# On the DoF of the 2-Antenna 3-User MISO BC with Alternating CSIT

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***Abstract*—In this paper, the 2-antenna 3-user multiple-input single-output broadcast channel is considered, where the channel state information at the transmitter (CSIT) for every receiver can be either perfect (P) or delayed (D), resulting thus in total in 8 possible CSIT states $I_1 I_2 I_3$, $I_i \in \{\mathbf{P}, \mathbf{D}\}$, $i \in \{1, 2, 3\}$. For this scenario, we show the achievability of the optimal degrees of freedom (DoF) for the setting where the CSIT states are restricted to take the following 5 values: PPP, PPD, PDP, PDD and DDD. The achievability is facilitated through the introduction of two novel constituent encoding schemes (CSs), in which joint encoding over the CSIT state pairs $(\mathbf{PPP}, \mathbf{PDD})$ and $(\mathbf{PDD}, \mathbf{DDD})$ is performed. After a careful assignment of the newly proposed and existing in the literature CSs to the available CSIT states, optimal DoF are achieved.**

## I. Introduction

The multiple-input single-output (MISO) broadcast channel (BC) is a network comprised of a single $M$-antenna transmitter and $K$ single-antenna receivers, where each receiver desires a private message. The work of [1] has shown the degrees of freedom (DoF) of the MISO BC to be $\min\{M, K\}$, where for the DoF achievability zero-forcing (ZF) encoding was applied. The result in [1] relied on the assumption of perfect and up-to-date channel state information at the transmitter (CSIT).

Contrary to having the current CSIT, the authors in [2] considered the MISO BC under the so-called delayed CSIT setting, in which the CSIT is completely outdated, excluding thus all possibilities to exploit channel time correlation. Despite the absence of the current CSIT, the work of [2] has shown the DoF to be greater than in case of completely absent CSIT given in [3]. The achievability in [2] was based on a novel transmission scheme, referred to in the following as MAT scheme. A more general CSIT setting has been considered in [4], [5] and [6], where at every channel use, the CSIT for each user can be either perfect (P), delayed (D) or not available (N), referred to as alternating CSIT. The complete DoF characterization for the 2-user MISO BC with alternating CSIT was given in [4]. The DoF of the MISO BC with delayed and imperfect CSIT were studied in [7] and [8], where the work of [8] completely characterized the DoF for the 2-user case.

In this paper, we consider the 2-antenna 3-user MISO BC with alternating CSIT, where the CSIT for each user can be either perfect (P) or delayed (D), resulting thus in total in 8 possible CSIT states $I_1 I_2 I_3$, $I_i \in \{\mathrm{P}, \mathrm{D}\}$, $i \in \{1, 2, 3\}$. For the overloaded $M < K$ MISO BC with delayed CSIT, it has been shown in [2] that additional DoF gains are possible by applying joint encoding over the whole set of users. However, optimal DoF have been achieved in [2] only for the case $M = 2$, $K = 3$.

*Related Work and Contribution:* Outer bounds for the $M$-antenna $K$-user MISO BC with alternating CSIT have been provided in [4] and [7]. As for the achievability, the work of [4] achieved the optimal $\min\{M, K\}$ DoF in a symmetric CSIT setting where at every channel use, at least for $M$ users perfect CSIT is available. For the case $M = 2$, $K = 3$, the result of [4] gives the optimal DoF for the case where admissible CSIT states are PPD, PDP and DPP. For the achievability, [4] relied on ZF. The work of [7] considered a modification of the CSIT setting in [4] by allowing in addition an alternation with the jointly delayed state. The authors in [7] proposed to apply MAT scheme for the jointly delayed CSIT state and ZF for the remaining states, where optimal DoF have been achieved for the case $M = 2$, $K = 3$. The work of [5] considered an overloaded $M = K - 1$ MISO BC where CSIT alternates between jointly perfect and jointly delayed states, for which a novel constituent encoding scheme (CS) was proposed. For the case $M = 2$, $K = 3$, the CS proposed by [5] achieved optimal DoF. In [9], a novel transmission scheme was proposed, which achieved optimal DoF for the $M = 2$, $K = 3$ MISO BC with a fixed PDD state.

In this paper, we characterize the optimal DoF for the $M = 2$, $K = 3$ MISO BC with alternating CSIT where the admissible CSIT states are PPP, PPD, PDP, PDD and DDD. Our result partially generalizes the findings on the DoF characterization obtained for the $M = 2$, $K = 3$ MISO BC with alternating CSIT in [2], [4], [5], [7] and [9]. In terms of the converse, we rely on the existing outer bound in [7]. As for the achievability, we first introduce two novel CSs in which joint encoding over the CSIT state pairs $(\mathrm{PPP}, \mathrm{PDD})$ and $(\mathrm{PDD}, \mathrm{DDD})$ is performed. Then, after a careful assignment of two newly proposed CSs, ZF, MAT scheme and the CS in [5] to the available CSIT states, optimal DoF are achieved.

## II. System Model

We consider a MISO BC depicted in Fig. 1, which is comprised of a 2-antenna transmitter Tx and 3 single-antenna receivers $\mathrm{Rx}_i$, $i \in \{1, 2, 3\}$. The signal received by $\mathrm{Rx}_i$ at the $t$-th channel use is given by

$$y_i(t) = \mathbf{h}_i^{\mathrm{T}}(t)\, \mathbf{x}(t) + z_i(t), \quad (1)$$

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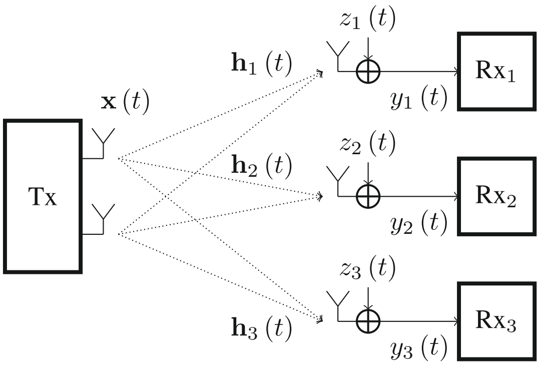


Fig. 1: The 2-antenna 3-user MISO BC

where $\mathbf{x}(t) \in \mathbb{C}^{2\times 1}$ is the vector of transmitted signals, $\mathbf{h}_i(t) \in \mathbb{C}^{2\times 1}$ is the vector of channel coefficients corresponding to $\mathrm{Rx}_i$, and $z_i(t) \sim \mathcal{CN}(0,1)$ is the additive white Gaussian noise at $\mathrm{Rx}_i$. Channel coefficients are drawn from continuous distributions, and are independent across transmit antennas, receivers and different channel uses. The transmitted signal is subject to the average transmit power constraint $\frac{1}{n}\sum_{t=1}^{n} \mathrm{E}\left\{\|\mathbf{x}(t)\|^2\right\} \leq P$, where $n$ is the communication duration.

