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MIMO Multi-Group Multi-Way Relaying: Interference Alignment in a Partially Connected Network

Daniel Papsdorf*, Sakthivel Velumani* and Anja Klein*
*Communications Engineering Lab, Technische Universität Darmstadt, Merkstrasse 25, 64283 Darmstadt, Germany
{d.papsdorf, a.klein}@nt.tu-darmstadt.de

***Abstract*—In this paper, a partially connected multi-group multi-way relay network is considered. Partially connected means that not all groups of nodes are connected to all relays in the network. However, any group is connected to at least one relay which serves this group of nodes. In such a network, each node wants to share a certain number of data streams with all other nodes in its group, but not with nodes in other groups. The most challenging part of such a network topology is the handling of the groups which are connected to multiple relays. This network topology can be represented as multiple subnetworks, where each of these contains one relay and all groups connected to this relay. In this paper, a new closed-form solution to achieve interference alignment is proposed. The relays are used to manipulate the effective channel in order to achieve interference alignment at each receiver. The concept of simultaneous signal and channel alignment is generalized in order to handle groups with more than two members. It is shown through simulation that the proposed interference alignment algorithm outperforms the reference algorithm in terms of sum rate and degrees of freedom.**

I. Introduction

In applications like video conferences, area monitoring, text based chats and sensor networks, multiple nodes form a group of interest. In such a network, each node inside a group wants to share its message with all other group members of this group. Group communiction without relays has been considered in [1], where a source transmits a common message to all destination nodes in each group. Due to physical propagation phenomena, e.g., path loss or shadowing, it is not meaningful to assume that all nodes inside a group are connected via direct links. Hence, we assume that the whole communication takes place via an intermediate relay. The two-way relaying protocol proposed in [2] is a well known relaying protocol to overcome the duplex loss of conventional relaying schemes in pair-wise communication networks. In a multi-group multi-way relaying network, a transmission scheme with in general several multiple access (MAC) phases and/or several broadcast (BC) phases is required.

A multi-way relay network where a single relay assists the communication of nodes inside multiple groups was introduced in [3], [4]. Nodes inside different groups do not want to exchange data. A pairwise communication network considered in [2], [5], [6] is a special case of the multi-group multi-way relay network. In a pairwise communication network, only two nodes want to exchange data streams and form therefore a group of two nodes. A single-group multi-way relaying network has been investigated in [7]. Algorithms which achieve interference alignment (IA) in a fully connected multi-group multi-way relaying network were proposed in [8] and [9]. In order to achieve IA, each node designs its transmit filter in such a way that all interference signals are aligned in an interference subspace which has to be linearly independent of the useful signal subspace at each receiver [10], [11]. An upper bound for the degrees of freedom (DoF) of a fully connected multi-group multi-way relay network was derived in [12].

In this paper, we propose an IA algorithm for partially connected multi-group multi-way relaying networks. If multiple relays assist the communication of multiple groups, it is not appropriate to assume that all groups are connected to only one serving relay or that all groups are connected to all relays. One reason for this could be the limited coverage of the relays. Hence, we propose a new algorithm in which groups connected to multiple relays will be served by multiple relays. The process of IA is decoupled into three linear problems: simultaneous group signal alignment (SGSA), simultaneous group channel alignment (SGCA) and transceive zero forcing. A similar algorithm was proposed in [5], but for pair-wise communication. Therefore, this algorithm cannot be applied to a multi-group multi-way relaying network.

The present paper is organized as follows: Section II introduces the underlying system model. In Section III, the proposed closed form solution is presented. In Section IV, the performance of the proposed algorithm is investigated. Section V concludes this paper.

Notation: In the following, lower case letters represent scalars, lower case bold letters represent vectors, and upper case bold letters represent matrices. $\mathbb{C}$ represents the set of complex numbers. $(.)^{\mathrm{H}}$, $(.)^{-1}$ and $(.)^{T}$ denote the complex conjugate transpose, the inverse and the transpose of the element inside the brackets, respectively. The Frobenious norm of $\mathbf{A}$ is denoted by $\|\mathbf{A}\|_{\mathrm{F}} = \sqrt{\mathrm{tr}\left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)}$. The trace of a matrix is denoted by $\mathrm{tr}(.)$. $\mathbb{E}[.]$ denotes the expectation of the element inside the brackets.

