Anam Tahir, Hussein Al-Shatri, Kilian Kiekenap and Anja Klein, ”Ultra-Reliable Low Latency Communication for Consensus Control in Multi-Agent Systems,” in *Proc. of the IEEE Wireless Communications and Networking Conference (WCNC)*, April 2019.

# Ultra-Reliable Low Latency Communication for Consensus Control in Multi-Agent Systems

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***Abstract*—A scenario of multiple agents working together to accomplish a common task is considered. Consensus control facilitates the coordination among the agents over time till they accomplish the task. In this paper, we consider formation control using consensus in which agents coordinate to form a circle. To make the coordination possible, agents need to periodically exchange their positions using orthogonal transmissions through a band-limited wireless channel which encounters different transmission delays on different links. To guarantee over all agents stability and convergence, we optimize the bandwidth allocation for equal rate transmission, which maintains the synchronization among agents. Moreover, we minimize the convergence time by optimizing the weights of the consensus algorithm. An ultra-reliable low latency communication between agents is guaranteed by transmitting short packets. The simulation results show that jointly optimizing the confidence weights and bandwidth allocation greatly reduces the convergence time as compared to conventional schemes.**

***Index Terms*—multi-agent systems, consensus control, URLLC, radio resource allocation, confidence weights.**

## I. Introduction

The field of multi-agent systems (MASs) is an emerging research direction which aims at bringing individual autonomous agents into a coordinated and synchronized system [1], [2]. Future systems define many use cases where MAS problems are the main challenge. For instance, the internet of things (IoT) considers a huge number of smart agents and the system necessitates some sort of coordination among the different agents in which distributed coordination is suitable [3], [4]. Industry 4.0 defines the concept of smart factories in which several stages of the production line coordinate with each other to ensure system stability and enhance the production efficiency [5]. In the fifth generation cellular systems (5G), car to car communications with autonomous driving is one of the main targeted use cases. In this use case, agents, i.e., autonomous cars, need to coordinate, negotiate and decide in real time for a safe, smooth and efficient traffic system [6]. Additionally, there are many different use cases of MAS for security, privacy, and public safety [4], [7].

Basically, consensus control in MAS means that all agents in the system reach an agreement regarding a quantity of interest, i.e., state of the agents [8]. In other words, a consensus algorithm is a distributed algorithm which defines the interaction rules that specify the information to be exchanged among agents to reach a common goal [8]. This means that the agreed quantity is not pre-calculated, but reached through a distributed negotiation process among the agents. To reach a consensus, agents have to be connected in a single graph [8]. In every step of the consensus algorithm, every agent calculates its new quantity of interest as a weighted average of its previous quantity and the quatities received from its neighbors. This way, the dynamics of this time discrete control system can be modelled using a system matrix with elements representing the confidence weights of every agent's quantity and the quantities received from other neighbouring agents [8]. Basically, a confidence weight assigned by an agent, e.g., position information received from its neighbour, determines how much this agent will consider the position of its neighbour in the next movement. The second largest eigenvalue of the system matrix is called algebraic connectivity of a graph [9] and it is a measure of the speed of convergence of a consensus algorithm, i.e., the smaller the eigenvalue, the faster the convergence to a consensus [10]. In consensus algorithms, it is mainly assumed that agents are synchronized in exchanging information and in taking decisions. However in reality, agents are exchanging information through a time variant radio channel, and thus, different transmission delays may occur on different links which may affect the system stability and convergence.

In consensus control literature, communication among agents is usually considered as a black box in which the consensus is investigated under abstract channel models, i.e., no sophisticated communications models are assumed considering for instance, different transmission modes, multiple antennas, power control, subcarrier allocation and/or orthogonal/non-orthogonal channel access. In particular, there are papers which abstractly model the communication delay and investigate the consensus. For instance, the authors of [11] studied the effect of communication delay on the consensus algorithm. They analyzed the impact of both time variant and invariant communication delays on the convergence rate of a consensus algorithm. In [12], a discrete time invariant MAS with equal constant delays on all links is considered. It was shown that the smaller the delay, the smaller the second eigenvalue of the system matrix, which results in faster convergence to a consensus. In [13], the influence of communication delay on the stability of consensus in MAS was studied assuming a constant communciation delay. The authors employed the frequency approach and Lyapunov-Krasovskii technique to study the stability and prove that the system remains stable, but the final agreed quantity may change due to the communication delay. Authors of [14] assumed an upper bound of the communication delay on all links and accordingly derived an upper bound on the convergence time. This bound is an explicit function of the delay's upper bound, system parameters and connectivity graph. The authors of [15] focused on the integrator dynamic model at agents. They studied both complete and loop shaped graphs and assumed a symmetrical communication delay between neighbouring agents in the system. Based on this, they derived the maximum allowable delay such that the agents can still converge to a consensus for both time variant and invariant delays. Furthermore, they investigated the cases where the delays in all links are either equal or unequal.

