# Transmit Antenna Selection and OSTBC in adaptive multiuser OFDMA systems with different user priorities and imperfect CQI

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*Abstract*— **In this paper, an adaptive multiuser OFDMA system in the downlink with multiple antennas at the transmitter and the receivers is investigated which serves two sets of users differing in their priority regarding channel access. At the transmitter either Transmit Antenna Selection (TAS) or Orthogonal Space Time Block Coding (OSTBC) is performed. In addition, Maximum Ratio Combining (MRC) is performed at each receiver side. A Weighted Proportional Fair Scheduling (WPFS) approach is applied using the different user priorities and the instantaneous Signal-to-Noise-Ratios (SNRs) of the corresponding equivalent Single Input Single Output (SISO) channels of the users as Channel Quality Information (CQI) to allocate the different subcarriers to the different users. These CQI values are assumed to be imperfect due to time delays and estimation errors. The joint impact of imperfect CQI and user priority on the system performance is analytically investigated for both transmission schemes. Numerical results show that serving users with different priorities comes at the expense of reduced system data rate and less robustness against imperfect CQI. Furthermore, it is beneficial to switch from TAS to OSTBC in scenarios for fast varying channels due to the additional exploitation of spatial diversity applying OSTBC.**

## I. Introduction

The Orthogonal Frequency Division Multiple Access (OFDMA) transmission scheme is a promising candidate for future mobile networks [1]. It allows an efficient adaptation to the channel conditions by performing time-frequency scheduling of the different subcarriers to the different users. In systems where users experience different channel conditions, Proportional Fair Scheduling (PFS) approaches provide a good trade-off between system throughput and fairness. OFDMA systems applying PFS as well as Weighted Proportional Fair Scheduling (WPFS) approaches are well discussed in the literature [2]-[7]. Both PFS and WPFS algorithms require instantaneous channel knowledge at the transmitter. However, in realistic scenarios, the channels are not perfectly known at the transmitter which also results in performance degradations compared to the case of perfect channel knowledge. In [8], the performance of an adaptive multiuser Single Input Single Output (SISO) OFDMA system with different user priorities is investigated in the presence of imperfect Channel Quality Information (CQI). In [9], an adaptive OFDMA system with multiple antennas applying Orthogonal Space Time Block Coding (OSTBC) with imperfect CQI is analyzed. In [10], an adaptive OFDMA system which applies Transmit Antenna Selection (TAS) with imperfect CQI is investigated. In the last two works, it is assumed that each user has the same priority concerning channel access. In the present paper, we extend the work of [8] considering an adaptive multiuser OFDMA system with different user priorities *and* with multiple antennas where either TAS or OSTBC is applied at the transmitter and Maximum Ratio Combining (MRC) is applied at the receiver. Assuming outdated and noisy estimated Channel Quality Information (CQI) at the transmitter, we analytically investigate the joint impact of imperfect channel knowledge and different user priorities on the performance of an OFDMA system applying WPFS. The remainder of this paper is organized as follows. In Section II, the system model and the assumption on imperfect CQI are presented. In Section III the WPFS approach considering user priorities is introduced together with an analytical investigation of the channel access probability. Section IV provides analytical closed form expressions of the average Bit Error Rate (BER) and user data rate taking into account imperfect CQI and user priorities. Finally, the impact of imperfect CQI and user priority on the achievable system data rate is illustrated and discussed in Section V.

## II. System model

### A. System assumptions

In this work, we consider a one cell OFDMA downlink scenario with $N$ subcarriers. One Base Station (BS) and $U$ Mobile Stations (MSs) are located in the cell, where the MSs are assumed to be uniformly distributed inside the cell. The BS is equipped with $n_T$ transmit antennas and each MS is equipped with $n_R$ receive antennas. Each user $u$ with $u = 1,..,U$ experiences a different average Signal-to-Noise-Ratio (SNR) $\bar{\gamma}_u$ depending on the pathloss. The entries of the $n_T \times n_R$ Multiple Input Multiple Output (MIMO) channel of each subcarrier are assumed to be uncorrelated as well as the channel realizations of different users and adjacent subcarriers. The transfer factor $H_u^{(i,j)}(n,k)$ of the channel from transmit antenna $i$ with $i = 1,\cdots,n_T$ to receive antenna $j$ with $i = 1,\cdots,n_R$ of each user $u$ at subcarrier with index

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$n$ at each time slot $k \in \mathbf{N}$ is modeled as a complex Gaussian distributed random process with variance one. From this, it follows that the instantaneous SNR $\gamma_u^{(i,j)}(n,k)$ of user $u$ of subcarrier with index $n$ in time slot $k$ from transmit antenna $i$ to receive antenna $j$ is modeled according to

$$\gamma_u^{(i,j)}(n,k) = \bar{\gamma}_u \cdot \left| H_u^{(i,j)}(n,k) \right|^2 . \tag{1}$$

### *B. Imperfect CQI*

In order to perform an adaptive transmission, channel knowledge at the BS is required. In this work, we use the instantaneous SNR values of (1) as CQI which are measured at the BS in a Time Division Duplex system. In a realistic scenario, the CQI available at the BS suffers from different error sources and, thus, cannot be assumed to be perfectly known. In the following, two sources of error together with the modelling are introduced. To ease the comprehensibility, the subcarrier and time indices $n$ and $k$ are omitted in the notation of the channel transfer function $H$.

*1) Noisy estimated CQI:* The CQI is assumed to be a noisy estimate due to channel estimation. The actual channel $H$ is then modeled as a superposition of the estimated channel $\hat{H}$ and an additional error term $E$ leading to $\hat{H} = H + E$, where $E$ is modeled as a complex Gaussian distributed random variable with zero mean and variance $\sigma_E^2$.

*2) Outdated CQI:* Since there always exists a time delay $T$ between the time instant when measuring the SNR and the actual time of data transmission, the CQI available at the BS is outdated. Assuming that the channel follows Jakes' model, the actual channel and the outdated channel are correlated with a correlation coefficient of $\rho = J_0(2\pi f_D T)$, with $J_0(x)$ denoting the 0th-order Bessel function of the first kind and $f_D$ the Doppler frequency [11].

