Relay-aided interference alignment for bi-directional communications in wireless networks

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Kurzfassung

In den letzten Jahren hat die Anzahl an drahtlosen Knoten exponentiell zugenommen und Interferenz zwischen den Kommunikationsverbindungen ist zum maßgeblich limitierenden Faktor in drahtlosen Kommunikationsnetzen geworden. Wenn die Signalleistung der Interferenz deutlich geringer ist als die Nutzsignalleistung, so können die Interferenzsignale als Rauschen betrachtet werden. Wenn die Signalleistung der Interferenz deutlich stärker ist als die Nutzsignalleistung, so können zunächst die Interferenzsignale dekodiert werden, während das Nutzsignal als Rauschen betrachtet wird. Im zweiten Schritt subtrahiert man die Interferenzsignale vom Empfangssignal und kann somit das Nutzsignal dekodieren. Jedoch besitzen Interferenz- und Nutzsignale häufig eine vergleichbare Leistung. In diesem Fall verwenden die Knoten bei einer herkömmlichen Übertragung orthogonale Ressourcen. Bei K Knoten erhält somit jeder Knoten nur $1/K$ der gesamten Bandbreite. In jüngster Zeit wurde Interference Alignment (IA) als ein effizientes Verfahren entwickelt, das Interferenz besonders bei hohen Signal-zu-Rausch-Abständen bewältigen kann. Mit IA wird der Empfangsraum in zwei Unterräume, den Nutzsignal-Unterraum und den Interferenz-Unterraum, aufgeteilt. Jeder Knoten verzerrt seine Datenströme derart vor, dass sich am vorgesehenen Empfänger alle Interenzsignale innerhalb des Interferenz-Unterraums zueinander ausrichten, während sich das Nutzsignal im interferenzfreien Nutzsignal-Unterraum befindet. Durch IA ist jeder Knoten in der Lage, mehr als $1/K$ der gesamten Bandbreite zu erhalten. Jedoch erfordert IA eine Vorverzerrung über mehrere Zeitschlitze, was zu großen Verzögerungen im System führt. Des Weiteren ist globale Kanalkenntnis an allen Knoten notwendig und es existiert keine generalisierte geschlossene IA Lösung. In dieser Arbeit wird gezeigt, wie man Relais dazu verwenden kann, die Verzögerung auf zwei Zeitschlitze zu reduzieren, IA nur mit lokaler Kanalkenntnis durchzuführen und eine geschlossene Lösung zu erhalten.

Der Fokus dieser Arbeit liegt auf bidirektionaler Kommunikation. Im Gegensatz zum herkömmlichen Gebrauch von Relais zur Steigerung der Reichweite wird in dieser Arbeit der Gebrauch von Relais zur Manipulation des effektiven Kanals zwischen Sendern und Empfängern vorgeschlagen, um so IA zu unterstützen. Hierbei unterstützen Q im Halbduplex-Modus arbeitende Relais die bi-direktionale Kommunikation zwischen K Knotenpaaren. Jeder Knoten besitzt N Antennen und möchte d Datenströme an seinen Kommunikationspartner senden. Da für eine bidirektionale Kommunikation Zwei-Wege Relaying spektral effizienter ist als Ein-Wege Relaying, wird Zwei-Wege Relaying als das zugrunde liegende Übertragungsprototkoll verwendet. Es wird hergeleitet, dass die Relais mindestens $QR \geq Kd$ Antennen besitzen müssen, um IA unterstützen zu

können. Ausgehend von dieser Bedingung werden abhängig von der Anzahl an Relais bzw. der Anzahl an Relaisantennen neue IA Algorithmen entwickelt. Bezüglich der erreichbaren Summenrate ist IA für hohe Signal-zu-Rausch-Abstände optimal. Für niedrige bis mittlere Signal-zu-Rausch-Abstände werden in dieser Arbeit ebenfalls neue Algorithmen zur Steigerung der Summenrate entwickelt.

Zunächst wird der Fall eines einzelnen Relais betrachtet. Es wird dabei gezeigt, dass $R \geq Kd$ eine notwendige Bedingung zur Durchführung von IA ist. Zu Beginn wird der Fall, bei dem das Relais die minimal erforderliche Anzahl $R = Kd$ Antennen besitzt, betrachtet. In einem Zwei-Wege Relais-Netzwerk ist IA ein trilineares Problem, da die Filter am Sender, am Relais und am Empfänger gemeinsam entworfen werden müssen. Es werden zwei neuartige Konzepte namens Signal Alignment (SA) und Channel Alignment (CA)vorgeschlagen, die zu einer Entkopplung der Filterentwürfe am Sender bzw. Empfänger und dem Relais führen. Durch SA werden die Signale der Knoten paarweise am Relais ausgerichtet. Mit Hilfe von CA erfolgt eine Ausrichtung des effektiven Kanals einschlielich des Kanals zwischen Relais und Empfänger und des Empfangsfilters mit dem effektiven Kanal seines Kommunikationspartners. Es zeigt sich, dass zum Ereichen von IA der Einsatz von SA und CA notwendig ist. Dabei wird IA in drei lineare Schritte aufgeteilt, nämlich SA, CA und Zero Forcing (ZF). Es wird eine geschlossene Lösung zum Erreichen von SA, CA und ZF vorgeschlagen. Für den Spezialfall $R = Kd$ wird gezeigt, dass zum Erreichen von IA paarweise Kanalkenntnis an den Knoten und globale Kanalkenntnis an den Relais ausreichend ist. Als Nächstes wird der Fall $R \geq Kd$ betrachtet, bei dem den Relais nun mehr Antennen als im ersten Fall zur Verfügung stehen. Neuartige Algorithmen wurden entwickelt, um entweder die zusätzlichen Antennen zur Steigerung der interferenzfrei übertragbaren Datenstöme, auch Freiheitsgrade (Degrees of Freedom) genannt, zu nutzen oder sie zur Reduzierung der minimal notwendigen Antennenzahl an den Knoten zur Durchführung von SA und CA zu verwenden. Für diesen Zweck werden SA und CA generalisiert und neue Konzepte namens Partial Signal Alignment (PSA) und Partial Channel Alignment (PCA) vorgeschlagen. Es wird gezeigt, dass IA durch PSA, PCA und ZF erreicht werden kann. Um untersuchen zu können, wie viele Antennen an den Knoten und dem Relais zum Erreichen von IA notwendig sind, werden die Eignungsbedingungen für die Lösbarkeit der IA Gleichungen hergeleitet. Falls die Anzahl an Variablen größer oder gleich der Anzahl an Gleichungen im System ist, so wird das System als geeignet klassifiziert, ansonsten als ungeeignet. Es wird gezeigt, dass die hergeleitete Eignungsbedingung eine Generalisierung der Bedingung bezüglich der Antennenanzahl an den Knoten für den Fall $R = Kd$ darstellt. PSA und PCA sind bilineare Probleme. Um PSA und PCA durchzuführen, wird ein iterativer Algorithmus vorgeschlagen. Außerdem wird für einen Spezialfall, dessen Bedingungen in der Arbeit angegeben sind, eine geschlossene Lösung präsentiert. Die Summenrate, die sich mit Hilfe des vorgeschlagenen IA-Algorithmus erreichen lässt, wird mit der eines Referenzalgorithmus ohne IA verglichen. Es zeigt sich, dass bei hohen Signal-zu-Rausch-Abständen der vorgeschlagene IA-Algorithmus eine höhere Summenrate als der Referenzalgorithmus erzielt.

Im zweiten Schritt wird der Fall mehrerer Relais betrachtet. Es wird gezeigt, dass $QR \geq Kd$ eine notwendige Bedingung für IA ist. Zunächst wird der Fall $QR = Kd$ untersucht. Für diesen Fall werden die Konzepte SA und CA, die für das Ein-Relais-Szenario entwickelt wurden, für ein Mehr-Relais-Szenario erweitert. Allerdings teilen sich die Relais in einem Mehr-Relais-Szenario nicht die Empfangssignale, d.h., das Empfangssignal eines Relais ist an den anderen Relais nicht verfügbar. Daraus folgt, dass die Relais-Verarbeitungsmatrix eine Block-Diagonalmatrix ist. Darum kann ZF nicht wie im Ein-Relais-Fall verwendet werden. Für den Fall $QR = Kd$ wird eine neue Methode namens Cooperative Zero Forcing (CZF) vorgeschlagen. Unter der Verwendung von CZF kooperieren die Knoten mit den Relais und wählen ihre SA- und CA-Ausrichtung derart, dass die Relais ZF mit einer Block-Diagonalmatrix durchführen können. Es wird die Eignungsbedingung hergeleitet und gezeigt, dass sie eine Generalisierung des für den Ein-Relais-Fall ($Q = 1$) hergeleiteten Ausdrucks darstellt. Es wird ein iterativer Algorithmus zum Erreichen von IA vorgeschlagen. Als Nächstes wird der Fall $QR \geq Kd$ betrachtet. Es wird gezeigt, dass die Generalisierung von SA bzw. PSA auf Mehr-Relais-Szenarien zu einem quad-linearen Problem führt und somit das trilineare IA Problem nicht vereinfacht. Darum wird ein neuer iterativer IA-Algorithmus präsentiert. In jeder Iteration werden nacheinander die Sende-, Relais- und Empfangsfilter entworfen während die anderen Filter konstant gehalten werden. Die Summenrate der vorgeschlagenen IA-Algorithmen wird mit dem Referenzalgorithmus ohne IA verglichen und es zeigt sich, dass bei hohen Signal-zu-Rausch-Abständen die vorgeschlagenen IA-Algorithmen höhere Summenraten als der Referenzalgorithmus erzielen.

