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On the Achievable Degrees of Freedom of the MIMO X-Channel with Delayed CSIT

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***Abstract*—We consider the two-user multiple-input multiple-output X-channel where the transmitters 1, 2 have M_1, M_2 antennas and the receivers 1, 2 have N_1, N_2 antennas, respectively, and study the achievable number of degrees of freedom (DoF) of this network under the assumption of delayed channel state information at the transmitters. For this scenario, Kao and Avestimehr proposed in [1] a transmission scheme, which was conjectured to achieve the number of DoF of this network under the assumption of linear encoding strategies at the transmitters. In our paper, we show that the number of DoF achieved using this scheme is less than previously reported due to the fact that the transmitted information symbols are not always decodable at the receivers. This is shown by performing linear independence analysis of the received linear combinations, where new decodability constraints on the parameters of the transmission scheme are identified for the region of antenna configurations where $N_1 + N_2 > \max\{M_1, M_2\}$, $M_1 + M_2 > \max\{N_1, N_2\}$ and $\min\{M_1, M_2\} > \min\{N_1, N_2\}$ hold. Based on the identified constraints, we propose a new transmission scheme, which for the case where the identified constraints are active, achieves the number of DoF greater than that achieved by the transmission scheme proposed in [1], where only the number of decodable information symbols is transmitted.**

I. Introduction

The number of degrees of freedom (DoF) of the multiple-input multiple-output (MIMO) X-channel (XC) has been established in [2] and [3], where a technique named interference alignment (IA) was used to achieve the number of DoF. One of the requirements of IA is the instantaneous channel state information at the transmitters (CSIT), which is difficult to ensure in real communication systems. Under independent and identically distributed (i.i.d.) fading, delays in CSIT make IA infeasible and the achievable number of DoF reduces to that achieved in absence of CSIT, which was evaluated in [4].

However, [5] has shown that even under i.i.d. fading, delayed CSIT can be used to increase the achievable number of DoF as compared to the case of absence of CSIT. For multiple-input single-output (MISO) broadcast channel (BC) with delayed CSIT, [5] evaluated the number of DoF, where the achievability was based on applying an innovative multi-phase transmission strategy. The DoF gains were achieved by retransmitting the previously overheard interference reconstructed using the delayed CSIT, which provided the receivers information about the desired information symbols and was cancelled at the unintended receivers.

The approach of [5] was later applied to the single-input single-output (SISO) XC with delayed CSIT in [6]–[8], where gains in the achievable number of DoF compared to the case of absence of CSIT were reported. The MIMO XC with delayed CSIT with the symmetric antenna configurations where $M_1 = M_2 = M$ and $N_1 = N_2 = N$ hold was considered in [9], where the number of DoF was evaluated except for the region of antenna configurations of $3/4 < M/N < 2$. [1] conjectured the complete DoF characterization of the MIMO XC with delayed CSIT, where for the DoF upper bound the assumption of linear encoding strategies at the transmitters was used.

In this paper, we perform the decodability analysis of the number of DoF achieving scheme proposed in [1]. We consider the antenna configurations where $N_1 + N_2 > \max\{M_1, M_2\}$ and $M_1 + M_2 > \max\{N_1, N_2\}$ hold, which corresponds to the case, where in [1] both transmitters are active during the transmission and employ for the transmission the delayed CSIT. Additionally, we restrict the antenna configurations to satisfy $\min\{M_1, M_2\} > \min\{N_1, N_2\}$. To perform the decodability analysis, we study linear independence of the received linear combinations at the receivers. We show that the information symbols transmitted using the transmission scheme of [1] are not always decodable, where we identify new decodability constraints on the parameters of the transmission scheme of [1]. Based on the identified constraints, we propose a new transmission scheme, which for the case where the identified constraints are active, achieves the number of DoF greater than that achieved by the scheme proposed in [1], where only the number of decodable information symbols is transmitted.

The rest of the paper is organized as follows. Section II describes the system model. In Section III, the transmission scheme proposed in [1] is introduced and in Section IV, the decodability analysis is performed. In Section V, we describe the proposed transmission scheme and give the achieved number of DoF in Section VI.

II. System Model

We consider the 2-user MIMO XC depicted in Fig. 1, where transmitter Tx_i has M_i antennas and receiver Rx_j has N_j antennas, $\forall i, j \in \{1, 2\}$. We assume that $N_1 + N_2 > \max\{M_1, M_2\}$, $M_1 + M_2 > \max\{N_1, N_2\}$ and $\min\{M_1, M_2\} > \min\{N_1, N_2\}$ hold. Additionally, without loss of generality, we assume $M_1 \geq M_2$.

