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# PAIR-AWARE TRANSCIVE BEAMFORMING FOR NON-REGENERATIVE MULTI-USER TWO-WAY RELAYING

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## ABSTRACT

In non-regenerative two-way relaying, two communicating nodes transmit simultaneously to the Relay Station (RS). The RS processes the received signal and forwards it to both nodes simultaneously. In order to obtain its partner's information, each node performs self-interference cancellation by subtracting its transmitted signal from its received signal. However, in multi-user two-way relaying, the interference at each node is not only the a priori self-interference, but also the interference from the other two-way pairs' nodes. Using Zero Forcing (ZF) transceive beamforming, the RS spends unnecessary energy to cancel the interference that can be cancelled by the nodes. Therefore, we propose to apply pair-aware transceive beamforming (PATB) at the RS to cancel only the other-pair interference and let each node cancels its self-interference. Two PATB schemes are proposed, namely, pair-aware (PA) matched filter (PA-MF) and PA semidefinite relaxation of maximising the minimum signal to noise ratio (PA-SDR). From sum rate analysis, PA-SDR outperforms PA-MF, and both outperform non-PA ZF.

**Index Terms**— multi-user, two-way relaying, transceive beamforming, non-regenerative

## 1. INTRODUCTION

Two-way relaying has emerged as a promising research topic. It defines how two nodes can communicate efficiently with the assistance of a Relay Station (RS) [1]. In the first phase, both nodes transmit simultaneously to the RS. In the second phase, the RS processes the superposed signal and sends the processed signal simultaneously to both nodes. In order to obtain its partner's messages, since each node knows its transmitted signal, it performs self-interference cancellation by subtracting its transmitted signal from its received signal. Considering the signal processing at the RS, it can either be regenerative [1, 2] or non-regenerative [1, 3]. Non-regenerative means that the RS simply performs linear signal processing without the necessity to decode and re-encode.

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A non-regenerative multi-antenna RS, which serves one two-way pair, is considered in [4] for single antenna nodes and in [3] for multi-antenna nodes. A multi-user two-way relaying, where a multi-antenna RS serves multiple two-way pairs, is treated in [5, 6, 7]. While [5] and [6] consider a regenerative RS, [7] considers a non-regenerative RS, where the RS transceiver processing based on Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) criteria is formulated. Different to the other previous mentioned publications, in [7] the bit error rate performance instead of the average sum rate performance is considered.

In this paper, we consider multi-user two-way relaying with a non-regenerative multi-antenna RS. In the first phase, all nodes transmit simultaneously to the RS. In the second phase, the RS performs transceive filtering to the received signal and transmits to all nodes. We propose to apply pair-aware transceive beamforming (PATB) at the RS to cancel the interference from the other two-way pairs and self-interference cancellation at each node to cancel the backpropagated own signal. Two PATB schemes are proposed, namely, pair-aware (PA) matched filter (PA-MF) and PA semidefinite relaxation of maximising the minimum signal to noise ratio (PA-SDR).

The contributions of this paper can be summarised as follows: 1. We derive the achievable sum rate and formulate the sum rate maximisation problem of non-regenerative multi-user two-way relaying. 2. We design PATB, which is practical but suboptimal, for non-regenerative multi-user two-way relaying. 3. We solve the sum rate maximisation problem numerically as an upper bound on the sum rate performance.

This paper is organised as follows. Section 2 describes the system model of non-regenerative multi-user two-way relaying. Section 3 explains the achievable sum rate. The design of the transceive beamforming is explained in Section 4. The sum rate performance analysis is given in Section 5. Finally, Section 6 provides the conclusion.

Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denotes scalar values. The superscripts  $(\cdot)^T$ ,  $(\cdot)^*$  and  $(\cdot)^H$  stand for matrix or vector transpose, complex conjugate and complex conjugate transpose, respectively. The operators  $E\{\mathbf{X}\}$  and  $\text{tr}\{\mathbf{X}\}$  denote the expectation and the trace of  $\mathbf{X}$ , respectively, and  $\mathcal{CN}(0, \sigma^2)$  denotes the zero-

mean complex normal distribution with variance  $\sigma^2$ .