## III. Adaptive transmission applying WPFS

In this section, an adaptive transmission scheme applying either OSTBC or TAS at the transmitter and MRC at the receiver is introduced in which the subcarriers are allocated to the different users following an WPFS approach. For both OSTBC and TAS, the resulting SNR values of the equivalent SISO channel are derived which are then employed by the WPF Scheduler to allocate the subcarriers. In order to consider different user priorities in the allocation process, for each user a weighting factor is introduced indicating the priority of the user. Further on, for both OSTBC and TAS, the channel access probability of a given user is derived analytically as a function of the priority factors of all users. Finally, it is shown how to adjust the weighting factors of the different users in order to fulfill certain channel access demands of the different users.

### *A. Resulting SNR of equivalent SISO channel*

First, we consider OSTBC at the transmitter using $n_T$ transmit antennas and MRC using $n_R$ receive antennas at each receiver. OSTBC leads to an averaging of the SNR values of the $n_T$ transmit antennas at each receive antenna where these averaged SNR values are then superimposed performing MRC. Hence, the resulting SNR of the equivalent SISO channel in time slot $k$ of subcarrier $n$ of user $u$ is modeled by

$$\gamma_u(n,k) = \frac{1}{n_T} \sum_{i=1}^{n_T} \sum_{j=1}^{n_R} \gamma_u^{(i,j)}(n,k). \tag{2}$$

which can be simplified to

$$\gamma_u(n,k) = \frac{1}{n_T} \sum_{i'=1}^{n_T \cdot n_R} \gamma_u^{(i')}(n,k) \tag{3}$$

with $i' = 1,..,n_T \cdot n_R$ and $\gamma_u^{(i')}(n,k) = \mathrm{vec}\{\gamma_u^{(i,j)}(n,k)\}$ where the operation $\mathrm{vec}\{\}$ stacks the columns of a matrix on top of each other to form a long vector.

Second, we consider TAS in combination with MRC at each receiver. Applying TAS, for each user, the transmit antenna which provides the highest SNR is chosen for transmission. Hence, the resulting SNR of the equivalent SISO channel in time slot $k$ of subcarrier $n$ of user $u$ is modeled by

$$\gamma_u(n,k) = \max_i \sum_{j=1}^{n_R} \gamma_u^{(i,j)}(n,k). \tag{4}$$

### *B. Weighted Proportional Fair Scheduling*

In this work, WPFS is performed to allocate the different subcarriers to the different users according to their priority applying the SNR measurements modeled by (3) and (4), respectively. Given the priority vector $\mathbf{p}$ of length $U$

$$\mathbf{p} = [p_1, \; \cdots, p_u, \; \cdots, p_U] \tag{5}$$

with $p_u \leq 1 \;\forall\; u = 1,..,U$ denoting the priority factor of user $u$, subcarrier $n$ in time slot $k$ is allocated to the user $u^\star(n,k)$ with the highest ratio between the weighted instantaneous SNR and the average SNR, leading to

$$u^\star(n,k) = \arg\max_u \left\{ \frac{p_u \cdot \gamma_u(n,k)}{\bar{\gamma}_u} \right\}. \tag{6}$$

### *C. Channel access probability in OSTBC systems*

In this section, the probability $P_{A,OSTBC}(u, \mathbf{p})$ for user $u$ getting access to a subcarrier applying OSTBC is derived as a function of the priority vector $\mathbf{p}$. In the following, we omit the indices $n$ and $k$ since the calculations are valid for each subcarrier and time slot. From (3) it can be shown that the Probability Density Function (PDF) of the resulting SNR $\gamma_u$ is a chi-square distribution with $2n_T \cdot n_R$ degrees of freedom [12] given by

$$p_{\gamma_u}(\gamma_u) = \left(\frac{n_T}{\bar{\gamma}_u}\right)^{n_T n_R} \cdot \frac{\gamma_u^{n_T n_R - 1}}{(n_T n_R - 1)!} \cdot \exp\left(-\frac{n_T \gamma_u}{\bar{\gamma}_u}\right). \tag{7}$$

Hence, the PDF of the weighted and normalized SNR $\gamma_\mathrm{w} = \frac{p_u \cdot \gamma_u}{\bar{\gamma}_u}$ is given by

$$p_{\gamma_\mathrm{w}}(\gamma_\mathrm{w}) = \left(\frac{n_T}{p_u}\right)^{n_T n_R} \cdot \frac{\gamma_\mathrm{w}^{n_T n_R - 1}}{(n_T n_R - 1)!} \cdot \exp\left(-\frac{n_T \gamma_\mathrm{w}}{p_u}\right). \tag{8}$$

In order to determine $P_{A,OSTBC}(u,\mathbf{p})$, the probability that user $u$ successfully competes against the other $U-1$ users has to be calculated given by

$$
\begin{aligned}
P_{A,OSTBC}(u,\mathbf{p}) &= \int_{y_1=0}^{\infty}\int_{y_2=0}^{y_1}\dots\int_{y_U=0}^{y_1} p_{\gamma_{\mathrm{w}}}(y_1) \\
&\quad \cdot p_{\gamma_{\mathrm{w}}}(y_2)\dots p_{\gamma_{\mathrm{w}}}(y_U)\; dy_1 dy_2 \dots dy_U \\
&= \int_0^{\infty}\left(\frac{n_T}{p_u}\right)^{n_T n_R}\cdot\frac{\gamma_1^{n_T n_R-1}}{(n_T n_R-1)!}\cdot\exp\left(-\frac{n_T\gamma_1}{p_u}\right) \\
&\quad \cdot\prod_{\substack{i=1\\ i\neq u}}^{U}\left(1-e^{-\frac{n_T y_1}{p_i}}\sum_{v=0}^{n_T n_R-1}\frac{1}{v!}\left(\frac{n_T y_1}{p_i}\right)^{v}\right)\, dy_1
\end{aligned}
\tag{9}
$$

applying [13, Eq. 3.381] and [13, Eq. 8.352.1]. Performing some transformations and applying [13, Eq. 3.381.4] and [13, Eq. 8.339.1] to (9), the channel access probability $P_{A,OSTBC}(u,\mathbf{p})$ of user $u$ can be written in closed form given by