The transmission is performed for the duration of T_Σ time slots, during which each of the transmitters Tx_i, $i \in \{1, 2\}$,

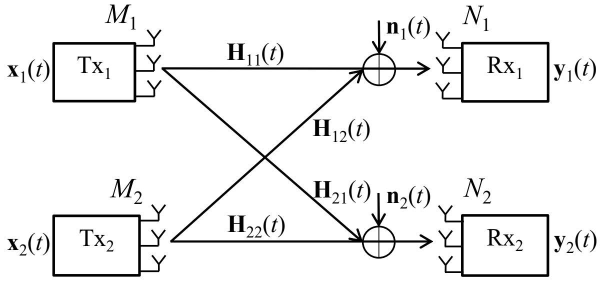

Fig. 1. The two-user MIMO XC

intends to communicate two vectors of information symbols $\mathbf{u}_{1i} \in \mathbb{C}^{b_{1i} \times 1}$ and $\mathbf{u}_{2i} \in \mathbb{C}^{b_{2i} \times 1}$ to Rx_1 and Rx_2, respectively, where b_{ji} denotes the number of the information symbols to be delivered from Tx_i to Rx_j, $j \in \{1,2\}$. Let $\mathbf{x}_i(t)$ be the signal transmitted by Tx_i in time slot t, $1 \leq t \leq T_\Sigma$. The signal received by Rx_j in time slot t is defined as

$$\mathbf{y}_j(t) = \mathbf{H}_{j1}(t)\,\mathbf{x}_1(t) + \mathbf{H}_{j2}(t)\,\mathbf{x}_2(t) + \mathbf{n}_j(t), \qquad (1)$$

where $\mathbf{H}_{ji}(t) \in \mathbb{C}^{N_j \times M_i}$ is the channel matrix between Tx_i and Rx_j in time slot t and $\mathbf{n}_j(t) \sim \mathcal{CN}\left(0, \mathbf{I}_{N_j}\right)$. The signal transmitted by Tx_i is subject to the average power constraint of $\frac{1}{T_\Sigma} \sum_{t=1}^{T_\Sigma} \mathrm{E}\left[\mathbf{x}_i^{\mathrm{H}}(t)\,\mathbf{x}_i(t)\right] \leq P$, where P is the maximum transmit power at a transmitter.

The entries of the channel matrix $\mathbf{H}_{ji}(t), \forall i,j \in \{1,2\}$, are drawn randomly from a continuous distribution and are i.i.d. across antennas, time slots and across different transmitter and receiver pairs. We assume that each receiver has the instantaneous global channel knowledge, i.e. in time slot t, $1 \leq t \leq T_\Sigma$, each receiver has the access to the sets of channel matrices $\{\mathbf{H}_{ji}(\tau)\}_{\tau=1}^{t}$, $\forall i,j \in \{1,2\}$. Each transmitter obtains the global channel knowledge with a single time slot delay, i.e. in time slot t, $2 \leq t \leq T_\Sigma$, it has the access to the sets of channel matrices $\{\mathbf{H}_{ji}(\tau)\}_{\tau=1}^{t-1}$, $\forall i,j \in \{1,2\}$.

We say that the number of DoF $d = \frac{1}{T_\Sigma} \sum_{j=1}^{2} \sum_{i=1}^{2} b_{ji}$ is achievable in the two-user MIMO XC, if the information symbol vector $\mathbf{u}_{ji}$ transmitted from Tx_i to Rx_j, $\forall i,j \in \{1,2\}$, is decodable with probability one.

III. Transmission Scheme

In this section, we describe the transmission scheme of [1], in order to perform its decodability analysis in Section IV.

A. Structure of Transmission Scheme

The overall transmission is comprised of three phases, where phase l, $l \in \{1,2,3\}$, has a duration of T_l time slots, $T_\Sigma = \sum_{l=1}^{3} T_l$. In phase 1, Tx_1 and Tx_2 simultaneously transmit $\mathbf{u}_{11}$ and $\mathbf{u}_{12}$ to Rx_1, where Rx_1 receives useful signal and Rx_2 overhears interference, which is useful for Rx_1. Phase 2 is similar to phase 1, where the transmitters transmit $\mathbf{u}_{21}$ and $\mathbf{u}_{22}$ to Rx_2, while Rx_1 overhears interference. In phase 3, the interference overheard at the unintended receivers in phases 1 and 2 is reconstructed using the delayed CSIT and retransmitted, where the receivers obtain useful information about the desired information symbols and the known interference is cancelled.