Schließlich werden für alle bisher betrachteten Szenarien Interferenzmanagement-Verfahren entwickelt, die nicht nur die Interferenzsignale, sondern auch die Nutzsignale berücksichtigen. IA ist bei hohen Signal-zu-Rausch Abständen optimal. Bei hohen Signal-zu-Rausch Abständen ist das Rauschen beinahe Null und nur die Interferenzsignale sind der limitierende Faktor. IA unterdrückt die Interferenzsignale vollständig und ist daher bei hohen Signal-zu-Rausch-Abständen optimal und erzielt folglich höhere Summenraten als die Referenzverfahren. Bei niedrigen bis mittleren Signal-zu-Rausch-Abständen, bei denen das Rauschen eine signifikante Rolle spielt, ist es jedoch vorteilhaft, die Nutzsignalleistung im Vergleich zur Rauschleistung zu verbessern. In dieser Arbeit werden zwei Algorithmen zur Verbesserung der Summenrate bei niedrigen bis mittleren Signal-zu-Rausch-Abständen vorgeschlagen. Der erste Algorithmus basiert dabei auf IA, und hat das folgende Ziel: aus allen möglichen IA Lösungen wird die Lösung gewählt, die den Signal-zu-Rausch-Abständ maximiert. Dieser Algorithmus ist immer anwendbar, wenn IA Lösungen in geschlossener Form erreicht werden können. Hierfür wird ein Gradienten-basierter Algorithmus vorgeschlagen, der mindestens ein lokales Maximum findet. Der zweite Algorithmus basiert auf der Minimierung des mittleren quadratischen Fehlers zwischen dem gesendeten und dem geschätzten Datensymbol unter Berücksichtigung der Einschränkung bzgl. der Sendeleistung an den Knoten und an den Relais. Es wird ein iterativer Algorithmus, der mindestens ein lokales Minimum findet, vorgeschlagen. Mittels Simulationen kann gezeigt werden, dass bei niedrigen bis mittleren Signal-zu-Rausch-Abständen diese zwei Algorithmen eine höhere Summenrate als der Referenzalgorithmus erzielen.

Abstract

In recent years, the number of wireless nodes increases exponentially and the interference between the communication links is the major limiting factor in wireless communication networks. If the interference signals power is considerably weaker than the useful signal power, then the interference signals can be treated as noise. If the interference signals power is considerably stronger than the useful signal power, then first the useful signal can be treated as noise and the interference signals can be decoded. Secondly, the interference signals can be subtracted from the received signal and the useful signal can be decoded. However, often the interference signals are of similar power as the useful signal. In this case, conventionally the nodes perform transmission using orthogonal resources. If there are K nodes, then each node gets only $\frac{1}{K}$ of the total bandwidth. Recently, interference alignment (IA) has been developed as an efficient technique to handle interference signals, especially at high signal to noise ratio (SNR). In IA, the receiver space is divided into two subspaces, namely, the useful subspace and the interference subspace. Each node precodes its data streams such that at the intended receiver, all the interference signals align with each other within the interference subspace and the useful signal is in the interference-free useful subspace. Through IA, each node is able to get more than $\frac{1}{K}$ of the total bandwidth. However, to perform IA, often precoding over multiple time slots is necessary which introduces large delays in the system. Furthermore, global channel knowledge is necessary at all the nodes and no generalized closed form solutions to perform IA are available. In this thesis, it is shown how relays can be utilized to reduce the delay to two time slots, to perform IA with local channel state information at the nodes, and to obtain closed form solutions.

In this thesis, the focus is on bidirectional communication. In contrast to the conventional use of relays, where the relays are used to improve the coverage, in this thesis, it is proposed to utilize the relays to manipulate the effective channel between the transmitters and the receivers in order to aid in the IA process. Q half-duplex relays with R antennas each aid in the bidirectional communication between K node pairs. Each node has N antennas and wants to transmit d data streams to its communication partner. For a bidirectional communication, two-way relaying is spectrally more efficient than one-way relaying and hence, two-way relaying is assumed as the underlying transmission protocol. It is assumed that the relays do not have enough antennas to spatially separate the data streams. It is derived that the relays need at least $QR \geq Kd$ antennas to aid in the IA process. Starting from this condition, depending on the number of relays and relay antennas, new algorithms to achieve IA are developed in this thesis. In terms of the sum rate achieved, IA is optimum at high SNR. For low and

medium SNR, new algorithms to improve the sum rate performance are also developed in this thesis.

First, a single relay is considered. It is shown that $R \geq Kd$ is a necessary condition to perform IA. Initially, the case when the relay has the minimum required number $R = Kd$ of antennas is considered. IA in a two-way relay network is a trilinear problem because the transmit, the relay and the receive filters have to be jointly designed. Two new concepts, namely signal alignment (SA) and channel alignment (CA), are proposed to decouple the design of the transmit and the receive filters, respectively, from the design of the relay filter. SA is the process through which the signals from the nodes align pair-wise at the relay. CA is the process of alignment of the effective channel including the channel between the relay and the receiver and the receive filter with the effective channel of its communication partner. It is shown that SA and CA are necessary steps to achieve IA. Thereby, the process of IA is decomposed into three linear steps, namely, SA, CA, and zero forcing (ZF). The number of antennas required at the nodes to perform SA and CA is derived. A closed form solution to achieve SA, CA, and ZF is proposed. It will be shown that, for the special case $R = Kd$, pair-wise channel knowledge at the nodes and global channel knowledge at the relays are sufficient to perform IA. Then, the case $R \geq Kd$ is considered. Now the relays have more antennas compared to the first case. New algorithms to use the additional antennas either to increase the number of interference free data streams defined as degrees of freedom or to reduce the minimum required number of antennas at the nodes to perform SA and CA have been proposed. For this purpose, SA and CA are generalized and new concepts namely, partial signal alignment (PSA) and partial channel alignment (PCA) are proposed. It is shown that IA is achieved through PSA, PCA, and ZF. In order to investigate how many antennas are needed at the nodes and at the relay to achieve IA, the properness conditions for the solvability of the IA equations are derived. If the number of variables is larger than or equal to the number of equations in the system, then the system is classified as proper, else as improper. It is shown that the derived properness condition is the generalization of the condition on the number of node antennas derived for the case $R = Kd$. PSA and PCA are bilinear problems. An iterative algorithm to perform PSA and PCA is proposed. Also, for a special case for which the conditions are given in this thesis, a closed form solution is also proposed. The sum rate achieved by the proposed IA algorithm is compared with a reference algorithm without IA. It is shown that the proposed IA algorithm has better sum rate than the reference algorithm at high SNR.

Secondly, multiple relays are considered. It is shown that $QR \geq Kd$ is a necessary condition to perform IA. Initially, the case $QR = Kd$ is investigated. In this case, the concepts of SA and CA developed for the single relay scenario are extended to the multiple relay scenario. However, in multiple relay case, the relays do not share their received signals i.e., the signal received at one relay is not available at the other relays. Hence, the relay processing matrix is a block diagonal matrix. Therefore, ZF cannot be performed as in the single relay case. A new method named cooperative zero forcing (CZF) is proposed for this case. In CZF, the nodes cooperate with the relays and choose their SA and CA directions such that the relays can perform ZF with a block diagonal matrix. The properness condition is derived. It is shown that the derived properness condition is a generalization of the expression derived for the single relay case with $Q = 1$. An iterative algorithm to achieve IA is proposed. Then, the case $QR \geq Kd$ is considered. It is shown that the generalization of SA or PSA to multiple relays leads to a quad-linear problem and therefore, does not simplify the trilinear IA problem. Hence, a new iterative IA algorithm is proposed. The properness condition is derived. An iterative algorithm to achieve IA is proposed. During each iteration, each of the transmit, the relay and the receive filters are designed one after another while keeping the other filters fixed. The sum rate performance of the proposed IA algorithms are compared with the reference algorithm without IA and it is shown that the proposed IA algorithm achieves better sum rate than the reference algorithm at high SNR.

