 5: while $\mathcal{W} \neq \emptyset$ do 6: foreach $m \in \mathcal{W}$ do 7: Select PN: $m = \arg \min_{u \in \mathcal{P}} \mathbf{D}(u, n)$ Store CN: $\mathcal{C}_m^s \leftarrow \mathcal{C}_m^s \cup n$ 8: 9: foreach $m \in \mathcal{P}^s$ do 10: 11: Select CN set with max-throughput: C_m^{s*} is served with rate r_m^s Update waiting set: $\mathcal{W} \leftarrow \mathcal{W} \setminus \mathcal{C}_m^{s*}$ Update PN set: $\mathcal{P}^{s+1} \leftarrow \mathcal{P}^s \cup \mathcal{C}_m^{s*}$ 12: 13: Compute transmission time of PN $m: t_m(\mathcal{C}_m^{s*}) \leftarrow B/r_m^s$ 14: 15: $s \leftarrow s + 1$ Compute slot-time: $t^s = \max_{m \in \mathcal{P}^s} t_m(\mathcal{C}_m^{s*})$ 16: 17: end Output multicast completion time $T = \sum_{s}^{S} t^{s}$ 18:

TABLE	I:	Channel	parameters.

Parameter	Value		
Free space path loss (α)	32.4dB		
Carrier frequency (f_c)	73GHz		
System Bandwidth (W)	1GHz		
Transmit power	14.9dBm [23]		
Noise figure	4dB@PN, 7dB@CN		
Thermal noise	-174dBm/Hz		
Path loss exponent (β)	2.0		
Standard deviation (σ)	1.9dB		
Shannon capacity (ρ)	$\rho = W \times \min\{\log_2\left(1 + 10^{0.1(\text{SNR} - \delta)}\right), \rho_{\text{max}}\}$		
	maximum spectral efficiency $\rho_{\text{max}} = 4.6 \text{bps/Hz}$		
	loss factor $\delta = 1.6$ dB		
Frame size (B)	1Gbits		

In each time slot, the time for each transmission is recorded as $t_m(\mathcal{C}_m^{s*})$, where \mathcal{C}_m^{s*} is the optimal. The multicast completion time thus can be calculated as $\sum_{s \in S} \max_{m \in \mathcal{N}} \{t_m(\mathcal{C}_m^{s*})\}$.

V. PERFORMANCE EVALUATION

This section evaluates the performance comparisons between the baseline and our proposed multicast algorithms.

A. Simulation Setup

We consider a uniform and randomly distributed nodes within a $200m \times 200m$ area with the source node (i.e., PN 0) located at the center. We adopt the mmWave path-loss model in [20], which is written as,

 $PL[dB] = \alpha + 10\beta \log_{10}(d_{m,n}) + 20 \log_{10}(f_c) + \chi_{\sigma}$, (11) where $d_{m,n}$ is the distance between the PN *m* and CN *n*, f_c is the carrier frequency, and χ_{σ} represents the shadow fading with zero-mean Gaussian random variable and standard deviation σ in dB. The received rate is computed using the Shannon capacity model in [21]. Table I summarizes the parameter values used in the simulator.

B. Benchmarked Algorithms

This subsection highlights the different algorithms used in the performance comparison.

ILP. This is based on solving the ILP presented in Section IV-A. It selects the transmission at each time slot, which globally maximizes the spatial sharing gain while achieving minimum completion time T. Therefore, it provides the lower bound for T. We solve the ILP by employing Gurobi³ along with CVX⁴ in MATLAB environment.

mmDiMu. This is our semi-distributed algorithm that considers both relaying and spatial sharing. While suboptimal, mmDiMu scales well regardless of the network density. The detail of the algorithm is as presented in Section IV-B. Unlike ILP, mmDiMu uses a semi-distributed approach, in which each PN makes the transmission decision autonomously, with only minimal channel state information between itself and the nodes that have not received the multicast data.

OMS [8]. This algorithm is a sub-category of multicast with an adaptive beamwidth scheduling algorithm. It capitalizes on opportunistic gains and provides optimal performance in conventional networks. Essentially, OMS sorts the nodes according to their channel SNR and serves the subset of nodes that maximizes the instantaneous sum throughput.