## 2. SYSTEM MODEL

We consider a multi-user two-way relaying scenario, where a non-regenerative multi-antenna RS supports multiple single-antenna-node two-way pairs. In the first phase, all nodes transmit simultaneously to the RS, and in the second phase, the RS transmits to the nodes. Two nodes in each pair perform an exclusive two-way communication. The  $K$  pairs, which consist of  $N = 2K$  nodes, are served by an RS with  $M \geq N$  antennas. A number  $M \geq N$  of antennas at the RS is required since we consider low complexity transceive beamforming.

In the following, let  $\mathcal{S}_N, \mathcal{S}_N = \{1, \dots, N\}$ , denotes the set of all nodes, with  $\mathcal{S}_N = \mathcal{S}_{\text{odd}} \cup \mathcal{S}_{\text{even}}$ ,  $\mathcal{S}_{\text{odd}}$  the set of odd numbered nodes and  $\mathcal{S}_{\text{even}}$  the set of even numbered nodes. Each node in  $\mathcal{S}_{\text{odd}}$  has a two-way partner node in  $\mathcal{S}_{\text{even}}$ . Let  $P^{(k)}, k \in \mathcal{K}, \mathcal{K} = \{1, \dots, K\}$  denotes the  $k$ -th two-way pair that consists of two nodes,  $S_{a_k} \in \mathcal{S}_{\text{odd}}$  and  $S_{b_k} \in \mathcal{S}_{\text{even}}$ , with  $a_k \in \mathbb{Z}_+, a_k = 2k - 1, k \in \mathcal{K}$  and  $b_k \in \mathbb{Z}_+, b_k = 2k, k \in \mathcal{K}$ .

In this work, we assume frequency-flat fading channels that are reciprocal and stationary within two communication phases. In the first phase, all nodes transmit simultaneously to the RS. The received signal at the RS is given by

$$\mathbf{y}_{\text{RS}} = \mathbf{H}\mathbf{Q}\mathbf{x} + \mathbf{z}_{\text{RS}}, \quad (1)$$

where  $\mathbf{H} \in \mathbb{C}^{M \times N} = [\mathbf{h}_1, \dots, \mathbf{h}_N]$  is the overall channel matrix, with  $\mathbf{h}_n \in \mathbb{C}^{M \times 1} = (h_{n,1}, \dots, h_{n,M})^T, n \in \mathcal{S}_N$ , the channel vector between node  $n$  and the RS. The channel coefficient  $h_{n,m}, m \in \mathcal{M}, \mathcal{M} = \{1, \dots, M\}$ , follows  $\mathcal{CN}(0, \sigma_h^2)$ . The diagonal matrix  $\mathbf{Q}$  has  $(\sqrt{q_1}, \dots, \sqrt{q_N})^T$  in its diagonal, with  $q_n$  the transmit power of node  $n$ . The vector  $\mathbf{x} \in \mathbb{C}^{N \times 1}$  is a vector of  $(x_1, \dots, x_N)^T$ , with  $x_n$  the signal of node  $n$  that follows  $\mathcal{CN}(0, \sigma_x^2)$ . The AWGN noise vector at the RS is denoted as  $\mathbf{z}_{\text{RS}} \in \mathbb{C}^{M \times 1} = (z_{\text{RS}1}, \dots, z_{\text{RS}M})^T$ , with  $z_{\text{RS}m}$  follows  $\mathcal{CN}(0, \sigma_{z_{\text{RS}}}^2)$ .