In our paper, we focus on the antenna configurations of $\min\{M_1, M_2\} > \min\{N_1, N_2\}$, where in phases 1 and 2 of the transmission scheme of [1], a special transmission strategy is applied for the case, where both transmitters have the numbers of antennas greater than the number of antennas at an unintended receiver. In our paper, we study the decodability of the information symbols transmitted using such transmission strategy by considering the transmission in phase 1 for the antenna configurations of $\min\{M_1, M_2\} > N_2$. In our study, we assume that the interference terms generated at Rx_2 in phase 1 are directly provided to Rx_1 without the transmission in phase 3. We conjecture, that the transmission of the interference terms in phase 3 can be omitted from the decodability analysis without loss of generality, since the transmission is performed through an i.i.d. channel.

According to [1], phase 1 is divided into k transmission periods, referred throughout the paper as transmission blocks, where the information symbols transmitted in each transmission block can be decoded independently from the information symbols transmitted in other transmission blocks. The transmission blocks have an equal duration of $T = T_1/k$ time slots, during which $b_i = b_{1i}/k$ information symbols are transmitted by Tx_i, $i \in \{1,2\}$, and q interference terms are overheard by Rx_2. Due to the identical structure of the transmission blocks, the decodability analysis is performed only for the first transmission block.

B. Transmission in Phase 1

The transmission of the first transmission block of phase 1 spans the time slots of $1 \leq t \leq T$, during which Tx_1 and Tx_2 transmit the information symbol vectors $\mathbf{u}_1 \in \mathbb{C}^{b_1 \times 1}$ and $\mathbf{u}_2 \in \mathbb{C}^{b_2 \times 1}$ to Rx_1, where $b_1 = M_1 T$ and $b_2 \geq N_2 T^{(1)}$ hold. The overall transmission is split into two parts comprising $T^{(1)}$ time slots of $1 \leq t \leq T^{(1)}$ and $T^{(2)} = T - T^{(1)}$ time slots of $T^{(1)} + 1 \leq t \leq T$ of the transmission block, which will be referred as part 1 and 2 of the transmission block, respectively.

Part 1: Let us denote the concatenation of the signal vectors transmitted by Tx_i, $i \in \{1,2\}$, during part 1 of the transmission block as

$$\mathbf{x}_i^{(1)} = \left[\mathbf{x}_i(1)^{\mathrm{T}} \quad \cdots \quad \mathbf{x}_i\left(T^{(1)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{M_i T^{(1)} \times 1}, \qquad (2)$$

where $\mathbf{x}_i(t) \in \mathbb{C}^{M_i \times 1}$ is the signal vector transmitted by Tx_i in time slot t. Let us denote the information symbol vector, which is comprised of the first $M_1 T^{(1)}$ elements of $\mathbf{u}_1$ by $\mathbf{u}_1^{(1)} \in \mathbb{C}^{M_1 T^{(1)} \times 1}$. According to [1], in each time slot, Tx_1 transmits a new information symbol of $\mathbf{u}_1^{(1)}$ from each antenna, where the transmitted signal vector is described as $\mathbf{x}_1^{(1)} = \mathbf{u}_1^{(1)}$.

Let us denote the concatenation of the random precoding matrices used by Tx_2 in part 1 of the transmission block as

$$\mathbf{C}_2^{(1)} = \left[\mathbf{C}_2(1)^{\mathrm{T}} \quad \cdots \quad \mathbf{C}_2\left(T^{(1)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{M_2 T^{(1)} \times b_2}, \qquad (3)$$

where $\mathbf{C}_2(t) \in \mathbb{C}^{M_2 \times b_2}$ is the precoding matrix used by Tx_2 in time slot t, the entries of which are i.i.d. and are taken from a continuous distribution. The signal vector transmitted by Tx_2 is calculated as $\mathbf{x}_2^{(1)} = \mathbf{C}_2^{(1)} \mathbf{u}_2$.