Finally, for all the scenarios considered above, interference management schemes which consider not only the interference signals but also the useful signals are proposed. IA is optimum at high SNR. At high SNR, noise is almost zero and the interference signals are the only limiting factor. IA completely suppresses the interference signals and is optimum at high SNR and, hence, achieves higher sum rate than the reference schemes at high SNR. However, at low and medium SNR, where the noise plays a significant role, it is beneficial to improve the useful signal power in comparison to the noise power. In this thesis, two algorithms are proposed to improve the sum rate performance at low and medium SNR. The first algorithm is based on IA. In this algorithm, the objective is as follows: out of all the available IA solutions, the one that maximizes the SNR is chosen. This algorithm is applicable whenever the IA solutions are obtained in closed form. A gradient based algorithm to find at least a local maximum is proposed. The second algorithm is based on minimization of the mean squared error between the transmitted and the estimated data symbols subject to the node power constraints and relay power constraint. An iterative algorithm to find at least a local minimum has been proposed. Through simulations it is shown that these two algorithms have better sum rate performance than the reference algorithm and the IA algorithms at low and medium SNR.

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Chapter 1

Introduction

1.1 Motivation

With the advancement in technology, smart nodes play an important role in every day activities [CIS14]. The number of mobile nodes used in day-to-day life increases rapidly [CIS14]. In a given wireless channel, the mobile nodes can share the medium by performing their transmissions during independent time slots. As the transmissions take place in independent time slots, the signal transmitted by one mobile node during its corresponding time slot does not interfere with the signal transmitted by another mobile node during another time slot [GPP07]. The different time slots are called orthogonal resources and this scheme of sharing the communication medium among the nodes is called time division multiple access (TDMA) [GPP07]. Similarly, the nodes can share the medium by performing transmission using different frequencies and different spatial dimensions called frequency division multiple access (FDMA) and space division multiple access (SDMA), respectively [GPP07]. However, the bandwidth available is limited [TV05]. If there are K nodes that want to transmit its signal to its corresponding receiver, then each node gets only $1/K$ of the total resource. As K increases to a very large number, the resource allocated to each node decreases to a very small fraction and this small fraction may not be sufficient for the node to transmit its signal to its receiver.

Since, the bandwidth is limited, in order to accomadate all the communications, several nodes have to simultaneously utilize the same orthogonal resources. Simultaneous transmission using the same resources results in interference among the signals received at the nodes. Figure 1.1 shows an example for an interference channel with three node pairs. There are three transmitters and three receivers. The solid green arrows represent the useful signals and the dotted red arrows correspond to the interference signals. If the power of the interference signal is weak compared to that of the useful signal, then it can be considered as noise [CJ08]. If the interference signal power is much stronger than the useful signal power, then considering the useful signal as noise, first the interference signal can be decoded. Secondly, from the received signal, the interference signal can be subtracted and then the useful signal can be obtained [CJ08]. However, often we come across scenarios where several devices are active within a small geographical area and the interference signals are as strong as the useful signals [CJ08].

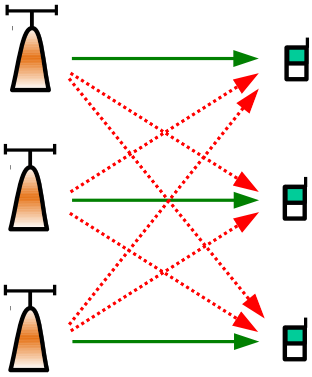

Figure 1.1. Three node pairs interference channel

New methods to handle interferences and utilize the available bandwidth effeciently need to be developed.

Recently, interference alignment (IA) [MMK06, JS08, CJ08] has been developed as an efficient technique to handle interferences. In IA, the receiver space is divided into two subspaces [CJ08], namely, the useful subspace (USS) and the interference subspace (ISS). Each of the transmitters chooses its transmit signal such that at each of the receivers, only the useful signal is within the USS and all the interference signals are within the ISS. By this, every node pair sacrifices one half of the total resources to align the interferences. The other half of the total resources is used for useful signal transmission [CJ08]. Thereby, each node pair obtains a share of one half of the total resources available in the network [CJ08]. The multi-dimensional receiver space can represent multiple time slots, multiple subcarriers, multiple antennas, multiple signal levels or combinations of them. In this thesis, the focus is on IA with multiple antennas.

Figure 1.2 shows an example for IA in spatial domain for three node pairs. Consider the transmission in a duration of one time slot and using one subcarrier. Each node has two antennas. Hence, two spatial dimensions or two spatial resources are available. Therefore, the transmit and receive signal spaces are two-dimensional spaces. According to the principle of IA, each transmitter utilizes one spatial dimension for transmission of the useful signal, i.e., each transmitter transmits one data stream in each time slot. In Figure 1.2, the directions of the coloured arrows denote the directions in which the useful signal is transmitted. The amplitude of the coloured arrows denotes the square root of the energy of the symbol being transmitted at that time instant. The transmission directions are chosen such that at each of the receivers, the interference signals are within the one-dimensional ISS and the useful signal is within the one-dimensional USS. This is illustrated in Figure 1.2 by the overlapping of the interference signals at each of the receivers. The aligned interferences shall be nullified by projecting the received signal to the orthogonal complement of the ISS. In this example, using conventional MIMO schemes [GPP07], one of the three node pair could transmit two data streams per time slot and per subcarrier. Using IA, all the three node pairs are able to achieve half of the total resource, which is one data stream per time slot and per subcarrier. For simplicity, in the following, the resource of one time slot and one subcarrier is defined as one channel use.

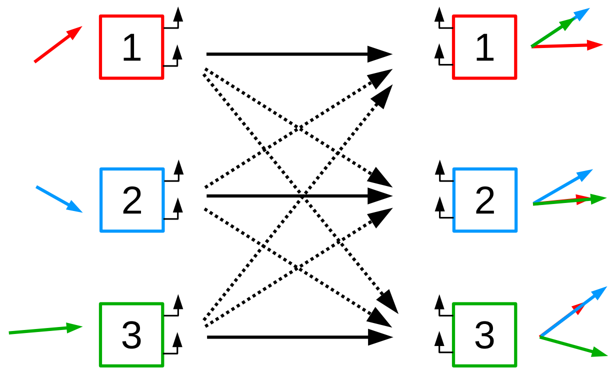

Figure 1.2. IA in a three node pairs interference channel using multiple antennas

Although IA is able to provide 1/2 of the total resources to each of the node pairs in the system, there are several limitations that need to be overcome. These limitations are given in the following:

In the example shown in Figure 1.2, through IA in spatial dimension, each node pair is able to transmit one data stream per channel use, which is half of the total resource. However, if a fourth node pair is added to this example, then IA in spatial dimension is not feasible [NSGS09, YGJK09, BCT11]. This is due to the fact that the transmission directions at each of the transmitters cannot be chosen to simultaneously align the interference at all the four receivers. However, precoding over multiple number of time slots can be performed to achieve IA and hence, each node pair can utilize half of the total resources [CJ08]. In general, for K-node pairs, coding over an infinite number of time slots is necessary for each node pair to utilize 1/2 of the total resources [CJ08].

The transmit filters define the direction along which the signals are received at the receivers. Hence, the ISS at each of the receivers is defined by all the transmit filters. Hence, all the transmit filters need to be jointly designed. Furthermore, the receive filters need to be designed to nullify the interference signal in the ISS. Hence, all the transmit and the receive filters need to be jointly designed. For the joint design of the transmit and the receive filters, either global channel knowledge is necessary at all the nodes [CJ08] or the interference covariance matrix needs to be estimated during each iterative steps of an distributed IA approach [CJ08]. Furthermore, for the joint design of the transmit and the receive filters, a generalized closed form solution is not known for IA [YGJK09, TGR09].

IA aims at completely removing the interference signals and does not care about the power of the useful signal at the receiver. At high SNR, the noise is almost zero and interference plays a major role in the performance of the system. However, for low and medium SNRs, it might be better to improve the useful signal power than to completely nullify the interference signals at the receiver. IA is capacity sub-optimum at low and medium SNRs. Furthermore, in case of multiple IA solutions available for a given scenario, IA arbitrarily chooses one of the several IA solutions and the useful signal power corresponding to the chosen solution might be very low at the receiver.

Relays shall be introduced into the network to overcome these limitations [GCJ08]. Introduction of relays results in additional variables in the system which shall be utilized to overcome the limitations mentioned above. In contrast to the conventional use of relays for range extension and signal amplification [RW07], in this thesis, relays are utilized to manipulate the effective channel between the transmitters and the receivers in order to aid in the process of IA. For instance, in [GCJ08], for the scenario with three single antenna node pairs, it has been shown that with the help of a single relay with a single antenna, coding over two time slots is sufficient for each of the node pairs to obtain a share of one half of the total resources. However, in the absence of the relay, coding over infinite time slots is necessary for each node pairs to obtain a share of one half of the total resources.