FHOMB [7]. Finite horizon opportunistic multicast beamforming (FHOMB) is designed to minimize the completion time when sending a finite number of packets to multicast receivers. At each time slot, a subset of nodes is selected, such that the estimated completion time is minimized. The estimated completion time is obtained by maximizing the minimum rate using a multi-lobe beam; this beam multicasts (usually at a low broadcast rate) to the remaining receivers. Adapt [2]. This is a scalable heuristic which groups the multicast nodes in subgroups using a hierarchical structure to construct the multicast tree. An example scheduling is as depicted in Fig. 1b. Once the subgroups/beam are determined, the source node serves each multicast subgroup sequentially through the beams; the node with the lowest SNR within each beam thus limits the transmit rate at each beam.

C. Evaluation Settings

To evaluate the performance of each algorithm, we examine the impact of two main parameters: (1) the number of nodes N and (2) the beamwidth $w = 360^{\circ}/L$ at the transceivers. Due to the high complexity of ILP, i.e., $O(2^N)$, N is restricted to 10 in scenarios where ILP is involved for comparison. The rest of the algorithms are evaluated for up to N = 100. We evaluate the performance for transmitter beamwidth w = $\{15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}\}$. Note that the transmit beamwidth whas an impact on the transmission gain [24], which we account for in the computation of the receiving rate. Unless mentioned otherwise, at the receiver side, we assume that it uses a quasiomnidirectional mode for receiving.

To ensure fair performance comparison between the algorithms, all algorithms use the same simulation setting. The minimum beamwidth is determined by w in each simulation scenario in Section V-D, and the beamwidth resolution is thus multiple of w for all the algorithms except Adapt. Since Adapt operates based on adapting its beamwidth to the multicast group, it can freely adjust its beamwidth as long as the minimum beamwidth is w. For instance, when the simulation has a setting of $w = 45^{\circ}$, Adapt could have any beamwidths between 45° and 360° while the other algorithms could only have beamwidths that are a multiple of 45° .

TABLE II: An example of multicast scheduling for ILP and mmDiMu for the scenario in Fig. 2.

Algorithm	ILP		mmDiMu	
Algorium	Transmission link	Time	Transmission link	Time
time slot, $s = 1$	$(0 \rightarrow (3)$	1	$(0 \rightarrow (3)$	1
time slot, $s = 2$	$0 \rightarrow 12$	3	$0 \rightarrow 12$	3
	$3 \rightarrow 5$	2	$(3) \rightarrow (4)$	1
time slot, $s = 3$	$(3) \rightarrow (4)$	1	$3 \rightarrow 5$	2
Completion time, T	5sec		6sec	

We implemented all the algorithms in Matlab and conducted the comparisons using the above settings. For each data point, we average the data over 200 simulation runs and compute the corresponding 95% confidence interval.

D. Simulation Results

As defined in (4) in Section III, the completion time T is the time required for all network nodes to finally receive the multicast data (by summing up the duration t^s at all time slots). Specifically, it is represented by the time at which the last multicast node receives its data.

1) Impact of the number of nodes N: Here, we evaluate the impact of different N, $N = \{2, 4, 6, 8, 10\}$, on the completion time T by fixing the transceivers beamwidth $w = 45^{\circ}$.

As a general trend, Fig. 3 shows that increasing the number of nodes N also increases the completion time T. When Nis large, the number of multicast slots required to transmit to all the nodes increases as well. ILP performs best as it picks the best policy that results in minimum T, as formulated in (4). It indeed only requires 23.54%, 3.79%, and 30.33% of the multicast completion time required by OMS, FHOMB, and Adapt, respectively, for N = 10. Specifically, ILP achieves a reduction in completion time by up to 96.21% as compared to the other algorithms. Our proposed algorithm mmDiMu also demonstrates a high gain in completion time. It achieves completion time reduction of up to 66.78%, 94.65%, and 62.77% over OMS, FHOMB, and Adapt, respectively.

Interestingly, while OMS performs well in conventional single-hop systems, it performs slightly worse than Adapt as N increases. As N increase, so as the SNR diversity of the nodes. In such a case, OMS will first opportunistically transmit to the node that has higher SNR. This behavior results in excluding the nodes with low SNR initially. As a result, it suffers from a low transmitting rate at a later time; it still has to serve the remaining nodes that have lower SNR. Unlike OMS, Adapt groups the nodes based on angular and then divides the group to minimize the transmission time and form a binary tree structure. Therefore, it refrains from the suboptimality that comes from greedily scheduling the nodes with better SNR. On the other hand, OMS performs better than Adapt for smaller N because the probability of having nodes at the edge is much smaller. Furthermore, OMS may use more than one (disjoint) beam to serve all the nodes, while this option is unavailable to Adapt. Therefore, sparse distribution of nodes - this mostly occurs when the node density is low (i.e., small N) – harms the performance of Adapt.

