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Consideration of Sectors for Direction of Arrival Estimation with Circular Arrays

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***Abstract*—A precise knowledge of the electromagnetic receiving characteristics of the antenna is mandatory for a precise determination of the directions of the impinging signals. For circular arrays the phase-mode description is an effective way of approximating the receiving characteristics of the antenna elements. For antenna elements, which are mounted in front of a cone-shaped reflector, angular ranges are defined to reduce the coupling and scattering influences. In these angular ranges, the receiving characteristics are more robust towards changes in the antenna structure and can be well approximated by a small number of phase-modes. The limitation of the angular range per element leads to the sector concept for DOA estimation. Especially the combination of the Virtual Array concept with the sector concept increases the robustness of the DOA estimation approaches against array imperfections.**

I. Introduction

High-resolution direction finding with antenna arrays is important in modern communication networks and for surveillance of the frequency spectrum, because it enables the resolution of multiple and closely spaced sources. Thanks to the circular symmetry, uniform circular antenna arrays (UCAs) are attractive antenna configurations in the context of DOA estimation. A lot of theoretical work has been done on DOA estimation with UCAs, but relatively few contributions take into account the actual electromagnetic characteristics of the antenna system. Neglecting scattering from the antenna structure and the surrounding as well as the coupling between the antenna elements can drastically degrade the system performance.

In [1][2][3] it was shown that the electromagnetic field radiated by cylindrical structures can be approximated by phase-modes, but the focus was on the description and compensation of mutual coupling effects between the elements. The receiving characteristics of complex antenna structures are considered in this paper. Therefore, an antenna configuration is selected where the elements are mounted in front of a cone-shaped reflector. The precise consideration of the receiving performance for different DOAs leads to the limitation of the azimuthal range per element to increase the robustness by reducing the coupling and scattering influences.

A concept of spatial selective DOA estimation was introduced in [4] to reduce the computational costs without greatly affecting the angular resolution. In this paper, the usage of sectors is motivated by the electromagnetic receiving characteristics of the antenna elements, because the limitation of the azimuthal range per element is suggested to increase the robustness against changes in the antenna structure.

Higher-order statistics are used to compensate for the reduced number of antenna elements in each sector. This improves the estimation performance and enables to resolve more sources. DOA estimation based on fourth-order cumulants was introduced in [5]. The Virtual Array concept illustrates the properties of higher-order cumulants for array processing [6][7][8].

In Section II the electromagnetic characteristics of the UCA are described and the approximation of the receiving characteristics based on phase-modes is given. In Section III the directional element performance is shown and the sector concept is introduced. Section IV illustrates the application of higher-order statistics in combination with the sector concept. In Section V some illustrative results are presented, proving that the combination of phase-modes, the sector concept and higher-order statistics increase the DOA estimation performance for the considered UCA.

II. Array signal model

A uniform circular array (UCA) as shown in Figure 1 consisting of N antenna elements mounted in front of a cone-shaped reflector is considered. The antenna elements operate over a broad frequency range, but for DOA estimation this frequency range is subdivided into narrowband parts. We assume that the antenna currents are only z-orientated and only vertically polarized components of incoming plane waves induce voltages over the feed impedances of the UCA. The antenna elements are distributed over a circle of radius R. The phase-center of each antenna element is located in the xy-plane, at azimuth angles $\phi_n = 2\pi(n-1)/N$ and elevation angle $\theta_n = 0$ with $n = 1, 2, ..., N$. If M uncorrelated plane waves impinge on the UCA, the superposition principle is applicable assuming a linear receiving system [9]. By defining a steering matrix in the presence of spatially white additive Gaussian noise $\mathbf{n}(t)$ we get the model commonly used in array processing

$$\mathbf{x}(t) = \mathbf{A}(\Theta)\mathbf{s}(\mathbf{t}) + \mathbf{n}(t), \quad (1)$$
$$\mathbf{A}(\Theta) = [\mathbf{a}(\Theta_1), ..., \mathbf{a}(\Theta_M)] \,\epsilon\mathbb{C}^{(\mathrm{N}\times\mathrm{M})},$$
$$\mathbf{s}(t) = [s_1(t), ..., s_M(t)]^T,$$

where $\mathbf{a}(\Theta_m) = \mathbf{a}(\theta_m, \phi_m)$ is the array steering vector for the impinging wave $m = 1, ..., M$ from direction (θ_m, ϕ_m),