$$
\begin{aligned}
P_{A,OSTBC}(u,\mathbf{p}) &= \sum_{v=1}^{U}\frac{(-1)^{v-1}}{p_u^{n_T n_R}}\sum_{|\eta|=v-1}\sum_{l=0}^{(v-1)\cdot(n_T n_R-1)} \\
&\quad \cdot\sum_{|\nu|=l}\left(\frac{1}{\left(\prod_{i=1}^{v-1}\nu_i!\right)}\right)\cdot\frac{\left(\sum_{i=1}^{v-1}\nu_i+n_T n_R-1\right)!}{(n_T n_R-1)!} \\
&\quad \cdot\frac{\prod_{i=1}^{v-1}\left(\frac{1}{p_{r(\eta,i)+1}}\right)^{\nu_i}}{\left(\frac{1}{p_u}+\sum_{i=1}^{U-1}\frac{\eta_i}{p_{i+1}}\right)^{\sum_{i=1}^{v-1}\nu_i+n_T n_R}}
\end{aligned}
\tag{10}
$$

with the multi-indices $\eta=[\eta_1,\eta_2,...,\eta_{U-1}]$ with $\eta_j\in\{0,1\}$ $\forall\ j=1,..,U-1$ and $\nu=[\nu_1,\nu_2,...,\nu_{v-1}]$ with $\nu_j\in\{0,1,..,n_T\cdot n_R-1\}$ $\forall\ j=1,..,v-1$. The function $r(\eta,i)$ returns the index of the $i$-th 1 in the multi-index $\eta$.

### *D. Channel access probability in TAS systems*

Using TAS instead of OSTBC at the transmitter, the channel access probability $P_{A,TAS}(u,\mathbf{p})$ for user $u$ also changes. In order to derive $P_{A,TAS}(u,\mathbf{p})$, the PDF $p_{\gamma_{\mathrm{w}}}(\gamma_{\mathrm{w}})$ in (9) has to be exchanged by the PDF $p_{\gamma_{\mathrm{w_{n_T}}}}(\gamma_{\mathrm{w_{n_T}}})$ of the best out of $n_T$ weighted and normalized SNR values resulting from transmitting with only one transmit antenna and performing MRC with $n_R$ receive antennas given by

$$
\begin{aligned}
p_{\gamma_{\mathrm{w_{n_T}}}}(\gamma_{\mathrm{w_{n_T}}}) &= \frac{n_T}{p_u^{n_R}}\cdot\frac{\gamma_{\mathrm{w_{n_T}}}^{n_R-1}}{(n_R-1)!}\cdot\exp\left(-\frac{\gamma_{\mathrm{w_{n_T}}}}{p_u}\right) \\
&\quad \cdot\left(1-\exp\left(-\frac{\gamma_{\mathrm{w_{n_T}}}}{p_u}\right)\sum_{v=0}^{n_R-1}\frac{1}{v!}\left(\frac{\gamma_{\mathrm{w_{n_T}}}}{p_u}\right)^{v}\right)^{n_T-1},
\end{aligned}
\tag{11}
$$

leading to

$$
\begin{aligned}
P_{A,TAS}(u,\mathbf{p}) &= \int_0^{\infty}\frac{n_T}{p_u^{n_R}}\cdot\frac{y_1^{n_R-1}}{(n_R-1)!}\cdot e^{-\frac{\gamma_1}{p_u}} \\
&\quad \cdot\left(1-e^{-\frac{y_1}{p_u}}\sum_{v=0}^{n_R-1}\frac{1}{v!}\left(\frac{y_1}{p_u}\right)^{v}\right)^{n_T-1} \\
&\quad \cdot\prod_{\substack{i=1\\ i\neq u}}^{U}\left(1-e^{-\frac{y_1}{p_i}}\sum_{v=0}^{n_R-1}\frac{1}{v!}\left(\frac{y_1}{p_i}\right)^{v}\right)^{n_T} dy_1
\end{aligned}
\tag{12}
$$

which can be rewritten as

$$
\begin{aligned}
P_{A,TAS}(u,\mathbf{p}) &= \int_0^{\infty} n_T\cdot\left(\frac{1}{p'_u}\right)^{n_R}\cdot\frac{y_1^{n_R-1}}{(n_R-1)!}\cdot e^{-\frac{\gamma_1}{p'_u}} \\
&\quad \cdot\prod_{\substack{i=1\\ i\neq u}}^{n_T\cdot U}\left(1-e^{-\frac{y_1}{p'_i}}\sum_{v=0}^{n_R-1}\frac{1}{v!}\left(\frac{y_1}{p'_i}\right)^{v}\right) dy_1
\end{aligned}
\tag{13}
$$

with the extended priority vector $\mathbf{p}'$ of length $n_T\cdot U$ given by

$$
\mathbf{p}'=\underbrace{[\mathbf{p}\quad\mathbf{p}\quad ...\quad\mathbf{p}]}_{n_T\ \text{times}}. \tag{14}
$$

Comparing (13) with (9), it can be seen that the integrals are similar besides the factor $n_T$ at the beginning. From this, it follows that the channel access probability in a TAS system can be calculated using the channel access probability of a OSTBC system with $\mathbf{p}'$, $U'=n_T\cdot U$, $n'_T=1$ and $n'_R=n_R$, given by

$$
P_{A,TAS}(u,\mathbf{p})=n_T\cdot P_{A,OSTBC}(u,\mathbf{p}',U',n'_T,n'_R). \tag{15}
$$