In relay-aided IA, since the relay is used to manipulate the effective channel between the transmitters and the receivers to aid in performing IA, the relay is not interested in the data streams transmitted through the channel. Hence, amplify and forward relays are assumed. Furthermore, in contrast to conventional relaying, in relay-aided IA, it is not necessary to have enough relays or relay antennas to spatially separate the data streams. In order to eliminate the interference between the signals received at the relays and the signals transmitted from the relays, half-duplex relays are assumed. There are two relaying protocols widely used in the literature namely, one-way relaying [BKW+09] and two-way relaying [RW07]. One-way relaying based relay aided IA has been considered in several works in literature including [GCJ08, NMK10, CC10, AGKW12a, AW11, LH12]. In many of the wireless networks, the nodes exchange information with their communication partners, i.e., bidirectional communication is performed. For bidirectional communication, two-way relaying is spectrally more efficient than one-way relaying [RW07]. Two-way relaying based relay aided IA is still a open problem.

1.2 State-of-the-art

1.2.1 Interference alignment

In this section, an overview of the publications related to IA in an interference channel with K node pairs and without relays are presented. All the IA algorithms proposed are for unidirectional communication. For bidirectional communication, the communication has to take place utilizing two time slots. IA can be performed in the dimensions of time [CJ08], frequency [SHMV08], spatial [GCJ08, PH09, TGR09, SSB$^+$09, SL10] or signal level [BPT10]. In this thesis, the focus is on IA in spatial dimensions and hence, in the following the publications investigating IA in spatial dimensions are described. Each of the K nodes have N antennas and wants to transmit $d \leq N$ data streams to its communication partner. First, publications that propose algorithms to achieve IA are given. Secondly, works considering the feasibility of IA are presented. Finally, publications that consider both interferences and useful signals to improve the sum rate performance at low and medium SNRs are presented. It is to be noted that at high SNR, where the noise is almost zero, these algorithms which consider both useful and interference signals typically result in IA solutions [GCJ08].

In order to achieve IA, the transmit and the receive filters need to be jointly designed which results in a bilinear problem. In [GCJ08], an iterative algorithm to achieve IA is proposed. Channel reciprocity is assumed in [GCJ08]. Hence, the subspace in which each node receives interferences from other nodes is also the same subspace along which this node will cause interference to other nodes [GCJ08]. Using this information, the transmit and receive filters are updated iteratively such that the interferences at the receivers are minimized [GCJ08]. In [PH09], the channel reciprocity requirement has been removed and an iterative algorithm based on alternating minimization has been proposed. Minimization of interference power is a non-convex problem and the iterative algorithm may converge to a local minimum in which case the residual interference will be non-zero. Due to the bilinear nature of the IA problem, a general closed form solution for IA is not available. However, in [TGR09] a closed form solution has been proposed for a special case. In [TGR09], it is assumed that each node has $N \geq (K-1)d$ antennas and a closed form solution is obtained.

The feasibility conditions for IA are investigated in [NSGS09, YGJK09, BCT11]. In [NSGS09, YGJK09], the feasibility of IA for a given network is given in terms of a properness condition. The properness condition is derived by counting the number M_{v} of variables and the number M_{c} of constraints in the system. If $M_{\mathrm{v}} \geq M_{\mathrm{c}}$, then the system is considered as proper, otherwise as improper. The properness condition is given by

$$N \geq \frac{(K+1)d}{2} \tag{1.1}$$

[NSGS09, YGJK09]. In [YGJK09], Bernstein's Theorem is used to verify if the proper systems are feasible by calculating the mixed volume of the polynomials. In [BCT11], using the concepts of algebraic geometry the authors have shown that the properness condition is also the feasibility condition.

IA aims at completely suppressing the interferences at the receiver. However, the useful signal power at the receiver is not considered in the IA process. Hence, IA is capacity optimum only at high SNR. However, at low and medium SNRs, it can be beneficial to allow a small amount of interference and increase the useful signal power at the receiver in comparision with the noise power. For this purpose, joint optimization of the transmit and the receive filters for maximizing other utility functions like Signal to Interference plus Noise Ratio (SINR) [GCJ08] or Minimizing Mean Square Error (MMSE) between the transmitted and the estimated data symbols [SSB+09, SL10] have been proposed in the literature. As described in the beginning of this subsection, at high SNR these algorithms result in IA solution [GCJ08].

1.2.2 Conventional relaying and relay-aided interference alignment

1.2.2.1 Introduction

In this section, an overview of the publications related to conventional relaying and relay aided IA are presented. The term conventional relaying refers to relaying without IA. Similar to the interference channel considered in Section 1.2.1, in the following, K node pairs are considered. Each node has N antennas and wants to transmit $d \leq N$ data streams to its communication partner. There are Q relays each with R antennas that assist in the communication. Two relaying schemes are well known in the literature namely, one-way relaying [BKW+09] and two-way relaying [RW07].

One-way relaying was originally developed for uni-directional communication. The transmission takes place in two time slots. In the first time slot, all the transmitters transmit the signals to the relay. The relay performs linear signal processing on the received signal. In the second time slot, the relay transmits the processed signal to the receivers. In addition to this, there is another variant of one-way relaying in which the direct link between the source and the destination nodes is also present. Hence, the transmitters transmit in both time slots, and the receivers also receive during both time slots. In one-way relaying, for bi-directional communication four time slots are necessary.

In two-way relaying, bi-directional communication takes place using only two time slots. In the first time slot, all the nodes transmit to the relay. The relay performs linear signal processing on the received signal and broadcast the processed signal to all the nodes in the second time slot.

In the following, first the references corresponding to conventional one-way relaying and one-way relay aided IA are investigated. Secondly, the literatures corresponding to conventional two-way relaying and two-way relay aided IA are investigated. In relay-aided IA, since the relays are used to manipulate the effective channel between the transmitters and receivers to aid IA, the number of relay antennas required is relatively smaller than in the conventional case where the relay antennas are used to spatially separate the data streams. Hence, in this section, a special emphasis is given to the number of relays and number of antennas at each relay.

1.2.2.2 One-way relaying

In this section, for one-way relaying, the publications related to conventional relaying and relay-aided IA are first presented for the case of a single relay and then for the case of multiple relays. Towards the end of this section, other interference management schemes that in addition to interference consider also the useful signal power are presented.

Conventional one-way relaying with a single relay is considered in [BKW+09]. The direct link is not present. It has been shown that a single relay with $R \geq K$ antennas can support the communication between K transmitters and K receivers with a single antenna each [BKW+09]. The relay performs multiuser beamforming [BKW+09]. This beamforming is a combination of receive beamforming in the first time slot and transmit beamforming in the second time slot [BKW+09]. The relay spatially separates the data streams based on the zero forcing (ZF) or minimum mean square error (MMSE) criterion [BKW+09]. Therefore, the relay needs at least K antennas [BKW+09]. If multiple antennas at the nodes are considered, then each source node can transmit $d \leq N$ data streams to its destination node. In order to spatially separate these Kd data streams, at least $R \geq Kd$ antennas are required at the relay.

One-way relay-aided IA with a single relay is considered in [GCJ08, NMK10, CC10, AGKW12a]. Single antenna nodes are considered in [GCJ08, NMK10] and multiple antenna nodes are considered in [CC10,AGKW12a]. In [GCJ08], it has been shown that a single antenna relay can aid the network with $N = 3$ node pairs to achieve 3/2 data streams per channel use. In this thesis, the total number of data streams per channel use that can be transmitted interference free in the network is defined as the degrees of freedom (DoF). In [GCJ08], in addition to the relay links, the direct links between the transmitters and receivers are also exploited. In comparison to the conventional case where at least $K = 3$ antennas are needed at the relay, for relay-aided IA the relay needs only a single antenna. In [NMK10], the general case with K single antenna nodes has been investigated. It has been shown that when the number R of antennas at the relay is larger than $\sqrt{(K-1)(K-2)}$, then $K/2$ DoF are achievable. In [CC10,AGKW12a], the nodes have N antennas each and transmit $d = N$ data streams each. In [CC10], the direct link is utilized only in the first time slot and it has been shown that when $R \geq (K-1)N$, $KN/2$ DoF are achievable in the system [CC10]. In [AGKW12a], the direct link is utilized in both time slots. By partially fixing the transmit and the receive filters, a closed form solution to find an IA solution has been proposed achieving $KN/2$ DoF [AGKW12a]. The relay needs $R \geq N\sqrt{K(K-2)}$ antennas in order to find a solution [AGKW12a]. From one perspective, in a K-user interference channel, the relay aids in the IA process: $KN/2$ DoF are achieved with a limited number of antennas at the nodes and with two time slot extensions required by the relaying protocol and a closed form solution for IA has been obtained [AGKW12a]. These come at the cost of introducing a relay which introduces new variables in the system that has been used to manipulate the channel to aid IA. From another perspective, in a one-way relay network, by performing IA the number of antennas required at the relay is relatively smaller than that in one-way relay network without IA [AGKW12a]. Depending on the perspective, either IA benefits from the introduction of the relay or one-way relaying benefits by applying the concept of IA.