Let $\mathbf{H}_{ji}^{(1)} \in \mathbb{C}^{N_j \times M_i}$ denote the diagonal concatenation of the channel matrices between Tx_i and Rx_j, $\forall i, j \in \{1, 2\}$, for part 1 of the transmission block, which is defined as

$$\mathbf{H}_{ji}^{(1)} = \begin{bmatrix} \mathbf{H}_{ji}(1) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{H}_{ji}\left(T^{(1)}\right) \end{bmatrix} \in \mathbb{C}^{N_j T^{(1)} \times M_i T^{(1)}}, \tag{4}$$

where $\mathbf{H}_{ji}(t)$ is the channel matrix between Tx_i and Rx_j in time slot t. The concatenation of the signal vectors received by Rx_j, $j \in \{1, 2\}$, in part 1 of the transmission block

$$\mathbf{y}_j^{(1)} = \left[\mathbf{y}_j(1)^{\mathrm{T}} \quad \cdots \quad \mathbf{y}_j\left(T^{(1)}\right)^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{C}^{M_j T^{(1)} \times 1} \tag{5}$$

can then be evaluated as

$$\mathbf{y}_j^{(1)} = \mathbf{H}_{j1}^{(1)} \mathbf{u}_1 + \mathbf{H}_{j2}^{(1)} \mathbf{C}_2^{(1)} \mathbf{u}_2 + \mathbf{n}_j^{(1)}, \tag{6}$$

where $\mathbf{y}_j(t) \in \mathbb{C}^{M_j \times 1}$ is the signal received by Rx_j in time slot t and $\mathbf{n}_j^{(1)} \sim \mathcal{CN}\left(0, \mathbf{I}_{N_j T^{(1)}}\right)$.

Part 2: Similarly to (2), let $\mathbf{x}_i^{(2)} \in \mathbb{C}^{M_i T^{(2)} \times 1}$ denote the concatenation of the signal vectors transmitted by Tx_i, $i \in \{1, 2\}$, in part 2 of the transmission block. Let $\mathbf{u}_1^{(2)} \in \mathbb{C}^{M_1 T^{(2)} \times 1}$ denote the symbol vector, which is comprised of the last $M_1 T^{(2)}$ elements of $\mathbf{u}_1$. The signal vector transmitted by Tx_1 in part 2 of the transmission block is calculated similarly to part 1 as $\mathbf{x}_1^{(2)} = \mathbf{u}_1^{(2)}$.

According to [1], in part 2 of the transmission block Tx_2 retransmits the interference, which was overheard in part 1 by Rx_2. Similarly to (3), let us denote the concatenation of the random precoding matrices used by Tx_2 in part 2 of the transmission block as $\mathbf{C}_2^{(2)} \in \mathbb{C}^{M_2 T^{(2)} \times N_2 T^{(1)}}$. The signal vector transmitted by Tx_2 can then be evaluated as $\mathbf{x}_2^{(2)} = \mathbf{C}_2^{(2)} \mathbf{H}_{22}^{(1)} \mathbf{C}_2^{(1)} \mathbf{u}_2$.

Similarly to (5), let us denote the concatenation of the signal vectors received by Rx_j, $j \in \{1, 2\}$, in part 2 of the transmission block as $\mathbf{y}_j^{(2)} \in \mathbb{C}^{N_j T^{(2)} \times 1}$. Similarly to (4), let $\mathbf{H}_{ji}^{(2)} \in \mathbb{C}^{N_j T^{(2)} \times M_i T^{(2)}}$ denote the diagonal concatenation of the channel matrices between Tx_i and Rx_j, $\forall i, j \in \{1, 2\}$, for part 2 of the transmission block. The signal vector received by Rx_j can then be calculated as

$$\mathbf{y}_j^{(2)} = \mathbf{H}_{j1}^{(2)} \mathbf{u}_1^{(2)} + \mathbf{H}_{j2}^{(2)} \mathbf{C}_2^{(2)} \mathbf{H}_{22}^{(1)} \mathbf{C}_2^{(1)} \mathbf{u}_2 + \mathbf{n}_j^{(2)}, \tag{7}$$

where $\mathbf{n}_j^{(2)} \sim \mathcal{CN}\left(0, \mathbf{I}_{N_j T^{(2)}}\right)$.

The concatenation of all signal vectors received during the transmission block by Rx_j, $j \in \{1, 2\}$, denoted as $\mathbf{y}_j = \left[\mathbf{y}_j^{(1)\mathrm{T}} \quad \mathbf{y}_j^{(2)\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{C}^{N_j T \times 1}$ can be evaluated as

$$\mathbf{y}_j = \begin{bmatrix} \mathbf{H}_{j1}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{j1}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{u}_1^{(1)} \\ \mathbf{u}_1^{(2)} \end{bmatrix} + \begin{bmatrix} \mathbf{H}_{j2}^{(1)} \mathbf{C}_2^{(1)} \\ \mathbf{H}_{j2}^{(2)} \mathbf{C}_2^{(2)} \mathbf{H}_{22}^{(1)} \mathbf{C}_2^{(1)} \end{bmatrix} \mathbf{u}_2 + \mathbf{n}_j, \tag{8}$$

where $\mathbf{n}_j \sim \mathcal{CN}\left(0, \mathbf{I}_{N_j T}\right)$.