### *E. Channel access gain*

Until now, we assumed that the priority vector $\mathbf{p}$ was given. In this section, it it shown how to adjust the weights in order to fulfill certain channel access demands of the different users. In the following, we introduce the channel access gain vector $\mathbf{g}$ of length $U$ given by

$$
\mathbf{g}=[g_1,\ \cdots,g_u,\ \cdots,g_U] \tag{16}
$$

with $g_u\geq 0\ \forall\ u=1,..,U$ denoting the increase of channel access probability for user $u$ applying WPFS compared to PFS where all users have the same priority and thus the same channel access probability given by $1/U$. From this, it follows that

$$
\sum_{u=1}^{U} g_u \stackrel{!}{=} U \tag{17}
$$

must hold, i.e. the maximum channel access gain a user $u$ can achieve is upper bounded by $g_u\leq U$. In the following, we assume that the users are sorted by their channel access gain in descending order, i.e. $g_{u-1}\geq g_u\geq g_{u+1}$. Now, for a given channel access gain vector $\mathbf{g}$ the corresponding priority vector $\mathbf{p}$ has to be found where the priority factor of the user with the lowest channel access gain $g_U$ is set to $p_U=1$ without loss of generality. Hence, the remaining $U-1$ priority factors $\tilde{\mathbf{p}}=[p_1,..,p_{U-1}]$ have to be determined such that $P_A(u,[\tilde{\mathbf{p}}\quad 1])=\frac{g_u}{U}\ \forall\ u$. This can be done by solving the following nonlinear optimization problem

$$
\tilde{\mathbf{p}}_{\mathbf{min}}=\arg\min_{\tilde{\mathbf{p}}}\left\{\sum_{u=1}^{U-1}\left|P_A(u,[\tilde{\mathbf{p}}\quad 1])-\frac{g_u}{U}\right|\right\} \tag{18}
$$

using for example the *fminsearch* function in MATLAB™. The corresponding priority vector $\mathbf{p}$ is then given by

$$
\mathbf{p}=[\tilde{\mathbf{p}}_{\mathbf{min}}\quad 1]. \tag{19}
$$

Figure 1 shows the interdependency of the channel access gain $\mathbf{g}$ and the corresponding priority factor $\mathbf{p}$ in a system with $U = 5$ users for different number of antennas. For brevity, we assume that $\mathbf{p} = [p\ 1\ 1\ 1\ 1]$, i.e., there is only one high priority user and 4 low priority users, leading to $\mathbf{g} = [g\ \frac{5-g}{4}\ \frac{5-g}{4}\ \frac{5-g}{4}\ \frac{5-g}{4}]$ . It can be seen that for different antenna constellations the priority factor $p$ has to be adjusted differently in order to guarantee a certain channel access gain $g$. For example, if the high priority user should get three times more channel resources compared to PFS ($g = 3$), the priority factor has to be set to $p = 3.75$ in a SISO system. In a $2 \times 1$ system, the priority factor has to be set to $p = 2.6$ applying TAS and to $p = 2.48$ applying OSTBC, where for a $2 \times 2$ system, $p = 2.0$ applying TAS-MRC and $p = 1.9$ applying OSTBC-MRC, respectively.

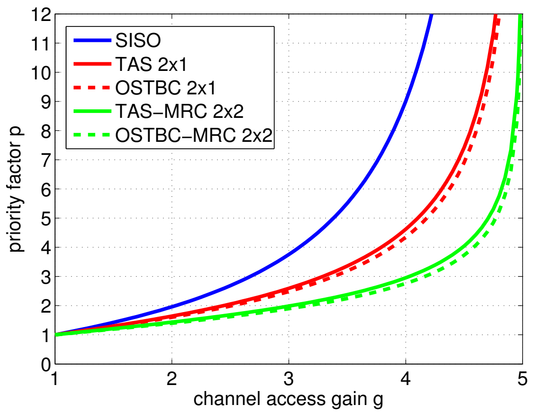


Fig. 1. Priority factor $p$ vs. channel access gain $g$

## IV. Impact of User Priority and Imperfect CQI on the System Performance

In this section, the impact of user priority and imperfect CQI on the performance of an OFDMA system applying WPFS is analyzed considering OSTBC and TAS. For both antenna techniques, the distribution of the SNR values of the selected users is derived. Subsequently, closed form expressions for the average data rate and BER are analytically derived taken into account user priority and imperfect CQI. Finally, the data rate is maximized subject to a target BER.

### A. SNR distribution considering user priority for OSTBC systems

To determine the PDF $p^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma})$ of the outdated and noisy estimated SNR $\hat{\gamma}$ of a scheduled user $u$, we have to calculate the marginal PDF by determining the integral over the joint PDF of the random variables $X_1, .., X_U$ which is given by

$$p_{X_1,..,X_U}(x_1, .., x_U) = p_{\hat{\gamma}_u}(x_1) \cdots p_{\hat{\gamma}_u}(x_U) \tag{20}$$

with

$$p_{\hat{\gamma}_u}(x) = \left(\frac{n_T}{\bar{\gamma}_{E,u}}\right)^{n_T n_R} \cdot \frac{x^{n_T n_R - 1}}{(n_T n_R - 1)!} \cdot \exp\left(-\frac{n_T \cdot x}{\bar{\gamma}_{E,u}}\right) \tag{21}$$

and $\bar{\gamma}_{E,u} = \bar{\gamma}_u \cdot (1 + \sigma^2_{E,u})$. Hence,

$$\begin{aligned} p^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma}) &= a_{OSTBC}(u) \cdot \\ &\underbrace{\int_0^{\frac{p_u}{p_1}\hat{\gamma}} \cdots \int_0^{\frac{p_u}{p_U}\hat{\gamma}}}_{U-1 \text{ times}} p_{X_1,..X_U}(\hat{\gamma}, y_1, .., y_{U-1})\, dy_1 \ldots dy_{U-1} \\ &= a_{OSTBC}(u) \cdot \left(\frac{n_T}{\bar{\gamma}_{E,u}}\right)^{n_T n_R} \cdot \frac{\hat{\gamma}^{n_T n_R - 1}}{(n_T n_R - 1)!} \cdot e^{-\frac{n_T \cdot \hat{\gamma}}{\bar{\gamma}_{E,u}}} \\ &\prod_{\substack{i=1 \\ i \neq u}}^{U} \left(1 - e^{-\frac{n_T p_u \hat{\gamma}}{p_i \bar{\gamma}_{E,u}}} \sum_{v=0}^{n_T n_R - 1} \frac{1}{v!} \left(\frac{n_T \cdot p_u \cdot \hat{\gamma}}{p_i \cdot \bar{\gamma}_{E,u}}\right)^v\right), \end{aligned} \tag{22}$$

where the factor $a_{OSTBC}(u)$ ensures that

$$\int_0^\infty p^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1. \tag{23}$$