In the second phase, the RS processes the received signal  $\mathbf{y}_{\text{RS}}$  and transmits the processed signal to the nodes. The received signal at the nodes can be written as

$$\mathbf{y}_{\text{nodes}} = \mathbf{H}^T \mathbf{G} (\mathbf{H}\mathbf{Q}\mathbf{x} + \mathbf{z}_{\text{RS}}) + \mathbf{z}_{\text{nodes}}, \quad (2)$$

where  $\mathbf{H}^T$  is the channel matrix between the RS and all nodes,  $\mathbf{G}$  is the transceive beamforming matrix and  $\mathbf{z}_{\text{nodes}} = (z_1, \dots, z_N)^T$ , with  $z_n$  follows  $\mathcal{CN}(0, \sigma_{z_{\text{nodes}}}^2)$ . The received signal at a receiving node  $r, r \in \mathcal{S}_N$ , when it receives the data streams from a transmitting node  $t, t \in \mathcal{S}_N$ , is given by

$$y_{r,t} = \underbrace{\mathbf{h}_r^T \mathbf{G} \mathbf{h}_t \sqrt{q_t} x_t}_{\text{useful signal}} + \underbrace{\sum_{\substack{i=1 \\ i \neq t}}^N \mathbf{h}_r^T \mathbf{G} \mathbf{h}_i \sqrt{q_i} x_i}_{\text{interference signals}} + \underbrace{\mathbf{h}_r^T \mathbf{G} \mathbf{z}_{\text{RS}}}_{\text{RS's propagated noise}} + \underbrace{z_r}_{\text{node } r \text{'s noise}}, \quad (3)$$

where (3) only holds for  $r = a_k$ , if and only if  $t = b_k$ , and for  $r = b_k$ , if and only if  $t = a_k$ .

## 3. ACHIEVABLE SUM RATE

In this section, we derive the achievable sum rate of non-regenerative multi-user two-way relaying. We start by defining the signal to interference and noise ratio (SINR) at each node, which is followed by the achievable sum rate.

Given the received signal as in (3), the SINR for the link between the receiving node  $r$  and the transmitting node  $t, \gamma_{r,t}$ , is given by

$$\gamma_{r,t} = \frac{S}{I + Z_{\text{RS}} + Z_r}, \quad (4)$$

with the useful signal power

$$S = \mathbb{E}\{|\mathbf{h}_r^T \mathbf{G} \mathbf{h}_t \sqrt{q_t} x_t|^2\} = |\mathbf{h}_r^T \mathbf{G} \mathbf{h}_t|^2 q_t \sigma_x^2, \quad (5)$$

the RS's propagated noise power

$$Z_{\text{RS}} = \mathbb{E}\{|\mathbf{h}_r^T \mathbf{G} \mathbf{z}_{\text{RS}}|^2\} = |\mathbf{h}_r^T \mathbf{G}|^2 \sigma_{z_{\text{RS}}}^2 \quad (6)$$

and the node  $r$ 's noise power

$$Z_r = \mathbb{E}\{|z_r|^2\} = \sigma_{z_{\text{nodes}}}^2. \quad (7)$$

The interference power,  $I$ , is given by  $I = I_s + I_o$ , with the self-interference power

$$I_s = \mathbb{E}\{|\mathbf{h}_r^T \mathbf{G} \mathbf{h}_r \sqrt{q_r} x_r|^2\} = |\mathbf{h}_r^T \mathbf{G} \mathbf{h}_r|^2 q_r \sigma_x^2, \quad (8)$$

and the other-pair interference power

$$I_o = \sum_{\substack{i=1 \\ i \neq r, i \neq t}}^N \mathbb{E}\{|\mathbf{h}_r^T \mathbf{G} \mathbf{h}_i \sqrt{q_i} x_i|^2\} = \sum_{\substack{i=1 \\ i \neq r, i \neq t}}^N |\mathbf{h}_r^T \mathbf{G} \mathbf{h}_i|^2 q_i \sigma_x^2, \quad (9)$$

The achievable sum rate of non-regenerative multi-user two-way relaying is given by

$$SR = \frac{1}{2} \sum_{r=1}^N R_r, \quad (10)$$

with

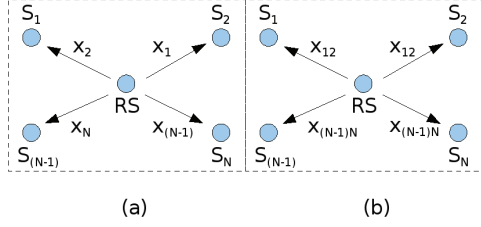
$$R_r = \log_2(1 + \gamma_{r,t}) \quad (11)$$

and  $\gamma_{r,t}$  in (4) for both conditions where (3) applies. The scaling factor  $\frac{1}{2}$  is due to two channel uses.

