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Multi-Group Multi-Way Relaying: When Analog Network Coding Finds Its Transceive Beamforming

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***Abstract*—In this paper, we introduce non-regenerative multi-group multi-way relaying. A half-duplex non-regenerative multi-antenna relay station (RS) assists multiple communication groups. In each group, there are multiple nodes that want to communicate with each other. Each node has a message and wants to decode the messages only from other nodes in its group. The required number of communication phases for each group is equal to the number of nodes in the group. In the first phase, all nodes transmit simultaneously to the RS and in the following phases the RS applies transceive beamforming and transmits to all nodes. We derive the achievable sum rate of non-regenerative multi-group multi-way relaying for asymmetric and symmetric traffic and for two different cases, namely, with and without analog network coding (ANC). We specially design an ANC-aware transceive beamforming for non-regenerative multi-group multi-way relaying. The ANC-aware transceive beamforming is designed using two criteria, namely, matched filter (MF) and semidefinite relaxation (SDR) of a maximisation of minimum signal to noise ratio problem. From sum rate analysis, it is shown that applying ANC at the RS improves the performance. Regarding the ANC-aware transceive beamforming, the ANC-SDR outperforms ANC-MF at the cost of computational complexity and additional signaling from the nodes to the RS.**

I. INTRODUCTION

The use of a Relay Station (RS) promises an increase in spectral efficiency and/or reliability of the communication networks. In a special case when two nodes want to communicate with each other via an RS, a two-way relaying protocol has been shown in [1] as an efficient protocol that improves the spectral efficiency of the networks. Two-way relaying adopts the idea of network coding [2], where the RS uses either analog network coding [1], [3] or digital network coding [4].

By applying analog network coding (ANC), the RS does not regenerate (decodes and reencodes) the data streams of the nodes. It simply transmits simultaneously to both nodes the superposition of their data streams. Each node performs self-interference cancellation prior to decoding in order to obtain the message from its partner. A non-regenerative RS has three advantages: no decoding error propagation, no delay due to decoding and deinterleaving, and transparency to the modulation and coding schemes that are used at the nodes [5].

It has been shown in many publications, e.g., [6], [7], that the use of multiple antennas improves the spectral efficiency and/or the reliability of the communication networks. Therefore, applying multiple antennas at the RS for two-way relaying is a logical step to have an even better performance. A non-regenerative multi-antenna RS that serves one two-way pair is considered in [5] for multi-antenna nodes and in [8] for single-antenna nodes. A multi-user scenario, where multiple two-way pairs are assisted by a non-regenerative multi-antenna RS, is treated in [9], [10]. While [9] considers linear transceive beamforming based on Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) criteria and measures the bit error-rate performance, [10] considers an optimum beamforming maximising the achievable sum rate and pair-aware transceive beamforming based on a semidefinite relaxation of a fair multicast beamforming and a matched filter.

In many applications in communication, multiple nodes want to communicate with each other. Examples of such applications are video conference and multi-player gaming. Until now, there are only few works on such multi-way channel, e.g., the work in [11], [12], where [13] is a special case when the number of nodes N is equal to two. In [14], the multi-way relay channel is considered, where an RS assists multiple communication groups. In each group, multiple nodes want to communicate with each other, but not with other nodes from other groups. A full-duplex communication is assumed and time division is used to separate the multi-way groups.

A single-group multi-way relaying protocol, where a half-duplex multi-antenna RS assists N nodes to communicate with each other, is proposed in [15] for non-regenerative RS and in [16] for regenerative RS. The required communication phases is only N phases, which consists of one multiple access (MAC) phase and $N-1$ broadcast (BC) phases. In each BC phase, the RS sends N data streams to N nodes simultaneously. Thus, each node receives an intended data stream from a specific node, while seeing other data streams as interference. Nevertheless, the interference can be cancelled by performing successive interference cancellation or by applying linear transceive beamforming that nullify the interference, e.g., ZF.

In this paper, we propose non-regenerative multi-group multi-way relaying, where a half-duplex multi-antenna RS assists L multi-way groups. In the l-th multi-way group, $l \in \mathcal{L}, \mathcal{L} = \{1, \cdots, L\}$, there are N_l nodes that want to communicate with each other, but not with other nodes from other multi-way groups. The required communication phases for the nodes in group l to communicate with each other is N_l. Since the RS is a half-duplex RS, the required communication phases for the multi-group multi-way relaying is defined by the highest $N_l, \forall l, l \in \mathcal{L}$. Our work is a generalisation of some of the above mentioned publications. If $L = 1$ and $N_1 = 2$, we have a non-regenerative two-way relaying as in [3], [5], [8]. If $L > 1$ and $N_l = 2, \forall l, l \in \mathcal{L}$, we have a non-regenerative multi-user two-way relaying as in [9] and [10], and if $L = 1$ and $N_1 \geq 2$, we have a non-regenerative multi-way relaying as in [15].