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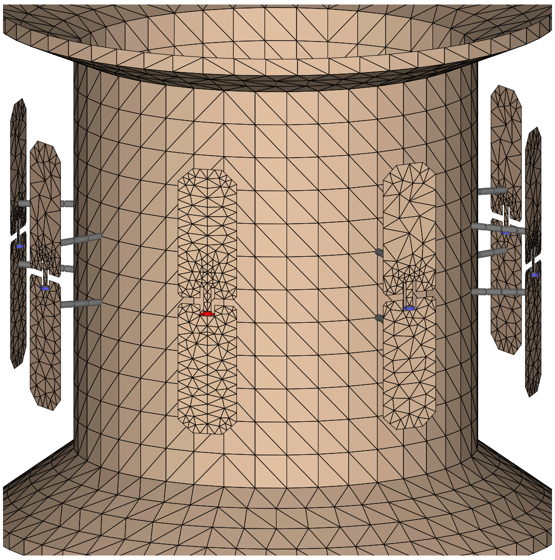


Fig. 1. UCA composed of N antenna elements in front of a cone-shaped reflector.

$\mathbf{s}(t)$ is the signal vector containing the complex envelopes of the impinging waves and $\mathbf{n}(t)$ is the noise vector. Based on the Maxwell equations, a phase-mode description for the steering vectors can be derived which contains all electromagnetic influences of the antenna structure. This model approximates the real steering vectors by a complex Fourier series [1][3][10]. Using the reciprocity theorem for antennas, the directional receiving characteristics of one element are equal to the radiation pattern of the antenna, if only this element is transmitting. Considering vertically polarized antenna elements, the radiation pattern is generated by z-oriented currents $I(R, \phi', z)$ on the structure and on the dipoles. With the angular frequency ω, the magnetic constant μ_0, the angular wavenumber k_0 and the cylindrical structure coordinates (R, ϕ', z), the θ-oriented radiation pattern $F(\theta, \phi)$ is related to the current distribution by

$$F(\theta,\phi) = -j\omega\mu_0 \sin(\theta) \cdot \int_{\phi'=0}^{2\pi} \int_z \Big[I(R,\phi',z) \times e^{jk_0(R\sin(\theta)cos(\phi-\phi')+zcos(\theta))} \Big] dz d\phi'. \qquad (2)$$

The radiation pattern and therewith the steering vectors can be decomposed into a limited number of phase modes. In this paper, we describe the receiving characteristic of one element $a_n(\theta, \phi)$ as a limited double Fourier series with the phase-mode coefficients f_{kl}

$$a_n(\theta,\phi) = \sum_{l=-L}^{+L} \sum_{k=-K}^{+K} f_{kl} e^{jk\phi} e^{jl\theta}, \qquad (3)$$

as suggested in [1]. The maximum number of significant phase-modes, determined by the electromagnetic dimensions of the UCA, is given by $K > k_0 R$ in azimuth direction and $L > k_0 \sqrt{R^2 + z_{max}^2} + 1$ in elevation direction, where z_{max} is the maximum height of the antenna structure in the axial direction. The electromagnetic receiving characteristics can be determined with electromagnetic simulation software and the phase-mode description can be based on this raw data.

III. Determination of Sectors

In the previous section, it was shown that the receiving characteristics of an UCA can be well approximated by phase-modes. In this section, the receiving characteristic of one selected UCA element is considered more precisely and angular ranges are determined where the receiving characteristic is less influenced by coupling and scattering effects. The coupling and scattering influences in the receiving characteristic of one element which is mounted in front of a cone-shaped reflector increase with the azimuthal or elevational distance from the broadside direction as shown in Figure 2. In elevation direction