The optimization problem for the sum rate maximisation can be written as

$$\begin{aligned} SR^{\text{opt}} &= \max_{\mathbf{Q}, \mathbf{G}} SR \\ \text{s.t. } &\text{tr}\{\mathbf{G}(\sigma_x^2 \mathbf{H}\mathbf{Q}\mathbf{Q}^H \mathbf{H}^H + \sigma_{\text{RS}}^2 \mathbf{I})\mathbf{G}^H\} = q_{\text{RS}} \\ &q_n \leq q_{\text{nodes}}, \forall n, n \in \mathcal{S}_N, \end{aligned} \quad (12)$$

where  $q_{\text{nodes}}$  and  $q_{\text{RS}}$  are the transmit power constraint of all nodes and the transmit power constraint of the RS, respectively. An optimum transceive beamforming  $\mathbf{G}^{\text{opt}}$  at the RS is needed to obtain the maximum sum rate. As finding  $\mathbf{G}^{\text{opt}}$  requires high computational complexity, we propose low complexity transceive beamforming in the following Section.



**Fig. 1.** Second phase of non-regenerative multi-user two-way relaying: (a). Non-PA Zero Forcing; (b). PA transceive beamforming.

#### 4. TRANSCEIVE BEAMFORMING

In this section, we explain the non-PA ZF transceive beamforming, which is the extension of [3] to the case of multi-user two-way relaying, and, afterwards, we explain the proposed PATB.

##### 4.1. Non-PA Zero Forcing

The non-PA ZF receive beamforming is given by

$$\mathbf{G}_{\text{Rx}} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \quad (13)$$

and the non-PA ZF transmit beamforming is given by

$$\mathbf{G}_{\text{Tx}} = \frac{1}{\alpha} \mathbf{H}^* (\mathbf{H}^T \mathbf{H}^*)^{-1}, \quad (14)$$

with

$$\alpha = \sqrt{\frac{\text{tr}\{(\mathbf{H}^H \mathbf{\Upsilon}_{\text{Rx}}^{-1} \mathbf{H})^{-1} (\mathbf{H}^T \mathbf{H}^*)^{-1}\}}{q_{\text{RS}}}} \quad (15)$$

and

$$\mathbf{\Upsilon}_{\text{Rx}} = \sigma_x^2 \mathbf{H} \mathbf{H}^H + \sigma_{\text{RS}}^2 \mathbf{I}. \quad (16)$$

The non-PA ZF transceive beamforming is given by

$$\mathbf{G} = \mathbf{G}_{\text{Tx}} \mathbf{\Pi} \mathbf{G}_{\text{Rx}}, \quad (17)$$

where the permutation matrix  $\mathbf{\Pi}$  is a block diagonal matrix with the  $(2 \times 2)$  anti-identity matrix in each block. For example, for  $K = 2$ ,  $\mathbf{\Pi} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ .

If the RS applies a non-PA ZF transceive beamforming, the self and other-pair interference will not appear at the nodes. Hence, the nodes do not need to perform self-interference cancellation. Figure 1.(a) shows the second phase of non-regenerative multi-user two-way relaying with non-PA ZF transceive beamforming.

##### 4.2. Pair-Aware Transceive Beamforming

By applying non-PA ZF transceive beamforming, the RS spends unnecessary energy to cancel the interference that can be cancelled by the nodes. Thus, we propose to apply PATB

at the RS to cancel only the other-pair interference and let each node cancel its self-interference. Figure 1.(b) shows the second phase of non-regenerative multi-user two-way relaying with PATB. Instead of separating  $x_{a_k}$  and  $x_{b_k}$ , the RS sends their superposition  $x_{a_k b_k}$  to the  $k$ -th two-way pair.