Similarly, FHOMB in [7] that performs well in single-hop multicasting performs poorly here. In FHOMB, a node receives

³ http://www.gurobi.com/ ⁴ http://cvxr.com/



Fig. 3: Completion time T for different N and $w = 45^{\circ}$.



Fig. 4: Total number of transmissions exploiting relay and spatial sharing for ILP and mmDiMu with $N = \{2, 4, 6, 8, 10\}$.

the complete frame over multiple fixed-length time slots. At each slot, the policy (i.e., the subset of nodes to transmit to) which gives the lowest estimated completion time (up to the time all nodes received the frame), is chosen. As mentioned, to determine the estimated completion time, the remaining nodes are served with broadcast. In mmWave networks, broadcasting in all direction results in a very low transmission rate, and thus the estimated completion time is significantly longer than a slot time. Here, a lower estimated time is favored since it provides a lower total transmission time. In most cases, this comes at the expense of a long slot duration t^s . As seen in Fig. 3, this results in high completion time.

As expected, mmDiMu performs worse than ILP because it autonomously schedules its transmission, disregarding the decision made by other PNs in the system. Let's consider the scenario in Fig. 2 and the corresponding schedule in Table II. The completion time of ILP is 1s lower than that of mmDiMu. Since mmDiMu sorts the nodes according to their SNR, the parent for (4) and (5) is (3), and (4) is served first. This results in $t^{s=3} = 2s$. However, ILP is aware that scheduling node (5) first results in optimal completion time. As N increases, the occurrence of this event increases as well. This reflects in the higher gain for ILP for larger N.

Remark: The low complexity mmDiMu only requires 29.15% additional completion time, in the worst-case N = 10, as compared to ILP. Nevertheless, this additional time is significantly lower than that required by other algorithms.

2) The importance of joint relaying and spatial sharing: The substantial gain in the completion time demonstrated by our proposed algorithms (i.e., ILP and mmDiMu) emphasizes the importance of leveraging the relaying and spatial sharing gains jointly in mmWave multicast networks. To shed light on this aspect, Fig. 4 and Fig. 5 depict the number and ratio,



Fig. 5: The fraction of relay and concurrent transmissions for ILP and mmDiMu with $N = \{2, 4, 6, 8, 10\}$.

respectively, of the relay and concurrent transmissions for ILP and mmDiMu. A transmission is a relay transmission if the transmitter is not the source node. A transmission pair is defined as a concurrent transmission if there is more than one transmission within the same time slot. For instance, in Table II, the number of relay transmission is 2 (i.e., $(3) \rightarrow (4)$ and $(3) \rightarrow (5)$), and the number of concurrent transmissions is 2 (i.e., $(0) \rightarrow (1)$ (2) and $(3) \rightarrow (5)$) for ILP.

In Fig. 4, the total number of relay and concurrent transmissions increases consistently with N. This increase is due to a higher communication diversity. We observe the total number of relay (in Fig. 4a) and concurrent (in Fig. 4b) transmissions of ILP is consistently higher than that of mmDiMu. This indeed contributes to ILP outperforming mmDiMu. Firstly, ILP has a precise view of the entire network and knows the optimal policy; it first transmits to the nodes that can transmit with a high rate to another node later while maximizing spatial sharing gain. Unlike ILP, at each slot, each PN in mmDiMu opportunistically transmits to the CN set that maximizes the instantaneous sum throughput; the set of selected CNs is usually those that are located nearer to the PN. As a result, the CN set may not necessarily be the optimal set to relay the data to the remaining nodes at a later time. Secondly, each CN in mmDiMu only selects one PN. That said, a CN does not choose a secondary PN even if it potentially allows concurrent transmissions. As a result, this reduces the number of relay and concurrent transmissions of mmDiMu, and thus resulting in a higher completion time (as shown in Fig. 3).

Further, we observe a high ratio of the relay (up to 70%) and concurrent (up to 80%) transmissions over the corresponding total number of transmission for both ILP and mmDiMu. Precisely, a high number of concurrent transmissions (in Fig. 4b) does not directly translate into a high number of the ratio (in Fig. 4b), but it highly depends on the total number of transmissions. This ratio confirms a large fraction of the performance gain roots from the exploitation of relaying and spatial sharing.

Remark: The gain achieved by ILP and mmDiMu mainly comes from the extensive exploitation of relaying and spatial sharing. This confirms the importance of leveraging these gains for mmWave multicast networks.

3) Impact of beamwidth w: This section evaluates the impact of $w = \{15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}\}$ on the completion time *T*, while fixing the number of nodes N = 8.