II. System Model

In this paper, we consider a partially connected multi-group multi-way relay network consisting of $L \geq 1$ groups which are served by $Q \geq 1$ amplify-and-forward half-duplex multi antenna relays, as shown in Figure 1. The l^{th} group, $l \in \mathcal{L} = \{1, ..., L\}$, contains K_l multi-antenna half-duplex nodes which want to communicate groupwise. The communication between the nodes inside a group takes place via at least one intermediate relay, because there are no direct links between the nodes themselves. It is assumed that the relays know which nodes belong to which group. Nodes in the first group are indicated with the set $\mathcal{G}_1 = \{1, ..., K_1\}$, nodes in the second group are indicated with the set $\mathcal{G}_2 = \{K_1+1, ..., K_1+K_2\}$, and so on. In general, the l^{th} group contains nodes with the indices in the set $\mathcal{G}_l = \{a_l, ..., b_l\}$, where $a_l = \sum_{j=1}^{l-1} K_j + 1$ and $b_l = \sum_{j=1}^{l} K_j$. Each node only wants to exchange data with nodes in its own group and belongs only to a single group, i.e., $\mathcal{G}_l \cap \mathcal{G}_k = \emptyset$, $\forall l \neq k$. The whole number of nodes in the entire network is given by $\mathcal{G} = \cup_{l=1}^{L} \mathcal{G}_l = \{1, ..., K\}$. Let $\mathcal{G}(q)$ denote the set of nodes which are connected to relay q and $\mathcal{R}(k)$ the set of relays which are connected to node k. Let L_q denote the number of groups connected to relay q. Node k in group l is equipped with N_k antennas and wants to share $d \leq N_k$ data streams with the other nodes in group l. For simplicity, it is assumed that all groups have the same number of nodes, i.e, $K_l = K$ and all relays are equipped with $R_q = L_q(K-1)d$ antennas. In order to exchange information between the K nodes in group l groupwise, a transmission scheme with one multiple access (MAC) phase and $K-1$ broadcast (BC) phases is considered.

Let $\mathbf{H}^{\text{m}}_{k,q} \in \mathbb{C}^{R_q \times N_k}$ and $\mathbf{H}^{\text{b}}_{k,q} \in \mathbb{C}^{N_k \times R_q}$ denote the MIMO channel matrix between node k and the relay q during the MAC phase and between relay q and node k in the BC phases, respectively. The channels are assumed to be constant over the BC phases. Further, let $\mathbf{d}_k \in \mathbb{C}^{d \times 1}$ and $\mathbf{V}_k \in \mathbb{C}^{N_k \times d}$ denote the data vector originating of node k and its precoding matrix, respectively. It is assumed that the transmit symbols are independent and identically distributed (i.i.d.), so that $\mathbb{E}[\mathbf{d}_k \mathbf{d}_k^{\text{H}}] = \mathbf{I}_d$, $\forall k \in \mathcal{G}$ holds. To satisfy the maximum transmit power constraint of the nodes, the precoders are normalized, i.e., $\|\mathbf{V}_k\|_F^2 \leq P_{\text{n,max}}$. The components of the noise vectors $\mathbf{n}_{\text{r},q} = \mathcal{CN}(0, \sigma^2_{\text{r},m}) \in \mathbb{C}^{R_q \times 1}$ at the relay and $\mathbf{n}_k = \mathcal{CN}(0, \sigma_k^2) \in \mathbb{C}^{d \times N_k}$ at the nodes are i.i.d. complex Gaussian random variables.

After the MAC phase, the signal received at relay q is given by

$$\mathbf{r}_q = \sum_{k \in \mathcal{G}(q)} \mathbf{H}^{\text{m}}_{k,q} \mathbf{V}_k \mathbf{d}_k + \mathbf{n}_{\text{r},q}. \tag{1}$$

The relay retransmits this received signal to all its connected nodes after performing linear signal processing. The processing matrix of the p^{th} BC Phase at relay q is denoted by $\mathbf{G}_q^p$ and it is normalized so that the transmit power constraint of relay q is fulfilled, i.e., $\|\mathbf{G}_q^p\|_F^2 \leq P_{\text{r,max}}$. For simplicity, it is assumed that all relays have the same transmit power.