C. Generation of Interference Terms

Let us consider the signal vector $\mathbf{y}_2$ received by Rx_2, which is defined in (8). Since the entries of $\mathbf{H}_{22}^{(1)}$ and $\mathbf{C}_2^{(1)}$ are distributed independently, the signal of Tx_2 will almost surely span $\left(T^{(1)} N_2\right)$-dimensional space at Rx_2. It means there exists a full rank matrix $\mathbf{W}_2 \in \mathbb{C}^{N_2 T^{(2)} \times N_2 T}$, for which

$$\mathbf{W}_2 \begin{bmatrix} \mathbf{I}_{N_2 T^{(1)}} \\ \mathbf{H}_{22}^{(2)} \mathbf{C}_2^{(2)} \end{bmatrix} = \mathbf{0}_{N_2 T^{(2)} \times N_2 T^{(1)}} \tag{9}$$

holds. Without loss of generality, we assume $\mathbf{W}_2 = \left[-\mathbf{H}_{22}^{(2)} \mathbf{C}_2^{(2)} \quad \mathbf{I}_{N_2 T^{(2)}} \right]$. By multiplying $\mathbf{y}_2$ with $\mathbf{W}_2$, Rx_2 obtains the vector of $q = N_2 T^{(2)}$ interference terms, which is evaluated as

$$\mathbf{u}_{1;2} = \mathbf{W}_2 \mathbf{y}_2 = \left[-\mathbf{H}_{22}^{(2)} \mathbf{C}_2^{(2)} \mathbf{H}_{21}^{(1)} \quad \mathbf{H}_{21}^{(2)} \right] \begin{bmatrix} \mathbf{u}_1^{(1)} \\ \mathbf{u}_1^{(2)} \end{bmatrix}, \tag{10}$$

where the noise term has been omitted since it does not influence the DoF analysis.

By omitting the noise, we define the vector

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{u}_{1;2} \end{bmatrix} = \mathbf{H}_1 \left[\mathbf{u}_1^{(1)\mathrm{T}} \quad \mathbf{u}_1^{(2)\mathrm{T}} \quad \mathbf{u}_2^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{C}^{\left(N_1 T + N_2 T^{(2)}\right) \times 1}, \tag{11}$$

which contains all linear combinations of $\mathbf{u}_1$ and $\mathbf{u}_2$, which will be used by Rx_1 for decoding, where $\mathbf{H}_1 \in \mathbb{C}^{\left(N_1 T + N_2 T^{(2)}\right) \times (M_1 T + b_2)}$ is the effective channel matrix. From (8) and (10), it follows that $\mathbf{H}_1$ has a form of

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{H}_{11}^{(1)} & \mathbf{0} & \mathbf{H}_{12}^{(1)} \mathbf{C}_2^{(1)} \\ \mathbf{0} & \mathbf{H}_{11}^{(2)} & \mathbf{H}_{12}^{(2)} \mathbf{C}_2^{(2)} \mathbf{H}_{22}^{(1)} \mathbf{C}_2^{(1)} \\ -\mathbf{H}_{22}^{(2)} \mathbf{C}_2^{(2)} \mathbf{H}_{21}^{(1)} & \mathbf{H}_{21}^{(2)} & \mathbf{0} \end{bmatrix}. \tag{12}$$

IV. Decodability Analysis

In this section, we perform the analysis of the decodability of the information symbol vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ by Rx_1. We suppose that Rx_1 obtains the number of the linear combinations which is equal to the number of the transmitted information symbols $N_1 T + N_2 T^{(2)} = M_1 T + b_2$, which is rewritten as

$$b_2 = (N_1 + N_2 - M_1) T - N_2 T^{(1)}. \tag{13}$$

In such case, the information symbols are decodable if and only if the linear combinations obtained by Rx_1 are linearly independent, i.e. $\mathbf{H}_1$ is full rank.