Fig. 6: Completion time T for different w, N = 8.



Fig. 7: Total number of relay and concurrent transmissions for ILP and mmDiMu with $w = \{15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}\}$.

Fig. 6 shows a general trend where increasing beamwidth w increases the completion time T. A larger w results in lower transmitting and receiving gains, which in turn results in a low data rate and a longer transmission time, and thus a high completion time T. This especially makes an impact on the algorithms that do not leverage relay. In particular, CNs located far away from the source node have to be served with very low transmission rates. Therefore, we observed an abrupt increase in the completion time of OMS, FHOMB and Adapt; the transmission time increases by 90.55s, 206.20s, and 72.74s, respectively, as w increases from 15° to 90° . By manipulating relay, these CNs are reachable through a closer relay PN, resulting in a higher transmission rate. Therefore, the increase in transmission time for ILP and mmDiMu is lesser, i.e., only 15.26s and 22.66s, respectively, as w increases from 15° to 90° . Although the increase seems insignificant, it is still non-negligible. A wider w improves the coverage of a PN to cover more CNs. As a result, the number of relay and concurrent transmissions reduces, and the completion time increases. This is evident from the decreasing number of these transmissions as w increases, as depicted in Fig. 7.

Remark: Although a wider beamwidth increases the completion time, ILP and mmDiMu are less impacted by it, as compared to the other algorithms.

4) Scalability: All previous results in this section only consider a maximum N of 10. This is due to the complexity and scalability issue of ILP. Nevertheless, it remains important as it provides insights into the algorithm performance difference to the optimal ones. Here, we demonstrate that even with a large N, our proposed mmDiMu algorithm achieves remarkable gains as oppose to OMS and Adapt. We omit FHOMB as it performs poorly even for cases



Fig. 9: Total number and ratio for relay and concurrent transmissions for mmDiMu with $N = \{20, 40, 60, 80, 100\}$.

with smaller N. We set $N = \{20, 40, 60, 80, 100\}$ and $w = 45^{\circ}$ for the following results.

Fig. 8 depicts a significant increase in the completion time T for OMS, but not for Adapt, as N increases. As N is large and continuously increases, the value of the lowest SNR does not change much, so does the transmit rate at each beam of the Adapt algorithm. However, OMS greatly suffers from its opportunistic decisions. mmDiMu experiences less increment in completion time than OMS. Increasing N increases the number of relay nodes and the opportunity of spatial sharing. This is evident from Fig. 9, where the number and ratio of relay and concurrent transmissions increase with N.

Remark: mmDiMu scales very well with the network density and achieves a significant reduction in the completion time as compared to OMS and Adapt.

5) Impact of interference: In theory, mmWave links mimic a pencil beam, and thus interference is negligible. However, the current off-the-shelf mmWave devices have a wider beam. In addition, the limitation in antenna design renders this assumption valid only in theory. Here, we evaluate the impact of transmit beamwidth w = $\{15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}\}$ for N up to 10 and 100 for ILP and mmDiMu, respectively, on the probability of mutual interference between concurrently transmitting pairs. We evaluate for interference characteristic for two types of antenna receiving modes: quasi-omnidirectional and directional, In the first case, the receiver suffers from interference as long as it is within the beam's coverage of the transmitter. This type of receiving mode is as employed by default in the existing off-the-shelf devices (i.e., TP-Link Talon AD7200 multi-band wifi router [25]). In a very recent work on improving beam alignment in the mmWave device [26], the authors are able to adaptively adjust the existing codebook available in the IEEE



Fig. 10: Omni-directional antenna receivers.



Fig. 11: Directional antenna receivers.

Fig. 12: Percentage of interference between concurrent transmission of ILP and mmDiMu for up to N = 10.

802.11ad devices and optimize the beam pattern to obtain a higher directionality beam. This shows the feasibility of implementing such receiving mode, and thus it is important to also evaluate for interference when the receiver has directional receiving mode. In this case, to cause interference, not only that the receiver has to be within the beam coverage of the interfering transmitter, but the transmitter must also be within the coverage area of the receiver's beam. Fig. 10 and Fig. 11 depict the percentage of mutual interference between the concurrent links as beamwidth w increases for the respective type of receiver. Note that, due to the complexity of OMS, the interference percentage is only shown for up to N = 10.