We consider an alternating CSIT setting in which the CSIT corresponding to each $\mathrm{Rx}_i$ at time $t$ is either perfect (P) or delayed (D). In such scenario, there are 8 possible joint CSIT states which are denoted by $I_1 I_2 I_3$, $I_i \in \{\mathrm{P}, \mathrm{D}\}$, $i \in \{1,2,3\}$. The joint CSIT state statistics are characterized by the joint probabilities $\lambda_{I_1 I_2 I_3}$, where $\sum_{I_1 I_2 I_3} \lambda_{I_1 I_2 I_3} = 1$ holds. In such case, the probability that the CSIT for $\mathrm{Rx}_i$ is in P state is described by the marginal probability $\lambda_i = \sum_{I_1 I_2 I_3, I_i = P} \lambda_{I_1 I_2 I_3}$. Without loss of generality, we assume the users to be ordered such that $\lambda_1 \geq \lambda_2 \geq \lambda_3$ holds. At every receiver, global instantaneous CSI is assumed.

We assume Tx has messages $W_1$, $W_2$ and $W_3$ intended to receivers $\mathrm{Rx}_1$, $\mathrm{Rx}_2$ and $\mathrm{Rx}_3$, respectively. Achievable rate tuples $(R_1(P), R_2(P), R_3(P))$ are defined in the standard Shannon theoretic sense. We define the DoF region $\mathcal{D}$ as the set of all achievable DoF tuples $(d_1, d_2, d_3)$, for which $d_i = \lim_{P\to\infty} \frac{R_i(P)}{\log_2(P)}$ holds. The maximum achievable sum-DoF (or simply DoF) is denoted by $d = \max_{(d_1,d_2,d_3)\in\mathcal{D}} d_1 + d_2 + d_3$.

## III. Main Results

We first state the DoF outer bound for the alternating CSIT which follows from the DoF outer bound for delayed and imperfect CSIT in [7].

**Theorem 1.** *For the 2-antenna 3-user MISO BC with alternating CSIT, the DoF region is outer bounded as*

$$2d_1 + d_2 + d_3 \leq 2 + \lambda_1, \tag{2a}$$

$$d_1 + 2d_2 + d_3 \leq 2 + \lambda_2, \tag{2b}$$

$$d_1 + d_2 + 2d_3 \leq 2 + \lambda_3, \tag{2c}$$

$$d_1 + d_2 + d_3 \leq 2, \tag{2d}$$

$$d_1, d_2, d_3 \leq 1. \tag{2e}$$

Below, we provide the analysis of the DoF outer bound given by Theorem 1. Depending on the set of active bounds in (2a)–(2e), we distinguish the following three regions of the CSIT configurations.

1) Region I: $3\lambda_1 - \lambda_2 - \lambda_3 \leq 2, \lambda_1 + \lambda_2 + \lambda_3 \leq 2$.

In this case, the bound (2d) is inactive. The optimal DoF tuple is given by $\mathbf{A}_{\mathrm{I}} = \left(\frac{1}{2} + \frac{3\lambda_1 - \lambda_2 - \lambda_3}{4}, \frac{1}{2} + \frac{3\lambda_2 - \lambda_1 - \lambda_3}{4}, \frac{1}{2} + \frac{3\lambda_3 - \lambda_1 - \lambda_2}{4}\right)$.

2) Region II: $3\lambda_1 - \lambda_2 - \lambda_3 > 2$.

In this case, the bounds (2a) and (2d) are inactive. The optimal DoF tuple is given by $\mathbf{A}_{\mathrm{II}} = \left(1, \frac{1}{3} + \frac{2\lambda_2 - \lambda_3}{3}, \frac{1}{3} + \frac{2\lambda_3 - \lambda_2}{3}\right)$.

3) Region III: $\lambda_1 + \lambda_2 + \lambda_3 > 2$.

In this case, all bounds are active. The are three optimal DoF tuples which are the corner points of the DoF region $\mathbf{A}_{\mathrm{III}}^{[1]} = (\lambda_1, \lambda_2, 2 - \lambda_1 - \lambda_2)$, $\mathbf{A}_{\mathrm{III}}^{[2]} = (\lambda_1, 2 - \lambda_1 - \lambda_3, \lambda_3)$ and $\mathbf{A}_{\mathrm{III}}^{[3]} = (2 - \lambda_2 - \lambda_3, \lambda_2, \lambda_3)$.

The examples of the shapes of the DoF regions are given in Fig 2. The corollary below summarizes our findings in a form of the DoF upper bound.

**Corollary 1.** *The DoF in the 2-antenna 3-user MISO BC with alternating CSIT are bounded from above as follows*

$$d \leq \begin{cases} \frac{3}{2} + \frac{1}{4}(\lambda_1 + \lambda_2 + \lambda_3) & \text{if} \quad \begin{array}{c} 3\lambda_1 - \lambda_2 - \lambda_3 \leq 2, \\ \lambda_1 + \lambda_2 + \lambda_3 \leq 2, \end{array} \\ \frac{5}{3} + \frac{1}{3}(\lambda_2 + \lambda_3) & \text{if} \quad 3\lambda_1 - \lambda_2 - \lambda_3 > 2, \\ 2 & \text{if} \quad \lambda_1 + \lambda_2 + \lambda_3 > 2. \end{cases} \tag{3}$$

The main result of the paper is then given by the following theorem.

**Theorem 2.** *For the 2-antenna 3-user MISO BC with alternating CSIT where $\lambda_{\mathrm{DPP}} = \lambda_{\mathrm{DPD}} = \lambda_{\mathrm{DDP}} = 0$, the DoF upper bound* (3) *is achievable.*

The proof of Theorem 2 is given in two sections. In Section IV, the set of the CSs necessary for the DoF achievability will be introduced. Then, the formal proof will be provided in Section V by assigning the CSs to the CSIT states.

## IV. Constituent Encoding Schemes

In this section, we describe the CSs which will be used to prove Theorem 2 in Section V. The schemes described in this section are summarized in Table I.

### A. Schemes Achieving 2 DoF

First, we consider three CSs which rely on ZF encoding. In each of the CSs, single symbols are delivered to two receivers over a single channel use.

1) $S_1^2$ achieves $(d_1, d_2, d_3) = (1,1,0)$ for $\lambda_{\mathrm{PPD}} = 1$.
2) $S_2^2$ achieves $(d_1, d_2, d_3) = (1,0,1)$ for $\lambda_{\mathrm{PDP}} = 1$.
3) $S_3^2$ achieves $(d_1, d_2, d_3) = (1,1,0)$ for $\lambda_{\mathrm{PPP}} = 1$.

In the following, we describe the first newly proposed CS in which joint encoding over PPP and PDD states is performed.