In the following, multiple relays are considered. Conventional one-way relaying with Q relays has been investigated e.g. in [RW07, BKW$^+$09, BW05, FSG09, OP06]. In [RW07, BKW$^+$09, BW05, FSG09], the nodes and the relays have a single antenna each. In [RW07, BKW$^+$09], the relay coefficients are chosen such that at the destination nodes, the inter-pair interference is completely suppressed. At least $Q > K(K-1)$ relays are required for an interference-free communication [RW07, BKW$^+$09]. The relay coefficients in [BW05] are used for minimizing the mean squared error and in

[FSG09] to minimize the relay power subject to a signal to interference plus noise ratio (SINR) constraint. In [OP06], the nodes have N antennas each and transmit $d = N$ data streams to the destination. Each of the Q relays requires $R \geq Kd$ antennas to completely remove the inter-pair interference at the destinations. In all the methods described in [RW07, BKW$^+$09, BW05, FSG09, OP06], a sufficient number of single antenna relays or a sufficient number of antennas at each relay are necessary to spatially separate the data streams and, hence, completely suppress interference at the destination nodes.

One-way relay-aided IA with multiple relays is investigated in [AW11, LH12]. Single antenna nodes are considered in [AW11] and multiple antenna nodes are considered in [LH12]. In [AW11], Q single antenna relays aid in performing IA. It has been shown that for $Q > K^2 - 3K^2 + 1$, $K/2$ DoF are achievable and the closed form solution for IA is also proposed. In [LH12], the nodes and the relays have multiple antennas and the special case $K = 2$, $Q = 2$ and $R = N$ is investigated. It is shown that even without channel state information (CSI) at the sources, $2N - 1$ DoF are achievable. The key idea is based on interference neutralization [GJW$^+$12]. In interference neutralization, in contrast to IA, the same interference signals received through the two different relays cancel each other at the receivers. The general case of $Q > 2$ is still open. However, the closed form solution proposed in [AGKW12a] for the single relay case using partially fixed transmit and receive filters can be extended for the general case $Q > 1$. For the case $Q > 1$, extending the method proposed in [AGKW12a], the number of antennas required at each relay is given by $R \geq \frac{N}{Q}\sqrt{K(K-2)}$ [AGKW12a]. Similar to the single relay case, for the multiple relay case from one perspective, the relay aids in IA and in another perspective, when performing IA in a one-way relay network, a smaller number of relays and/or relay antennas are required compared to conventional one-way relaying.

In addition to the above mentioned one-way relay aided IA schemes, several works have investigated IA based methods to improve the system performance at low and medium SNRs [ALG$^+$13, AGKW12b, MXF$^+$10]. In [ALG$^+$13], for the case when there are multiple IA solutions, an algorithm to find the IA solution that minimizes the mean squared error has been proposed. In [AGKW12b], for the case when no IA solution is feasible, an algorithm to find an imperfect IA solution has been proposed. The term imperfect IA implies that the interferences are not completely aligned within the ISS and are partially in the USS [AGKW12b]. However, the sum of the power of the interference signals within the USS is minimized. In [MXF$^+$10], an algorithm that iteratively designs the transmit, relay and receive filters to minimize the mean square error between the transmitted and the estimated data symbols has been proposed. At high SNR where the noise power is very low, the solution obtained by the iterative MMSE algorithm converges towards IA solution.

1.2.2.3 Two-way relaying

In the following, conventional two-way relaying schemes and two-way relay-aided IA schemes will be analysed. Note that the references described below are the works published before the investigations performed in this thesis. Related works performed in parallel to this thesis are cited in the respective chapters 3 to 5 of the thesis.

In the following, first a single relay is considered. Conventional two-way relaying with a single relay with single and multiple antenna nodes is considered in [BKW+09, YZGK10a,CY10,AK10,LDLG11,SFXK11] and in [JS10,XPW+11,DK12], respectively. The relay filters are designed based on different objectives. In [BKW+09, YZGK10b], the relay filters are designed to zero force, i.e., to nullify the interferences at the receivers. In both references [BKW+09, YZGK10a], the relay needs $R \geq 2K$ antennas to spatially separate the data streams. Assuming that each node can perfectly cancel the self-interference, in [CY10, AK10, LDLG11], self-interference aware beamforming methods are proposed. As self-interference is cancelled at the receivers, the relay needs only $R \geq 2K - 1$ antennas. In [SFXK11], signal to interference plus noise ratio (SINR) and minimum mean squared error (MMSE) are taken as the design criteria. In [JS10,XPW+11,DK12], multiple antenna nodes are considered. In [JS10,XPW+11], each node transmits $d = 1$ data stream, whereas in [DK12] each node transmits $d = N$ data streams. In [XPW+11], the multiple antennas at the nodes are used to maximize the effective channel gain and ZF is performed at the relay. In [JS10], the transmit and receive filters are designed to maximize the received signal power and the relay filters are designed based on MMSE and ZF criteria. In [DK12], the relay and receive filters are calculated iteratively, to minimize the MSE. In all three references [JS10, XPW+11, DK12], the multiple antennas at the nodes are not designed to avoid or nullify the unknown interferences at the receivers and the relay needs enough antennas to spatially separate the signals received from the nodes. In [JS10], $R \geq 2Kd$ antennas are necessary. In [XPW+11, DK12], utilizing the fact that the nodes can cancel the self-interference, the relay needs only $R \geq (2K - 1)d$ antennas.

Two-way relay aided IA with a single relay is considered in the following. In contrast to conventional two-way relaying, in two-way relay-aided IA, the transmit, relay and receive filters are jointly designed to suppress the unknown interferences at the receivers. Hence, for two-way relay-aided IA, the relays do not need to spatially separate the data streams and in comparison with conventional two-way relaying, it is expected that a relatively smaller number of antennas is sufficient for interference-free communication. At the same time, since the direct link between the transmitters and receivers is not utilized, a minimum number of relay antennas is required for the communication. The number of antennas required at the relay and at the nodes are still open questions. Furthermore, an algorithm to jointly design the transmit, relay and receive filters is yet to be developed. In general, two-way relaying aided IA is an open problem.

Multiple relays are considered in the following. Conventional two-way relaying with multiple single antenna relays supporting multiple single antenna node pairs is considered in [RW07, WCY$^+$11, WNZE11]. In [RW07] and [WCY$^+$11], the relay coefficients are designed to suppress the interferences at the receivers. In [RW07], the power of the useful signal at the receivers is not considered and the self-interferences at the receivers are not suppressed by the relay. It is shown that $Q \geq 2K^2 - 2K + 1$ relays are required. In contrast, in [WCY$^+$11], the useful signal power at the receivers are constrained to be one and the self-interferences are also suppressed by the relays. It is shown in [WCY$^+$11] that the relay needs $Q \geq K^2 + K$ antennas. In case the number of relays is not sufficient to suppress the interferences, a least squares solution to minimize the interferences at the receivers is described in [WCY$^+$11]. In [WNZE11], the relay coefficients are designed to satisfy given SINR contraints at the receivers and to minimize the transmit power at the relays.

Two-way relay aided IA with multiple relays is an open problem. An algorithm to perform IA with the aid of multiple relays is still open. Also, the number of relays and relay antennas necessary for the feasibility of IA is not known.

Furthermore, for the cases where there are multiple IA solutions in a two-way relay network, how to choose an IA solution that in addition to suppressing the interferences, also maximizes the useful signal power at the receiver is still an open question. Also, other interference management schemes that aim at improving the performance at low and medium SNRs by making a trade-off between strengthening the useful signal power and nullifying interferences is open.

1.3 Open issues

In this section, first the scenario considered in this thesis is briefly introduced. Then the open issues addressed in this thesis are described.

In this thesis, bidirectional communication between K node pairs is considered. Each of the $2K$ nodes has N antennas and wants to transmit d data streams to its communication partner. Q relays each with R antennas aid in performing IA at the receivers.

The open issues addressed in this thesis are the following:

In this thesis, two-way relaying is considered as the relaying protocol. The nodes and the relays are assumed to be half-duplex. Hence, the direct link cannot be utilized. The signals from all the $2K$ nodes should go through the relays. Therefore two-way relay channel is a multiple key-hole channel [FSSY11]. Furthermore, it is assumed that the relays do not have enough antennas to spatially separate the data streams. However, the relays need a minimum number of antennas so that the useful and interference signals are separable at the receiver.

1. What is the minimum total number of antennas required at the relays so that IA is feasible in the considered multi-pair two-way relay network?

The minimum required number of antennas at the relays gives a lower bound on the number of antennas at the relays. For a fixed number of relay antennas, assuming this fixed number to be larger than the lower bound, the corresponding minimum number of antennas required at the nodes needs to be derived and IA algorithms to determine the transmit, relay and receive filters need to be developed. If the nodes and the relays have more than the corresponding minimum required number of antennas, then IA is feasible. Firstly, a single relay is considered.

2. How can a single relay with minimum required number of antennas aid in performing IA and how many antennas are required at the nodes?
3. How to perform IA if a single relay with more than the minimum required number of antennas is available and how to utilize the additional antennas available at the relay?