The PATB is based on the ZF Block Diagonalization (ZFBD), which has been proposed in [8] for downlink spatial multiplexing beamforming. Firstly, we use ZFBD to compute the equivalent channel for each two-way pair, which is free from other-pair interference. Secondly, we compute the single-pair filter for each pair based on the equivalent channel. Thirdly, we compute the receive and transmit beamforming matrix for all pairs. Advantageously, we can reduce the computational complexity by using the receive beamforming matrix to compute the transmit beamforming matrix.

Let  $\mathbf{H}_k^T \in \mathbb{C}^{2 \times M}$  and  $\tilde{\mathbf{H}}_k^T \in \mathbb{C}^{(N-2) \times M}$  denote the channel matrix of pair  $P^{(k)}$  and the channel matrix of all other pairs  $P^{(l)}$ ,  $\forall l, l \in \mathcal{K}, l \neq k$ , respectively. Given the singular value decomposition

$$\tilde{\mathbf{H}}_k^T = \tilde{\mathbf{U}}_k \tilde{\mathbf{S}}_k [\tilde{\mathbf{V}}_k^{(1)}, \tilde{\mathbf{V}}_k^{(0)}], \quad (18)$$

we compute the equivalent channel matrix of pair  $P^{(k)}$ ,  $\mathbf{H}_k^{(\text{eq})} \in \mathbb{C}^{2 \times (N-\tilde{r}_k)} = \mathbf{H}_k^T \tilde{\mathbf{V}}_k^{(0)}$ , which assures that the interference signals from (and to) the other two-way pairs  $P^{(l)}$ ,  $\forall l, l \in \mathcal{K}, l \neq k$ , are suppressed. The matrix  $\tilde{\mathbf{V}}_k^{(0)} \in \mathbb{C}^{M \times (N-\tilde{r}_k)}$  contains the right singular vectors of  $\tilde{\mathbf{H}}_k^T$ , with  $\tilde{r}_k$  denoting the rank of matrix  $\tilde{\mathbf{H}}_k^T$ .

Since there is no other-pair interference and assuming all nodes perform self-interference cancellation, we compute PA receive beamforming for each pair  $k$ ,  $\mathbf{m}_k$ , individually. In this work we consider two single pair beamformers. The first one is based on a matched filter, where  $\mathbf{m}_k = \mathbf{H}_k^{(\text{eq})H} \mathbf{1}_2$ , with  $\mathbf{1}_2 = [1, 1]^T$ . Hence, we name it PA-MF. The second one is obtained by solving

$$\begin{aligned} \max_{\mathbf{m}_k} \min_r \left\{ \left| \frac{\mathbf{m}_k \mathbf{h}_r^{(\text{eq})}}{\sigma_{\text{nodes}}^2} \right|^2 \right\}_{r=a_k}^{r=b_k}, \quad (19) \\ \text{s.t. } \|\mathbf{m}_k\|_2^2 \leq 1 \end{aligned}$$

which is a fair single-pair beamformer. Such an optimisation problem as in (19) has been solved in [9] using semidefinite relaxation (SDR). Thus, we name the second one PA-SDR.

The PA receive beamforming matrix is given by

$$\mathbf{G}_{\text{Rx}} = \left[ \left( \mathbf{m}_1^T \tilde{\mathbf{V}}_1^{(0)T} \right)^T, \dots, \left( \mathbf{m}_K^T \tilde{\mathbf{V}}_K^{(0)T} \right)^T \right]^T, \quad (20)$$

and the PA transmit beamforming is given by

$$\mathbf{G}_{\text{Tx}} = \mathbf{G}_{\text{Rx}}^T \mathbf{\Gamma}^{\frac{1}{2}}, \quad (21)$$

with the power loading matrix  $\mathbf{\Gamma} \in \mathbb{R}^{K \times K}$  given by

$$\mathbf{\Gamma} = \text{diag}(\text{mean}(|\Gamma_1|), \dots, \text{mean}(|\Gamma_K|))^{-1}, \quad (22)$$