To better illustrate, Figure 1 shows examples of multi-group multi-way relaying with $L = 2$ communication groups, $N_1 = 3$ nodes in the first group and $N_2 = 2$ nodes in the second group. The first group consists of S0, S1 and S2, and the second group consists of S3 and S4. Figure 1.(a) is a direct extension of [15] to the case of multi-group. In the first phase, MAC phase, all nodes transmit simultaneously to the RS. In the following BC phases, the RS performs transceive beamforming to the received signal and transmits to each node an intended data stream. In the second phase, the RS sends x_1 to S0, x_2 to S1, x_0 to S2, x_4 to S3 and x_3 to S4. For the second group, after two phases, the two nodes S3 and S4 receive the data stream from their partner. In the third phase, the RS only sends x_2 to S0, x_0 to S1 and x_1 to S2. It does not send anything to S3 and S4 because they have already completed their communication. After three phases, all nodes in the first group obtain the data streams from their partners.

Different from Figure 1.(a), in Figure 1.(b) an ANC is used. In the second phase, the RS sends x_{01} to all nodes in the first group and x_{34} to all nodes in the second group. The transmitted data stream x_{01} is a superposition of the data streams from S0 and S1, and x_{34} is the superposition of the data streams from S3 and S4. For the second group, S3 and S4, they perform self-interference cancellation by subtracting x_{34} with their a priori known transmitted signal to obtain their partner's data stream. For the first group, both S0 and S1 perform self-interference cancellation, so that S0 obtains x_1 and S1 obtains x_0. Node S2 cannot yet perform self-interference cancellation, since x_{01} does not contain its data stream. In the third phase, the RS transmits x_{02} to all nodes in the first group. Both nodes S0 and S2 perform self-interference cancellation so that S0 obtains x_2 and S2 obtains x_0. Since S1 knows x_0 from the second phase, it performs known-interference cancellation to obtain x_2. For S2, since it knows x_0 from the third phase, it obtains x_1 by performing known-interference cancellation to the received data stream in the second phase, namely x_{01}. Thus, S2 needs to wait until it receives the data stream containing its data stream. After performing self-interference cancellation, it performs known-interference cancellation to obtain the other data stream. After three phases, all nodes in the first group obtain the data streams from their partners.

By using ANC, for each group l, the RS needs to superpose two data streams out of N_l data streams. The RS has to separate these two data streams from other received data streams, which resulting in a superposed data stream of only those two data streams. The superposed data stream needs to be transmitted simultaneously only to N_l nodes. Therefore, we specially design an ANC-aware transceive beamforming to implement ANC in non-regenerative multi-group multi-way relaying.

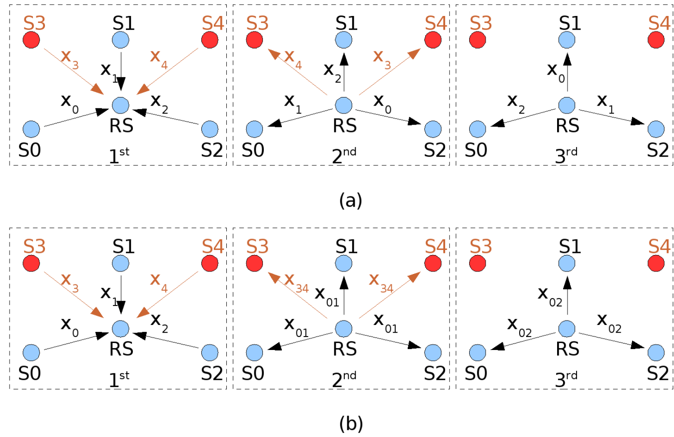

Fig. 1. Multi-group multi-way relaying: (a). Without analog network coding (ANC); (b). With ANC.

The contribution of this paper can be summarised as follows: 1. We propose a non-regenerative multi-group multi-way protocol, where a multi-antenna RS serves multiple multi-way groups. 2. We extend the work of [15] to the case of multi-group. 3. We propose the use of ANC for multi-group multi-way relaying. 4. We derive the achievable sum rate for asymmetric and symmetric traffic cases for contributions 2 and 3. 5. We specially design an ANC-aware transceive beamforming for multi-group multi-way relaying with ANC.

This paper is organised as follows. Section II describes the protocol and the system model of non-regenerative multi-group multi-way relaying. Section III explains the achievable sum rate. The transceive beamforming for contributions 2 and 3 is explained in Section IV. The sum rate performance analysis is given in Section V. Finally, Section VI provides the conclusion.

Notations: Boldface lower and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{*}$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate, and complex conjugate transpose, respectively. The operators $\mathrm{mod}_N(x)$, $\mathrm{E}\{\mathbf{X}\}$ and $\mathrm{tr}\{\mathbf{X}\}$ denote the modulo N of x, the expectation and the trace of $\mathbf{X}$, respectively, and $\mathcal{CN}(0, \sigma^2)$ denotes the zero-mean complex normal distribution with variance σ^2.