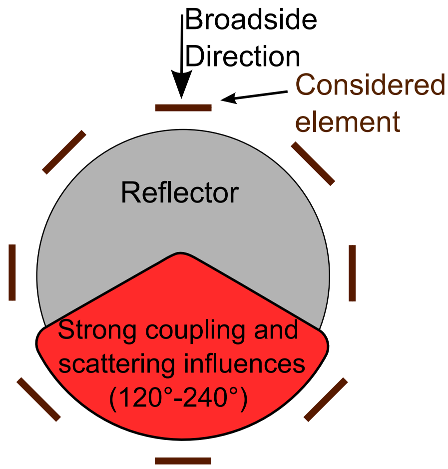
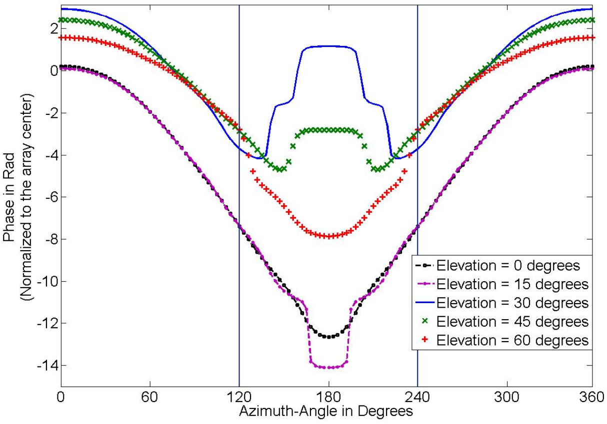
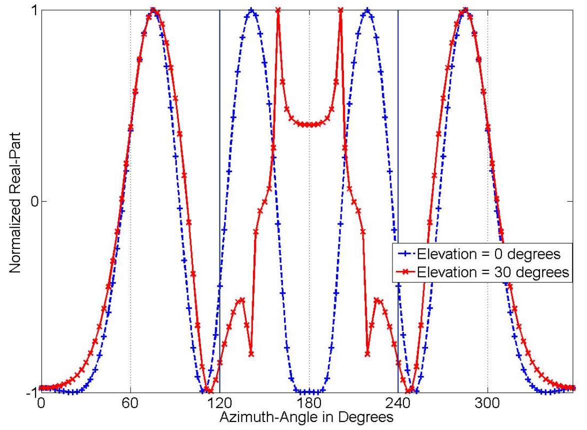


Fig. 2. Azimuthal range with good receiving properties for one element in front of a cone shaped reflector.

only signals out of a limited spatial range are of interest, which is usually limited to $\pm 40°$ from broadside direction. In this paper, an elevation range from $\pm 60°$ is considered. Especially for azimuth angles close to $180°$ from broadside the coupling due to the structure has a high influence on the receiving characteristic of a single element, which furthermore increases with the elevation displacement from the horizontal plane. For sectorized DOA estimation techniques, which are described in the next chapter, it is not necessary to know the receiving characteristics of each element for the whole azimuth plane and azimuthal ranges can be defined in which the receiving characteristics of the elements have to be known. Therefore, the phase-modes can also be concentrated on these azimuthal ranges and less Fourier coefficients are required to describe the receiving characteristics of each element, because the coupling and scattering influences are reduced. In Figure 3(a) the phase of the receiving characteristic of one UCA element normalized to the array center is shown. If the azimuth angles between $120°$ and $240°$ are neglected, the phase over the azimuthal range has a cosine characteristic for all elevation angles. Therefore, we define the azimuthal sector for which the receiving characteristics should be approximated from $-120°$



(a) Phase of the receiving characteristic of one UCA element in front of a cone shaped reflector.



(b) Real part of the receiving characteristic of one UCA element in front of a cone shaped reflector.

Fig. 3. Receiving characteristic of one UCA element.

to $+120°$ from the broadside direction of each element. The phase-modes are an approximation for the real and imaginary part of the receiving characteristics in this sector. Figure 3(b) shows that the real part which performs similar to the imaginary part has cosine shape in this sector. Furthermore, the differences in the azimuthal receiving characteristics between different elevation planes are small. This is not the case in the region of the excluded azimuth angles. Due to the cosine shape of the real and the imaginary part and the small differences in the receiving characteristics between different elevation planes, the receiving characteristics in the azimuthal range from $\pm 120°$ can be well approximated by phase-modes.

Limiting the azimuthal range of each element increases the robustness of the receiving characteristics against changes in the antenna structure, because the angular areas where strong coupling and scattering influences mainly determine the receiving characteristics are excluded. For UCAs where the elements are not mounted in front of a reflector, the allocation of the elements to different azimuthal sectors for DOA estimation can increase the robustness as well. For the determination of the angular areas, the element performance has to be considered precisely and the angles with strong coupling and scattering effects have to be excluded.

IV. DOA Estimation in Sectors

In the previous section, angular areas were defined where the elements are less influenced by coupling and scattering effects. This variation in the performance of the antenna elements for DOA estimation over the azimuthal plane should be considered in this section. Therefore, the azimuthal plane can be divided into different sectors. In every azimuthal sector only a reduced number of elements is used for DOA estimation. UCAs have rotational symmetry and it is therefore a natural choice to subdivide the azimuthal plane into N equal sectors and assign the elements to the sectors depending on their performances as shown in Figure 4. The reduction of

