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A Mathematical Model for User Traffic in Coverage and Capacity Optimization of a Cellular Network

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***Abstract*—A promising approach to optimize coverage and capacity in the cellular network is the adjustment of antenna azimuth orientation and tilt. Tuning both antenna parameters without considering a realistic user traffic distribution in the network might result in a cellular layout having cells covering large areas when small areas are expected and vice-versa. In real life scenarios, mobile users are distributed in the network where some areas are concentrated more than others. Therefore, the adjustment of antenna parameters should yield a cellular layout that is compatible with the distribution of the user traffic in the network. In this work, a new mathematical model for user traffic is presented in coverage and capacity optimization. The antenna azimuth orientations and tilts are configured jointly for a predefined user traffic using an optimization procedure based on Taguchi's method applying nearly orthogonal array. The proposed model for user traffic is validated in Long Term Evolution downlink where results show that coverage and capacity are optimized and the resulting network layouts are fully compliant with the assumed user traffic distributions.**

***Index Terms*—Model for user traffic, LTE, joint optimization method, antenna parameters, Taguchi's method.**

I. Introduction

The primary aim of coverage and capacity optimization is to improve the wireless connections of cell edge user equipments (UEs) and increase, if possible, the throughputs of others. It has been shown in [1] that antenna parameters have a great impact on coverage and capacity of Long Term Evolution (LTE) networks. Therefore, the adjustment of the tilt of the antennas and their azimuth orientations has become a promising approach to increase the coverage and capacity.

In [2], the tilts and azimuth orientations of the antennas are optimized independently using an iterative procedure based on Taguchi's method that uses a so-called orthogonal array (OA) [3], [4] which is not to be confused with orthogonal antenna array. The optimization is carried out offline in a network planning environment. Moreover, in [5], a joint optimization of the antenna azimuth orientation and tilt is enabled by using a nearly orthogonal array (NOA) that offers more flexibility than an OA regarding the number of configuration parameters [6]. However, these two optimization methods do not consider a specific and given user traffic distribution in the network. Not considering this might yield a cellular layout that is incompatible with occurring user densities. For instance, the tilts of the antennas located in areas having high user densities are expected to have higher tilt values than those that are located in areas having low user densities and vice-versa. This result is not necessarily obtained if no particular assumption is made on user traffic distribution.

The main contribution of this work is developing a new mathematical model for a realistic user traffic in coverage and capacity optimization. In addition, we will show the degradation in the network performance if the occurring user densities are not considered in the optimization. The mathematical model is generic and can be applied to any optimization method. In this paper, the proposed approach is validated using the algorithm developed in [5] that configures the antenna azimuth orientation and tilt jointly. Moreover, we propose four optimization functions: Two optimization functions that are determined based on cell-specific performance measures and two others based on network-wide measures.

The paper is organized as follows. The mathematical model for user traffic and the four optimization functions are presented in section II. The system model of LTE downlink is discussed in section III. In section IV, the proposed approach is tested in LTE downlink mode. The paper is then concluded in section V.

II. A mathematical model for User Traffic

In this section, the mathematical model for user traffic in coverage and capacity optimization is explained in detail. We consider a pixel-based model for an LTE network consisting of C cells where each pixel represents a potential UE location.

A. Relative User Density Map

Besides the shadowing map, the network has an additional relative user density map. Every pixel $i = 1, \ldots, N_{\text{pixels}}$ with N_{pixels} being the total number of pixels in the network has a relative value d_i which describes how much its user density on average is greater than those of other pixels. For instance, Fig. 1 depicts an illustrative relative user density map of a network. The highest user density occurs in the center of the network (red color) and reduces gradually as we move away till we reach the areas with the lowest user densities (blue color) near the network borders. With properly planned enhanced Node B (eNodeB) locations, this user density map is compatible with the deployment scenario where eNodeBs