II. Protocol and System Model

In this section, the communication protocol and the system model of multi-group multi-way relaying are described. There are L multi-way groups. In each group l, there are N_l single antenna nodes that want to communicate with each other, but not with other nodes in other groups. The communication between the nodes in each group can only be performed through a half-duplex multi-antenna RS with M antenna elements. Although in this paper we only consider single antenna nodes, our work can be readily extended to the case of multi-antenna nodes. It is assumed that $M \geq \sum_{l=1}^{L} N_l$, so that the RS can spatially separate the data streams received from and transmitted to all nodes by applying low complexity linear transceive beamforming.

The l-th group consists of nodes $\mathrm{S}i_l, i_l \in \mathcal{I}_l$, where $\mathcal{I}_l$ is the set of nodes' indexes in group l given by $\mathcal{I}_l = \{a_l, \cdots, b_l - 1\}$, with $a_l = \sum_{q=0}^{l-1} N_q, b_l = \sum_{q=0}^{l} N_q$ and $N_0 = 0$. The total number of nodes is $N = \sum_{l=1}^{L} N_l$.

In the first phase, all nodes transmit simultaneously to the RS. The received signal at the RS is given by

$$\mathbf{y}_{\mathrm{RS}} = \mathbf{H}\mathbf{x} + \mathbf{z}_{\mathrm{RS}}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{M\times N} = [\mathbf{h}_0, \cdots, \mathbf{h}_{(N-1)}]$ is the overall channel matrix, with $\mathbf{h}_i \in \mathbb{C}^{M\times 1} = (h_{i,1}, \cdots, h_{i,M})^{\mathrm{T}}, i \in \mathcal{I}, \mathcal{I} = \bigcup_{l=1}^{L} \mathcal{I}_l$, the channel vector between node $\mathrm{S}i$ and the RS. The channel coefficient $h_{i,m}, m \in \mathcal{M}, \mathcal{M} = \{1, \cdots, M\}$, follows $\mathcal{CN}(0, \sigma_h^2)$. The vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is a vector of $(x_0, \cdots, x_{(N-1)})^{\mathrm{T}}$, with x_i the transmit signal of node $\mathrm{S}i$ that follows $\mathcal{CN}(0, \sigma_x^2)$. The AWGN noise vector at the RS is denoted as $\mathbf{z}_{\mathrm{RS}} \in \mathbb{C}^{M\times 1} = (z_{\mathrm{RS}1}, \cdots, z_{\mathrm{RS}M})^{\mathrm{T}}$ with $z_{\mathrm{RS}m}$ follows $\mathcal{CN}(0, \sigma_{z_{\mathrm{RS}}}^2)$.

Let in the following p, $p \in \mathcal{P}, \mathcal{P} = \{2, \cdots, P\}$, denotes the index of BC phase, with $P = \max(N_l), \forall l, l \in \mathcal{L}$, and $\max(N_l)$ the highest number of nodes N_l among all multi-way groups. Assuming reciprocal and stationary channels in P phases, the downlink channel from the RS to the nodes is simply the transpose of the uplink channel $\mathbf{H}$. Since the RS is a non-regenerative RS, it will neither decode nor reencode the data streams. In the p-th phase, the RS simply performs transceive beamforming, which spatially separates the signals received from and transmitted to all nodes. The received signal vector of all nodes in the p-th phase can be written as

$$\mathbf{y}_{\mathrm{nodes}}^{p} = \mathbf{H}^{\mathrm{T}}\mathbf{G}^{p}(\mathbf{H}\mathbf{x} + \mathbf{z}_{\mathrm{RS}}) + \mathbf{z}_{\mathrm{nodes}}, \tag{2}$$

and, accordingly, the received signal at node $\mathrm{S}r_l, r_l \in \mathcal{I}_l$, while receiving the data stream from node $\mathrm{S}t_l, t_l \in \mathcal{I}_l \setminus r_l$, in the p-th phase is given by

$$\begin{aligned} y_{r_l,t_l}^{p} = &\underbrace{\mathbf{h}_{r_l}^{\mathrm{T}}\mathbf{G}^{p}\mathbf{h}_{t_l}x_{t_l}}_{\text{useful signal}} + \underbrace{\sum_{\substack{j=0\\ j\neq t_l}}^{N-1} \mathbf{h}_{r_l}^{\mathrm{T}}\mathbf{G}^{p}\mathbf{h}_{j}x_{j}}_{\text{BC's interference signals}} + \\ &\underbrace{\mathbf{h}_{r_l}^{\mathrm{T}}\mathbf{G}^{p}\mathbf{z}_{\mathrm{RS}}}_{\text{RS' propagated noise}} + \underbrace{z_{r_l}}_{\text{node } r_l\text{'s noise}}, \end{aligned} \tag{3}$$

where $\mathbf{G}^p$ is the p-th phase transceive beamforming matrix and $\mathbf{z}_{\mathrm{nodes}} = \left(z_0, \cdots, z_{(N-1)}\right)^{\mathrm{T}}$, with z_{r_l} follows $\mathcal{CN}(0, \sigma_{z_{r_l}}^2)$.