On the contrary, research in communications focuses on achieving ultra-reliable low latency communications (URLLC) without considering the dynamics of the MAS. In particular, URLLC has many potential use cases in automation, rail and car to car communications [16], [17]. In URLLC, the objective is to achieve a highly reliable communication link with a significantly low bit error rate (BER) between $10^{-5}$ and $10^{-7}$. At the same time, the maximum latency requirement of the communication is tight, i.e. typically around $0.25$ – $0.3$ ms/packet [18]. To achieve the requirements of low latency and high reliability for URLLC, new protocols, short frame/packet structure and algorithms for fast baseband signal processing with minimum signalling need to be developed [16], [19], [20].

In this paper, we aim at modelling URLLC considering the dynamics of MAS for a consensus control. In other words, a joint modelling and optimization of communication and consensus control is the main contribution of this work. To elaborate this, we study the formation control problem using consensus in which agents want to form a circle. Our contributions can be summarized as follows:

- We write an extended system matrix of the dynamics which includes the impact of allocated bandwidth of every link.
- To guarantee stability and convergence [12], we optimize the bandwidth allocation for uniform delay over links in every time slot.
- We optimize the confidence weights, i.e., the entries of the extended system matrix, such that the consensus convergence time is minimized.
- We analyse the performance of jointly optimizing both communication and consensus control parameters against optimizing either one of them only and show the potential performance gain by considering both fields in a single model.

This paper is organized as follows. In Section II, the system model is discussed which includes the consensus model and the communication model. In Section III, the problem is stated and the extended system matrix is derived. Section IV explains our proposed algorithm for minimizing the convergence time. Simulation results are shown and discussed in Section V. In Section VI, the conclusions are drawn.

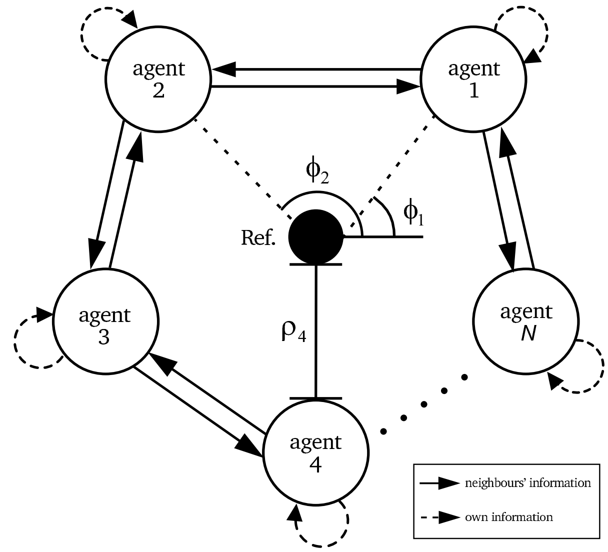



Fig. 1: A MAS scenario with $N$ agents forming a circle.

## II. System Model

In this section, the joint model of the communication and consensus dynamics is introduced. A scenario consisting of $N$ autonomous agents with equal capabilities is considered. Initially, agents are randomly located in an area and they need to coordinate and move to form a circle around a common reference point. It is assumed that every agent knows its location with respect to a reference point and it moves in a two-dimensional plane. Moreover, reliable communication can be established all the time among neighbour agents. The model of the dynamics for this MAS scenario is time discrete, which means that an agent $i$ exchanges information with its neighbours, takes a decision and moves in every time slot $k$. In every time slot $k$, every agent $i$ knows its position in two-dimensional polar coordinates $(\rho_i(k), \phi_i(k))$ where $\rho_i(k)$ is the distance of agent $i$ to the reference point at time slot $k$ and $\phi_i(k)$ is the angle with respect to the reference angle at time slot $k$.