 

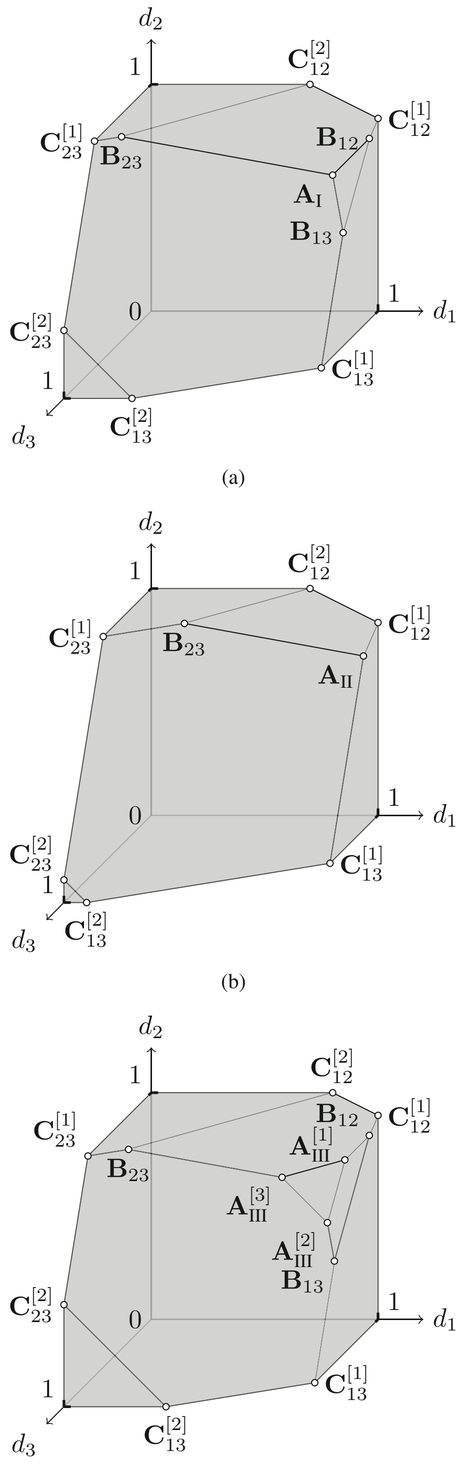


Fig. 2: Shapes of the DoF regions for $2\lambda_1 - \lambda_2 \geq 2$ and $2\lambda_2 - \lambda_3 \geq 2$: (a) Region I, (b) Region II and (c) Region III. The non-optimal DoF tuples are given by $\mathbf{C}_{12}^{[1]} = \left(1, \frac{1+\lambda_2}{2}, 0\right)$, $\mathbf{C}_{12}^{[2]} = (1, \lambda_2, 0)$, $\mathbf{C}_{13}^{[1]} = \left(1, 0, \frac{1+\lambda_3}{2}\right)$, $\mathbf{C}_{13}^{[2]} = (1, 0, \lambda_3)$, $\mathbf{C}_{23}^{[1]} = \left(0, 1, \frac{1+\lambda_3}{2}\right)$, $\mathbf{C}_{23}^{[2]} = (0, 1, \lambda_3)$, $\mathbf{B}_{12} = (1, 1 - \lambda_1 + \lambda_2, 2\lambda_1 - \lambda_2 - 1, )$, $\mathbf{B}_{13} = (1, 2\lambda_1 - \lambda_3 - 1, 1 - \lambda_1 + \lambda_3)$ and $\mathbf{B}_{23} = (2\lambda_2 - \lambda_3 - 1, 1, 1 - \lambda_2 + \lambda_3)$

TABLE I: Summary of the constituent encoding schemes.

| CS | CSIT state fractions | DoF tuple | Achievability |
|---|---|---|---|
| $S_1^2$ | $\lambda_{\mathrm{PPD}} = 1$ | $(1, 1, 0)$ | ZF |
| $S_2^2$ | $\lambda_{\mathrm{PDP}} = 1$ | $(1, 0, 1)$ | ZF |
| $S_3^2$ | $\lambda_{\mathrm{PPP}} = 1$ | $(1, 1, 0)$ | ZF |
| $S_4^2$ | $(\lambda_{\mathrm{PPP}}, \lambda_{\mathrm{PDD}}) = \left(\frac{1}{2}, \frac{1}{2}\right)$ | $\left(1, \frac{1}{2}, \frac{1}{2}\right)$ | Proposed |
| $S_5^2$ | $(\lambda_{\mathrm{PPP}}, \lambda_{\mathrm{DDD}}) = \left(\frac{2}{3}, \frac{1}{3}\right)$ | $\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$ | [5] |
| $S^{5/3}$ | $(\lambda_{\mathrm{PDD}}, \lambda_{\mathrm{DDD}}) = \left(\frac{2}{3}, \frac{1}{3}\right)$ | $\left(1, \frac{1}{3}, \frac{1}{3}\right)$ | Proposed |
| $S^{3/2}$ | $\lambda_{\mathrm{DDD}} = 1$ | $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ | MAT [2] |

4) $S_4^2$ achieves $(d_1, d_2, d_3) = \left(1, \frac{1}{2}, \frac{1}{2}\right)$ for $(\lambda_{\mathrm{PPP}}, \lambda_{\mathrm{PDD}}) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

The transmission spans two channel uses: $t = 1$ corresponding to PDD state and $t = 2$ corresponding to PPP state. During the transmission, the symbols $u_1^{[1]}$ and $u_1^{[2]}$ are delivered to $\mathrm{Rx}_1$, the symbol $u_2$ is delivered to $\mathrm{Rx}_2$ and the symbol $u_3$ is delivered to $\mathrm{Rx}_3$.

At $t = 1$, the symbol vector $\mathbf{u}_1 = \left[u_1^{[1]} \quad u_1^{[2]}\right]^{\mathrm{T}}$ is transmitted using random precoding and the symbols $u_2$ and $u_3$ are transmitted using ZF to ensure that no interference is overheard at $\mathrm{Rx}_1$. The signal transmitted at $t = 1$ writes as $\mathbf{x}(1) = \mathbf{C}_1(1)\,\mathbf{u}_1 + \mathbf{c}_{23}(1)\,(u_2 + u_3)$, where $\mathbf{C}_1(1) \in \mathbb{C}^{2\times 2}$ is a random matrix with independent entries taken from continuous distributions and $\mathbf{c}_{23} \in \mathbb{C}^{2\times 1}$ is a precoding vector satisfying $\mathbf{h}_1^{\mathrm{T}}(1)\,\mathbf{c}_{23}(1) = 0$. By omitting the receive noise term, the signal received by $\mathrm{Rx}_1$ writes as $y_1(1) = \mathbf{h}_1^{\mathrm{T}}(1)\,\mathbf{C}_1(1)\,\mathbf{u}_1$, which contains a useful linear combination of $u_1^{[1]}$ and $u_1^{[2]}$. The signals $y_i(1) = \mathbf{h}_i^{\mathrm{T}}(1)\,(\mathbf{C}_1(1)\,\mathbf{u}_1 + \mathbf{c}_{23}(1)\,(u_2 + u_3))$, $j \in \{2, 3\}$, are comprised of useful signal and interference.