In this work, we extend the work of [15] to the case of multi-group as well as proposing ANC for multi-group multi-way relaying. In the following, we explain the relationship of the receiver index r_l, the transmitter index t_l and the BC phase index p for both cases, with and without ANC.

Without ANC, the relationship is defined by

$$t_l = a_l + \mathrm{mod}_{N_l}(r_l + p - a_l - 1), \tag{4}$$

where for each particular group l, (4) holds only when $p \le N_l$, since when $p > N_l$, the RS is not transmitting again to the nodes in group l. Figure 1.(a) shows the example when $L = 2$, $N_1 = 3$ and $N_2 = 2$.

With ANC, in each BC phase, each node needs to know which data streams from two nodes have been superposed by the RS. This will increase the signalling in the network. Thus, assuming each node knows its own and its partners' indexes, we propose a method for choosing data streams to be network coded by the RS that does not need any signalling. For each group l, we choose the data stream from the lowest index node $\mathrm{S}v_l$, $v_l = a_l$, and network coded this data stream with one data stream from another node $\mathrm{S}w_l, w_l \in \mathcal{I}_l \setminus v_l$, in the group, which is selected successively based on the relationship defined by $w_l = v_l + p - 1$. As in the case of without ANC, for group l, the relationship holds only when $p \le N_l$. In the p-th phase, the RS sends $x_{v_l w_l}$ to the nodes in group l. Node $\mathrm{S}r_l = v_l$ receives the data stream from node $\mathrm{S}t_l = w_l$ and it simply performs self-interference cancellation to obtain x_{w_l}. The same thing happen to node $\mathrm{S}r_l = w_l$, it simply performs self-interference cancellation to obtain x_{v_l}. For node $\mathrm{S}v_l$, it needs to perform only self-interference cancellation in each BC phase to obtain other nodes' data streams. For the other $N - 1$ nodes $\mathrm{S}w_l, w_l \in \mathcal{I}_l \setminus v_l$, they need to perform self-interference cancellation once they receive the data stream containing their data stream and, after knowing x_{v_l}, they perform known-interference cancellation by cancelling x_{v_l} from each of the received data streams that are received in the other BC phases.

Remarks 1: Even though x_{v_l} is transmitted $N-1$ times to the nodes, it does not increase the information rate of x_{v_l} at the other $N-1$ nodes. Once x_{v_l} is decoded and known by the nodes, there is no uncertainty of x_{v_l} in the other data streams.

III. Achievable Sum Rate

In this section, we derive the achievable sum rate of multi-group multi-way relaying. We start by defining the signal to interference and noise ratio (SINR) for the case of with and without ANC. The achievable sum rate of multi-group multi-way relaying for asymmetric and symmetric traffics are explained afterwards. Asymmetric traffic refers to the situation where we allow all nodes in the group to transmit with different rate. Each node transmits with a rate that ensures that in the following $N_l - 1$ consecutive BC phases, all $N_l - 1$ intended receive nodes can decode the data stream correctly. Symmetric traffic is when all nodes in the group have to transmit simultaneously with the same rate that is defined by the lowest rate among all possible link combinations of receive and transmit node (r_l, t_l).

A. Signal to Interference and Noise Ratio

Given the received signal in (3) and without ANC, the SINR for the link between node $\mathrm{S}r_l$ and $\mathrm{S}t_l$, γ_{r_l,t_l}, is given by

$$\gamma_{r_l,t_l}^{p} = \frac{S}{I_{\mathrm{s}} + I_{\mathrm{sgos}} + I_{\mathrm{ogs}} + Z_{\mathrm{RS}} + Z_{r_l}}, \tag{5}$$

with the useful signal power

$$S = \mathrm{E}\{|\mathbf{h}_{r_l}^{\mathrm{T}}\mathbf{G}^{p}\mathbf{h}_{t_l}x_{t_l}|^2\} = |\mathbf{h}_{r_l}^{\mathrm{T}}\mathbf{G}^{p}\mathbf{h}_{t_l}|^2\sigma_x^2, \tag{6}$$

the self-interference power

$$I_{\mathrm{s}} = \mathrm{E}\{|\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_{r_l} x_{r_l}|^2\} = |\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_{r_l}|^2 \sigma_x^2, \quad (7)$$

the self-group other-stream interference power

$$I_{\mathrm{sgos}} = \sum_{\substack{j=a_l \\ j\neq r_l \\ j\neq t_l}}^{b_l-1} \mathrm{E}\{|\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_j x_j|^2\} = \sum_{\substack{j=a_l \\ j\neq r_l \\ j\neq t_l}}^{b_l-1} |\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_j|^2 \sigma_x^2, \quad (8)$$

the other group streams interference power

$$I_{\mathrm{ogs}} = \sum_{d\notin\mathcal{I}_l} \mathrm{E}\{|\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_d x_d|^2\} = \sum_{d\notin\mathcal{I}_l} |\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_d|^2 \sigma_x^2, \quad (9)$$