Fig. 1 shows how $\rho_i(k)$ and $\phi_i(k)$ are calculated. It is assumed that agents are numbered based on their initial angles $\phi_i(0)$, $\forall i$ to the reference angle, see Fig. 1. For instance, agent $i$ with the smallest initial position angle is labelled as agent $i = 1$ and agent $i = N$ is the agent with the largest initial position angle. Since agents aim at forming a circle around the reference point, they communicate with each other using the cyclic pursuit strategy [21]. This strategy defines the neighbours of each agent from which they will receive the data. In our system model, each agent $i$ has two neighbours, agent $i-1$ and agent $i+1$. Since agents aim at forming a circle, the first and last agents are neighbours. Let $\mathcal{N}_i$ denote the set of neighbours of agent $i$. This way, a single connected graph is defined initially and this graph remains the same till the convergence. Accordingly, agent $i$ can communicate with its neighbour agents $j \in \mathcal{N}_i$ all the time, till they reach a consensus. Agent $i$ calculates the Euclidean distance to its neighbour $j \in \mathcal{N}_i$ using

$$d_{ij} = \sqrt{\rho_i^2 + \rho_j^2 - 2\rho_i\rho_j \cos\left(\phi_i - \phi_j\right)}. \tag{1}$$

### A. Communication Model

It is assumed that each agent is equipped with a wireless transceiver in which the transmission and reception are accomplished simultaneously in different disjoint frequency bands. Furthermore, every agent $i$ transmits to its two neighbours in $\mathcal{N}_i$ using two simultaneous unicast transmissions in different frequency bands, and thus, the interference between different links is avoided. This means, all agents can transmit simultaneously using frequency division multiple access (FDMA) mode with a total of $2N$ orthogonal transmissions. Let $B_{ij}(k)$ be the bandwidth reserved for the transmission from agent $j$ to agent $i$ in time slot $k$. The total bandwidth over all concurrent transmissions is upper bounded as

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} B_{ij}(k) \leq B_{\text{tot}}. \tag{2}$$

Let $h_{ij}(k) \in \mathbb{C}$ denote the channel coefficient between the transmit agent $j$ and the receive agent $i$ in time slot $k$. Thus, the channel gain of this link is calculated as $g_{ij}(k) = |h_{ij}(k)|^2$. Throughout the paper, we will keep the time slot index $k$ if the formula contains different time slots and omit it elsewhere. The receiver noise at an agent $i$ is modelled as additive white Gaussian noise with zero mean and variance $N_o B_{ij}$ where $N_o$ denotes the single sided noise density per Hertz. Let $p_{ij}$ be the transmit power of agent $j$ to transmit through the link to agent $i$, $\forall i, j \in \mathcal{N}_i$. In URLLC, it is assumed that short packets will be employed to transmit the position information to neighbours with frame duration $D$ in seconds. The achieved data rate at agent $i$ for the transmission of agent $j$ is calculated according to [22] as

$$R_{ij} = \frac{B_{ij}}{\ln(2)}\left(\ln\left(1 + \frac{g_{ij}p_{ij}}{N_o B_{ij}}\right) - \sqrt{\frac{V_{ij}}{DB_{ij}}}Q^{-1}(\epsilon)\right), \tag{3}$$

where $\epsilon$ is the target bit error probability, $Q^{-1}(.)$ is the inverse Gaussian-Q function and $V_{ij}$ is the channel dispersion which is calculated as

$$V_{ij} = 1 - \frac{1}{\left(1 + \frac{g_{ij}p_{ij}}{N_o B_{ij}}\right)^2}. \tag{4}$$

The transmission delay, which is defined as the time duration needed for a packet of a fixed size $b$ to be transferred from agent $j$ to agent $i$, is calculated as

$$\tau_{ij} = \frac{b}{R_{ij}}. \tag{5}$$

Since it is assumed that there is always a reliable communication link among neighbour agents, i.e., $R_{ij} > 0$, the communication delay is finite, i.e., $\tau_{ij} < \infty$. Using the time slot duration $T$, the delay $\tau_{ij}$ can be expressed in terms of the number $m_{ij}$ of time slots as

$$m_{ij} = \left\lceil \frac{\tau_{ij}}{T} \right\rceil. \tag{6}$$

### B. Consensus Model

Agents use consensus laws to form a circle. Basically, these laws are a predefined guideline which helps agents to decide where they have to move in the next time slot $k+1$. At every time slot $k$, each agent $i$ knows its own current position $(\rho_i(k), \phi_i(k))$ and positions of its neighbours with some communication delay $\tau_{ij}$. Let $\gamma$ be the step size which is a scaler multiplied by the change of the position between the current and next time slot. So, it represents how much an agent will change it position inthe next time slot. Basically, $0 < \gamma < 1/\Delta$ where $\Delta$ is the maximum degree of a network graph of the MAS. Basically, the maximum degree of a graph is the maximum number of edges incident to every vertex, i.e., $\Delta = 2$ in our case because every agent has two neighbours. Moreover, $a_{ij}$ is a weighting factor given by agent $i$ to the position of its neighbouring agent $j$ for $j \neq i$. The new position of agent $i$ in time slot $k+1$ equals the current position of node $i$ in time slot $k$ plus a weighted sum of the difference of the current position of node $i$ and the outdated positions of its neighbours $j$, $\forall j \in \mathcal{N}_i$. This weighted sum of differences is scaled by the step size $\gamma$. Mathematically, every agent $i$ decides on its position in the next time slot $k+1$ as follows: the distance to the reference point is updated as