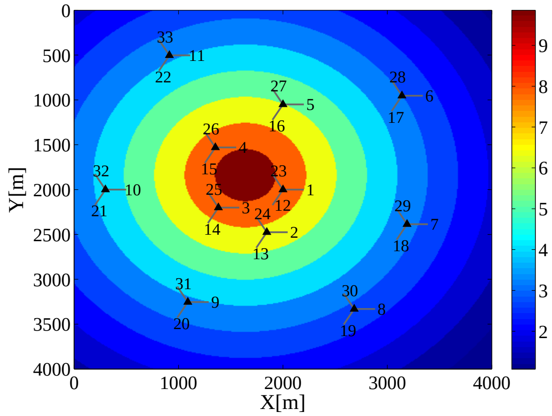


Fig. 1. An illustrative relative user density map of the network.

are close to each other in areas having high user densities and apart in the other areas having low user densities.

B. General Definitions

Let us denote the total number of UEs in the network by N_{users}. We define the average number of UEs per pixel i as

$$p_i = d_i \cdot \frac{N_{\text{users}}}{\sum_{i=1}^{N_{\text{pixels}}} d_i}. \tag{1}$$

This number can be considered as an average over time, e.g., week or month period. Typically, a pixel will actually be occupied by a UE only in a fraction of time. Hence, p_i is in general smaller than 1, i.e., $0 < p_i \leq 1$. We also denote the connection function that assigns each pixel i to a single cell $c = 1, \ldots, C$ based on the strongest reference signal received power (RSRP) level in downlink by $\mathrm{X}(i) = c$ [7].

The total number of pixels connected to a cell c, denoted by $\mathcal{A}_c$, is calculated as follows

$$\mathcal{A}_c = \sum_{i|\mathrm{X}(i)=c} 1. \tag{2}$$

$\mathcal{A}_c$ can be also thought of as the area covered by cell c. The number of UEs in cell c, denoted by N_c, is computed as

$$N_c = \sum_{i|\mathrm{X}(i)=c} p_i. \tag{3}$$

In this work, we assume a resource fair scheduler, and a full buffer traffic model for every UE, i.e, a UE has always data to transmit. Therefore, each UE connected to a cell c gets the same number of physical resource blocks (PRBs) computed as

$$N_{\text{PRB},c} = \min\left\{\frac{N_{\text{total}}}{N_c}, N_{\text{total}}\right\} \tag{4}$$

where N_{total} is the total number of PRBs for each sector. Typically, $N_c \geq 1$ is guaranteed if we have a reasonable number of N_{users} in the network.

C. Calculation of UE Throughput at a Pixel

In our network, we assume a full load system where a pixel receives interference from every other neighboring cell. This is a direct consequence of a resource fair scheduler with a sufficiently large number of UEs. The signal-to-interference-noise-ratio (SINR) at a pixel i is denoted by SINR_i. Having the SINR calculated at each pixel, the throughput R_i of a UE located at pixel i can be approximated using Shannon's equation as

$$R_i = N_{\text{PRB},c} \cdot \left[W_{\text{eff}} \cdot B \cdot \log_2\left(1 + \frac{\text{SINR}_i}{S_{\text{eff}}}\right)\right] \tag{5}$$

where W_{eff} and S_{eff} are the bandwidth and SINR efficiency factors [8], respectively, and B is the bandwidth occupied by one PRB in kHz.

D. Optimization Functions Based on Cell-Specific Measures

In this subsection, two optimization functions based on cell-specific performance measures are presented. The user traffic distribution in the network is considered implicitly in the definitions of the optimization functions.

1) Optimization Function Based on Cell-Edge Throughput: The first optimization function is based on the calculation of a percentile level of the UE throughput distribution in a cell. Each pixel i in the network has a different average number of users p_i. In order to have a fair performance evaluation, the weighted x percentile (x%-tile) of the UE throughput distribution in a cell c, denoted by $\delta_{c,x}$, is computed. This can be done by plotting first the weighted cumulative distribution function (WCDF) of $R_{i|X(i)=c}$ using $p_{i|X(i)=c}$ as a weight and then computing the x%-tile.