the RS's propagated noise power

$$Z_{\mathrm{RS}} = \mathrm{E}\{|\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{z}_{\mathrm{RS}}|^2\} = |\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p|^2 \sigma_{z_{\mathrm{RS}}}^2 \quad (10)$$

and the node r_l's noise power

$$Z_{r_l} = \mathrm{E}\{|z_{r_l}|^2\} = \sigma_{z_{r_l}}^2. \quad (11)$$

In every p-th BC phase, node Sr_l may perform interference cancellation. It subtracts the a priori self-interference as well as the a priori self-group other-stream interference from the previous BC phases. Once the nodes have decoded other nodes' data streams in the previous BC phases, they may use it to perform known-interference cancellation in a similar fashion to self-interference cancellation. With interference cancellation, the SINR can be rewritten as

$$\gamma_{r_l,t_l}^p = \frac{S}{I_l^{\mathrm{canc}} + I_{\mathrm{ogs}} + Z_{\mathrm{RS}} + Z_{r_l}}, \quad (12)$$

where

$$I_l^{\mathrm{canc}} = \sum_{\substack{j=a_l \\ j\neq r_l \\ j\neq t_l \\ j\notin\mathcal{B}_l}}^{b_l-1} |\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_j|^2 \sigma_x^2, \quad (13)$$

is the BC's interference signals without self-interference and self-group other-stream interference that have been decoded in the previous BC phases, with $\mathcal{B}_l = \{\hat{t}_l \mid \hat{t}_l = a_l + \mathrm{mod}_{N_l}(r_l + o - a_l - 1), \forall o, o \in \mathcal{O}, \mathcal{O} = \{2, \cdots, (p-1)\}\}$.

When the RS is applying ANC, the self-group other-stream interference will not appear and the γ_{r_l,t_l}^p is given by

$$\gamma_{r_l,t_l}^p = \frac{S}{I_{\mathrm{sok}} + I_{\mathrm{oganc}} + Z_{\mathrm{RS}} + Z_{r_l}}, \quad (14)$$

with the other-group ANC streams interference power

$$I_{\mathrm{oganc}} = \sum_{s\in\mathcal{L}\setminus l} \left(|\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_{v_s}|^2 \sigma_x^2 + |\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_{w_s}|^2 \sigma_x^2\right). \quad (15)$$

Regarding I_{sok}, it may take form either as self-interference power as in (7) or known-interference power given by

$$I_{\mathrm{k}} = \mathrm{E}\{|\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_{v_l} x_{v_l}|^2\} = |\mathbf{h}_{r_l}^{\mathrm{T}} \mathbf{G}^p \mathbf{h}_{v_l}|^2 \sigma_x^2. \quad (16)$$

Since I_{sok} can be cancelled at each node, the γ_{r_l,t_l}^p with self- and known-interference cancellation is given by

$$\gamma_{r_l,t_l}^p = \frac{S}{I_{\mathrm{oganc}} + Z_{\mathrm{RS}} + Z_{r_l}}. \quad (17)$$

B. Asymmetric Traffic

The achievable sum rate of non-regenerative multi-group multi-way relaying without ANC is given by

$$SR = \frac{1}{P} \sum_{l=1}^{L} \left((N_l - 1) \sum_{t_l\in\mathcal{I}_l} R_{t_l} \right), \quad (18)$$

where

$$R_{t_l} = \min\left(R_{r_l,t_l}\right), \forall r_l, r_l \in \mathcal{I}_l, r_l \neq t_l, \quad (19)$$

is the minimum rate among all nodes r_l in group l when they receive the data streams from node t_l, with

$$R_{r_l,t_l} = \log_2(1 + \gamma_{r_l,t_l}) \quad (20)$$

and γ_{r_l,t_l} is given in (5) for the case of without interference cancellation or (12) otherwise. We have (19) since all nodes transmit only once. The factor $N_l - 1$ is due to in $N_l - 1$ consecutive BC phases, in the group l, there are $N_l - 1$ nodes that receive the same data streams from a certain transmit node t_l. The scaling factor $\frac{1}{P}$ is due to P channel uses for multi-group multi-way relaying.

With ANC, the achievable sum rate is given also by (18) where (19) and (20) are applied. However, the γ_{r_l,t_l} can only be the one with self- and known-interference cancellation as in (17). After knowing x_{v_l}, node S$r_l, r_l \in \mathcal{I}_l \setminus v_l$, performs known-interference cancellation to the received data stream $x_{v_l w_l}, w_l \neq r_l \neq v_l$, to obtain x_{w_l}, i.e., cancels (16).

Remarks 2: One note regarding the achievable sum rate with ANC is that by having (19) for transmitting node Sv_l, we ensure that node Sv_l transmits x_{v_l} with the rate that can be decoded correctly by all other $N_l - 1$ nodes in group l. Thus, having known x_{v_l}, all other $N_l - 1$ nodes can use it to perform known-interference cancellation in a similar fashion to their self-interference cancellation.