In the p^{th} BC phase, the signal received by node k is given by

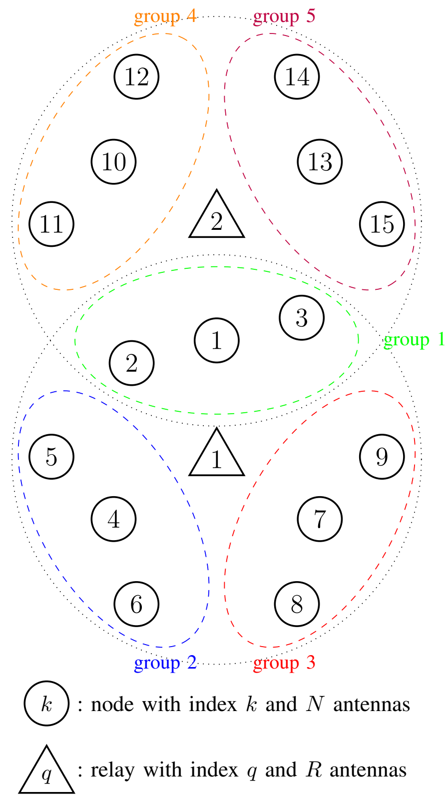

Fig. 1. Partially connected multi-group multi-way relay network with $L = 5$ groups, $K = 3$ nodes and $Q = 2$ relays.

$$\begin{aligned}\mathbf{y}_k^p = &\sum_{q \in \mathcal{R}(k)} \sum_{\substack{j \in \mathcal{G}_l, \\ j \neq k}} \mathbf{H}^{\text{b}}_{k,q} \mathbf{G}_q^p \mathbf{H}^{\text{m}}_{j,q} \mathbf{V}_j \mathbf{d}_j \\ &+ \sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\text{b}}_{k,q} \mathbf{G}_q^p \mathbf{H}^{\text{m}}_{k,q} \mathbf{V}_k \mathbf{d}_k \\ &+ \sum_{q \in \mathcal{R}(k)} \sum_{i \in \mathcal{G}(q) \backslash \mathcal{G}_l} \mathbf{H}^{\text{b}}_{k,q} \mathbf{G}_q^p \mathbf{H}^{\text{m}}_{i,q} \mathbf{V}_i \mathbf{d}_i \\ &+ \sum_{q \in \mathcal{R}(l)} \mathbf{H}^{\text{b}}_{k,q} \mathbf{G}_q^p \mathbf{n}_{\text{r},q} + \mathbf{n}_k, \quad \forall k \in \mathcal{G}_l. \end{aligned} \tag{2}$$

The first and second term of (2) are the useful and self-interference signal, respectively. The third term represents the interference from other groups and the last two terms represent the effective noise at node k. It is assumed that the self-interference can be completely canceled.

To achieve an IA solution, the useful and interference signals have to be in linearly independent subspaces at the receivers. Hence, the filters at the nodes and relays are designed in such a way that all interference signals are aligned within a $N_k - d$ dimensional interference subspace at receiving node k and the useful signals are within a d-dimensional useful signal subspace disjoint from the interference subspace during each of the $K-1$ BC phases. At the receiving node, a two stage receive filter is considered. Before the second stage receive filter, the useful signals are in a subspace disjoint from the interference subspace. Let $\mathbf{U}_k^{\mathrm{H}} \in \mathbb{C}^{d\times N_k}$ denote the first stage receive filter at node k. The output of the receive filter after the p^{th} BC phase at node k is given by