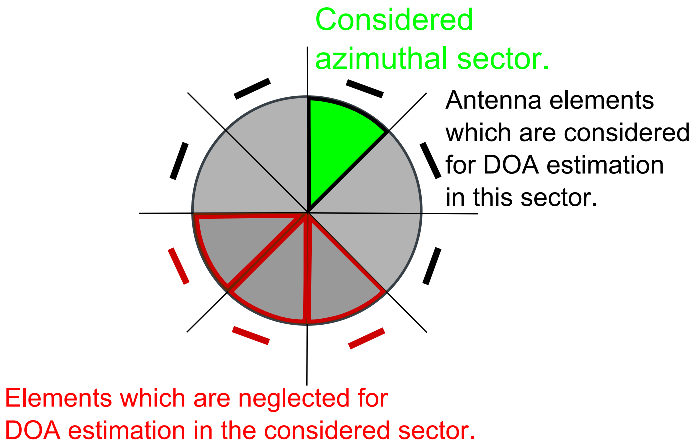


Fig. 4. Example for the allocation of antenna elements in front of a cone shaped reflector to a sector.

the number of considered elements per sectors decreases the number of sources which can be resolved by high-resolution DOA estimation approaches. To overcome these limitations, higher-order statistics can be used for the resolution of non-Gaussian signals like digital communication signals (e.g., M-PSK, M-QAM signals). Using higher-order statistics for DOA estimation is well known. The main interest relies on the increase of both the effective aperture and the effective number of antenna elements of the considered array, which can be illustrated by the Virtual Array concept [8]. Higher-order statistics allow a better resolution and the processing of more sources than antenna elements. The ideal higher-order cumulants of the array output are insensitive to Gaussian noise, making it possible to devise consistent parameter estimates without the knowledge of the noise covariance.

The elements are assigned to the sectors depending on their performance and therefore only elements with robust characteristics are used to estimate the DOAs in the considered sector. The virtual antenna elements using optimal fourth-order cumulants combined with the sector concept can be seen in Figure 5. The Virtual Array consists of an eight element

UCA with center element end twelve additional elements. All sensors of the Virtual Array have a robust characteristic for the considered azimuthal sector and the strong coupling and scattering influences which were caused by the cone-shaped reflector are eliminated in the Virtual Array. Higher-Order

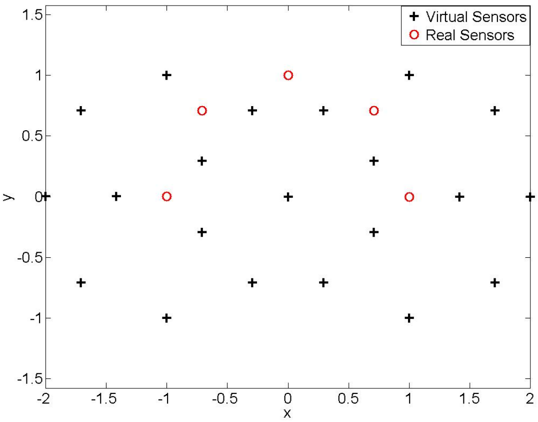


Fig. 5. Fourth-Order virtual array for five elements from a sectorized eight element UCA.

statistics can for example be combined with the Multiple Signal Classification (MUSIC) algorithm as suggested in [11] or with the Capon Beamformer [12]. For the combination with the Capon Beamformer, diagonal loading of the cumulant matrix shall be used to increase the robustness. The robustness of the DOA estimation against modeling errors increases with using higher-order statistics.
The combination of a beamformer for a rough determination of the sectors where signals are received with a high resolution algorithm like MUSIC can be used for the reduction of computational costs as suggested in [4].

V. Simulation Results

For the comparison of the sector concept with the consideration of the whole azimuthal plane per antenna element, a performance measure named Mean Correction Error (MCE) is defined

$$MCE(f) = \frac{f_{LF}}{N_{sec} \cdot I \cdot f} \sum_{i=1}^{I} ||\widehat{a}(\Theta_i) - a(\Theta_i)||_F, \qquad (4)$$

where I is the number of angles at which the deviation between the real and the estimated steering vectors is evaluated. The MCE is dependent on the considered frequency for broadband antennas, because the coupling and scattering effects change over the frequency range. The directional characteristics of the UCA increase with frequency and therefore the MCE is normalized to the ratio between the lowest operating frequency f_{LF} and the considered frequency f of the antenna. The MCE is furthermore normalized to the number N_{sec} of considered antenna elements per sector to be a measure for the deviation per considered element. The MCE for considering the five nearest antenna elements resulting in an azimuthal range of approximately $\pm 120°$ per element is compared with the MCE using all elements for DOA estimation in Figure 6. The nominal model is a phase-mode representation of a modified UCA, where the feed impedances and the radome parameters differ from the considered UCA. The nominal model shows the robustness against changes in the antenna structure using a limited or a full azimuthal range per element. The limitation of the azimuthal range reduces the MCE over the whole frequency range of the antenna. An example for