Secondly, multiple relays are considered. In case of multiple relays, the relays do not share the received signal. Hence, this introduces new challenges in comparison to the single relay case.

4. How can multiple relays with minimum required total number of antennas aid in performing IA and how many antennas are required at the nodes so that IA can be achieved with minimum total number of antennas at the relays?

5. How to perform IA when the total number of antennas at the relays is larger than the minimum required number and how to utilize the additional antennas available at the relays?

After performing IA, the aligned interference signals are nullified by projecting the received signal in a subspace orthogonal to ISS. This projection leads to noise amplifications.

6. How to perform IA so that the ratio of the desired signal power and the noise power after the receive filter is maximized?

IA aims at completely suppressing the interference signals. At low and medium SNRs, it is beneficial to allow some interference in the useful subspace and improve the ratio of the desired signal power and the noise power.

7. How to design transmit, relay, and receive filters without IA such that the data rate is improved at low and medium SNRs compared to the design with IA?

1.4 Contributions and overview

In this section, the overview of the thesis and the contributions which solve the open issues mentioned in Section 1.3 are given. In the following, the contents along with the main contributions of each chapter are briefly described.

In Chapter 2, the system model for the multi-pair two-way relay network is given. First, an equivalent low pass discrete signal model is introduced and the IA conditions are derived. Secondly, the minimum number of antennas required at the relays in order to satify the derived IA conditions is determined. This solves open issue 1. Finally the expression for the achievable sum rate for the considered multi-pair two-way relay network is derived.

In Chapter 3, a single relay is considered. First, the case when the relay has the minimum required number of antennas is considered. In this case, it is shown that IA can be decoupled into three linear problems. The closed form solutions for each of the three linear problems are presented. The number of antennas required at the nodes to perform IA is also derived. These solve open issues in 2. Secondly, the case when the relay has more than the minimum required number of antennas is considered. In this case, it is shown that IA can be decoupled into two bilinear problems and one linear problem. An iterative algorithm to solve the bilinear problems and a closed form solution for the linear problem are given. The required number of antennas at the nodes is derived in terms of the properness condition. From the properness condition, it is shown that the additional antennas at the relay can be used to reduce the required number of antennas at the nodes in comparison to the case where the relay has only the minimum required number of antennas. These solve open issues in 3. In addition, for a special case, it is shown that the two bilinear problems can be linearized and a closed form solution is presented. The condition for the possibility of linearization is also derived.

In Chapter 4, multiple relays are considered. First, the case when the relays have the minimum required number of antennas is considered. Compared to the case of a single relay with the minimum required number of antennas, now we have the same total number of antennas at the relays, but the relays do not share the received signals. Hence, the method developed for the single relay case cannot be used for the multiple relays case. New IA algorithms are developed. Here, it is shown that the IA problem can be decoupled into two linear and one bilinear problem. The required number of antennas at the nodes is derived in terms of the properness condition. A closed form solution and an iterative solution are proposed to solve the corresponding linear and bilinear problems, respectively. These solve open issues in 4. Secondly, the case when the total number of relay antennas is larger than the minimum required number is considered. In this case, the transmit, relay and receive filters are iteratively optimized. The properness condition is derived. An iterative algorithm to design the transmit, relay and receive filters is proposed. From the properness condition derived it is observed that the additional antennas at the relays can be used to reduce the required number of antennas at the nodes in comparison to the case where the relays have minimum required total number of antennas. These solve open issues in 5.

In Chapter 5, interference management schemes to improve the system performance at low and medium SNRs are considered. IA is optimum only at high SNR. In this chapter, new algorithms either with or without IA are developed to improve the performance at low and medium SNRs. First, the case when there are multiple IA solutions and the solutions are available in closed form is considered. In this case, we want to find the solution that maximizes the sum SNR at the receivers. This problem is derived and it is a non-convex optimization problem. A gradient based algorithm which finds at least a local maximum is proposed. This solves open issue 6. Secondly, an MMSE based iterative algorithm is proposed. In contrast to IA the schemes which aim at completely suppressing the interferences, the MMSE scheme minimizes the mean squared error of the estimated data symbols. Designing the transmit, relay and receive filters to minimize the MSE subject to the node and the relay power constraints is a non-convex problem. Fixing two of the three filters, the problem becomes either an unconstrained quadratic minimization problem or a quadratically constrained quadratic minimization problem. These two problems are convex and are solved in closed form using Karush Kuhn Tucker conditions. This solves open issue 7.

The performances of all the proposed IA and MMSE algorithms are evaluated based on the achievable sum rate in each of the Chapters 3, 4, and 5.

Finally, the main conclusions of the thesis are summarized in Chapter 6.

Chapter 2

System model

2.1 Introduction

In this chapter, the general system model for the considered multi-pair two-way relaying with multiple relays is introduced. First, the considered scenario and the assumptions regarding the nodes, relays, and channels are introduced in Section 2.2. Secondly, the equivalent low pass frequency domain system model of the considered multi-pair two-way relay network is introduced in Section 2.3. Following this, the IA conditions are described in Section 2.4. In this thesis, the relays are used to manipulate the effective channel from the transmitters to the receivers to aid in the IA process. In Section 2.5, assuming that the nodes have a sufficient number of antennas, the minimum number of antennas at the relays required to perform intereference-free communication is derived. Throughout this thesis, the achievable sum rate achieved is considered as the performance measure. The achievable sum rate expression is derived in Section 2.6.

2.2 Considered scenario and assumptions

In this section, the considered multi-pair two-way relay network and the assumptions regarding the problem investigated in this thesis are introduced. In this thesis, a bi-directional pair-wise communication in an ad-hoc network is considered. There are K node pairs as shown in Figure 2.1. The nodes displayed using the same colour are the communication partners. Each node wants to exchange information with its

Table 2.1. Variables in the considered scenario

K	Number of node pairs
Q	Number of relays
N	Number of node antennas
R	Number of relay antennas
d	Number of data streams per node

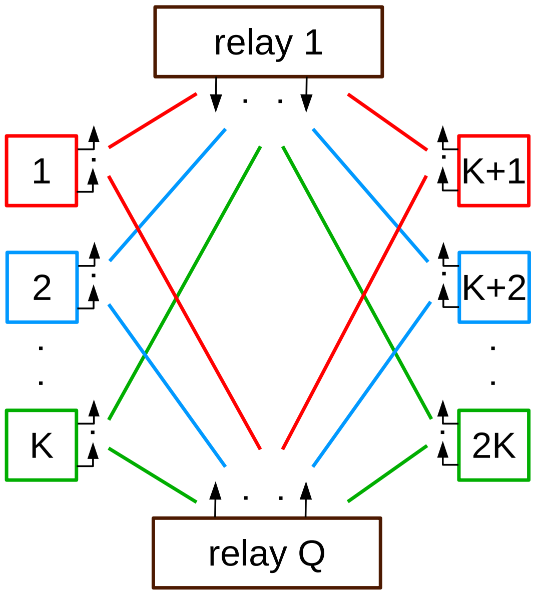

Figure 2.1. K-pair two-way relay network

communication partner. There are Q relays that assist in the communication. The nodes and relays have multiple antennas. This scenario also represents a special case of a cellular network with each base station communicating bi-directionally with only one mobile station through multiple relay stations. The objective of this thesis is to design the transmit, relay and receive filters such that IA is achieved at the receivers. The following is assumed about the system and the communication protocol:

- For simplicity of the notation, all the nodes are assumed to have the same number N of antennas and all the relays are assumed to have the same number R of antennas. Furthermore, it is assumed that each node wants to exchange the same number d of data streams with its communication partners. However, the properness conditions derived and the IA algorithms proposed in this thesis can be extended to the generalized case where each node and each relay has different number of antennas and each node wants to exchange different number of data streams with its communication partner.

- In terms of implementation, half-duplex nodes and relays are more realistic and simpler than full-duplex nodes and relays. Therefore, the nodes and relays are assumed to be half-duplex.

- For bi-directional communication, two-way relaying [RW07] is spectrally more efficient than one-way relaying. Hence, two-way relaying is assumed to be the relaying protocol. In the first time slot, all the nodes transmit to the relay as shown in Figure 2.2. This is called medium access (MAC) phase. In the second time slot, after linear signal processing of the signals received from the nodes, the relays broadcast the processed signals to the nodes as shown in Figure 2.3. This is called broadcast (BC) phase.