Quasi-omnidirectional receiver: Fig. 10 shows a general trend in which the percentage of interference increases with beamwidth w and the number of nodes N. As w increases, the coverage area increases, and thus increasing the probability of interfering with the nearby nodes. As N increases, so does the density of the network. That said, the probability that one or more receiving node falling within the coverage area of a transmitter is higher, and thus the percentage of interference. Although increasing beamwidth causes higher interference, it only leads to a maximum percentage of interference of up to 6% (see Fig. 10) in the largest N scenario; there is no interference in most scenarios. Since the source of interference is due to the frequency of concurrent transmissions, mmDiMu experiences a slightly higher interference than ILP. mmDiMu indeed has a higher ratio of concurrent transmission as compared to ILP (refer Fig. 5b). As shown in the example



Fig. 13: Interference analysis for N = 10 and $w = 60^{\circ}$. The difference color represents the transmission at different transmission slots.

scenario in Fig. 13, ILP and mmDiMu has 7 and 10 total transmissions, respectively. Out of those, ILP and mmDiMu has 5 and 9 links, respectively, involved in the concurrent transmission, which results in a ratio of 71.43% and 90%, respectively. While ILP has no interference among the communication links, transmission from node (1) of mmDiMu (see Fig. 13b) causes interference to node (7) when $(4) \rightarrow (7)$ and $(0) \rightarrow (1)$ occur simultaneously. Nevertheless, even with omnidirectional receiving mode, the percentage of interference in both algorithms is kept below 6%.

Directional receiver: When the receiver uses directional receiving mode, the percentage of interference becomes smaller (see Fig. 11). This is due to the reason that interference only occurs when the transmitter is within the beam of the receiver's beam, and the receiver's beamwidth w limites the beam coverage. For instance, while simultaneous transmission of $(0 \rightarrow 1)$ and $(4 \rightarrow 7)$ causes interference in the quasi-omnidirectional receiver's case, here, nodes (1) and (7) use directional reception, and thus avoiding interference from nodes (4) and (0), respectively; the interfering nodes (4) and (0) are not within the directional receiving beam of nodes (1) and (7), respectively. Therefore, we observed drops in the percentage of interference by up to 2.4% (i.e., when N = 10, $w = 90^{\circ}$ for mmDiMu). The general performance trend is as seen in Fig. 10 for the same reasons explained above.

Scenario with N up to 100: In Fig. 14, we show the interference's percentage of mmDiMu for up to N = 100 in order to provide some insight onto implementation setup for higher-density scenarios. As seen, using directional instead of omni-directional receiving mode clearly provides a much lower percentage of interference; the reduction is up to 25.01% for N = 100 and $w = 90^{\circ}$. If the receiver's location is known, and the accuracy of beam alignment is high, using narrow beamwidth such as 15° only has a percentage of interference of up to 0.13% in the worst case; many practical research work on mmWave use horn antenna with $w = 7^{\circ}$ [27].

Remark: Even for beamwidth as wide as $w = 45^{\circ}$, mmDiMu manages to keep the interference's percentage below 5% for N = 100. We foresee future mmWave devices with highly directional and adjustable beam, in which, given any scheduling decision, the interference between the concurrently transmitting pairs in dense networks can be further minimized.



Fig. 14: Percentage of interference of mmDiMu for omnidirectional (omni) and directional (dir) receiver.

VI. CONCLUSION

In this paper, we investigate the challenge of multicasting in mmWave networks. We jointly leverage relay transmission to improve the reachability and link rate and spatial gain by enabling simultaneous unicast and/or multicast communications. We formulate the problem with an ILP and provide a semi-distributed solution called mmDiMu. The ILP solution generates optimal scheduling decisions while suffering from poor scalability. mmDiMu performs closely to the optimal and can scale to large networks with very dense settings due to its semi-distributed nature. We show through extensive simulation that our proposed ILP and semi-distributed mmDiMu solutions provide significant gain over the multicast scheduling methods designed for sub-6GHz networks, in which we achieve up to 96.21% reduction in completion time. In comparison with the adaptive beamwidth algorithm (namely Adapt) proposed for mmWave multicasting, we gain up to 78.22% in completion time. Furthermore, although we exclude interference reduction from the objective function, we achieve an impressively low (5%) interference in high-density scenarios even with wide beam.

There are still interesting open problems, such as studying the impact of user mobility, blockage, the tradeoff between efficiency, and complexity in asynchronous scheduling. We leave these for future work.

VII. ACKNOWLEDGEMENTS

This work has been performed in the context of the DFG Collaborative Research Center (CRC) 1053 MAKI - projects A3, B3, C1, C7, and T1, the Center emergenCITY of the LOEWE initiative (Hessen, Germany), and the DFG Joint Sino-German Research Grant No. 392046569 (NSFC: 61761136014).

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