Performing the substitution of the variable $y_1 = \frac{\hat{\gamma} \cdot p_u}{\gamma_{E,u}}$ in the integral of (9), it can be seen that the integrals in (9) and (23) are identical except for the factor $a_{OSTBC}(u)$, leading to

$$a_{OSTBC}(u) = \frac{1}{P_{A,OSTBC}(u, \mathbf{p})}. \tag{24}$$

Finally, the Cumulative Density Function (CDF) $F^{(u)}_{\hat{\gamma}}(\hat{\gamma})$ of the outdated and noisy estimated SNR of a scheduled user $u$ is determined by integrating (22) resulting in

$$\begin{aligned} F^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma}) &= \frac{a_{OSTBC}(u)}{p_u^{n_T \cdot n_R}} \cdot \sum_{v=1}^{U} (-1)^{v-1} \sum_{|\eta|=v-1} \\ &\cdot \sum_{l=0}^{(v-1)\cdot(n_T n_R - 1)} \sum_{|\nu|=l} \cdot \frac{\left(\sum_{i=1}^{v-1} \nu_i + n_T n_R - 1\right)!}{(n_T n_R - 1)!} \\ &\cdot \frac{\left(\frac{1}{(\prod_{i=1}^{v-1} \nu_i!)}\right) \cdot \left(\prod_{i=1}^{v-1} \left(\frac{1}{p_{r(\eta,i)+1}}\right)^{\nu_i}\right)}{\Lambda(\mathbf{p}, \eta)^{\sum_{i=1}^{v-1} \nu_i + n_T n_R}} \cdot [1- \\ &e^{-\frac{n_T p_u \hat{\gamma} \cdot \Lambda(\mathbf{p},\eta)}{\bar{\gamma}_{E,u}}} \sum_{\kappa=0}^{\sum_{i=1}^{v-1} \nu_i + n_T n_R - 1} (\kappa!)^{-1} \left(\frac{n_T p_u \hat{\gamma} \Lambda(\mathbf{p}, \eta)}{\bar{\gamma}_{E,u}}\right) \Bigg] \end{aligned} \tag{25}$$

with $\Lambda(\mathbf{p}, \eta) = \frac{1}{p_u} + \sum_{i=1}^{U-1} \frac{\eta_i}{p_{i+1}}$ and $\eta$, $\nu$ and $r(\eta, i)$ as defined in (10).

### B. SNR distribution considering user priority for TAS systems

To determine the PDF and CDF of the outdated and noisy estimated SNR of a scheduled user in a TAS system, the same derivation steps shown in (20) to (25) have to be done. However, PDF $p_{\hat{\gamma}_u}(x)$ has to be exchanged by the PDF $p_{\hat{\gamma}_{u_{n_T}}}(x)$ given by

$$\begin{aligned} p_{\hat{\gamma}_{u_{n_T}}}(x) &= \frac{n_T}{\bar{\gamma}_{E,u}^{n_R}} \cdot \frac{x^{n_R - 1}}{(n_R - 1)!} \cdot \exp\left(-\frac{x}{\bar{\gamma}_{E,u}}\right) \\ &\cdot \left(1 - \exp\left(-\frac{x}{\bar{\gamma}_{E,u}}\right) \sum_{v=0}^{n_R - 1} \frac{1}{v!} \left(\frac{x}{\bar{\gamma}_{E,u}}\right)^v\right)^{n_T - 1}. \end{aligned} \tag{26}$$

Hence, the PDF $p^{(u)}_{TAS,\hat{\gamma}}(\hat{\gamma})$ of the outdated and noisy estimated SNR of the scheduled user $u$ results in

$$p^{(u)}_{H,TAS,\hat{\gamma}}(\hat{\gamma}) = a_{TAS}(u) \cdot \frac{n_T \cdot \hat{\gamma}^{n_R-1}}{(n_R-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \left(1 - e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \sum_{v=0}^{n_R-1} \frac{1}{v!} \left(\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}\right)^v\right)^{n_T-1} \prod_{\substack{i=1\\i\neq u}}^{U} \left(1 - e^{-\frac{p_u \hat{\gamma}}{p_i \bar{\gamma}_{E,u}}} \cdot \sum_{v=0}^{n_R-1} \frac{1}{v!} \left(\frac{p_u \cdot \hat{\gamma}}{p_i \cdot \bar{\gamma}_{E,u}}\right)^v\right)^{n_T}, \tag{27}$$

which can be rewritten as

$$p^{(u)}_{H,TAS,\hat{\gamma}}(\hat{\gamma}) = a_{TAS}(u) \cdot \frac{n_T \cdot \hat{\gamma}^{n_R-1}}{(n_R-1)!} \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{E,u}}} \cdot \prod_{\substack{i=1\\i\neq u}}^{nT\cdot U} \left(1 - e^{-\frac{p'_u \hat{\gamma}}{p'_i \bar{\gamma}_{E,u}}} \sum_{v=0}^{n_R-1} \frac{1}{v!} \left(\frac{p'_u \cdot \hat{\gamma}}{p'_i \cdot \bar{\gamma}_{E,u}}\right)^v\right) \tag{28}$$

with $\mathbf{p}'$ as defined in (14). Again, the factor $a_{TAS}(u)$, which ensures that

$$\int_0^\infty p^{(u)}_{TAS,\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1, \tag{29}$$

can be determined by performing a substitution of the variable $y_1 = \frac{\hat{\gamma} \cdot p_u}{\gamma_{E,u}}$ in the integral of (13). It can be seen that the integrals in (13) and (29) are identical except for the factor $n_T \cdot a_{TAS}(u)$, leading to

$$\begin{aligned} a_{TAS}(u) &= \frac{1}{n_T \cdot P_{A,TAS}(u, \mathbf{p})} \\ &= \frac{1}{n_T \cdot P_{A,OSTBC}(u, \mathbf{p}', U', n'_T, n'_R)}. \end{aligned} \tag{30}$$

Comparing (28) and (30) with (22) and (24), it can be seen that the PDF $p^{(u)}_{H,TAS,\hat{\gamma}}(\hat{\gamma})$ can be determined using the PDF $p^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma})$ given by

$$p^{(u)}_{TAS,\hat{\gamma}}(\hat{\gamma}) = p^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma}, \mathbf{p}', U', n'_T, n'_R) \tag{31}$$

with $U' = n_T \cdot U$, $n'_T = 1$, $n'_R = n_R$ and $\mathbf{p}'$ as defined in (14).