$$\begin{aligned}\mathbf{s}_k^p &= \mathbf{U}_k^{\mathrm{H}} \sum_{q\in\mathcal{R}(k)} \sum_{\substack{j\in\mathcal{G}_l,\\ j\neq k}} \mathbf{H}_{k,q}^{\mathrm{b}}\mathbf{G}_q^p\mathbf{H}_{j,q}^{\mathrm{m}}\mathbf{V}_j\mathbf{d}_j \\ &+ \mathbf{U}_k^{\mathrm{H}} \sum_{q\in\mathcal{R}(k)} \sum_{i\in\mathcal{G}(q)\setminus\mathcal{G}_l} \mathbf{H}_{k,q}^{\mathrm{b}}\mathbf{G}_q^p\mathbf{H}_{i,q}^{\mathrm{m}}\mathbf{V}_i\mathbf{d}_i \\ &+ \mathbf{U}_k^{\mathrm{H}}\Bigg(\sum_{q\in\mathcal{R}(k)} \mathbf{H}_{k,q}^{\mathrm{b}}\mathbf{G}_q^p\mathbf{n}_{\mathrm{r},q} + \mathbf{n}_k\Bigg), \quad \forall k\in\mathcal{G}_l. \end{aligned} \tag{3}$$

It is worth to mention that the first stage receive filter nullifies the interference signal at the receivers by a projection of the received signal to a subspace orthogonal to the aligned interference signals. The useful signal in the d-dimensional subspace is not nullified by the first stage filter. These conditions for $k\in\mathcal{G}_l$ are given by

$$\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{k,q}^{\mathrm{b}}\mathbf{G}_q^p\mathbf{H}_{i,q}^{\mathrm{m}}\mathbf{V}_i = 0, \quad \forall\ i\in\mathcal{G}(q)\setminus\mathcal{G}_l, \tag{4}$$

$$\mathrm{rank}\Big(\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{k,q}^{\mathrm{b}}\mathbf{G}_q^p\mathbf{H}_{j,q}^{\mathrm{m}}\mathbf{V}_j\Big) = d, \quad \forall\ j\in\mathcal{G}_l, j\neq k. \tag{5}$$

In the d-dimensional useful subspace, there are $(K-1)d$ useful signals. These $(K-1)d$ useful signals cannot be spatially separated using a single BC phase. Hence, the relays have to transmit d linearly independent linear combinations of the $(K-1)d$ useful signals in the $K-1$ BC phases in order spatially separate the useful signals via joint possessing over all BC phase. Therefore, the second stage receive filter $\mathbf{Q}_{k,q}^{\mathrm{H}}$ is applied to the output signal of the first stage receive filter concatenated over $K-1$ phases. Then, $\hat{\mathbf{d}}_k$ contains all $(K-1)d$ estimated symbols from different nodes within the same group and is given by

$$\hat{\mathbf{d}}_k = \mathbf{Q}_{k,q}^{\mathrm{H}}\big[\mathbf{s}_k^{1\mathrm{T}}\dots\mathbf{s}_k^{(K-1)\mathrm{T}}\big]^{\mathrm{T}}. \tag{6}$$

III. Proposed algorithm

In partially connected networks, it is challenging to handle nodes which are connect to multiple relays. Nodes which are only connected to a single relay can perform GSA and GCA at this relay, as proposed for fully connected networks in [8]. The closed-form solution proposed in this paper generalizes GSA and GCA in order to achieve an IA solution also at nodes connected to multiple relays. This new technique is called SGSA and SGCA. It is assumed that groups are connected to multiple relays or single relays, not single nodes out of a group. For simplicity, an intersection of only two subnetworks is considered.

A. MAC-Phase: Simultaneous Group Signal Alignment

In the MAC phase, each node belonging to the set $\mathcal{G}(q)$ transmits d data streams to relay q. However, relay q has only $L_q(K-1)d$ antennas and hence a $L_q(K-1)d$ dimensional relay space. In a $L_q(K-1)d$ dimensional relay space, L_qKd data streams cannot be spatially separated. However, the transmit filters of the nodes are designed in such a way that all Kd data streams from the nodes in group l are within a $(K-1)d$-dimensional subspace at relay q. Therefore, the first SGSA condition is given by the following system of linear equations:

$$\underbrace{\begin{bmatrix}\mathbf{H}_{a_l,q}^{\mathrm{m}} & \mathbf{H}_{a_l+1,q}^{\mathrm{m}} & \dots & \mathbf{H}_{b_l,q}^{\mathrm{m}}\end{bmatrix}}_{\mathbf{H}_{l,q}^{\mathrm{eff,m}}} \underbrace{\begin{bmatrix}\mathbf{A}_{a_l}\\ \mathbf{A}_{a_l+1}\\ \vdots\\ \mathbf{A}_{b_l}\end{bmatrix}}_{\mathbf{\Delta}_l} = 0. \tag{7}$$