$T^{(1)}$ can be chosen by considering the decodability constraint of $b_2 \le M_2 T^{(1)}$, which is rewritten as

$$T^{(1)} \ge \frac{N_1 + N_2 - M_1}{M_2 + N_2} T \tag{14}$$

using (13). Since b_2 is inversely proportional to $T^{(1)}$, in order to maximize b_2, $T^{(1)}$ has to be chosen as a minimum satisfying (14), which will yield the parameters of the scheme of [1]. The following example shows that in such case $\mathbf{H}_1$ is not always full rank.

Example: Suppose $M_1 = M_2 = 4$, $N_1 = 6$ and $N_2 = 1$, where according to [1], $T = 5$, $T^{(1)} = 3$ and $T^{(2)} = 2$. In part 2 of the transmission block, Tx_1 transmits $M_1 T^{(2)} = 8$ symbols and Tx_2 retransmits $N_2 T^{(1)} = 3$ terms. The length of $\mathbf{y}_1^{(2)}$ is $N_1 T^{(2)} = 12 > M_1 T^{(2)} + N_2 T^{(1)} = 11$, hence the linear combinations comprising $\mathbf{y}_1^{(2)}$ are linearly dependent.

The following theorem helps us to identify the constraints on $T^{(1)}$ which are necessary for $\mathbf{H}_1$ to be full rank.

Theorem 1*:* $\mathbf{H}_1$ is rank deficient if

$$N_2 \min\left\{T, 2T^{(1)}\right\} + M_1 T^{(2)} < (N_1 + N_2)\, T^{(2)}. \tag{15}$$

Proof: Let us consider the matrix

$$\mathbf{H}_1' = \begin{bmatrix} \mathbf{0} & \mathbf{H}_{11}^{(2)} & \mathbf{H}_{12}^{(2)}\mathbf{C}_2^{(2)}\mathbf{H}_{22}^{(1)}\mathbf{C}_2^{(1)} \\ -\mathbf{H}_{22}^{(2)}\mathbf{C}_2^{(2)}\mathbf{H}_{21}^{(1)} & \mathbf{H}_{21}^{(2)} & \mathbf{0} \end{bmatrix}, \tag{16}$$

which is comprised of the last $N_1 T^{(2)} + N_2 T^{(2)}$ rows of $\mathbf{H}_1$. We show the rank deficiency of $\mathbf{H}_1$ by showing the rank deficiency of $\mathbf{H}_1'$, which has at most a rank of

$$\operatorname{rank}(\mathbf{H}_1') \le (N_1 + N_2)\, T^{(2)}. \tag{17}$$

Using the rank property of horizontally concatenated matrices, we upper bound the rank of $\mathbf{H}_1'$ by the sum of the ranks of the matrices constituting $\mathbf{H}_1'$ as

$$\operatorname{rank}(\mathbf{H}_1') \le \operatorname{rank}\left(\begin{bmatrix}\mathbf{H}_{11}^{(2)\mathrm{T}} & \mathbf{H}_{21}^{(2)\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\right) + \operatorname{rank}\left(\mathbf{H}_{22}^{(2)}\mathbf{C}_2^{(2)}\mathbf{H}_{21}^{(1)}\right) + \operatorname{rank}\left(\mathbf{H}_{12}^{(2)}\mathbf{C}_2^{(2)}\mathbf{H}_{22}^{(1)}\mathbf{C}_2^{(1)}\right). \tag{18}$$

The terms constituting the right hand side of (18) can be upper bounded using the rank property of matrix products as

$$\operatorname{rank}\left(\mathbf{H}_{22}^{(2)}\mathbf{C}_2^{(2)}\mathbf{H}_{21}^{(1)}\right) \le N_2 \min\left\{T^{(1)}, T^{(2)}\right\}, \tag{19}$$

$$\operatorname{rank}\left(\mathbf{H}_{12}^{(2)}\mathbf{C}_2^{(2)}\mathbf{H}_{22}^{(1)}\mathbf{C}_2^{(1)}\right) \le \min\left\{N_1 T^{(2)}, M_2 T^{(2)}, N_2 T^{(1)}\right\}. \tag{20}$$

By inserting (19), (20) in (18) and using the properties of $\operatorname{rank}\left(\begin{bmatrix}\mathbf{H}_{11}^{(2)\mathrm{T}} & \mathbf{H}_{21}^{(2)\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\right) \le M_1 T^{(2)}$ and (17), one obtains

$$\operatorname{rank}(\mathbf{H}_1') \le N_2 \min\left\{T, 2T^{(1)}\right\} + M_1 T^{(2)}, \tag{21}$$

which, given (15) holds, states that $\mathbf{H}_1'$ and $\mathbf{H}_1$ are rank deficient.