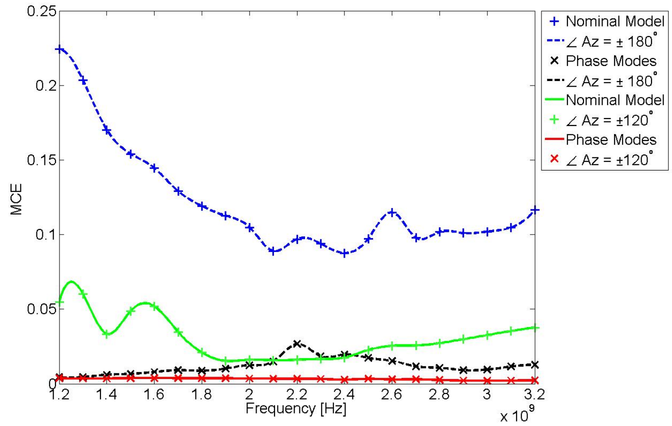
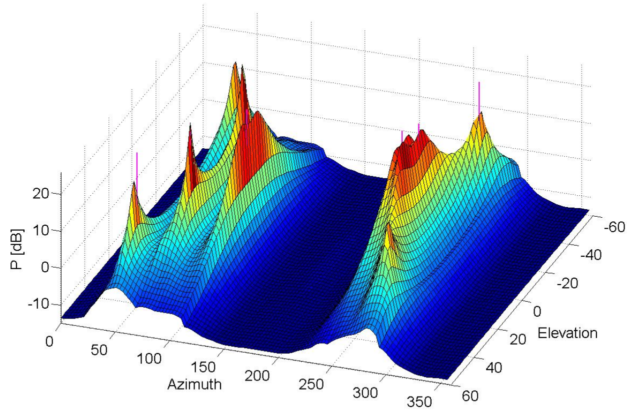


Fig. 6. MCE, where the nominal model is a phase-mode representation of an UCA with different feed impedance and radome settings.

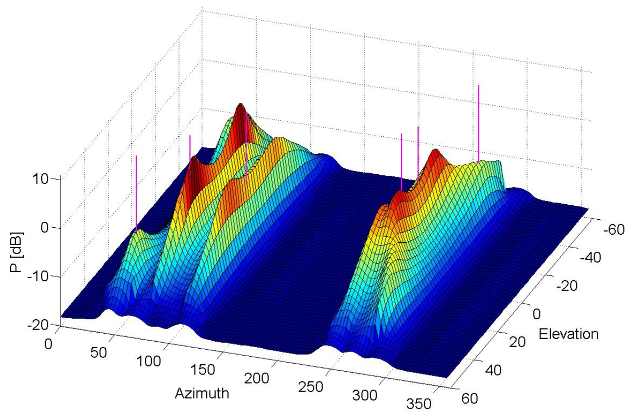
the influence of this MCE reduction on the DOA estimation performance is shown in Figure 7. The DOA estimation performance is considerably increased by limiting the azimuthal range per antenna element, although the number of antenna elements considered in every sector is reduced. Therefore, it is shown that the sector concept, which results in a limitation of the azimuthal range per element, improves the robustness against changes in the antenna structure and improves the performance of the DOA estimation algorithms. Figure 6 also shows the MCE for the phase-mode approximation of the UCA. The MCE is low with and without using the sector concept, but without the limitation of the azimuthal range of the elements it is slightly increased due to the sharp edges of the real and imaginary part in the angular area between $120°$ and $240°$, which cannot be approximated by phase-modes.

VI. Conclusion

The electromagnetic receiving characteristics of an UCA can be well approximated by phase-modes, especially if the sector concept is used. The usage of the sector concept decreases the number of antenna elements, which are considered in each azimuthal sector by only considering the elements with the best performances for the DOA estimates. It therefore increases the robustness against changes in the antenna structure and furthermore improves the DOA estimates. Higher-order statistics can be combined with the sector concept to increase the number of sources which can be resolved and the resolution at all. The combination of higher-order statistics



(a) Azimuthal range $= \pm 120°$ per element $\longrightarrow$ Five elements per sector.



(b) Azimuthal range $= \pm 180°$ per element $\longrightarrow$ All (eight) elements per sector.

Fig. 7. 4th-Order MUSIC DOA estimates (normalized amplitudes, $f = 1.4$ GHz) using a phase-mode model for a modified UCA - Source positions are marked by magenta lines.

with the sector concept yields robust high-resolution DOA estimates.

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