The solution space of $\mathbf{\Delta}_l$ is determined by taking the null space of $\mathbf{H}_{l,q}^{\mathrm{eff,m}}$,

$$\begin{bmatrix}\mathbf{V}_{a_l}\\ \mathbf{V}_{a_l+1}\\ \vdots\\ \mathbf{V}_{b_l}\end{bmatrix} \subseteq \begin{bmatrix}\mathbf{A}_{a_l}\\ \mathbf{A}_{a_l+1}\\ \vdots\\ \mathbf{A}_{b_l}\end{bmatrix} = \mathrm{null}(\mathbf{H}_{l,q}^{\mathrm{eff,m}}). \tag{8}$$

$\mathbf{V}_k \forall k\in\mathcal{G}(q)$ needs to be of full rank d so that the d data streams transmitted by node k in $\mathcal{G}(q)$ span a d dimensional subspace at relay q. For groups which are only connected to a single relay, this first condition is sufficient, e.g., the groups $2,3,4,5$ in Figure 1. Hence, the first SGSA condition is the same as the GSA condition in [8].

In addition to the first condition, groups which are connected to relay q and $q\prime$ have to fulfill also the second SGSA condition, e.g., group 1 in Figure 1. This condition is given by

$$\underbrace{\begin{bmatrix}\mathbf{H}_{a_l,q\prime}^{\mathrm{m}} & \mathbf{H}_{a_l+1,q\prime}^{\mathrm{m}} & \dots & \mathbf{H}_{b_l,q\prime}^{\mathrm{m}}\end{bmatrix}}_{\mathbf{H}_{l,q\prime}^{\mathrm{eff,m}}} \underbrace{\begin{bmatrix}\mathbf{A}\prime_{a_l}\\ \mathbf{A}\prime_{a_l+1}\\ \vdots\\ \mathbf{A}\prime_{b_l}\end{bmatrix}}_{\mathbf{\Delta}\prime_l} = 0. \tag{9}$$

If the two solution spaces $\mathbf{\Delta}_l$ and $\mathbf{\Delta}\prime_l$ have an intersection, i.e., $\mathbf{\Delta}_l\cap\mathbf{\Delta}\prime_l\neq 0$, it is possible to achieve SGSA at the two relays q and $q\prime$ simultaneously. This common solution space is given by

$$\mathbf{V}_l \subseteq \tilde{\mathbf{\Delta}}_l = \mathrm{null}\left(\begin{bmatrix}\mathbf{H}_{l,q}^{\mathrm{eff,m}}\\ \mathbf{H}_{l,q\prime}^{\mathrm{eff,m}}\end{bmatrix}\right), \tag{10}$$

where $\mathbf{V}_l = \big[\mathbf{V}_{a_l}\mathbf{V}_{a_l+1}\cdots\mathbf{V}_{b_l}\big]^T$ is a subset of the intersection of the null spaces of $\mathbf{H}_{l,q}^{\mathrm{eff,m}}$ and $\mathbf{H}_{l,q\prime}^{\mathrm{eff,m}}$. The precoding filters $\mathbf{V}_l$ are chosen out of the solution space $\tilde{\mathbf{\Delta}}_l$.

B. BC-Phase: Simultaneous Group Channel Alignment

In order to achieve IA in each of the BC phases, SGCA is performed at the receiving nodes and transceive zero forcing at the relays. All nodes design their first stage receive filter such that the effective channels of all K nodes in group l span a $(K-1)d$ dimensional subspace in the corresponding $L(K-1)d$ dimensional relay space. Therefore, the interference signals will be in an $N_k - d$ dimensional interference subspace orthogonal to the d dimensional subspace spanned by the columns of $\mathbf{U}_k^{\mathrm{H}}$. The first SGCA condition is given by