$$\rho_i(k+1) = \rho_i(k) + \gamma\left(\sum_{j\in\mathcal{N}_i} a_{ij}\left(\rho_j\left(k - m_{ij}\right) - \rho_i\left(k\right)\right)\right), \tag{7}$$

and the angle with respect to the reference angle is updated as

$$\phi_i(k+1) = \phi_i(k) + \gamma\left(\sum_{j\in\mathcal{N}_i} a_{ij}\left(\phi_j(k - m_{ij}) - \phi_i(k)\right)\right). \tag{8}$$

Based on this, consensus is achieved when all $\rho_i(K)$, $\forall i$ are equal and $|\phi_i(K) - \phi_j(K)| = \frac{2\pi}{N}$, $\forall i, j \in \mathcal{N}_i$ where $K$ is the time slot index at convergence, hence, agents are at equal distance from the reference point and evenly spaced on a circle.

## III. Extended System Matrix

In this section, we will derive an extended system matrix of the dynamics which represents the impact of optimizing the confidence weights and bandwidth allocation on the dynamics of MAS. In fact, consensus algorithms are proposed for MAS in which the communication delay is given and cannot be adapted. Moreover, uniform delay among all links has the properties of maintaining stability and guaranteeing convergence [10], [12]. Accordingly, we optimize the bandwidth allocation such that uniform delay in all links is achieved, i.e., $\tau(k) = \tau_{ij}(k)$, $\forall i, j \in \mathcal{N}_i$. This way, agents are synchronized in every time slot with equal delay $\tau$ in seconds or $m$ in time slots. For the rest of the paper, we assume that agents are synchronized by optimizing the bandwidth allocation and thus, we denote the delay by $m$ with no indices. Let the MAS introduced in Section II be modelled as an undirected graph $\mathrm{G} = (\mathcal{V}, \mathcal{E}, \mathbf{A})$, where the vertices $\mathcal{V} = \{v_1, \ldots, v_N\}$ represent the set of agents, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the graph connectivity among the agents where $e_{ij}$ denotes the edge between agent $i$ and agent $j$. Finally, $\mathbf{A} = [a_{ij}]_{N\times N}$ is the adjacency matrix representing the weighting factors among the agents such that

$$a_{ij} = \begin{cases} 0, & \text{if } e_{ij} \notin \mathcal{E} \\ > 0, & \text{if } e_{ij} \in \mathcal{E}. \end{cases} \tag{9}$$

Because the communication between neighbouring agents is bidirectional, the graph G is modelled undirected. The degree matrix $\mathbf{D} \in \mathbb{R}^{N\times N}$ of the graph G is a diagonal matrix [8] where diagonal elements indexed $i = 1, \ldots, N$ equal $d_i = \sum_{j\neq i} a_{ij}$, $\forall i$, which are the degree of each vertex in the network graph. Accordingly, the Laplacian matrix [8] is calculated as $\mathbf{L} = \mathbf{D} - \mathbf{A}$. In other words, every element in $\mathbf{L}$ equals

$$l_{ij} = \begin{cases} \sum\limits_{j\neq i} a_{ij}, & \text{if } i = j \\ -a_{ij}, & \text{if } i \neq j. \end{cases} \tag{10}$$

Let $\boldsymbol{\rho}^{(i)}(k-m)$ be a vector containing $\rho_i(k)$ as the $i$-th element and $\rho_j(k-m)$, $j \neq i$ elsewhere. Similarly $\boldsymbol{\phi}^{(i)}(k-m)$ denotes a vector with $\phi_i(k)$ in the $i$-th element and $\phi_j(k-m)$ for $j \neq i$. Using the Laplacian matrix, the discrete time consensus model defined in (7) and (8) can be rewritten as

$$\begin{aligned} \rho_i(k+1) &= \mathbf{W}\ \boldsymbol{\rho}^{(i)}(k-m), \\ \phi_i(k+1) &= \mathbf{W}\ \boldsymbol{\phi}^{(i)}(k-m), \end{aligned} \tag{11}$$