- The channels are assumed to be constant for at least one symbol duration.
- Orthogonal Frequency Division Multiplexing [NP00] is assumed to be the underlying transmission scheme.
- Without loss of generality, we focus on only one subcarrier. The methods developed in this thesis shall be applied independently to all the available subcarriers. All the nodes use the single available subcarrier simultaneously for the transmission. Hence, the signals from all the $2K$ nodes interfere with each other at the relay. Similarly, in the second time slot the relays transmit the processed signals to all the nodes using the same subcarrier as in the first time slot.
- It is assumed that the relays do not have enough antennas to spatially separate all the data streams at the relays. However, the relays have at least a minimum number of antennas so that the useful and interference signals can be separated at the receivers.
- Since the objective of this thesis is to utilize the relays to choose their filter coefficients in order to assist the nodes in performing IA at the receivers, the relays do not decode the received signals. Hence, amplify and forward relaying is assumed.
- It is assumed that the signals from all the nodes reach the relay at the same time. It is also assumed that the signals from all the relays reach the receiver node at the same time.
- Linear signal processing is assumed at all the nodes and relays. Non-linear signal processing can lead to a better performance than linear signal processing, but involves higher computational complexity.
- In order to guarantee that the d data streams transmitted from any of the $2K$ nodes are linearly independent of each other, the number of antennas at the nodes is assumed to be larger than or equal to d, i.e., $N \geq d$.
- Each of the $2K$ nodes has an individual maximum power constraint and the relays have a maximum total power constraint. The term maximum power constraint implies that the nodes and the relays can transmit with a power level less than or equal to the corresponding maximum transmit power.

- Additive white Gaussian noise (AWGN) with mean zero and variance σ_{1q}^2 and σ_{2k}^2 is assumed at nodes and relays, respectively. Here, k and q are the node and the relay indices, respectively. The indices 1 and 2 are used to differentiate the noise variance at relays and at nodes, respectively.

- In [NM93], it has been shown that the data rate achieved for given transmit, relay and receive filters is maximized when the input symbols are zero mean circularly symmetric complex Gaussian distributed. Hence, in this thesis the input symbols are assumed to be zero mean circularly symmetric complex Gaussian distributed. Thereby, the data rate expression derived in [PNG08] can be used to derive the sum rate expression in this thesis.

- Perfect global CSI is assumed to be available at the nodes and at the relays. However, in some special cases of the considered scenario, the proposed IA algorithms require only partial CSI at the nodes and global CSI at the relays.

- Discrete equivalent low pass signals are considered in the thesis.

- It is assumed that the self interference can be perfectly cancelled

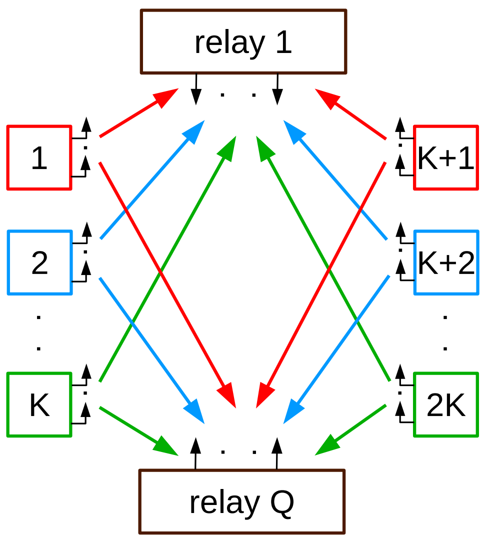

Figure 2.2. MAC phase

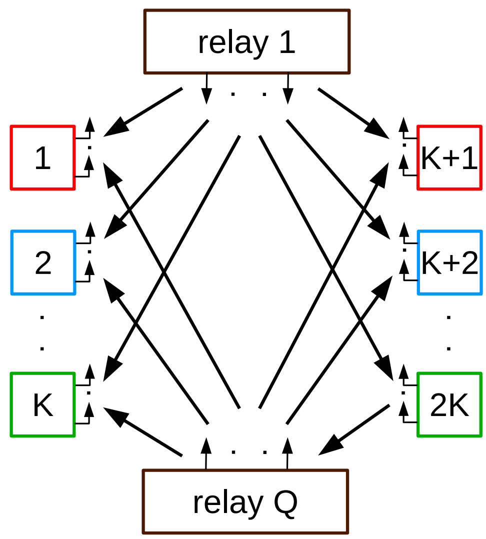

Figure 2.3. BC phase

In this thesis, we use lower case letters for scalars. Column vectors and matrices are denoted by lower case bold letters and upper case bold letters, respectively. $(.)^{\mathrm{T}}$, $(.)^{\mathrm{H}}$ denote the transpose and the complex conjugate transpose operations, respectively. $\mathrm{E}\{.\}$ denotes the expectation of the element within the brackets. We define two subspaces to be disjoint if no non-zero vector in one subspace can be expressed as a linear combination of the basis vectors of the other subspace. In this thesis, the term achievable sum rate of a proposed method indicates the sum of the data rate achieved by all the nodes that can be obtained using the transmit, relay and receive filters designed using the corresponding method and for zero mean circularly symmetric complex Gaussian input symbols.

2.3 Discrete low pass signal model

In this section, the discrete equivalent low pass signal model [Pro01, GPP07] for the considered multi-pair two-way relay network is introduced. Figure 2.1 shows the K-pair two-way relay network with Q amplify and forward half-duplex relays. There are K node pairs. Without loss of generality, in this thesis, we assume that node j and node k are communication partners with the following relation: For $j = 1, \ldots, 2K$, the communication partner index k is given by

$$k = \begin{cases} j + K & \text{if } j \leq K \\ j - K & \text{if } j > K. \end{cases} \tag{2.1}$$

Node j wants to exchange d data streams with its communication partner k.

In this thesis, two-way relaying [RW07] is considered to be the relaying protocol. As explained in Section 2.2, in the MAC phase, all the $2K$ the nodes transmit their signals to the relays. Let the column vector $\mathbf{d}_j$ denote the data symbols that node j wants to transmit to node k. The covariance matrix of $\mathbf{d}_j$ is given by

$$\mathbf{R}_{\mathbf{d}_j} = \mathrm{E}\left\{\mathbf{d}_j \mathbf{d}_j^{\mathrm{H}}\right\}. \tag{2.2}$$

In this thesis, as explained in Section 2.2, linear signal processing is assumed at all the nodes and at all the relays. Hence, the filtering operation can be modelled by a matrix multiplication with the input signal vector. Let $\mathbf{V}_j$ denote the transmit filter matrix of node j. The transmit filter can be designed to achieve different objectives, for instance to achieve IA, to maximize the signal to noise ratio or to minimize the mean squared error at the receiver. Each of the $2K$ nodes has a maximum transmit power of P_{node}. The transmit filter matrix $\mathbf{V}_j$ is normalized to satisfy the transmit power constraint given by

$$\mathrm{Trace}\left(\mathbf{V}_j^{\mathrm{H}} \mathbf{V}_j\right) \leq P_{\text{node}} \quad j = 1, 2, \ldots, 2K. \tag{2.3}$$

In the equivalent low pass signal model, in the frequency domain, the channel between any transmit antenna and any receive antenna is given by a complex coefficient. Hence, in the MAC phase, the channel between any node with N antennas and any relay with R antennas is given by an $R \times N$ matrix [PNG08]. Similarly, in the BC phase, the channel between any relay with R antennas and any node with N antennas is given by an $N \times R$ matrix [PNG08]. If the antennas are separated by a distance larger than half the wavelength of the electromagnetic waves, then the channel coefficients can be approximated to be independent of each other and the resulting channel matrix will be almost surely of full rank [TV05].

Let $\mathbf{H}_{jq}^{\mathrm{sr}}$ denote the MIMO channel matrix between node j and relay q in the MAC phase. Let $\mathbf{n}_{1q}$ denote the noise vector at relay q. As described in Section 2.2, the components of the noise vectors are i.i.d. complex Gaussian random variables. Hence, $\mathbf{n}_{1q}$ is zero mean circularly symmetric complex Gaussian distributed with variance σ_{1q}^2 [PNG08]. The signal received at relay q is given by

$$\mathbf{r}_q = \sum_{i=1}^{2K} \mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i \mathbf{d}_i + \mathbf{n}_{1q}. \tag{2.4}$$

It is assumed that the signal processing performed at the relays is linear. Hence, the linear signal processing at the relay q can be modeled as the multiplication of the received signal vector $\mathbf{r}_q$ with the matrix $\mathbf{G}_q$. For simplicity of the notation, we concatenate the signals received at all the Q relays and denote it by $\mathbf{r} = \begin{bmatrix} \mathbf{r}_1^{\mathrm{T}} & \cdots & \mathbf{r}_Q^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$. Similarly, the concatenation of the matrices representing the linear signal processing performed by all the relays is denoted by the matrix $\mathbf{G}$. In this thesis, it is assumed that the relays do not share their received signals. Therefore, the relays filter matrix $\mathbf{G}$ is block diagonal as follows:

$$\mathbf{G} = \begin{pmatrix} \mathbf{G}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \cdots & \mathbf{0} \\ \vdots & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{G}_Q \end{pmatrix}. \tag{2.5}$$