At $t = 2$, the symbols $u_2$ and $u_3$ are retransmitted using ZF to ensure that no interference is received at $\mathrm{Rx}_1$. Additionally, perfect CSIT available for $\mathrm{Rx}_2$ and $\mathrm{Rx}_3$ is employed for the design of the precoding vectors to ensure that $\mathrm{Rx}_2$ and $\mathrm{Rx}_3$ overhear at $t = 2$ the interference identical to that at $t = 1$. The signal transmitted at $t = 2$ is given by $\mathbf{x}(2) = \mathbf{C}_1(2)\,\mathbf{u}_1 + \mathbf{c}_{23}(2)\,(\gamma_2 u_2 + \gamma_3 u_3)$, where $\mathbf{c}_{23}(2) \in \mathbb{C}^{2\times 1}$ is a precoding vector satisfying $\mathbf{h}_1^{\mathrm{T}}(2)\,\mathbf{c}_{23}(2) = 0$. $\mathbf{C}_1(2) \in \mathbb{C}^{2\times 2}$ is the precoding matrix which has to fulfill

$$\begin{bmatrix} \mathbf{h}_2^{\mathrm{T}}(2) \\ \mathbf{h}_3^{\mathrm{T}}(2) \end{bmatrix} \mathbf{C}_1(2) = \begin{bmatrix} \mathbf{h}_2^{\mathrm{T}}(1) \\ \mathbf{h}_3^{\mathrm{T}}(1) \end{bmatrix} \mathbf{C}_1(1), \tag{4}$$

which is ensured by setting

$$\mathbf{C}_1(2) = \begin{bmatrix} \mathbf{h}_2^{\mathrm{T}}(2) \\ \mathbf{h}_3^{\mathrm{T}}(2) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{h}_2^{\mathrm{T}}(1) \\ \mathbf{h}_3^{\mathrm{T}}(1) \end{bmatrix} \mathbf{C}_1(1). \tag{5}$$

$\gamma_2, \gamma_3 \in \mathbb{C}$ are the precoding scalars which have to fulfill

$$\mathbf{h}_3^{\mathrm{T}}(2)\,\mathbf{c}_{23}(2)\,\gamma_2 = \mathbf{h}_3^{\mathrm{T}}(1)\,\mathbf{c}_{23}(1), \tag{6}$$

$$\mathbf{h}_2^{\mathrm{T}}(2)\,\mathbf{c}_{23}(2)\,\gamma_3 = \mathbf{h}_2^{\mathrm{T}}(1)\,\mathbf{c}_{23}(1), \tag{7}$$

which is ensured by setting

$$\gamma_2 = \frac{\mathbf{h}_3^{\mathrm{T}}(1)\,\mathbf{c}_{23}(1)}{\mathbf{h}_3^{\mathrm{T}}(2)\,\mathbf{c}_{23}(2)}, \gamma_3 = \frac{\mathbf{h}_2^{\mathrm{T}}(1)\,\mathbf{c}_{23}(1)}{\mathbf{h}_2^{\mathrm{T}}(2)\,\mathbf{c}_{23}(2)}. \tag{8}$$

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At $t=2$, $\mathrm{Rx}_1$ receives $y_1(2)=\mathbf{h}_1^{\mathrm{T}}(2)\,\mathbf{C}_1(2)\,\mathbf{u}_1$ containing the second remaining linear combination of $u_1^{[1]}$ and $u_1^{[2]}$ which guarantees the decodability of both symbols. $\mathrm{Rx}_2$ and $\mathrm{Rx}_3$ cancel the interference in the received signals as $y_i(1)-y_i(2)$, $i\in\{2,3\}$, where $u_2$ and $u_3$ are decoded from the obtained interference-free signals.

5) $S_5^2$ achieves $(d_1,d_2,d_3)=\left(\frac{2}{3},\frac{2}{3},\frac{2}{3}\right)$ for $(\lambda_{\mathrm{PPP}},\lambda_{\mathrm{DDD}})=\left(\frac{2}{3},\frac{1}{3}\right)$.

For $S_5^2$ we refer to the CS in [5], in which 2 symbols are delivered to every receiver over a single DDD and two PPP states.

*B. Scheme Achieving $\frac{5}{3}$ DoF*

$\frac{5}{3}$ DoF have been achieved in the fixed PDD setting in [9]. The CSIT requirement in [9] can be relaxed to $(\lambda_{\mathrm{PDD}},\lambda_{\mathrm{DDD}})=\left(\frac{5}{6},\frac{1}{6}\right)$ by substituting the PDD states which do not exploit perfect CSIT by DDD states. In the following we describe the second newly proposed CS which improves upon the CS in [9] by achieving the same $\frac{5}{3}$ DoF while further relaxing the CSIT requirement to $(\lambda_{\mathrm{PDD}},\lambda_{\mathrm{DDD}})=\left(\frac{2}{3},\frac{1}{3}\right)$.

1) $S^{5/3}$ achieves $(d_1,d_2,d_3)=\left(1,\frac{1}{3},\frac{1}{3}\right)$ for $(\lambda_{\mathrm{PDD}},\lambda_{\mathrm{DDD}})=\left(\frac{2}{3},\frac{1}{3}\right)$.

In $S^{5/3}$, joint encoding over 6 PDD and 3 DDD states is performed. During the transmission, nine symbols $\{u_1^{[k]}\}_{k=1}^{9}$ are delivered to $\mathrm{Rx}_1$, three symbols $\{u_2^{[k]}\}_{k=1}^{3}$ are delivered to $\mathrm{Rx}_2$ and three symbols $\{u_3^{[k]}\}_{k=1}^{3}$ are delivered to $\mathrm{Rx}_3$.

The transmission is split into two phases. Phase 1 comprises the first five channel uses $t=1,2,3,4,5$, each having PDD state, during which the original information symbols are transmitted. From the interference terms overheard in phase 1, five terms useful for pairs of receivers, $u_{2,3}$, $u_{1,2}^{[1]}$, $u_{1,2}^{[2]}$, $u_{1,3}^{[1]}$ and $u_{1,3}^{[2]}$, referred to as order-2 symbols, are generated. The transmission of the generated order-2 symbols is performed in phase 2 which comprises the remaining four channel uses $t=6,7,8,9$, three of which have DDD state and one has PDD state. The summary of the transmission is given in Table II, where the overheard interference terms are marked in red. The detailed description of the transmission is provided below.

*Phase 1:* Phase 1 is split into $(2,3)$-stage, $(1,2)$-stage and $(1,3)$-stage, during which the order-2 symbols useful for different pairs of receivers are generated.

$(2,3)$-*stage:* $t=1$. In $(2,3)$-stage, the order-2 symbol $u_{2,3}$ is generated.

At $t=1$, the symbol $u_1^{[1]}$ is transmitted using random precoding and the symbols $u_2^{[1]}$ and $u_3^{[1]}$ are transmitted using ZF to ensure that no interference is overheard by $\mathrm{Rx}_1$. The signal transmitted at $t=1$ is given by

$$\mathbf{x}(1)=\mathbf{c}_1(1)\,u_1^{[1]}+\mathbf{c}_{23}(1)\,(u_2^{[1]}+u_3^{[1]}),\tag{9}$$

where $\mathbf{c}_1(1)\in\mathbb{C}^{2\times1}$ is a random precoding vector and $\mathbf{c}_{23}(1)\in\mathbb{C}^{2\times1}$ is a precoding vector satisfying $\mathbf{h}_1^{\mathrm{T}}(1)\,\mathbf{c}_{23}(1)=0$. At $t=1$, $\mathrm{Rx}_1$ receives an interference-free signal $y_1(1)=\mathbf{h}_1^{\mathrm{T}}(1)\,\mathbf{c}_1(1)\,u_1^{[1]}$, which allows $\mathrm{Rx}_1$ to decode $u_1^{[1]}$.