$$\underbrace{\begin{bmatrix}\mathbf{H}_{a_l,q}^{\mathrm{bH}} & \mathbf{H}_{a_l+1,q}^{\mathrm{bH}} & \dots & \mathbf{H}_{b_l,q}^{\mathrm{bH}}\end{bmatrix}}_{\mathbf{H}_{l,q}^{\mathrm{eff,b}}} \cdot \mathbf{B}_l = 0 \tag{11}$$

and the second by

$$\underbrace{\begin{bmatrix}\mathbf{H}_{a_l,q\prime}^{\mathrm{bH}} & \mathbf{H}_{a_l+1,q\prime}^{\mathrm{bH}} & \dots & \mathbf{H}_{b_l,q\prime}^{\mathrm{bH}}\end{bmatrix}}_{\mathbf{H}_{l,q\prime}^{\mathrm{eff,b}}} \cdot \mathbf{B}\prime_l = 0. \tag{12}$$

From (7), (9) and (11), (12), it can be seen that SGSA and SGCA are dual problems. Hence, determining the solution space of SGCA is dual to determining the SGSA solution space. Hence, the SGCA solution space is given by

$$\mathbf{U}_l \subseteq \tilde{\mathbf{B}}_l = \mathrm{null}\left(\begin{bmatrix}\mathbf{H}_{l,q}^{\mathrm{eff,b}} \\ \mathbf{H}_{l,q\prime}^{\mathrm{eff,b}}\end{bmatrix}\right). \tag{13}$$

The receive filters $\mathbf{U}_l = \begin{bmatrix}\mathbf{U}_{a_l}\mathbf{U}_{a_l+1}\dots\mathbf{U}_{b_l}\end{bmatrix}^T$ are chosen from the solution space $\tilde{\mathbf{B}}_l$.

C. Properness Condition

In this section, the properness condition which has to be fulfilled to perform SGSA and SGCA is derived. The number of antennas at relay q is $R_q = L_q(K-1)$. The signal space of a node has to be large enough such that all nodes in a group can select a common subspace in the desired signal spaces at relay q, if the group is inside the set $\mathcal{G}(q)$, or at the relays q and $q\prime$ if the group is inside the set $\mathcal{G}(q) \cap \mathcal{G}(q\prime)$. If a group is inside the set $\mathcal{G}(q)$, the columns of matrix (8) span a $\left(\sum_{k=1}^{K_l} N_k - R_q\right)$-dimensional solution space. Hence, the first SGSA condition is fulfilled if and only if

$$\sum_{k=1}^{K_l} N_k \geq R_q + d, \quad \forall l \in \mathcal{L}. \tag{14}$$

If a group is inside the set $\mathcal{G}(q) \cap \mathcal{G}(q\prime)$, the columns of matrix (10) span a $\left(\sum_{k=1}^{K_l} N_k - \sum_{j\in\mathcal{R}(l)} R_j\right)$-dimensional solution space. Hence, the first and the second SGSA condition are simultaneously fulfilled if and only if

$$\sum_{k=1}^{K_l} N_k \geq R_q + R_{q\prime} + d, \quad \forall l \in \mathcal{L}. \tag{15}$$

D. Transceive Zero Forcing

In this section, the design of the relay processing matrix is described. After SGSA and SGCA, there are $L_q(K-1)d$ effective data streams and $L_q(K-1)d$ effective channels at relay q. The relays perform transceive zero forcing in order to transmit these $L_q(K-1)d$ effective data streams to all nodes in the set $\mathcal{G}(q)$. The effective channels of the MAC and BC phase are given by

$$\mathbf{H}_{\mathrm{eff},l,q}^{\mathrm{MAC}} = \begin{bmatrix}\mathbf{H}_{a_l,q}^{\mathrm{m}}\mathbf{V}_{a_l} & \dots & \mathbf{H}_{b_l,q}^{\mathrm{m}}\mathbf{V}_{b_l}\end{bmatrix}, \tag{16}$$

$$\mathbf{H}_{\mathrm{eff},l,q}^{\mathrm{BC}} = \left[\left(\mathbf{U}_{a_l}\mathbf{H}_{a_l,q}^{\mathrm{b}}\right)^T \quad \dots \quad \left(\mathbf{U}_{b_l}\mathbf{H}_{b_l,q}^{\mathrm{b}}\right)^T\right]^T, \tag{17}$$