Due to (31), the CDF $F^{(u)}_{TAS,\hat{\gamma}}(\hat{\gamma})$ can also be determined using the CDF $F^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma})$ given by

$$F^{(u)}_{TAS,\hat{\gamma}}(\hat{\gamma}) = F^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma}, \mathbf{p}', U', n'_T, n'_R) \tag{32}$$

with $U' = n_T \cdot U_L$, $n'_T = 1$, $n'_R = n_R$ and $\mathbf{p}'$ as defined in (14).

### C. Average data rate

The average data rate is defined as sum rate of the different number of bits per symbol according to the applied modulation schemes weighted by their probability. Assuming that there are $M$ modulations schemes available, $\gamma^{(u)} = [\gamma^{(u)}_0, \gamma^{(u)}_1, ..., \gamma^{(u)}_M]^{\mathrm{T}}$ denotes the threshold vector of user $u$ which contains the SNR threshold values determining the interval in which a particular modulation scheme is applied, where $\gamma^{(u)}_0 = 0$ and $\gamma^{(u)}_M = \infty$ for all users. Thus, the average data rate $\bar{R}^{(u)}_{OSTBC,H/L}$ of user $u$ for high and low priority users can be formulated as

$$\bar{R}^{(u)}_{OSTBC} = r_{n_T} \cdot \sum_{m=1}^{M} \int_{\gamma^{(u)}_{m-1}}^{\gamma^{(u)}_m} b_m \cdot p^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma})\, d\hat{\gamma} \tag{33}$$

with $b_m$ denoting the number of bits per symbol corresponding to the applied modulation scheme and $r_{n_T}$ denoting the data rate of the Space Time Block Code as a function of $n_T$. Using (25), the average user data rate for OSTBC systems is given by

$$\bar{R}^{(u)}_{OSTBC} = r_{n_T} \cdot \sum_{m=1}^{M} b_m \cdot \left(F^{(u)}_{OSTBC,\hat{\gamma}}(\gamma^{(u)}_m) - F^{(u)}_{OSTBC,\hat{\gamma}}(\gamma^{(u)}_{m-1})\right). \tag{34}$$

In order to determine the average user data rate for TAS systems, (33) can also be used, however, $r_{n_T}$ is set to $r_{n_T} = 1$ since no Space Time Coding is applied, resulting in

$$\bar{R}^{(u)}_{TAS} = \sum_{m=1}^{M} b_m \cdot \left(F^{(u)}_{TAS,\hat{\gamma}}(\gamma^{(u)}_m) - F^{(u)}_{TAS,\hat{\gamma}}(\gamma^{(u)}_{m-1})\right). \tag{35}$$

### D. Average BER

Using the approximation for the instantaneous BER for M-QAM and M-PSK modulation introduced in [14] given by

$$BER_m(\gamma) = 0.2 \cdot \exp(-\beta_m \gamma) \tag{36}$$

with $m = 1, .., M$, where $\beta_m = \frac{1.6}{2^{b_m}-1}$ for M-QAM modulation and $\beta_m = \frac{7}{2^{1.9b_m}+1}$ for M-PSK modulation, the average BER is defined as the sum of the number of bit errors of the different modulation constellations divided by the average bit rate [15]. To determine the average BER, we introduce the conditional PDF $p^{(u)}_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ of the actual SNR $\gamma$ and the outdated and noisy estimated SNR $\hat{\gamma}$ of user $u$ when applying OSTBC at the transmitter side and MRC at the receiver side given by

$$p^{(u)}_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) = \frac{n_T}{\bar{\gamma}_u \sigma^2_{r,u}} \cdot \exp\left(-\frac{\mu_u^2 \cdot \hat{\gamma} + \gamma}{\bar{\gamma}_u \sigma^2_{r,u}}\right) \cdot \left(\frac{\gamma}{\mu_u^2 \hat{\gamma}}\right)^{(n_T n_R - 1)/2} \cdot I_{n_T n_R - 1}\left(\frac{2 n_T \mu_u \sqrt{\gamma \cdot \hat{\gamma}}}{\bar{\gamma}_u \sigma^2_{r,u}}\right), \tag{37}$$

with $\mu_u = \frac{\rho_u}{1+\sigma^2_{E,u}}$, $\sigma^2_{r,u} = \frac{1+\sigma^2_{E,u}-\rho^2}{1+\sigma^2_{E,u}}$ and $I_n(x)$ denoting the $n$th-order modified Bessel function of the first kind. For OSTBC systems, the average BER of user $u$ is then given by

$$\overline{BER}^{(u)}_{OSTBC} = \frac{r_{n_T}}{\bar{R}^{(u)}_{OSTBC}} \cdot \sum_{m=1}^{M} \int_{\gamma^{(u)}_{m-1}}^{\gamma^{(u)}_m} b_m\, p^{(u)}_{OSTBC,\hat{\gamma}}(\hat{\gamma}) \cdot \left[\int_0^\infty BER_m(\gamma) \cdot p^{(u)}_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})\, d\gamma\right] d\hat{\gamma}. \tag{38}$$

Inserting (22), (36) and (37) in (38) and introducing the functions

$$\Upsilon(m, \eta) = \left(1 + \sum_{i=1}^{U-1} \frac{p_u \cdot \eta_i}{p_{i+1}}\right) \cdot (n_T + \beta_m \bar{\gamma}_u \sigma^2_{r,u}) + \bar{\gamma}_{E,u} \beta_m \mu_u^2. \tag{39}$$

and

$$\Psi(m) = n_T + \beta_m \bar{\gamma}_u \sigma_{r,u}^2 \tag{40}$$

and with $\eta$, $\nu$ and $r(\eta, i)$ as defined in (10), (38) can be written in closed form as