The signal received by $\mathrm{Rx}_j$, $j\in\{2,3\}$, is given by $y_j(1)=\mathbf{h}_j^{\mathrm{T}}(1)\,(\mathbf{c}_1(1)\,u_1^{[1]}+\mathbf{c}_{23}(1)\,(u_2^{[1]}+u_3^{[1]}))$, which can be written for $\mathrm{Rx}_2$ as $y_2(1)=\beta_2^{[1]}u_1^{[1]}+\beta_2^{[2]}u_2^{[1]}+\beta_2^{[3]}u_3^{[1]}=\beta_2^{[2]}u_2^{[1]}+L_2^{[1]}\left(u_1^{[1]},u_3^{[1]}\right)$ and for $\mathrm{Rx}_3$ as $y_3(1)=\beta_3^{[1]}u_1^{[1]}+\beta_3^{[2]}u_2^{[1]}+\beta_3^{[3]}u_3^{[1]}=\beta_3^{[3]}u_3^{[1]}+L_3^{[1]}\left(u_1^{[1]},u_2^{[1]}\right)$. From the interference terms overheard by $\mathrm{Rx}_2$ and $\mathrm{Rx}_3$, an order-2 symbol

$$\begin{aligned}u_{2,3}&=\beta_2^{[1]}\beta_3^{[1]}u_1^{[1]}+\beta_2^{[1]}\beta_3^{[2]}u_2^{[1]}+\beta_3^{[1]}\beta_2^{[3]}u_3^{[1]}\\&=\beta_2^{[1]}\beta_3^{[2]}u_2^{[1]}+\beta_3^{[1]}L_2^{[1]}(u_1^{[1]},u_3^{[1]})\\&=\beta_3^{[1]}\beta_2^{[3]}u_3^{[1]}+\beta_2^{[1]}L_3^{[1]}(u_1^{[1]},u_2^{[1]})\end{aligned}\tag{10}$$

is generated. The delivery of $u_{2,3}$ is to allow $\mathrm{Rx}_2$ and $\mathrm{Rx}_3$ to cancel the interference in the received signals and decode $u_2^{[1]}$ and $u_3^{[1]}$, respectively.

$(1,2)$-*stage:* $t=2,3$. In $(1,2)$-stage, the order-2 symbols $u_{1,2}^{[1]}$ and $u_{1,2}^{[2]}$ are generated.

At $t=2$, the symbol vector $\mathbf{u}_1^{[2,3]}=\left[u_1^{[2]}\quad u_1^{[3]}\right]^{\mathrm{T}}$ is transmitted using random precoding and the symbol $u_2^{[2]}$ is transmitted using ZF to ensure that no interference is overheard by $\mathrm{Rx}_1$. The signal transmitted at $t=2$ is

$$\mathbf{x}(2)=\mathbf{C}_1(2)\,\mathbf{u}_1^{[2,3]}+\mathbf{c}_2(2)\,u_2^{[2]},\tag{11}$$

where $\mathbf{C}_1(2)\in\mathbb{C}^{2\times2}$ is a random precoding matrix and $\mathbf{c}_2(2)\in\mathbb{C}^{2\times1}$ is a precoding vector satisfying $\mathbf{h}_1^{\mathrm{T}}(2)\,\mathbf{c}_2(2)=0$. At $t=2$, $\mathrm{Rx}_1$ receives $y_1(2)=\mathbf{h}_1^{\mathrm{T}}(2)\,\mathbf{C}_1(2)\,\mathbf{u}_1^{[2,3]}$, which contains the useful linear combination of the elements of $\mathbf{u}_1^{[2,3]}$. $\mathrm{Rx}_2$ receives $y_2(2)=\mathbf{h}_2^{\mathrm{T}}(2)\,(\mathbf{C}_1(2)\,\mathbf{u}_1^{[2,3]}+\mathbf{c}_2(2)\,u_2^{[2]})=\beta_2^{[4]}u_2^{[2]}+L_2^{[2]}(\mathbf{u}_1^{[2,3]})$, where from the interference term overheard by $\mathrm{Rx}_2$, an order-2 symbol $u_{1,2}^{[1]}=L_2^{[2]}(\mathbf{u}_1^{[2,3]})$ is generated. The delivery of $u_{12}^{[1]}$ is to allow $\mathrm{Rx}_1$ and $\mathrm{Rx}_2$ to decode $\mathbf{u}_1^{[2,3]}$ and $u_2^{[2]}$, respectively.

At $t=3$, the new symbol vector $\mathbf{u}_1^{[4,5]}=\left[u_1^{[4]}\quad u_1^{[5]}\right]^{\mathrm{T}}$ and the new symbol $u_2^{[3]}$ are transmitted. The signal transmitted at $t=3$ is given by $\mathbf{x}(3)=\mathbf{C}_1(3)\,\mathbf{u}_1^{[4,5]}+\mathbf{c}_2(3)\,u_2^{[3]}$, where $\mathbf{C}_1(3)\in\mathbb{C}^{2\times2}$ is a random precoding matrix and $\mathbf{c}_2(3)\in\mathbb{C}^{2\times1}$ is precoding vector satisfying $\mathbf{h}_1^{\mathrm{T}}(3)\,\mathbf{c}_2(3)=0$. From the interference term overheard by $\mathrm{Rx}_2$, the order-2 symbol $u_{1,2}^{[2]}=L_2^{[3]}(\mathbf{u}_1^{[4,5]})$ is generated.

$(1,3)$-*stage:* $t=4,5$. In $(1,3)$-stage, the order-2 symbols $u_{1,3}^{[1]}$ and $u_{1,3}^{[2]}$ are generated.