$\forall q;\ l \in \mathcal{G}(q)$. Let $\mathbf{G}_q^{\mathrm{rx\ H}}$ and $\mathbf{G}_q^{\mathrm{tx}}$ denote the receive and transmit zero forcing matrices of relay q, respectively. These are given by the following two equations:

$$\mathbf{G}_q^{\mathrm{rx\ H}} = \begin{bmatrix}\mathbf{H}_{\mathrm{eff},1,q}^{\mathrm{MAC}} & \mathbf{H}_{\mathrm{eff},2,q}^{\mathrm{MAC}} & \dots & \mathbf{H}_{\mathrm{eff},L_q,q}^{\mathrm{MAC}}\end{bmatrix}^{-1}, \tag{18}$$

$$\mathbf{G}_q^{\mathrm{tx}} = \begin{bmatrix}(\mathbf{H}_{\mathrm{eff},1,q}^{\mathrm{BC}})^{\mathrm{T}} & (\mathbf{H}_{\mathrm{eff},2,q}^{\mathrm{BC}})^{\mathrm{T}} & \dots & (\mathbf{H}_{\mathrm{eff},L_q,q}^{\mathrm{BC}})^{\mathrm{T}}\end{bmatrix}^{(-1)\mathrm{T}}. \tag{19}$$

The matrices of the right-hand side of (18) and (19) are square matrices, which are non-singular with probability one. The entire relay processing matrix in the p^{th} BC phase is given by

$$\mathbf{G}_q^p = \mathbf{G}_q^{\mathrm{rx\ H}}\mathbf{P}_p\mathbf{G}_q^{\mathrm{tx}} \tag{20}$$

where $\mathbf{P}_p$ is a block diagonal precoding matrix. The matrix $\mathbf{P}_p$ has to be chosen such that in $K-1$ BC phases, $(K-1)d$ linearly independent linear combinations are received. Any block diagonal matrices arbitrarily chosen will almost surely be a valid solution.

E. Group Signal Separation

In this section, the second stage receive filter is designed. After applying the first stage receive filter, the interference will be zero and the useful signals will be in a d-dimensional subspace at each node. Let $\mathbf{H}_{k,j,q}^{\mathrm{p(eff)}} = \mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{k,q}^{\mathrm{b}}\mathbf{G}_q^p\mathbf{H}_{j,q}^{\mathrm{m}}\mathbf{V}_j$ denote the effective channel between nodes j and k of group l connected to relay q in the BC phase p. Hence, the effective channels from all $(K-1)$ nodes in group l to node k is given by

$$\mathbf{H}_{k,q}^{\mathrm{(p)eff}} = \begin{bmatrix}\mathbf{H}_{k,1,q}^{\mathrm{(p)eff}} & \dots & \mathbf{H}_{k,i,q}^{\mathrm{(p)eff}} & \dots & \mathbf{H}_{k,K,q}^{\mathrm{(p)eff}}\end{bmatrix}_{i\neq k}, \ \forall q. \tag{21}$$

The effective channel from all the $K-1$ nodes to node k in group l over all $(K-1)$ BC phases is given by

$$\mathbf{H}_{k,q}^{\mathrm{eff}} = \left[\left(\mathbf{H}_{k,q}^{\mathrm{(1)eff}}\right)^{\mathrm{T}} \quad \left(\mathbf{H}_{k,q}^{\mathrm{(2)eff}}\right)^{\mathrm{T}} \quad \dots \quad \left(\mathbf{H}_{k,q}^{\mathrm{(K-1)eff}}\right)^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{22}$$

The second stage receive filter separates the useful signals received from the nodes within the group and is designed as a zero forcing filter in order to spatially separate the $(K-1)d$ data streams:

$$\mathbf{Q}_{k,q}^{\mathrm{H}} = \left(\mathbf{H}_{k,q}^{\mathrm{eff}}\right)^{-1}. \tag{23}$$

The matrix on the right-hand side of (23) is a square matrix of full rank. Nodes which are connected to multiple relays have to apply all q second stage filters in order separate the useful signals.