where $\mathbf{W} = \mathbf{I}_N - \gamma\mathbf{L}$ and $\mathbf{I}_N$ is the identity matrix with size $N \times N$. Since we aim at optimizing the confidence weights in every time slot, the system matrix $\mathbf{W}(k)$ is time variant. Furthermore, the system matrix $\mathbf{W}(k)$ is of size $N \times N$ and has the following structure: the diagonal elements are the self confidence weights $w_{ii}$, $\forall i$ whereas the off-diagonal elements are the confidence weights of the information received from the neighbours. Since the graph G is undirected, balanced, circular and fixed, two off-digonal elements in every row of $\mathbf{W}(k)$ are nonzero which corresponds to the links to the two neighbours. Moreover, the system matrix $\mathbf{W}(k)$ is a non-negative, row stochastic matrix which means that every row-sum of this matrix equals 1, and thus, this matrix has a trivial eigenvalue of 1 and all the eigenvalues are in a unit circle. Because the graph G is balanced, the system matrix is doubly stochastic [8]. With a system matrix of the dynamics, the convergence speed can be optimized. However, the system matrix $\mathbf{W}(k)$ of the dynamics does not include the communication delay.

Let us assume that in every time slot $k$, the delay can be $m^{(0)}$ for no delay and up to $m^{(T)}$ for maximum delay. Then, an extended system matrix of the dynamics with communication delay can be written as

$$\boldsymbol{\Theta}(k) = \begin{pmatrix} \mathbf{W}_\mathrm{d}(k) + \mathbf{W}_\mathrm{od}(k, m^{(0)}) & \mathbf{W}_\mathrm{od}(k, m^{(1)}) & \cdots & \mathbf{W}_\mathrm{od}(k, m^{(T)}) \\ \mathbf{I}_N & \mathbf{0}_N & \cdots & \mathbf{0}_N \\ & \ddots & \ddots & \vdots \\ \mathbf{0}_N & \cdots & \mathbf{I}_N & \mathbf{0} \end{pmatrix} \tag{12}$$

where $\mathbf{0}_N$ is an $N \times N$ zero matrix and $\mathbf{W}_\mathrm{d}(k)$ is a diagonal matrix whose diagonal elements are those of $\mathbf{W}(k)$ and

$$\mathbf{W}_\mathrm{od}(k, m^{(t)}) = \begin{cases} \mathbf{W}(k) - \mathbf{W}_\mathrm{d}(k), & \text{if } m^{(t)} = m_\mathrm{opt} \\ \mathbf{0}_N, & \text{otherwise,} \end{cases} \tag{13}$$

where $m_\mathrm{opt}$ is the optimum minimum delay at all links and $t = 0, 1, 2, \ldots, T$ is the delay index. With $\boldsymbol{\rho}(k) = (\rho_1(k), \ldots, \rho_N(k))^\mathrm{T}$ and $\boldsymbol{\phi}(k) = (\phi_1(k), \ldots, \phi_N(k))^\mathrm{T}$, the system dynamics in (11) can be written as

$$\begin{aligned} \boldsymbol{\rho}^\mathrm{ex}(k+1) &= \boldsymbol{\Theta}(k)\ \boldsymbol{\rho}^\mathrm{ex}(k), \\ \boldsymbol{\phi}^\mathrm{ex}(k+1) &= \boldsymbol{\Theta}(k)\ \boldsymbol{\phi}^\mathrm{ex}(k), \end{aligned} \tag{14}$$

where $\boldsymbol{\rho}^\mathrm{ex}(k) = \left(\boldsymbol{\rho}(k)^\mathrm{T}, \ldots, \boldsymbol{\rho}(k-m)^\mathrm{T}\right)^\mathrm{T}$ and $\boldsymbol{\phi}^\mathrm{ex}(k) = \left(\boldsymbol{\phi}(k)^\mathrm{T}, \ldots, \boldsymbol{\phi}(k-m)^\mathrm{T}\right)^\mathrm{T}$.