At $t=4$, the symbol vector $\mathbf{u}_1^{[6,7]}=\left[u_1^{[6]}\quad u_1^{[7]}\right]^{\mathrm{T}}$ is transmitted using random precoding and the symbol $u_3^{[2]}$ is transmitted using ZF. The signal transmitted at $t=4$ is given by $\mathbf{x}(4)=\mathbf{C}_1(4)\,\mathbf{u}_1^{[6,7]}+\mathbf{c}_3(4)\,u_3^{[2]}$, where $\mathbf{C}_1(4)\in\mathbb{C}^{2\times2}$ is a random precoding matrix and $\mathbf{c}_3(4)\in\mathbb{C}^{2\times1}$ is a precoding vector satisfying $\mathbf{h}_1^{\mathrm{T}}(4)\,\mathbf{c}_3(4)=0$. At $t=5$, the transmission is repeated with the new symbol vector $\mathbf{u}_1^{[8,9]}=\left[u_1^{[8]}\quad u_1^{[9]}\right]^{\mathrm{T}}$ and the symbol $u_3^{[3]}$. The signal transmitted at $t=5$ is given by $\mathbf{x}(5)=\mathbf{C}_1(5)\,\mathbf{u}_1^{[8,9]}+\mathbf{c}_3(5)\,u_3^{[3]}$, where $\mathbf{C}_1(5)\in\mathbb{C}^{2\times2}$ is a random precoding matrix and $\mathbf{c}_3(5)\in\mathbb{C}^{2\times1}$ is the precoding

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TABLE II: Summary of the scheme $S^{5/3}$

| $t$ | State | $\text{Rx}_1$ | $\text{Rx}_2$ | $\text{Rx}_3$ | Generated Symbol |
|---|---|---|---|---|---|
| 1 | PDD | $\beta_1^{[1]}u_1^{[1]}$ | $\beta_1^{[1]}u_2^{[1]}+L_2^{[1]}(u_1^{[1]},u_3^{[1]})$ | $\beta_2^{[1]}u_3^{[1]}+L_3^{[1]}(u_1^{[1]},u_2^{[1]})$ | $u_{2,3}=\beta_2^{[1]}\beta_3^{[2]}u_2^{[1]}+\beta_3^{[1]}L_2^{[1]}(u_1^{[1]},u_3^{[1]})$ $=\beta_3^{[1]}\beta_2^{[3]}u_3^{[1]}+\beta_2^{[1]}L_3^{[1]}(u_1^{[1]},u_2^{[1]})$ |
| 2 | PDD | $L_1^{[1]}(\mathbf{u}_1^{[2,3]})$ | $\beta_2^{[4]}u_2^{[2]}+L_2^{[2]}(\mathbf{u}_1^{[2,3]})$ | - | $u_{1,2}^{[1]}=L_2^{[2]}(\mathbf{u}_1^{[2,3]})$ |
| 3 | PDD | $L_1^{[2]}(\mathbf{u}_1^{[4,5]})$ | $\beta_2^{[5]}u_2^{[3]}+L_2^{[3]}(\mathbf{u}_1^{[4,5]})$ | - | $u_{1,2}^{[2]}=L_2^{[3]}(\mathbf{u}_1^{[4,5]})$ |
| 4 | PDD | $L_1^{[3]}(\mathbf{u}_1^{[6,7]})$ | - | $\beta_3^{[4]}u_3^{[2]}+L_3^{[2]}(\mathbf{u}_1^{[6,7]})$ | $u_{1,3}^{[1]}=L_3^{[2]}(\mathbf{u}_1^{[6,7]})$ |
| 5 | PDD | $L_1^{[4]}(\mathbf{u}_1^{[8,9]})$ | - | $\beta_3^{[5]}u_3^{[3]}+L_3^{[3]}(\mathbf{u}_1^{[8,9]})$ | $u_{1,3}^{[2]}=L_3^{[3]}(\mathbf{u}_1^{[8,9]})$ |
| 6 | DDD | $L_1^{[5]}(\mathbf{u}_{1,2})$ | $L_2^{[4]}(\mathbf{u}_{1,2})$ | $L_3^{[4]}(\mathbf{u}_{1,2})$ | $u_{1,2;3}=L_3^{[4]}(\mathbf{u}_{1,2})$ |
| 7 | DDD | $L_1^{[6]}(\mathbf{u}_{1,3})$ | $L_2^{[5]}(\mathbf{u}_{1,3})$ | $L_3^{[5]}(\mathbf{u}_{1,3})$ | $u_{1,3;2}=L_2^{[5]}(\mathbf{u}_{1,3})$ |
| 8 | DDD | $L_1^{[7]}(u_{1,2;3},u_{1,3;2})$ | $\beta_2^{[6]}u_{1,2;3}$ | $\beta_3^{[6]}u_{1,3;2}$ | - |
| 9 | PDD | $L_1^{[8]}(u_{1,2;3},u_{1,3;2})$ | $L_2^{[6]}(u_{2,3},u_{1,2;3})$ | $L_3^{[6]}(u_{2,3},u_{1,3;2})$ | - |

vector satisfying $\mathbf{h}_1^{\mathrm{T}}(5)\,\mathbf{c}_3(5)=0$. From the interference terms overheard at $t=4,5$ by $\text{Rx}_3$, the order-2 symbols $u_{1,3}^{[1]}=L_3^{[2]}(\mathbf{u}_1^{[6,7]})$ and $u_{1,3}^{[2]}=L_3^{[3]}(\mathbf{u}_1^{[8,9]})$ are generated.

*Phase 2:* In the first two channel uses $t=6,7$, each having a DDD state, the order-2 symbols $u_{1,2}^{[1]}$, $u_{1,2}^{[2]}$, $u_{1,3}^{[1]}$, $u_{1,3}^{[2]}$ are transmitted, where from the interference terms overheard at the unintended receivers, two terms $u_{1,2;3}$ and $u_{1,3;2}$ useful for two receivers and known at the remaining third receiver, referred to as order-(2,1) symbols, are generated. In the remaining two channel uses $t=8$ having DDD state and $t=9$ having PDD state, the freshly generated order-(2,1) symbols $u_{1,2;3}$ and $u_{1,3;2}$ and the remaining order-2 symbol $u_{2,3}$ are delivered to the receivers which desire them.

At $t=6$, the order-2 symbol vector $\mathbf{u}_{1,2}=\begin{bmatrix}u_{1,2}^{[1]} & u_{1,2}^{[2]}\end{bmatrix}^{\mathrm{T}}$ is transmitted using random precoding. The signal transmitted at $t=6$ is given by

$$\mathbf{x}(6)=\mathbf{C}_{1,2}(6)\,\mathbf{u}_{1,2}, \tag{12}$$

where $\mathbf{C}_{1,2}(6)\in\mathbb{C}^{2\times 2}$ is a random precoding matrix. The signal received at $t=6$ by $\text{Rx}_i$, $i\in\{1,2,3\}$, is given by $y_i(6)=\mathbf{h}_i^{\mathrm{T}}(6)\,\mathbf{C}_{1,2}(6)\,\mathbf{u}_{1,2}$, in which $\text{Rx}_1$ and $\text{Rx}_2$ receive useful linear combinations of the elements of $\mathbf{u}_{1,2}$ and $\text{Rx}_3$ overhears an interference term. From the interference term overheard by $\text{Rx}_3$, an order-(2,1) symbol $u_{1,2;3}=y_3(6)=L_3^{[4]}(\mathbf{u}_{1,2})$ useful for $\text{Rx}_1$ and $\text{Rx}_2$ and known at $\text{Rx}_3$ is generated. The delivery of $u_{1,2;3}$ is to allow $\text{Rx}_1$ and $\text{Rx}_2$ to decode $\mathbf{u}_{1,2}$.