IV. Performance analysis

This section presents the degrees of freedom (DoF) and sum-rate performance of the proposed algorithm. Since a DoF analysis is only valid for an asymptotically high SNR, the sum rate over a larger SNR range is simulated in order to assess the performance as shown in Figure 2. For the simulation, a network with $L = 5$ groups, $K = 4$ nodes in each group

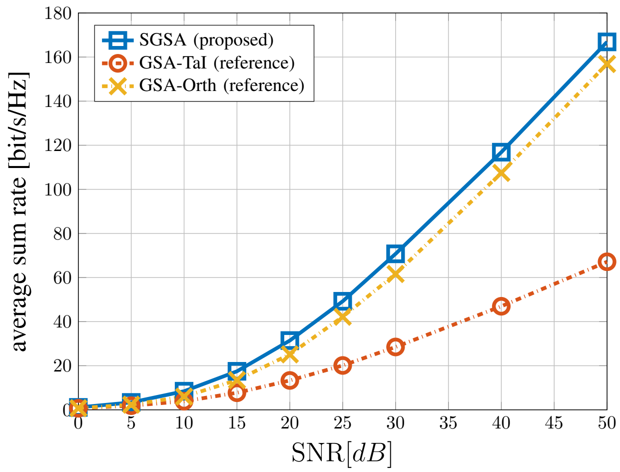

Fig. 2. Sum rate performance of a partially connected multi-group multi-way relay network with $L = 5$ groups, $K = 4$ nodes and $Q = 2$ relays.

and $Q = 2$ relays is considered. A single group of the entire network is connected to both relays. The other groups are only connected to a single relay and equally distributed to them. It is assumed that the channels are random i.i.d. Rayleigh fading channels which are normalized such that the average received signal power is the same as the average transmit signal power. The noise power at each node and at each relay is assumed to be the same for the simulation, i.e., $\sigma^2_{\mathrm{r},m} = \sigma^2_k$. The transmit power at each relay is adjusted to $P_{\mathrm{r,max}} = \frac{1}{Q}KLP_{\mathrm{n,max}}$.

The reference algorithms are based on the algorithm proposed in [8], i.e., the nodes and the relays design the filters accordingly. In "GSA-TaI", the group located inside the intersection area of two subnetworks is only served by one connected relay, the other relay cannot assist the communication and treats the signals from nodes of this group as interference. In "GSA-Orth", the communication of the groups belonging to a subnetwork takes place one after the other, i.e., the subnetworks utilize orthogonal resources. This will avoid interference between the subnetworks.

The proposed algorithm "SGSA" as well as the reference algorithm "GSA-Orth" achieve an interference free communication and 15 DoF, which is the same number of DoF as a fully connected multi-group multi-way relay network with the same number of groups and nodes would achieve by applying the algorithm proposed in [8]. In terms of DoF, the reference algorithm "GSA-TaI" is not able to achieve the same DoF. The reason for this behavior is the relay which does not assist the communication and therefore suffers from interference.

From Figure 2, it can be seen that the proposed algorithm "SGSA" achieves a better sum rate performance than the reference algorithms. The reason for that is firstly the interference-free communication of the proposed algorithm in comparison to "GSA-TaI" and secondly lower number of requierd time slots in comparison to "GSA-Orth". The fully connected reference algorithm would require more antennas at the nodes in each group and considers only a single relay compared to the partially connected algorithms. Hence, a direct comparison is not appropriate and therefore the sum rate performance of the fully connected network is not shown in Figure 2. The other way around the proposed algorithm can be applied to fully connected networks.

V. Conclusion

In this paper, a partially connected multi-group multi-way relay network is investigated. Partially connected means that not all groups of nodes are connected to all relays in the entire network. The communication between the multi antenna nodes inside a group takes place via at least one intermediate amplify-and-forward half-duplex multi antenna relay. A new closed-form solution has been proposed to achieve interference alignment at each receiver. The properness conditions for the proposed solution are derived as well. It has been shown that the proposed interference alignment algorithm outperforms the reference algorithm in terms of sum rate and degrees of freedom.

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