The extended system matrix $\boldsymbol{\Theta}(k)$ of the dynamics is a $N(m^{(T)}+1) \times N(m^{(T)}+1)$ row stochastic and primitive matrix, and hence, the dynamics in (14) will converge to a consensus [12]. However, $\boldsymbol{\Theta}(k)$ is not a doubly stochastic matrix [8], therefore, it will not converge to the average of the initial states [12], i.e., $\rho_i(K) \neq \sum_{i=1}^{N} \rho_i(0)/N$. The overall dynamics can be written as

$$\begin{aligned} \boldsymbol{\rho}^\mathrm{ex}(K) &= \prod_{k=0}^{K} \boldsymbol{\Theta}(k)\ \boldsymbol{\rho}^\mathrm{ex}(0), \\ \boldsymbol{\phi}^\mathrm{ex}(K) &= \prod_{k=0}^{K} \boldsymbol{\Theta}(k)\ \boldsymbol{\phi}^\mathrm{ex}(0). \end{aligned} \tag{15}$$

The second largest eigenvalue modulus (SLEM) of the product $\prod_{k=0}^{K} \boldsymbol{\Theta}(k)$ of the extended system matrices determines the convergence speed [8]. In other words, minimizing SLEM of $\prod_{k=0}^{K} \boldsymbol{\Theta}(k)$ will minimize the convergence time.

## IV. Minimizing Convergence Time

Since agents know only causal information of channel gains and positions but not the future ones, the bandwidth allocation and confidence weights are optimized on a time slot by time slot basis. In every time slot $k$, the convergence time minimization problem with the variables of bandwidths $B_{ij}(k)$ and confidence weights $w_{ij}$, $\forall i, j \in \mathcal{N}_i$, will be solved such that the SLEM of $\prod_{l=0}^{k} \boldsymbol{\Theta}(l)$ is minimized. Since the SLEM of a row stochastic matrix is always less than or equal to 1, the SLEM of $\prod_{l=0}^{k} \boldsymbol{\Theta}(l)$ is monotonically non-increasing with increasing $k$.

As described in the previous section, uniform delay among agents maintains the stability and guarantees the convergence of consensus in MAS [12]. Thus, we solve the convergence time minimization problem in two steps. First, the bandwidth allocation is optimized such that the transmissions among neighbouring agents will experience equal delays, i.e., the communication among agents is synchronized. Second, the confidence weights in the resulting extended system matrix $\tilde{\mathbf{\Theta}}(k)$ with minimum delay are optimized for the minimum SLEM of $\tilde{\mathbf{\Theta}}(k)$.

### A. Bandwidth Allocation

Since it is assumed that the packet sizes $b$ are equal, finding equal minimum delay is equivalent to finding equal maximum data rates, see (5). Therefore, the optimization problem for optimizing the bandwidth allocation can be stated as

$$\underset{B_{ij}}{\text{argmax}} \left\{ \min_{i,j\in\mathcal{N}_i} R_{ij}(B_{ij}) \right\} \tag{16}$$

subject to

$$\sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} B_{ij} \leq B_{\text{tot}}. \tag{17}$$

Since the solution of this problem leads to equal rates at all links, this problem can be solved using the bisection method:

1: Initialize $R_{\text{lb}} = 0$, $R_{\text{ub}}$ = calculate (3) with $g_{ij} = \max_{x,y\in\mathcal{N}_x} g_{xy}$ and $B_{ij} = B_{\text{tot}}$.
2: If $R_{\text{ub}} - R_{\text{lb}} \leq \xi$, then terminate.
3: Set $R = \frac{R_{\text{ub}}+R_{\text{lb}}}{2}$.
4: Calculate $B_{ij}$, $\forall i,j \in \mathcal{N}_i$ for achieving $R$ using (3).
5: If $\sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} B_{ij} > B_{\text{tot}}$, then $R_{\text{ub}} = R$, else $R_{\text{lb}} = R$.
6: Go to step 2.

This algorithm returns the maximum rate and the optimum bandwidth allocation where the bandwidth constraint in (2) holds with equality. Accordingly, the corresponding minimum delay can be calculated using (5) and (6). The extended system matrix $\tilde{\mathbf{\Theta}}(k)$ can be constructed using (12) and (13).