At $t=7$, the order-2 symbol vector $\mathbf{u}_{1,3}=\begin{bmatrix}u_{1,3}^{[1]} & u_{1,3}^{[2]}\end{bmatrix}^{\mathrm{T}}$ is transmitted using random precoding. The signal transmitted at $t=7$ is given by $\mathbf{x}(7)=\mathbf{C}_{1,3}(7)\,\mathbf{u}_{1,3}$, where $\mathbf{C}_{1,3}(7)\in\mathbb{C}^{2\times 2}$ is a random precoding matrix. From the interference term overheard by $\text{Rx}_2$, an order-(2,1) symbol $u_{1,3;2}=L_2^{[5]}(\mathbf{u}_{1,3})$ useful for $\text{Rx}_1$ and $\text{Rx}_3$ and known at $\text{Rx}_2$ is generated.

At $t=8,9$, $u_{1,2;3}$ and $u_{1,3;2}$ are transmitted using random precoding and $u_{2,3}$ is transmitted using ZF. The signals transmitted at $t=8,9$ are given by

$$\mathbf{x}(8)=\mathbf{c}_{1,2;3}(8)\,u_{1,2;3}+\mathbf{c}_{1,3;2}(8)\,u_{1,3;2},$$

$$\mathbf{x}(9)=\mathbf{c}_{1,2;3}(9)\,u_{1,2;3}+\mathbf{c}_{1,3;2}(9)\,u_{1,3;2}+\mathbf{c}_{2,3}(9)\,u_{2,3} \tag{13}$$

where $\mathbf{c}_{1,2;3}(8),\mathbf{c}_{1,3;2}(8),\mathbf{c}_{1,2;3}(9),\mathbf{c}_{1,3;2}(9)\in\mathbb{C}^{2\times 1}$ are random precoding vectors and $\mathbf{c}_{2,3}(9)\in\mathbb{C}^{2\times 1}$ is a precoding vector satisfying $\mathbf{h}_1^{\mathrm{T}}(9)\,\mathbf{c}_{2,3}(9)=0$. At $t=8,9$, $\text{Rx}_1$ receives $y_1(t)=\mathbf{h}_1^{\mathrm{T}}(t)\left(\mathbf{c}_{1,2;3}(t)\,u_{1,2;3}+\mathbf{c}_{1,3;2}(t)\,u_{1,3;2}\right)$, from which both desired order-(2,1) symbols are decoded. The signals received by $\text{Rx}_2$ and $\text{Rx}_3$ are given by $y_j(8)=\mathbf{h}_j^{\mathrm{T}}(8)\left(\mathbf{c}_{1,2;3}(8)\,u_{1,2;3}+\mathbf{c}_{1,3;2}(8)\,u_{1,3;2}\right)$ and $y_j(9)=\mathbf{h}_j^{\mathrm{T}}(9)\left(\mathbf{c}_{1,2;3}(9)\,u_{1,2;3}+\mathbf{c}_{1,3;2}(9)\,u_{1,3;2}+\mathbf{c}_{2,3}(9)\,u_{2,3}\right)$, $j\in\{2,3\}$, from which $\text{Rx}_2$ and $\text{Rx}_3$ decode the desired symbols after subtracting the known order-(2,1) symbols.

### C. Scheme Achieving $\frac{3}{2}$ DoF

1) $S^{3/2}$ achieves $(d_1,d_2,d_3)=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ for $\lambda_{\text{DDD}}=1$.

Here, we refer to the scheme for delayed CSIT in [2], in which 4 symbols are delivered to every receiver in 8 time slots.

## V. Proof of Theorem 2

In this section, we provide the assignment of the CS introduced in Section IV to the available CSIT states, which results in the achievability of the DoF upper bound (3).

The encoding over PPD and PDP states is performed independently using $S_1^2$ and $S_2^2$, respectively, where the CS fractions are given by $\lambda_{S_1^2}=\lambda_{\text{PPD}}$ and $\lambda_{S_2^2}=\lambda_{\text{PDP}}$. The encoding over the remaining PPP, PDD and DDD states is performed jointly. Initially, $S_4^2$ is applied for joint encoding over PPP and PDD states. Depending on whether $\lambda_{\text{PDD}}$ is greater or smaller than $\lambda_{\text{PPP}}$, two cases are distinguished.

*1) Case A:* $\lambda_{\text{PDD}}\geq\lambda_{\text{PPP}}$. In this case, PPP state can be fully exhausted using $S_4^2$ with the CS fraction $\lambda_{S_4^2}=2\lambda_{\text{PPP}}$. The remaining PDD state fraction $\lambda_{\text{PDD}}^*=\lambda_{\text{PDD}}-\lambda_{\text{PPP}}$ is alternated with DDD state using $S^{5/3}$. Depending on whether $2\lambda_{\text{DDD}}$ is greater or smaller than $\lambda_{\text{PDD}}^*$, two sub-cases are distinguished.

A.1. $2\lambda_{\text{DDD}}\geq\lambda_{\text{PDD}}^*$: The remaining PDD state fraction $\lambda_{\text{PDD}}^*$ can be fully exhausted using $S^{5/3}$ with the CS fraction $\lambda_{S^{5/3}}=\frac{3}{2}\lambda_{\text{PDD}}^*$. Over the remaining fraction of DDD state, encoding using $S^{3/2}$ is performed with the CS fraction $\lambda_{S^{3/2}}=\lambda_{\text{DDD}}-\frac{\lambda_{\text{PDD}}^*}{2}$.

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TABLE III: Case A.1: achieving $\frac{3}{2}+\frac{1}{4}(\lambda_1+\lambda_2+\lambda_3)$ DoF for Region I, $\lambda_{\text{PDD}} \geq \lambda_{\text{PPP}}$

| CS | CSIT state fractions | DoF tuple | CS fraction |
|---|---|---|---|
| $S_1^2$ | $\lambda_{\text{PPD}}=1,$ | $(1,1,0)$ | $\lambda_{\text{PPD}}$ |
| $S_2^2$ | $\lambda_{\text{PDP}}=1,$ | $(1,0,1)$ | $\lambda_{\text{PDP}}$ |
| $S_4^2$ | $(\lambda_{\text{PPP}},\lambda_{\text{PDD}})=(\frac{1}{2},\frac{1}{2})$ | $(1,\frac{1}{2},\frac{1}{2})$ | $2\lambda_{\text{PPP}}$ |
| $S^{5/3}$ | $(\lambda_{\text{PDD}},\lambda_{\text{DDD}})=(\frac{2}{3},\frac{1}{3})$ | $(1,\frac{1}{3},\frac{1}{3})$ | $\frac{3}{2}\lambda^*_{\text{PDD}}$ |
| $S^{3/2}$ | $\lambda_{\text{DDD}}=1$ | $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ | $\lambda_{\text{DDD}}-\frac{\lambda^*_{\text{PDD}}}{2}$ |

TABLE IV: Case A.2: achieving $\frac{5}{3}+\frac{1}{3}(\lambda_2+\lambda_3)$ DoF for Region II

| CS | CSIT state fractions | DoF tuple | CS fraction |
|---|---|---|---|
| $S_1^2$ | $\lambda_{\text{PPD}}=1,$ | $(1,1,0)$ | $\lambda_{\text{PPD}}$ |
| $S_2^2$ | $\lambda_{\text{PDP}}=1,$ | $(1,0,1)$ | $\lambda_{\text{PDP}}$ |
| $S_4^2$ | $(\lambda_{\text{PPP}},\lambda_{\text{PDD}})=(\frac{1}{2},\frac{1}{2})$ | $(1,\frac{1}{2},\frac{1}{2})$ | $2\lambda_{\text{PPP}}$ |
| $S^{5/3}$ | $(\lambda_{\text{PDD}},\lambda_{\text{DDD}})=(\frac{2}{3},\frac{1}{3})$ | $(1,\frac{1}{3},\frac{1}{3})$ | $\lambda^*_{\text{PDD}}+\lambda_{\text{DDD}}$ |

A.2. $2\lambda_{\text{DDD}} < \lambda^*_{\text{PDD}}$: DDD state can be fully exhausted using $S^{5/3}$. Over all available PDD and DDD states, joint encoding is performed with the CS fraction $\lambda_{S^{5/3}} = \lambda^*_{\text{PDD}}+\lambda_{\text{DDD}}$.