$\delta_{c,x}$ is used in this work to evaluate the performance of the UEs in a cell c. The value of x has a prominent role in steering the optimization toward coverage or capacity maximization. If a low value of x is chosen, more emphasis is given to the performance of the cell edge UEs and the optimization aims primarily at increasing the network coverage. On the other hand, a high value of x lessens the impact of the performance of cell edge UEs and the optimization aims at maximizing the network capacity. In this work, we will set x to 5 and 25 and compare their performance in the simulation result section.

Among the antenna configuration parameters, there exist some interactions. For instance, adjusting the tilt or the azimuth orientation of sector's antenna j does not only affect $\delta_{j,x}$ but also the performance measures $\delta_{c\neq j,x}$ of all its neighbors. To account for the interactions existing among the azimuth orientations and tilts of different sectors [2], the performance measures of all cells are bundled into one optimization function.

The definition of the optimization function is the key to achieve the desired network performance. The aim of the optimization is to improve the performance measure $\delta_{c,x}$ for each cell c while keeping fair user experience (outage probabilities) among cells. The intention is to avoid solutions that improve the performance in some cells on the expense of others. Moreover, it is necessary to apply different weights for the cells as those having higher number of UEs than others should have more impact on the overall optimization function. This weighting is essential to have a fair overall performance evaluation. For these reasons, the first optimization function is defined to be the weighted harmonic mean (WHM) of $\delta_{c,x}$, i.e.,

$$\text{WHM}(\delta_{c,x}) = \frac{N_{\text{users}}}{\sum_{c=1}^{C} \frac{N_c}{\delta_{c,x}}}. \tag{6}$$

The choice of WHM rather than weighted arithmetic mean is because WHM mitigates more the impact of outliers. The WHM aggravates the impact of small $\delta_{c,x}$ values and lessens the impact of large ones, which in-turn provides a more homogeneous user experience in the network. The usage of harmonic mean rather than arithmetic mean has already been discussed more extensively in [2].

2) Optimization Function Based on Average UE Throughput: Instead of computing a percentile level of the UE throughput distribution in a cell, we calculate the average of the UE throughput in a cell c, denoted by λ_c, as

$$\lambda_c = \frac{\sum_{i|\text{X}(i)=c} p_i \cdot R_i}{N_c}. \tag{7}$$

As in section II-D1, the second optimization function is defined by taking the WHM of λ_c as

$$\text{WHM}(\lambda_c) = \frac{N_{\text{users}}}{\sum_{c=1}^{C} \frac{N_c}{\lambda_c}}. \tag{8}$$

In this case, cells having higher number of UEs impact the overall optimization function more than others.

E. Optimization Functions Based on Network-Wide Measures

We define two other optimization functions that are determined using network-wide measures in contrast to those presented in II-D.

1) Percentile-Based Optimization Function: The third optimization function β_x is the $x\%$-tile level of the UE throughput distribution in the network and not specifically in a cell as in II-D1. β_x is computed by plotting the WCDF of $R_{i|\forall i}$ using $p_{i|\forall i}$ as a weight and then taking the $x\%$-tile level.

2) WHM of UE Throughput in the Network: The WHM of UE throughput in the network denoted by γ is computed as

$$\gamma = \frac{N_{\text{users}}}{\sum_{i=1}^{N_{\text{pixel}}} \frac{p_i}{R_i}}. \tag{9}$$

Thus, R_j of a UE located at pixel j has more impact on the optimization function than R_k of a UE located at pixel k if $p_j > p_k$.