$$\begin{aligned}\overline{BER}_{OSTBC}^{(u)} = \frac{a_{OSTBC}(u) \cdot r_{n_T}}{5 \cdot \bar{R}_{OSTBC}^{(u)}} \cdot \sum_{m=1}^{M} b_m \quad (41)\\
\cdot \sum_{v=1}^{U} (-1)^{v-1} \sum_{|\eta|=v-1} \sum_{l=0}^{(v-1)\cdot(n_T n_R - 1)} \sum_{|\nu|=l}\\
\cdot \left( \frac{1}{(\prod_{i=1}^{v-1} \nu_i!)} \right) \cdot \frac{\left( \sum_{i=1}^{v-1} \nu_i + n_T n_R - 1 \right)!}{(n_T n_R - 1)!}\\
\cdot \left( \prod_{i=1}^{v-1} \left( \frac{1}{p_{r(i)+1}} \right)^{\nu_i} \right) \cdot \left( \frac{p_u \cdot \Psi(m)}{\Upsilon(m,\eta)} \right)^{\sum_{i=1}^{v-1} \nu_i}\\
\cdot \left( \frac{n_T}{\Upsilon(m,\eta)} \right)^{n_T n_R} \cdot \sum_{\kappa=0}^{\sum_{i=1}^{v-1} \nu_i + n_T n_R - 1} (\kappa)^{-1}\\
\left[ e^{\frac{-\gamma_{m-1}^{(u)} n_T \Upsilon(m,\eta)}{\bar{\gamma}_{E,u} \Psi(m)}} \left( \frac{\gamma_{m-1}^{(u)} n_T \Upsilon(m,\eta)}{\bar{\gamma}_{E,u} \Psi(m)} \right)^{\kappa} \right.\\
\left. - e^{\frac{-\gamma_{m}^{(u)} n_T \Upsilon(m,\eta)}{\bar{\gamma}_{E,u} \Psi(m)}} \left( \frac{\gamma_{m}^{(u)} n_T \Upsilon(m,\eta)}{\bar{\gamma}_{E,u} \Psi(m)} \right)^{\kappa} \right].\end{aligned}$$

Exploiting (31), the average BER applying TAS can be written as

$$\overline{BER}_{TAS}^{(u)} = \overline{BER}_{OSTBC}^{(u)}(\mathbf{p}', U', n_T', n_R', r_{n_T}') \tag{42}$$

with $U' = n_T \cdot U$, $n_T' = 1$, $n_R' = n_R$, $r_{n_T}' = 1$ and $\mathbf{p}'$ as defined in (14).

### *E. Optimizing data rate*

In the following, we search for the optimal modulation scheme threshold vector $\gamma^{(u)}$ of user $u$ which maximizes the average data rate subject to a target BER $BER_T$, i.e., the following optimization problem has to be solved:

$$\begin{aligned}\bar{R}_{opt}^{(u)} &= \max_{\gamma^{(u)}} \left( \bar{R}^{(u)}(\gamma^{(u)}) \right) \quad (43)\\
&\text{subject to}\\
&\overline{BER}^{(u)}(\gamma^{(u)}) \leq BER_T.\end{aligned}$$

Since the calculation of the average user data rate and BER are similar for OSTBC and TAS systems as shown in the previous section, the solution of (43) is only derived for OSTBC systems omitting the notation OSTBC. As shown in [8], (43) can be solved performing a Lagrange multiplier approach with the objective function $\Phi^{(u)}(\gamma)$ is given by

$$\begin{aligned}\Phi^{(u)}(\gamma^{(u)}) = \bar{R}^{(u)}(\gamma^{(u)}) + \lambda \quad (44)\\
\cdot \left( \bar{R}^{(u)}(\gamma^{(u)}) \overline{BER}^{(u)}(\gamma^{(u)}) - \bar{R}^{(u)}(\gamma^{(u)}) BER_T \right)\end{aligned}$$

where $\lambda$ denoting the Lagrange multiplier. Differentiating $\Phi^{(u)}(\gamma^{(u)})$ with respect to the elements of $\gamma^{(u)}$, where

$$\frac{\partial \Phi^{(u)}(\gamma_{opt}^{(u)})}{\partial \gamma_m^{(u)}} = 0 \tag{45}$$

for all $m = 1, .., M-1$ must hold true, results in $M-1$ equations given by

$$\begin{aligned}\frac{(1 - \lambda BER_T)}{\lambda} = \frac{1}{b_{m+1} - b_m} \Big( \zeta^{(u)}(m, \gamma_m^{(u)}, \sigma_{E,u}^2, \rho_u) \quad (46)\\
\cdot b_m - \zeta^{(u)}(m+1, \gamma_m^{(u)}, \sigma_{E,u}^2, \rho_u) \cdot b_{m+1} \Big)\end{aligned}$$

with

$$\begin{aligned}\zeta^{(u)}(\hat{\gamma}, m, \sigma_{E,u}^2, \rho_u) = 0.2 \cdot \left( \frac{n_T}{n_t + \beta_m \bar{\gamma}_u \sigma_{r,u}^2} \right)^{n_T n_r} \quad (47)\\
\cdot \exp\left( - \frac{\hat{\gamma} n_T \mu_u^2 \beta_m}{n_T + \beta_m \bar{\gamma}_u \sigma_{r,u}^2} \right)\end{aligned}$$

denoting the solution of the inner integral of (38).

From (46) it can be seen that each element $\gamma_m^{(u)}$ of the optimal threshold vector $\gamma_{opt}^{(u)}$ can be calculated using an initial value $\gamma_1^{(u)}$. Thus, each threshold vector $\gamma^{(u)}$ is a function of the initial value $\gamma_1^{(u)}$, i.e., $\gamma^{(u)} = f(\gamma_1^{(u)})$. Determining the maximum average data rate subject to the target BER, we have to find the optimal initial value $\gamma_{1,opt}^{(u)}$ which fulfills

$$\overline{BER}^{(u)}(f(\gamma_{1,opt}^{(u)})) \leq BER_T, \tag{48}$$

which can be done numerically using for example the *fzero* function in MATLAB™.