■

To obtain the conditions which are necessary for $\mathbf{H}_1$ to be full rank, the condition where (15) does not hold is rewritten as two inequalities involving $T^{(1)}$ as

$$T^{(1)} \ge \frac{N_1 + N_2 - M_1}{N_1 + 3N_2 - M_1} T, \tag{22}$$

$$T^{(1)} \ge \frac{N_1 - M_1}{N_1 + N_2 - M_1} T. \tag{23}$$

In cases where the right hand sides of (22) or (23) are greater than the right hand side of (14), the information symbols transmitted using the transmission scheme of [1] are not decodable, where the exact regions will be given in Section V.

TABLE I

REGIONS OF ANTENNA CONFIGURATIONS WHERE PROPOSED TRANSMISSION IS APPLIED

	$\min\{M_1, M_2\} > N_2$	$\min\{M_1, M_2\} > N_1$	$\min\{M_1, M_2\} > \max\{N_1, N_2\}$
Phase 1	Proposed	[1]	Proposed
Phase 2	[1]	Proposed	Proposed

To maximize b_2 while ensuring the decodability, $T^{(1)}$ has to be chosen as a minimum satisfying (14), (22) and (23). The following theorem states the decodability of the transmitted information symbols.

Theorem 2*:* Given $T^{(1)}$ is a minimum satisfying (14), (22), (23), matrix $\mathbf{H}_1$ is full rank almost surely.

The proof of Theorem 2 is omitted due to space limitation.

V. Proposed Transmission Scheme

In this section, we design the proposed transmission scheme by modifying phases 1 and 2 of the transmission scheme of [1], where the antenna configurations where the modification is applied are shown in Table I. The modified transmission in phase 1 is based on the decodability analysis of Section IV, where the modified transmission in phase 2 can be performed similarly by swapping the receiver indices.

According to Section IV, we set $T^{(1)}$ as a minimum satisfying (14), (22) and (23). Next, we identify three regions of antenna configurations, where either (14), (22) or (23) are active.

Region 1: (14) is active. (14) overrides (22) and (23) when

$$M_1 + M_2 \le N_1 + 2N_2,$$
$$(N_1 + N_2 - M_1)^2 - (M_2 + N_2)(N_1 - M_1) \ge 0 \tag{24}$$

hold, respectively. To satisfy (14), we choose

$$T = M_2 + N_2,\ T^{(1)} = N_1 + N_2 - M_1,$$
$$b_2 = M_2 (N_1 + N_2 - M_1). \tag{25}$$

In Region 1, the parameters of the transmission scheme are identical to that of [1].

Region 2: (22) is active. (22) overrides (14) and (23) when

$$M_1 + M_2 > N_1 + 2N_2,\ N_1 - M_1 \le N_2 \tag{26}$$

hold, respectively. To satisfy (22), we choose

$$T = N_1 + 3N_2 - M_1,\ T^{(1)} = N_1 + N_2 - M_1,$$
$$b_2 = (N_1 + 2N_2 - M_1)(N_1 + N_2 - M_1). \tag{27}$$

Region 3: (23) is active. (23) overrides (14) and (22) when

$$(N_1 + N_2 - M_1)^2 - (M_2 + N_2)(N_1 - M_1) < 0,$$
$$N_1 - M_1 > N_2 \tag{28}$$

hold, respectively. To satisfy (23), we choose

$$T = N_1 + N_2 - M_1,\ T^{(1)} = N_1 - M_1,$$
$$b_2 = (N_1 + N_2 - M_1)^2 - N_2 (N_1 - M_1). \tag{29}$$

Region 1, 2 and 3 are shown in Fig. 2, where in Region 2 and 3 the information symbols transmitted using the transmission scheme of [1] are not decodable.