III. LTE System Model

In the section, the system model for LTE downlink is presented along with the simulation parameters. The cellular network is composed of $C = 33$ cells located in an area of 4×4 km, see Fig. 1. The eNodeB positions follow the proposal of [9]. Every cell c is served by one of the three sectors of an eNodeB. The maximum eNodeB transmission power is 40 W or equivalently 29 dBm per PRB, i.e., 10 MHz system bandwidth with $N_{\text{total}} = 50$ PRBs. The path loss offset and exponent are set to 128.1 dB and 3.76, respectively. The penetration loss is assumed to be 20 dB and the thermal noise power is -114 dBm on a single PRB. The standard deviation of the shadowing is set to 8 dB and the decorrelation distance to 50 m. The transmit antennas of eNodeBs are mounted at height $h_{\text{BS}} = 30$ m whereas a pixel is assumed to be located at ground, i.e., UE height is zero. Moreover, the pixels are separated from each other by a distance of 10 m, and W_{eff}, S_{eff} and B are set to 0.88, 1.25 and 180 kHz, respectively. The transmit antenna pattern of an eNodeB is modeled in 3-dimensions (3-D). It is approximated using the model defined in [10] by summing up the azimuth and vertical patterns. In addition, the antenna parameters are according to those recommended by 3GPP in [7], [11].

IV. Simulation Results

In this section, the antenna tilts and azimuth orientations maximizing each optimization function are obtained using the iterative algorithm proposed in [5]. Moreover, we will apply these optimized antenna settings to the network and compare the resulting network layouts assuming a non-uniform user traffic as in Fig. 1, a uniform user traffic and an unspecific user traffic where the number of UEs per cell is assumed to be constant irrespective of the cell size (see subsection IV-C3). In all simulations, the total number of UEs in the network is assumed to be $N_{\text{users}} = C \cdot 10 = 330$ UEs.

A. Evaluation Methodology

For coverage evaluation, we plot the WCDF of the 5%-tile $\delta_{5\%}$ of the UE throughput distribution in a cell. Therefore, $\delta_{j,5\%}$ of cell j has more weight than $\delta_{k,5\%}$ of cell k if $N_j > N_k$. Similarly, for capacity evaluation, we plot the WCDF of the mean λ of the UE throughput in a cell.

B. Impact of the Optimization Function on the Performance

In this subsection, we compare the network performance for different optimization functions assuming the non-uniform user traffic depicted in Fig. 1. The WCDF of $\delta_{5\%}$ is shown in Fig. 2 for different optimization functions. It can be observed that $\text{WHM}(\delta_{c,5\%})$ achieves the best performance in terms of coverage. This is because $\text{WHM}(\delta_{c,5\%})$ reflects more the performance of the cell edge UEs than others. Moreover, we notice that γ yields a slight degradation in performance if compared to $\text{WHM}(\delta_{c,5\%})$. In constrast, $\text{WHM}(\lambda_c)$ leads to a degradation in coverage performance if compared to others as it aims primarily at improving the capacity rather than coverage.

In Fig. 3, the WCDF of λ is shown for different optimization functions. According to the figure, WHM(λ_c) yields the best performance in terms of capacity whereas WHM$(\delta_{c,5\%})$ leads to a capacity loss if compared to others. The other optimization functions have almost the same performance and their WCDFs lie between those of WHM$(\delta_{c,5\%})$ and WHM(λ_c). Thus, the definition of the optimization function has a key role in steering the optimization toward coverage or capacity maximization.

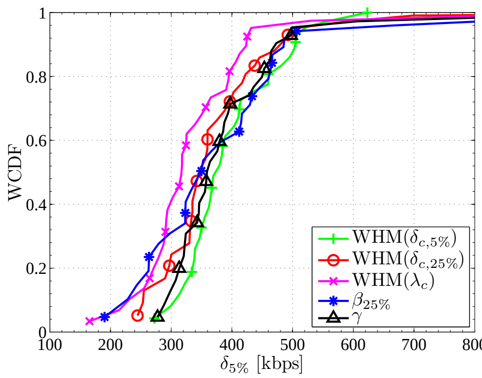


Fig. 2. WCDF of $\delta_{5\%}$ assuming a non-uniform user traffic for different optimization functions.