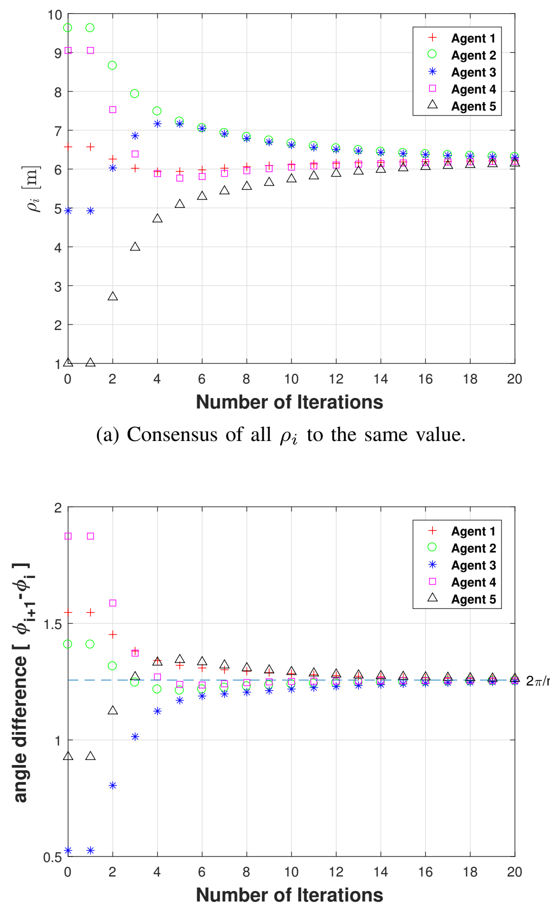
### B. Optimizing Confidence Weights

After finding the optimum transmission rate, the confidence weights are optimized using the following optimization problem

$$\underset{\tilde{\mathbf{\Theta}}}{\text{argmin}} \left\{ \text{tr}\left(\tilde{\mathbf{\Theta}}\right) \right\} \tag{18}$$

subject to

$$\tilde{\mathbf{\Theta}} \succeq 0, \tag{19}$$
$$\tilde{\mathbf{\Theta}}\mathbf{1}_N = \mathbf{1}_N, \tag{20}$$
$$w_{ij} = 0, \quad e_{ij} \notin \mathcal{E}, \ i \neq j, \tag{21}$$
$$w_{ii} \leq w_{ij}, \quad \forall i,j \in \mathcal{N}_i, \tag{22}$$
$$w_{ij} = (1 - w_{ii}) \frac{d_{ij}}{\sum_{j\in\mathcal{N}_i} d_{ij}}, \ i \neq j, \tag{23}$$
$$0 < w_{ij} < 1, \tag{24}$$




(a) Consensus of all $\rho_i$ to the same value.

(b) Convergence of all angle differences to $2\pi/n$.

Fig. 2: Agent locations as a function of iteration number.

where $w_{ij}$, $\forall i,j$ are the elements of $\tilde{\mathbf{\Theta}}$ and $\mathbf{1}_N$ is a $N \times 1$ vector of ones. Since the extended system matrix of the dynamics is row stochastic, its highest eigenvalue equals 1 and all other eigenvalues are less than 1. Therefore, minimizing the trace of $\tilde{\mathbf{\Theta}}$ in (18) minimizes the summation of all eigenvalues except the highest, and thus, SLEM is minimized. Constraints of (19) and (20) ensure that the extended system matrix is positive semi-definite and row stochastic, respectively. Constraint (23) gives higher confidence weights to the information of neighbouring agents with large distances as compared to the weights of its own information and neighbours with closer distances. This way, remote agents will move faster than closeby agents. This problem is convex and can be solved using conventional semidefinite programming methods [23].

## V. Performance Evaluation

In this section, the performance of the proposed algorithm is investigated. A scenario of 5 agents randomly placed in a square $200 \times 200$ m$^2$ area is considered. The reference point is at the origin. The communication simulation parameters are set according the 3GPP standard [24] and [25] and summarized in Table I. To assess the performance of the proposed