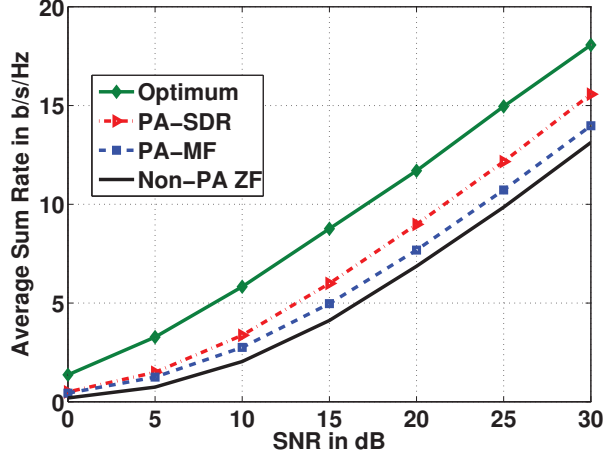


Fig. 2. Sum rate performance:  $K = 2$  two-way pairs

where the modulus operator  $|\cdot|$  is assumed to be applied element-wise and the mean function returns the average mean of a vector, with  $\Gamma_k = \mathbf{H}_k^T \tilde{\mathbf{V}}_k^{(0)} \mathbf{m}_k$ . In order to satisfy the transmit power constraint, a normalisation factor  $\beta \in \mathbb{R}$  is needed, with

$$\beta = \sqrt{\frac{q_{RS}}{\text{tr} \{ (\mathbf{G}_{Tx} \mathbf{G}_{Rx} (\sigma_x^2 \mathbf{H} \mathbf{H}^H + \sigma_{RS}^2 \mathbf{I}) \mathbf{G}_{Rx}^H \mathbf{G}_{Tx}^H) \}}} \quad (23)$$

Finally, the PATB is given by

$$\mathbf{G} = \beta \mathbf{G}_{Tx} \mathbf{G}_{Rx}. \quad (24)$$

## 5. SUM RATE NUMERICAL ANALYSIS

In this section, we analyse the sum rate performance of non-regenerative multi-user two-way relaying in a scenario where  $K = 2$  and  $M = 4$ . We set  $\sigma_{RS}^2 = 1$ ,  $\sigma_{z_{nodes}}^2 = 1$ ,  $\sigma_x^2 = 1$ ,  $q_{RS} = 1$ ,  $q_{nodes} = 1$  and  $\text{SNR} = \sigma_h^2$ . Unless for the optimum beamforming case, we set  $q_n = 1, \forall n, n \in \mathcal{S}_N$ .

Figure 2 shows the simulation results for the considered scenario. The PATB outperforms non-PA ZF transceive beamforming. The PA-MF transceive beamforming gains about 1.5 dB while PA-SDR gains about 4 dB compared to non-PA ZF. Even though PA-MF performs worse than PA-SDR, it has lower computational complexity. The maximum sum rate in (12) is computed numerically using `fmincon` from MATLAB and it provides an upper bound for the sum rate performance. The PA-SDR transceive beamforming is only 4 dB worse than the optimum transceive beamforming in (12). It can be seen that the optimum transceive beamforming and the non-PA ZF have the same slope, which shows that both achieve the same spatial multiplexing gain. Nevertheless, the non-PA ZF suffers about 8 dB power penalty. Similar phenomena for one hop downlink transmission has been shown in [10] when comparing the capacity achieving beamforming (Dirty Paper Coding) with linear beamforming ZF.

## 6. CONCLUSION

In this paper, we consider multi-user two-way relaying with non-regenerative multi-antenna RS. We derive the achievable sum rate and formulate the maximum sum rate problem. We propose two PATB schemes, namely, PA-MF and PA-SDR, to be applied at the RS to cancel the interference from (and to) the other two-way pairs, and let each node performs self-interference cancellation. The PA-SDR outperforms PA-MF and both outperform non-PA ZF transceive beamforming.

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