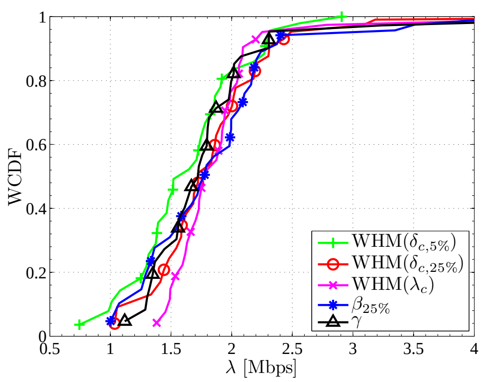


Fig. 3. WCDF of λ assuming a non-uniform user traffic for different optimization functions.

C. Network Layouts

The network layout resulting from applying the optimized azimuth orientations and tilts is depicted in Fig. 4 without considering shadowing for the optimization function WHM$(\delta_{c,25\%})$ with the user traffic assumption as a parameter. The network layouts of other optimization functions are not shown as they all have the same trends explained in the following.

1) Non-Uniform User Traffic: The network layout obtained by assuming the non-uniform user traffic depicted in Fig. 1 is shown in Fig. 4(a). It can be seen that the area covered by a cell gradually increases as we move away from the center of the network. Indeed, this network layout is fully compliant with the assumed non-uniform user traffic. Hence, we are able using the mathematical approach to optimize the azimuth orientations and tilts jointly taking into account the user traffic distribution in the network.

2) Uniform User Traffic: A uniform user traffic corresponds to the case where each pixel in the network has the same relative user density, i.e., $d_i = 1 \ \forall i$. This uniform user traffic assumption does not really comply with the deployment scenario of eNodeBs which inherently assumes a user traffic model similar to Fig. 1. The resulting network layout is depicted in Fig. 4(b). According to the figure, the inner cells have smaller coverage areas in general if compared to others. Moreover, the network layout resembles that obtained assuming a non-uniform user traffic.

3) Unspecific User Traffic: We refer to unspecific user traffic in the case where we inherently assume that each cell has the same number of UEs irrespective of the cell size and all pixels of a cell c have the same average number of UEs, i.e., $p_i = p_c \ \forall i|\mathrm{X}(i) = c$. Using the first assumption, we can express the number of UEs in cell c as

$$N_c = \sum_{i|\mathrm{X}(i)=c} p_i = \frac{N_{\text{users}}}{C} = N \ \forall \ c. \tag{10}$$

By using the second assumption, the average number p_c of UEs per pixel can be derived as

$$\sum_{i|\mathrm{X}(i)=c} p_i = p_c \cdot \sum_{i|\mathrm{X}(i)=c} 1 \quad = \quad \frac{N_{\text{users}}}{C} \tag{11}$$

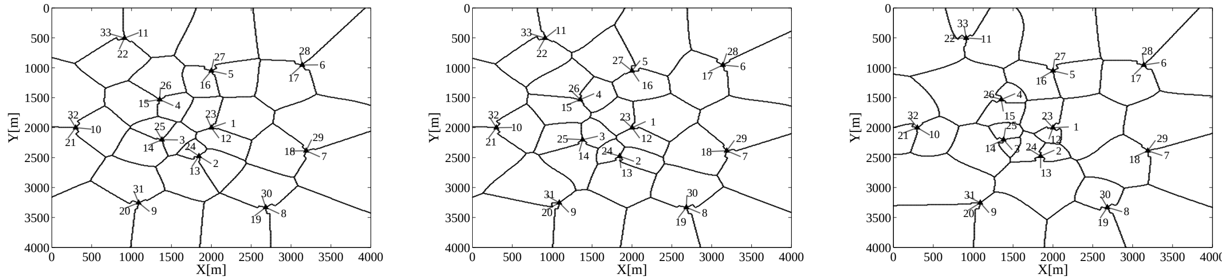
$$p_c \quad = \quad \frac{N_{\text{users}}}{C \cdot \mathcal{A}_c}. \tag{12}$$