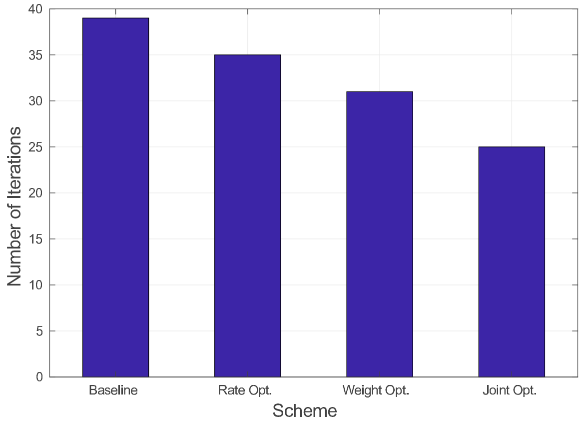



Fig. 3: Number of iterations taken by each scheme to reach consensus

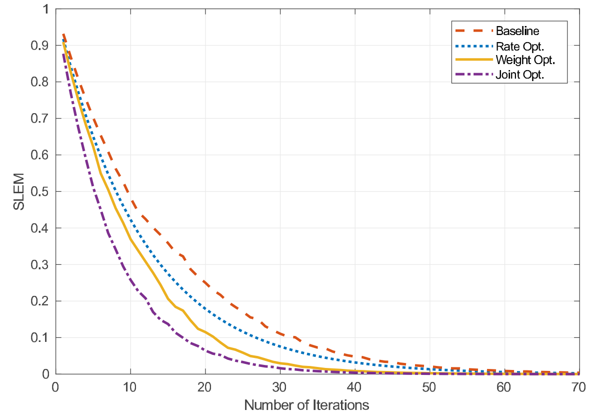



Fig. 4: SLEM of the product of system matrices for each scheme

scheme termed joint optimum, three benchmark schemes are employed. The first is the baseline scheme in which neither bandwidth allocation nor confidence weights are optimized. They are fixed and equal, i.e., $B_{ij} = B_{\text{tot}}/2N$, $w_{ii} = 1/3$ and $w_{ij} = 1/3$, $\forall i, j \in \mathcal{N}_i$. The second benchmark scheme is rate optimum in which the bandwidth allocation is performed using the proposed bisection method in Section IV-A. However, the weights in this scheme are fixed and equal, $w_{ii} = 1/3$ and $w_{ij} = 1/3$, $\forall i, j \in \mathcal{N}_i$. Finally, the weight optimum scheme, where the weights are optimized in every time slot using the optimization problem (18)–(24) with equal bandwidth allocation, is considered.

For a single snapshot and using the proposed joint optimum scheme, Fig. 2a and Fig. 2b show the locations of each agent as a function of the number of iterations. Although the transmissions among agents experience a delay, the consensus is achieved in around 20 iterations. It can be noticed that the convergence value of $\rho_i(K)$ does not equal the average of the initial values $\sum_{i=1}^{N} \rho_i(0)/N$ because of the communication delay. However, the angle difference between neighbouring agents converges to $2\pi/N$ because agents form a complete circle.

To assess the convergence of the proposed scheme against the benchmark schemes, we ran 1000 Monte-Carlo simulations. Fig. 3 compares the average convergence speed for the different schemes. Obviously, the baseline scheme is the slowest as neither the bandwidth nor the confidence weights are optimized. The rate optimum scheme is faster than the baseline scheme by around $10.3\%$ but it is slower than the weight optimum because it only ensures that the communication among agents are synchronized. However, by optimizing the confidence weights, far away agents will get higher weights which result in moving faster towards the neighbours. Therefore, the weight optimum scheme achieves around $20.5\%$ gain as compared to the baseline scheme. Finally, optimizing both bandwidths and confidence weights converges the fastest because the communication is synchronized and larger distances between agents result in faster movement towards each other. The joint optimum scheme achieves $36\%$ gain as compared to the baseline scheme. Furthermore, the joint optimum scheme achieves a performance gain of $19.4\%$ and $29\%$ over the weight optimum scheme and rate optimum scheme, respectively.

As described in Section III, the speed of convergence to a consensus is measured by SLEM of the extended system matrix. In Fig. 4, the SLEM as a function of the number of iterations is shown. It is clear that the SLEM is monotonically non-increasing as a function of the number of iterations. Also, SLEM for all schemes converges asymptotically towards zero. Jointly optimizing bandwidth and confidence weight leads to the fastest minimization of SLEM.

TABLE I: Simulation Parameters

| | | |
|---|---|---|
| Number of agents | $N$ | 5 |
| Number of links | $2N$ | 10 |
| Total bandwidth | $B_{\text{tot}}$ | 25 kHz |
| Distance between agents | $d_{ij}$ | 10 – 200 m |
| Channel gain | $g_{ij}$ | $-15.3 - 37.6 \log(d_{ij})$ |
| Transmit power | $p_{ij}$ | 23 dBm |
| Frame duration | $D$ | 1 ms |
| Noise spectral density | $N_o$ | $-173$ dBm/Hz |
| Bit error probability | $\epsilon$ | $10^{-7}$ |
| Packet size | $b$ | 160 bits |

## VI. Conclusion

In this paper, consensus in MAS is studied. In particular, we aim at modelling and optimizing the communication and consensus control together to minimize the convergence time. For a scenario of multiple agents aiming at forming a circle, a communication model which analyses the transmission delay as a function of bandwidth is considered. We propose a consensus algorithm in which the confidence weights and the transmission bandwidths are optimized. We wrote an extended system matrix which represents the impact of optimizing the bandwidth and confidence weights on the dynamics of MAS. URLLC is employed in which short packets are exchanged among the agents with high reliability and low latency. The results show that our proposed scheme converges significantly faster than other conventional schemes.

## Acknowledgment

This work has been performed in the context of the DFG funded CRC 1053 MAKI. The authors would like to thank Dr. Volker Willert and Dr. Tatiana Tatarenko for the fruitful discussions on consensus control.

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