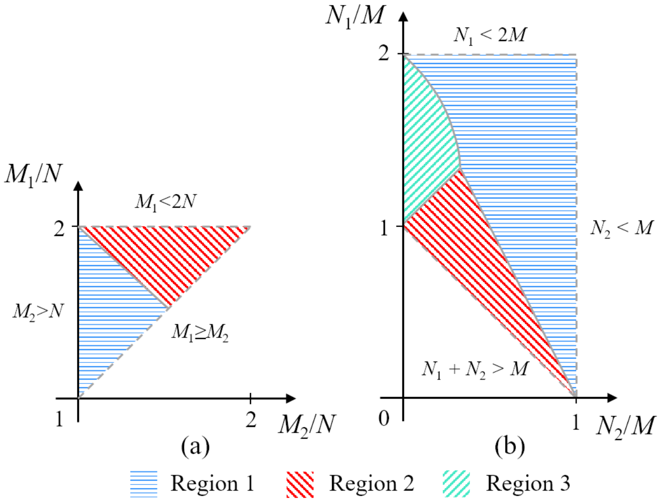

Fig. 2. Region 1, 2 and 3 for the cases of (a) $N_1 = N_2 = N$ and (b) $M_1 = M_2 = M$.

VI. Achieved Number of DoF

In this section, we evaluate the number of DoF achieved using the proposed transmission scheme. For arbitrary antenna configurations, the achieved number of DoF of the proposed transmission scheme can be evaluated according to [1], where in our paper, due to space limitation, we calculate the achieved number of DoF only for the symmetric antenna configurations where $M_1 = M_2 = M$ and $N_1 = N_2 = N$ hold. In such case, phases 1 and 2 have identical parameters, where Region 3 is empty and Region 1 and 2 correspond to the regions of $1 < \frac{M}{N} \leq 3/2$ and $\frac{3}{2} < \frac{M}{N} < 2$, respectively. By choosing $k = 1$ and $T_3 = \frac{q}{N}$, we evaluate the achieved number of DoF of the proposed transmission scheme using (25) and (27) as

$$d = \begin{cases} \frac{6MN}{4M+N}, & \text{if } 1 < \frac{M}{N} \leq \frac{3}{2}, \\ \frac{N(6N-M)}{5N-M}, & \text{if } \frac{3}{2} < \frac{M}{N} < 2, \end{cases} \tag{30}$$

which for $1 < \frac{M}{N} \leq \frac{3}{2}$ is identical to that of [1].

In the region of $\frac{3}{2} < \frac{M}{N} < 2$, we compare the proposed transmission scheme to the transmission scheme of [1], where only the number of decodable information symbols is transmitted. For phases 1 and 2, we calculate the number of the decodable information symbols using the left hand side of (15) as $b_1 + b_2 = MT + (3N - M)T^{(1)}$ and the number of the retransmitted interference terms as $q = b_1 + b_2 - NT = (M - N)T + (3N - M)T^{(1)}$. The achieved number of DoF of [1] in the region of $\frac{3}{2} < \frac{M}{N} < 2$ is then evaluated as $d = \frac{2N\left(2M^2-4MN+6N^2\right)}{2M^2-3MN+7N^2}$.

In Fig. 3, we compare the number of DoF achieved using the proposed transmission scheme to the number of DoF achieved using the transmission scheme of [1], where additionally the transmission scheme of [9] is added to the comparison. In the region of antenna configurations of $\frac{3}{2} < \frac{M}{N} < 2$, the number of DoF achieved using the proposed transmission scheme is greater than that achieved by the schemes of [1] and [9].

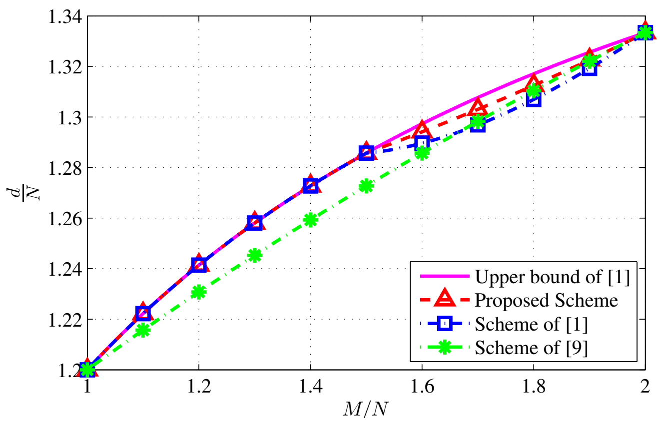

Fig. 3. The number of DoF of the symmetrical two-user MIMO XC.

VII. Conclusion

The achievable number of DoF of the two-user MIMO XC with delayed CSIT has been studied. Based on the linear independence analysis we identified new decodability constraints on the parameters of the DoF achieving scheme of [1]. Based on the identified constraints, a new transmission scheme has been proposed, which for the case where the identified constraints are active, achieves the number of DoF greater than that achieved by the scheme of [1], where only the number of decodable information symbols is transmitted.

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