*2) Case B:* $\lambda_{\text{PDD}} < \lambda_{\text{PPP}}$. In this case, PDD state can be fully exhausted using $S_4^2$ with the CS fraction $\lambda_{S_4^2} = 2\lambda_{\text{PDD}}$. The remaining PPP state fraction $\lambda^*_{\text{PPP}} = \lambda_{\text{PPP}} - \lambda_{\text{PDD}}$ is alternated with DDD state using the scheme $S_5^2$. Depending on whether $2\lambda_{\text{DDD}}$ is greater or smaller than $\lambda^*_{\text{PPP}}$, two sub-cases are distinguished.

B.1. $2\lambda_{\text{DDD}} \geq \lambda^*_{\text{PPP}}$: The remaining PPP state fraction $\lambda^*_{\text{PPP}}$ can be fully exhausted using $S_5^2$ with the CS fraction $\lambda_{S_5^2} = \frac{3}{2}\lambda^*_{\text{PPP}}$. Over the remaining fraction of DDD state, encoding using $S^{3/2}$ is performed with the CS fraction $\lambda_{S^{3/2}} = \lambda_{\text{DDD}} - \frac{\lambda^*_{\text{PPP}}}{2}$.

B.2. $2\lambda_{\text{DDD}} < \lambda^*_{\text{PPP}}$: DDD state can be fully exhausted using $\lambda_{S_5^2}$ with the CS fraction $\lambda_{S_5^2} = 3\lambda_{\text{DDD}}$. Over the remaining fraction of PPP state, encoding using $S_3^2$ is performed with the CS fraction $\lambda_{S_3^2} = \lambda^*_{\text{PPP}} - 2\lambda_{\text{DDD}}$.

*Relationship to the DoF upper bound* (3)*:* For $\lambda_{\text{DPP}} = \lambda_{\text{DPD}} = \lambda_{\text{DDP}} = 0$, the relationship

$$\begin{aligned}\lambda_1 &= \lambda_{\text{PPP}}+\lambda_{\text{PPD}}+\lambda_{\text{PDP}}+\lambda_{\text{PDD}},\\ \lambda_2 &= \lambda_{\text{PPP}}+\lambda_{\text{PPD}},\\ \lambda_3 &= \lambda_{\text{PPP}}+\lambda_{\text{PDP}},\end{aligned} \tag{14}$$

holds. In such case, $2\lambda_{\text{DDD}} \geq \lambda^*_{\text{PDD}}$ is equivalent to $3\lambda_1-\lambda_2-\lambda_3 \leq 2$ and $2\lambda_{\text{DDD}} \geq \lambda^*_{\text{PPP}}$ is equivalent to $\lambda_1+\lambda_2+\lambda_3 \leq 2$. Hence, Cases A.1 and B.1 correspond to Region I with $\lambda_{\text{PDD}} \geq \lambda_{\text{PPP}}$ and $\lambda_{\text{PDD}} < \lambda_{\text{PPP}}$, respectively, and Cases A.2 and B.2 correspond to Regions II and III, respectively. The calculations of the achieved DoF for each of the cases are given in Tables III, IV, V and VI.

TABLE V: Case B.1: achieving $\frac{3}{2}+\frac{1}{4}(\lambda_1+\lambda_2+\lambda_3)$ DoF for Region I, $\lambda_{\text{PDD}} < \lambda_{\text{PPP}}$

| CS | CSIT state fractions | DoF tuple | CS fraction |
|---|---|---|---|
| $S_1^2$ | $\lambda_{\text{PPD}}=1$ | $(1,1,0)$ | $\lambda_{\text{PPD}}$ |
| $S_2^2$ | $\lambda_{\text{PDP}}=1,$ | $(1,0,1)$ | $\lambda_{\text{PDP}}$ |
| $S_4^2$ | $(\lambda_{\text{PPP}},\lambda_{\text{PDD}})=(\frac{1}{2},\frac{1}{2})$ | $(1,\frac{1}{2},\frac{1}{2})$ | $2\lambda_{\text{PDD}}$ |
| $S_5^2$ | $(\lambda_{\text{PPP}},\lambda_{\text{DDD}})=(\frac{2}{3},\frac{1}{3})$ | $(\frac{2}{3},\frac{2}{3},\frac{2}{3})$ | $\frac{3}{2}\lambda^*_{\text{PPP}}$ |
| $S^{3/2}$ | $\lambda_{\text{DDD}}=1$ | $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ | $\lambda_{\text{DDD}}-\frac{\lambda^*_{\text{PPP}}}{2}$ |

TABLE VI: Case B.2: achieving 2 DoF for Region III

| CS | CSIT state fractions | DoF tuple | CS fraction |
|---|---|---|---|
| $S_1^2$ | $\lambda_{\text{PPD}}=1,$ | $(1,1,0)$ | $\lambda_{\text{PPD}}$ |
| $S_2^2$ | $\lambda_{\text{PDP}}=1,$ | $(1,0,1)$ | $\lambda_{\text{PDP}}$ |
| $S_4^2$ | $(\lambda_{\text{PPP}},\lambda_{\text{PDD}})=(\frac{1}{2},\frac{1}{2})$ | $(1,\frac{1}{2},\frac{1}{2})$ | $2\lambda_{\text{PDD}}$ |
| $S_5^2$ | $(\lambda_{\text{PPP}},\lambda_{\text{DDD}})=(\frac{2}{3},\frac{1}{3})$ | $(\frac{2}{3},\frac{2}{3},\frac{2}{3})$ | $3\lambda_{\text{DDD}}$ |
| $S_3^2$ | $\lambda_{\text{PPP}}=1$ | $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ | $\lambda^*_{\text{PPP}}-2\lambda_{\text{DDD}}$ |

## VI. Conclusion

In this paper, the 2-antenna 3-user MISO BC with alternating CSIT was considered. We showed the achievability of the optimal DoF for the CSIT setting in which the admissible CSIT states are PPP, PPD, PDP, PDD and DDD. To accomplish this, two novel CSs were proposed. After a careful assignment of the newly proposed and existing in the literature CSs to the available CSIT states, optimal DoF were achieved.


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