Remarks 3: Regarding the scaling factor $\frac{1}{P}$, although each group l needs only N_l phases, nevertheless, since there is only one MAC phase and due to the half-duplex constraint, all groups suffer the same penalty. Hence, it is better to perform multi-group multi-way relaying only for multi-way groups with the same number of nodes in each multi-way group. For different number of nodes in each multi-way group, the free $P - N_l$ BC phases can be used to serve group l with advanced transmission, e.g., space-time block codes, which might be taken into account for future work.

C. Symmetric Traffic

In certain scenarios, there may be a requirement to have a symmetric traffic between all nodes. All nodes communicate with the same traffic rate defined by the minimum of $R_{t_l}, \forall t_l, t_l \in \mathcal{I}_l$. The achievable sum rate for symmetric traffic for both with and without ANC becomes

$$SR_{symm} = \frac{1}{P} \sum_{l=1}^{L} (N_l - 1) N_l \left(\min\left(R_{t_l}\right)\right). \quad (21)$$

IV. Transceive Beamforming

In this section, the design of low complexity transceive beamformers for non-regenerative multi-group multi-way relaying without ANC is explained. It is followed by the ANC-aware transceive beamforming.

A. Non-ANC transceive beamforming

It has been shown in [15] that ZF and MMSE outperform maximisation of signal to noise ratio (MSNR) beamformer. Even though MMSE outperforms ZF, there is a need of extra signalling effort from the nodes to the RS as the RS needs the information of noise variances of all nodes to compute the transmit beamforming. Thus, for non-ANC transceive beamforming, we consider only ZF. The ZF receive and transmit beamforming for two-way relaying is derived in [5] and it has been extended in [15] to the case of multi-way relaying. Here, we extend them to the case of multi-group multi-way relaying.

The ZF receive beamforming is given by

$$\mathbf{G}_{\mathrm{Rx}} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{H}} \tag{22}$$

and the ZF transmit beamforming is given by

$$\mathbf{G}_{\mathrm{Tx}} = \frac{1}{p_{\mathrm{ZF}}}\mathbf{H}^{*}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}^{*}\right)^{-1}, \tag{23}$$

with

$$p_{\mathrm{ZF}} = \sqrt{\frac{\mathrm{tr}\left\{\left(\mathbf{H}^{\mathrm{H}}\boldsymbol{\Upsilon}_{\mathrm{Rx}}^{-1}\mathbf{H}\right)^{-1}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}^{*}\right)^{-1}\right\}}{E_{\mathrm{RS}}}} \tag{24}$$

and

$$\boldsymbol{\Upsilon}_{\mathrm{Rx}} = \sigma_x^2\mathbf{H}\mathbf{H}^{\mathrm{H}} + \sigma_{\mathrm{RS}}^2\mathbf{I}_M \tag{25}$$

where E_{RS} is the transmit power of the RS.

We use the permutation matrix $\boldsymbol{\Pi}_l^p$ to define the relationship between receiving index r_l, transmitting index t_l and the corresponding phase index p for each group l. The $\boldsymbol{\Pi}_l^p$ is given by the operation $\mathrm{colperm}(\mathbf{I}_{N_l},(p-1))$ with $\mathbf{I}_{N_l}$ an identity matrix of size N_l. The $\mathrm{colperm}(\mathbf{I}_{N_l},(p-1))$ permutates circularly to the right the columns of the identity matrix $(p-1)$ times. The overall permutation matrix is a block diagonal matrix consisting all the permutation matrix $\boldsymbol{\Pi}_l^p$ in its diagonal. For example, for Figure 1, the permutation matrix $\boldsymbol{\Pi}^2 = \begin{pmatrix} 0&1&0&0&0\\0&0&1&0&0\\1&0&0&0&0\\0&0&0&0&1\\0&0&0&1&0 \end{pmatrix}$ and $\boldsymbol{\Pi}^3 = \begin{pmatrix} 0&0&1&0&0\\1&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&0&0 \end{pmatrix}$.

Finally, the p-th phase transceive beamforming for the non-regenerative multi-group multi-way relaying is defined as

$$\mathbf{G}^p = \mathbf{G}_{\mathrm{Tx}}\boldsymbol{\Pi}^p\mathbf{G}_{\mathrm{Rx}}. \tag{26}$$

B. ANC-aware Transceive Beamforming

Let in the p-th phase $\mathbf{H}_l^{\mathrm{T}^p} \in \mathbb{C}^{2\times M}$ and $\tilde{\mathbf{H}}_l^{\mathrm{T}^p} \in \mathbb{C}^{(N-2)\times M}$ denote the channel matrix of two nodes $\mathrm{S}v_l$ and $\mathrm{S}w_l$, and the channel matrix of all other nodes, respectively. Given the matrix $\tilde{\mathbf{V}}_l^{(0)^p} \in \mathbb{C}^{M\times(N-\tilde{f}_l)}$ which contains the right singular vectors of $\tilde{\mathbf{H}}_l^{\mathrm{T}}$, with $\tilde{f}_l$ denoting the rank of matrix $\tilde{\mathbf{H}}_l^{\mathrm{T}^p}$, we compute the equivalent channel matrix $\mathbf{H}_l^{(\mathrm{eq})^p} \in \mathbb{C}^{2\times(N-\tilde{f}_l)} = \mathbf{H}_l^{\mathrm{T}^p}\tilde{\mathbf{V}}_l^{(0)^p}$ that assures that the other signals coming from the other $N-2$ nodes are lies on the orthonormal basis of $\mathbf{H}_l^{\mathrm{T}^p}$.