## V. Numerical Results

In the following, we consider an OFDMA scheme applying WPFS with $U = 25$ users where 3 users have high priority and 22 users low priority, i.e, $\mathbf{g} = [g\ g\ g\ \frac{25-3\cdot g}{22}\ \cdots\ \frac{25-3\cdot g}{22}]$ with $1 \leq g \leq 8.33$. The BS is equipped with $n_T = 2$ transmit antennas and each MS is equipped with $n_R = 2$ receive antennas. For simplicity, we assume that the average SNR in the system is $\bar{\gamma} = 8$ dB for all users. The target BER is set to $BER_T = 10^{-3}$. The CQI is assumed to be outdated expressed by the normalized time delay $f_D T$, where $f_D$ denotes the Doppler frequency which is assumed to be the same for each user. Furthermore, the SNR values are noisy estimates with a fixed error variance $\sigma_E^2 = 0.15$. In Fig. 2, the average system data rate applying TAS at the BS and MRC at the MSs indicated by different colors is depicted as a function of the time delay $f_D T$ and the channel access gain $g$. As one can see, the achievable data rate is high for small time delays and low channel access gains. When increasing $g$ for a given $f_D T$, the system data rate decreases since favoring high priority users even if they are in bad channel conditions results in a performance degradation. When increasing $f_D T$ for a given $g$, the data rate also decreases, since a more robust modulation scheme is required to cope with the outdated CQI. It can be seen that for higher channel access gains $g$, the transmission becomes more vulnerable to outdated CQI. For example in

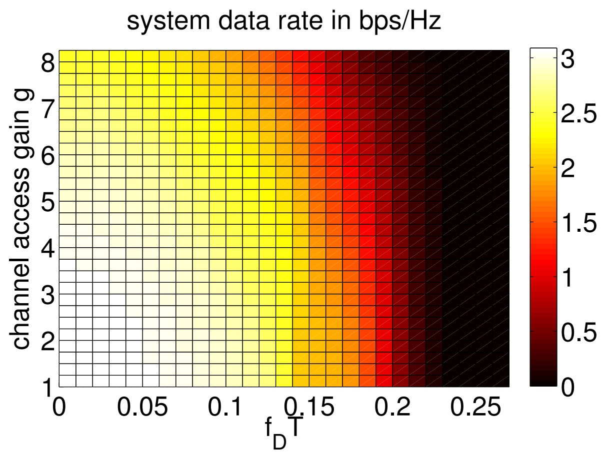


Fig. 2. $2 \times 2$ Transmit Antenna Selection - MRC system data rate vs. time delay $f_D T$ and priority gain $g$

Fig. 2, if $g = 1$, a system data rate of $\bar{R}_{sys} = 2.5$ bps/Hz can be achieved up to a delay of $f_D T = 0.15$. If $g = 5$, $\bar{R}_{sys}$ can only be achieved up to a delay of $f_D T = 0.075$. In Fig. 3, the same analysis is shown applying the well known Alamouti OSTBC scheme [16] at the BS and MRC at the MSs. Comparing the system performance applying OSTBC and TAS, TAS clearly outperforms OSTBC in the region of small time delays $f_D T$ as shown in Fig. 4 where the system date rate switching between TAS and OSTBC is depicted with the black line separating the regions in which either TAS or OSTBC is applied. The reason why TAS outperforms OSTBC for small time delays $f_D T$ is the averaging effect of OSTBC on the SNR values, i.e. applying OSTBC, the probability for high SNR values decreases. However, when increasing the time delay $f_D T$, OSTBC outperforms TAS since now OSTBC is more robust against outdated CQI due to the exploitation of spatial diversity.

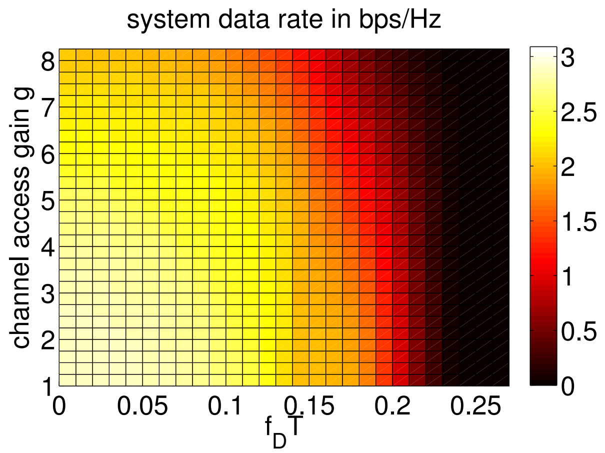


Fig. 3. $2 \times 2$ Alamouti - MRC system data rate vs. time delay $f_D T$ and priority gain $g$

## VI. Conclusions

In this paper, we analyze the performance of an adaptive multiple antenna OFDMA system with different user priorities in the presence of imperfect CQI. At the BS either TAS or OSTBC is performed where MRC is performed at each receiver. WPFS is applied to allocate the resources according to the users priority and channel quality. It is shown analytically how to adjust the weighting in order to fulfill certain channel access demands of the different users. Further on, for both TAS and OSTBC closed form expressions for the average user data rate and BER are analytically derived taking into account the joint impact of imperfect CQI and user priority. Based on these expressions, it is derived analytically how to adjust the SNR thresholds for the modulation schemes in order to achieve a maximum data rate subject to a given BER requirement depending on the number $U$ of users, the channel access gain vector $\mathbf{g}$, the estimation error variance $\sigma_E^2$, the normalized time delay $f_D T$ and the number of transmit and receive antennas $n_T$ and $n_R$. From the numerical results one can conclude that serving users with different priorities comes at the expense of reduced system data rate and less robustness against outdated CQI. Furthermore, it is beneficial to switch from TAS to OSTBC in scenarios with fast varying channels due to the additional exploitation of spatial diversity applying OSTBC.

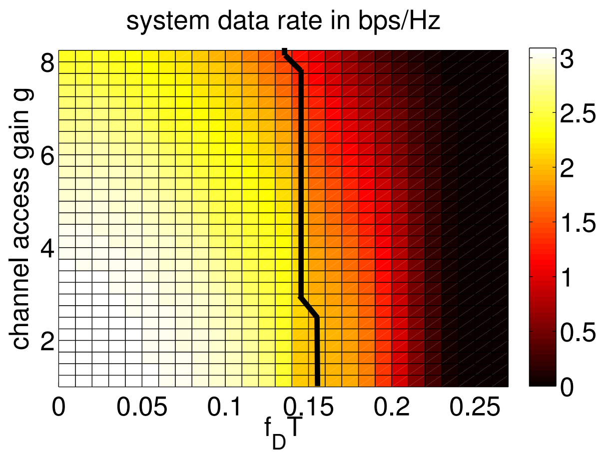


Fig. 4. Combined system data rate vs. time delay $f_D T$ and priority gain $g$

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