According to Eq. (12), p_c is inversely proportional to the area $\mathcal{A}_c$. The obtained network layout is shown in Fig. 4(c). We can notice that the network layout is incompatible with the deployment scenario of eNodeBs. Some sectors serve relatively small areas where they are expected to cover much larger area and vice-versa. This is because we are inherently assuming that each cell has the same number N of UEs. For clarity, consider two cells k and j having the same number of UEs. Using Eq. (12), the relationship among p_k, p_j, $\mathcal{A}_k$ and $\mathcal{A}_j$ can be derived as

$$\frac{p_k}{p_j} \quad = \quad \frac{\mathcal{A}_j}{\mathcal{A}_k}. \tag{13}$$

According to Eq. (13), if $\mathcal{A}_j < \mathcal{A}_k$, then we are inherently assuming that $p_j > p_k$ and vice-versa. Therefore, cells having small coverage areas in Fig. 4(c) are assumed to have higher user densities than those having large coverage areas. This situation can be avoided by taking into consideration the occurring user densities in the network.

To visualize the impact of assuming a uniform or unspecific user density on the network performance, their corresponding optimized antenna azimuth orientations and tilts are applied to the network having non-uniform user density depicted in Fig. 1. The WCDFs of $\delta_{5\%}$ and λ are shown in Fig. 5 and Fig. 6, respectively, for the optimization function



(a) Non-uniform user traffic (b) Uniform user traffic (c) Unspecific user traffic

Fig. 4. Network layout obtained by using WHM($\delta_{c,25\%}$) as an optimization function with the user traffic assumption as a parameter.

WHM($\delta_{c,25\%}$) with the user traffic assumption as a parameter. According to the figures, it can be observed that the optimized antenna azimuths and tilts obtained by assuming a uniform and, in particular, unspecific user density lead to a degradation in network performance if compared to those obtained by assuming a non-uniform user density. Thus, considering the user densities is essential in coverage and capacity optimization to avoid any degradation in network performance that may result from a possible mismatch between the assumed and occuring user traffic distributions.

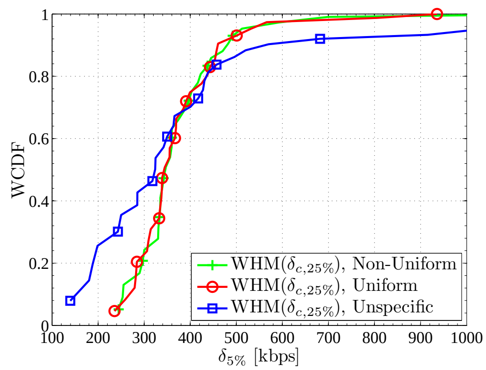


Fig. 5. WCDF of $\delta_{5\%}$ for the optimization function WHM($\delta_{c,25\%}$) with the user traffic assumption as a parameter.

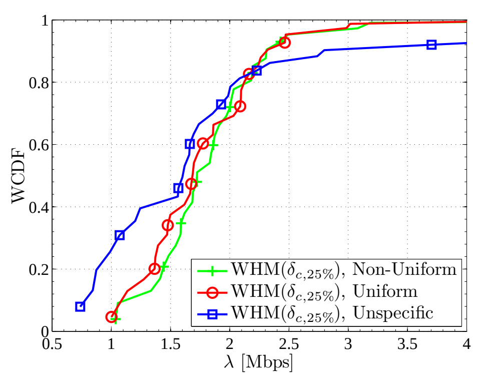


Fig. 6. WCDF of λ for the optimization function WHM($\delta_{c,25\%}$) with the user traffic assumption as a parameter.

V. Conclusion

In this paper, a mathematical model for user traffic is presented in coverage and capacity optimization. The model is validated by configuring the antenna azimuth orientations and tilts jointly for different optimization functions and user traffic assumptions. LTE downlink simulations show that, by using the mathematical model, the antenna parameters are adjusted such that the coverage and capacity are optimized, and the cellular layout is fully compliant with the assumed user traffic. If no particular assumption is made on the user traffic, we may get a degradation in the network performance due to the mismatch between the assumed and occurring user densities.

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