For ANC-aware transceive beamforming, first we compute the receive beamforming based on $\mathbf{H}_l^{(\mathrm{eq})^p}$. The ANC-aware receive beamforming separates two data streams, x_{v_l} and x_{w_l}, from other data streams and superposes them as a single data stream. In this paper, we propose two methods for computing ANC receive beamforming. The first one is based on a matched filter, where the receive beamforming vector for group l is $\mathbf{m}_l = \mathbf{H}_l^{(\mathrm{eq})^{\mathrm{p}}\mathrm{H}}\mathbf{1}_2$, with $\mathbf{1}_2 = [1,1]^T$, and we name it ANC-MF. The second one is obtained by solving

$$\max_{\mathbf{m}_l}\ \min_{i_l}\left\{\left|\frac{\mathbf{m}_l\mathbf{h}_{i_l}^{(\mathrm{eq})^{\mathrm{p}}}}{\sigma_{z_{\mathrm{RS}}}^2}\right|^2\right\}_{i_l=v_l}^{i_l=w_l}, \quad \text{s.t. } \|\mathbf{m}_l\|_2^2 \leq 1 \tag{27}$$

which leads to a fair single-pair beamformer. Such an optimisation problem as in (27) is also treated in [17] for single-group multicast beamforming. It is claimed to be NP-hard and it has been solved in [17] using semidefinite relaxation (SDR). Thus, we name our second one ANC-SDR. The ANC-aware receive beamforming matrix is given by

$$\mathbf{G}_{\mathrm{Rx}}^p = \left[\left(\mathbf{m}_1^{\mathrm{T}}\tilde{\mathbf{V}}_1^{(0)^{\mathrm{T}}}\right)^{\mathrm{T}},\cdots,\left(\mathbf{m}_L^{\mathrm{T}}\tilde{\mathbf{V}}_L^{(0)^{\mathrm{T}}}\right)^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{28}$$

As now the RS has L data streams to be transmitted to L groups, we apply multi-group multicast beamforming. A multi-group multicast beamforming is already treated in some works, e.g., [18], [19]. In a similar fashion with the way we compute ANC-aware receive beamforming, we firstly need to compute the equivalent downlink channel matrix. By having in the p-th phase the channel matrix of all nodes in group l, $\mathbf{H}_{l_{\mathrm{DL}}}^{\mathrm{T}^p} \in \mathbb{C}^{N_l\times M}$, and the channel matrix of all other nodes in the other multi-way groups $s, s\in\mathcal{L}\setminus l$, $\tilde{\mathbf{H}}_{l_{\mathrm{DL}}}^{\mathrm{T}^p} \in \mathbb{C}^{(N-N_l)\times M}$, we can compute the equivalent downlink channel matrix $\mathbf{H}_{l_{\mathrm{DL}}}^{(\mathrm{eq})^p} \in \mathbb{C}^{N_l\times(N-\tilde{f}_{l_{\mathrm{DL}}})}$. For ANC-aware transmit beamforming, the ANC-MF is defined by $\mathbf{m}_{l_{\mathrm{DL}}} = \mathbf{H}_{l_{\mathrm{DL}}}^{(\mathrm{eq})^{\mathrm{p}}\mathrm{H}}\mathbf{1}_{N_l}$. The ANC-SDR transmit beamforming is obtained by solving

$$\max_{\mathbf{m}_{l_{\mathrm{DL}}}}\ \min_{i_{l_{\mathrm{DL}}}}\left\{\left|\frac{\mathbf{m}_{l_{\mathrm{DL}}}\mathbf{h}_{i_{l_{\mathrm{DL}}}}^{(\mathrm{eq})^{\mathrm{p}}}}{\sigma_{z_{i_{l_{\mathrm{DL}}}}}^2}\right|^2\right\}_{i_{l_{\mathrm{DL}}}=a_l}^{i_{l_{\mathrm{DL}}}=b_l-1} \quad \text{s.t. } \|\mathbf{m}_{l_{\mathrm{DL}}}\|_2^2 \leq 1. \tag{29}$$

The ANC-aware transmit beamforming is given by

$$\mathbf{G}_{\mathrm{Tx}}^p = \left[\tilde{\mathbf{V}}_{1_{\mathrm{DL}}}^{(0)}\mathbf{m}_{1_{\mathrm{DL}}},\cdots,\tilde{\mathbf{V}}_{L_{\mathrm{DL}}}^{(0)}\mathbf{m}_{L_{\mathrm{DL}}}\right]\boldsymbol{\Gamma}^{\frac{1}{2}}, \tag{30}$$

with the power loading matrix $\boldsymbol{\Gamma} \in \mathbb{R}^{L\times L}$ given by

$$\boldsymbol{\Gamma} = \mathrm{diag}\left(\mathrm{mean}\left(|\Gamma_1|\right),\cdots,\mathrm{mean}\left(|\Gamma_L|\right)\right)^{-1}, \tag{31}$$

where the modulus operator $|\cdot|$ is assumed to be applied element-wise and the mean function returns the average mean of a vector, with $\Gamma_l = \mathbf{H}_{l_{\mathrm{DL}}}^{\mathrm{T}^p}\tilde{\mathbf{V}}_{\mathrm{l_{DL}}}^{(0)}\mathbf{m}_{l_{\mathrm{DL}}}$. In order to satisfy the

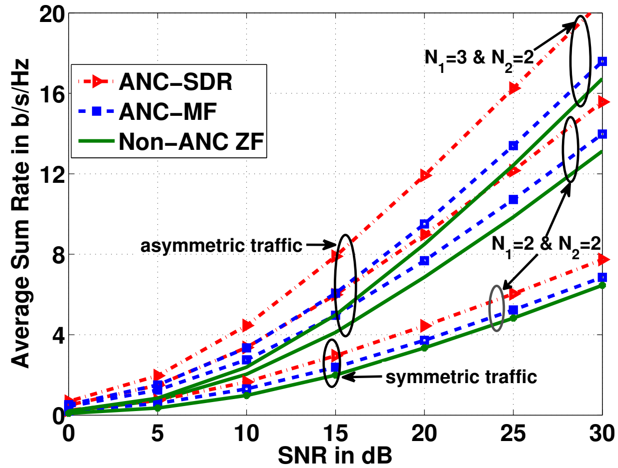

Fig. 2. Sum rate performance of non-regenerative multi-group multi-way relaying.

transmit power constraint at the RS, a normalisation factor $\beta \in \mathbb{R}$ is needed, with

$$\beta = \sqrt{\frac{E_{\mathrm{RS}}}{\mathrm{tr}\left\{\mathbf{G}^p_{\mathrm{Tx}}\mathbf{G}^p_{\mathrm{Rx}}(\sigma_x^2\mathbf{H}\mathbf{H}^{\mathrm{H}} + \sigma^2_{\mathrm{RS}}\mathbf{I})\mathbf{G}^{p^{\mathrm{H}}}_{\mathrm{Rx}}\mathbf{G}^{p^{\mathrm{H}}}_{\mathrm{Tx}}\right\}}}. \quad (32)$$

Finally, the ANC-aware transceive beamforming is given by

$$\mathbf{G}^p = \beta\mathbf{G}^p_{\mathrm{Tx}}\mathbf{G}^p_{\mathrm{Rx}}. \quad (33)$$

V. Performance Analysis

In this section, the sum rate performance is analysed. The first scenario is with $L = 2$, $N_1 = 2$, $N_2 = 2$ and $M = N = N_1 + N_2 = 4$. The second scenario is with $L = 2$, $N_1 = 3$, $N_2 = 2$ and $M = N = N_1 + N_2 = 5$. We set $\sigma^2_{z_{\mathrm{RS}}} = \sigma^2_{z_i} = 1, \forall i, i \in \mathcal{I}, \sigma^2_x = 1$, $E_{\mathrm{RS}} = 1$ and $\mathrm{SNR} = \sigma^2_h$.

Figure 2 shows the sum rate performance of multi-group multi-way relaying for the given scenarios. For the first scenario, the asymmetric and symmetric traffic is compared. The asymmetric traffic results in a higher sum rate than the symmetric traffic and, for both trafic, the ANC outperforms the non-ANC. In the first and the second scenarios for asymmetric traffic, the performance of ANC with ANC-aware transceive beamforming is better than without ANC. The ANC-SDR performs the best followed by ANC-MF. Both outperform non-ANC ZF and the gain is getting higher the higher the N. The drawbacks of ANC-SDR are the computational complexity and the signaling to provide the RS with the noise variance of the nodes, which are used to compute the transmit beamforming.

VI. Conclusion

In this paper, we introduce non-regenerative multi-group multi-way relaying. A multi-antenna RS assists L communication groups, where N_l nodes in group l want to communicate with each other. The number of communication phases is equal to the highest N_l among all L multi-way groups. We derive the achievable sum rate for non-regenerative multi-group multi-way relaying for asymmetric and symmetric traffic, with and without ANC. We propose non-ANC ZF transceive beamforming for multi-group multi-way relaying without ANC and we specially design an ANC-aware transceive beamforming for applying ANC. From the sum rate analysis, ANC improves the performance of the networks. Regarding the transceive beamforming, ANC-SDR performs the best followed by ANC-MF and non-ANC ZF.

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