The relays have a total transmit power P_{relay} available for transmission. Let

$$\mathbf{s} = \mathbf{G}\mathbf{r} \tag{2.6}$$

denote the concatenated signal containing the signals transmitted from all Q relays. $\mathbf{G}$ is normalized such that the relay power constraint

$$\mathrm{E}\left[\mathrm{Trace}\left(\mathbf{s}\mathbf{s}^{\mathrm{H}}\right)\right] \leq P_{\mathrm{relay}} \tag{2.7}$$

is satisfied. (2.7) can be rewritten as

$$\text{Trace}\left(\mathbf{G}\sum_{q=1}^{Q}\left(\sum_{i=1}^{2K}\mathbf{H}_{iq}^{\text{sr}}\mathbf{V}_i\mathbf{V}_i^{\text{H}}\mathbf{H}_{iq}^{\text{srH}}+\sigma_{1q}^2\mathbf{I}\right)\mathbf{G}^{\text{H}}\right)\leq P_{\text{relay}}. \tag{2.8}$$

In the second time slot called broadcast (BC) phase, the relays broadcast the signals to the nodes. Let $\mathbf{H}_{qk}^{\text{rd}}$ denote the MIMO channel matrix between relay q and node k in the BC phase. Let $\mathbf{n}_{2k}$ denote the noise vector at node k. $\mathbf{n}_{2k}$ is zero mean circularly symmetric complex Gaussian distributed with variance σ_{2k}^2. The received signal $\mathbf{y}_k$ at node k is given by

$$\mathbf{y}_k=\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\text{rd}}\mathbf{G}_q\mathbf{H}_{jq}^{\text{sr}}\mathbf{V}_j\mathbf{d}_j+\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\text{rd}}\mathbf{G}_q\mathbf{H}_{kq}^{\text{sr}}\mathbf{V}_k\mathbf{d}_k+\sum_{\substack{i=1,\\ i\neq j,k}}^{2K}\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\text{rd}}\mathbf{G}_q\mathbf{H}_{iq}^{\text{sr}}\mathbf{V}_i\mathbf{d}_i+\widetilde{\mathbf{n}}_k \tag{2.9}$$

where

$$\widetilde{\mathbf{n}}_k=\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\text{rd}}\mathbf{G}_q\mathbf{n}_{1q}+\mathbf{n}_{2k} \tag{2.10}$$

is the effective noise at receiver k. In (2.9), the first term corresponds to the useful signal. The second term corresponds to the self interference. It is assumed that the self interference can be perfectly cancelled. The third term corresponds to the unknown interference.

Let $\mathbf{U}_k^{\text{H}}$ denote the receive filter matrix of node k. Then the estimated data vector $\hat{\mathbf{d}}_j$ at receiver k are given by

$$\hat{\mathbf{d}}_j=\mathbf{U}_k^{\text{H}}\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\text{rd}}\mathbf{G}_q\mathbf{H}_{jq}^{\text{sr}}\mathbf{V}_j\mathbf{d}_j+\mathbf{U}_k^{\text{H}}\sum_{\substack{i=1,\\ i\neq j,k}}^{2K}\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\text{rd}}\mathbf{G}_q\mathbf{H}_{iq}^{\text{sr}}\mathbf{V}_i\mathbf{d}_i+\mathbf{U}_k^{\text{H}}\widetilde{\mathbf{n}}_k. \tag{2.11}$$

Note that in the above equation, the self interference signal has been subtracted before applying the receive filter.

2.4 Interference alignment conditions

In this section, the IA conditions for the multi-pair two-way relay network are introduced. As introduced in Chapter 1, in contrast to the conventional methods where the the total resources are shared among the node pairs using TDMA, FDMA or SDMA [GPP07], in IA, each of the node pairs utilizes half of the total resources for useful signal transmission. For instance, if there are $K = 10$ node pairs, using conventional methods like TDMA, FDMA or SDMA each node pair obtains a share of 10%

of the total resources. However, using IA, each node pair is able to obtain a share of 50%. In IA, independent of the number of node pairs, each node pair will be able to obtain 50% of the total resource [CJ08]. IA in a K node pairs interference channel without relay has been introduced in Section 1.1. In the following, the basic idea of performing IA in a multi-pair two-way relay network is introduced. Similar to the K node pairs interference channel, for the multi-pair two-way relay network, the receiver space is divided into two disjoint subspaces, namely USS and ISS. In the considered multi-pair two-way relay network, in addition to the transmit filters, also the relay filters are designed such that at the receiver, the useful signals are within the USS and the interference signals from all the interfering node pairs are within the ISS. After performing IA, the intererence signals in ISS are nullified by projecting the received signal to a subspace orthogonal to ISS. This projection is termed receive ZF. Since the USS and the ISS are disjoint, the useful signals are not nullified during the receive ZF.

In this thesis, IA is performed along spatial dimensions. The nodes have N antennas and, hence, the receive space is N dimensional. d dimensions are reserved for the useful signals. Therefore, our objective is to align all the interferences within an $N-d$ dimensional ISS and to ensure that the useful signals fully occupy a d dimensional USS which is disjoint from ISS. This objective can be expressed as follows:

$$\operatorname{rank}\left(\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i\right) \leq N-d \quad \text{for } i \neq j,k, \tag{2.12}$$

$$\operatorname{rank}\left(\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{jq}^{\mathrm{sr}}\mathbf{V}_j\right) = d, \tag{2.13}$$

$$\left(\operatorname{span}\left(\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{jq}^{\mathrm{sr}}\mathbf{V}_j\right)\right) \cap \left(\operatorname{span}\left(\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i\right)\right) = \{0\} \quad \text{for } i \neq j,k. \tag{2.14}$$

(2.12) ensures that the interference signals are within an $N-d$ dimensional subspace at the receiver k. (2.13) ensures that the useful signal fully occupies a d dimensional USS. (2.14) ensures that the USS and ISS are disjoint. The receive filter nullifies the ISS and, hence, the subspace spanned by the columns of the receive filter is orthogonal to the ISS. Thus, the above conditions can be rewritten as

$$\mathbf{U}_k^{\mathrm{H}}\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i = \mathbf{0} \quad \text{for } i \neq j,k, \tag{2.15}$$

$$\operatorname{rank}\left(\mathbf{U}_k^{\mathrm{H}}\sum_{q=1}^{Q}\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{jq}^{\mathrm{sr}}\mathbf{V}_j\right) = d. \tag{2.16}$$

The subspace spanned by the columns of the matrix $\mathbf{U}_k$ is d-dimensional. (2.15) guarantees that the interference signals are within an $N-d$ dimensional subspace so that they are all nullified after the receive filter. (2.16) guarantees that the useful signals span a d dimensional subspace after the receive filter. The receive filter nullifies ISS, but not USS and, hence, ISS and USS are disjoint.

2.5 Minimum required number of relay antennas

In this section, the minimum required number of antennas at the relay is derived so that IA is feasible in the system. In the following, it is assumed that the nodes have enough number N of antennas so that N is not a limiting factor in performing IA. The relay interference channel is a multiple key-hole channel [FSSY11]. This can be clearly seen by inspecting the effective channel matrix from all the transmitters to all the receivers.

Let $\mathbf{H}_j^{\mathrm{sr}}$ denote the concatenation of the MIMO channel matrices from node j to all the Q relays. Similarly, let $\mathbf{H}_j^{\mathrm{rd}}$ denote the concatenation of the MIMO channel matrices from all the Q relays to the node j. Then $\mathbf{H}_j^{\mathrm{sr}}$ and $\mathbf{H}_j^{\mathrm{rd}}$ are given by

$$\mathbf{H}_j^{\mathrm{sr}} = \begin{bmatrix} \mathbf{H}_{j1}^{\mathrm{sr}} \\ \vdots \\ \mathbf{H}_{jQ}^{\mathrm{sr}} \end{bmatrix} \tag{2.17}$$

and

$$\mathbf{H}_j^{\mathrm{rd}} = \begin{bmatrix} \mathbf{H}_{1j}^{\mathrm{rd}} & \cdots & \mathbf{H}_{Qj}^{\mathrm{rd}} \end{bmatrix}, \tag{2.18}$$

respectively. Let $\widetilde{\mathbf{H}}$ denote the effective channel from all the transmitters to all the receivers. Then $\widetilde{\mathbf{H}}$ is given by

$$\widetilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_1^{\mathrm{rd}} \\ \vdots \\ \mathbf{H}_{2K}^{\mathrm{rd}} \end{bmatrix} \mathbf{G} \begin{bmatrix} \mathbf{H}_1^{\mathrm{sr}} & \cdots & \mathbf{H}_{2K}^{\mathrm{sr}} \end{bmatrix}. \tag{2.19}$$

Recollect from Section 2.2 that the nodes are assumed to be able to perfectly cancel the self interference. Furthermore, in this section, it is assumed that the number of antennas at the nodes is not a limiting factor. Therefore, the signals from the communication partners can be allowed to overlap with each other at the relays and hence, the signals from a single node pair can be treated as signal from a single node. Therefore, without loss of generality, there are Kd effective data streams that are transmitted through the relays.

From Section 2.2, the channel matrices $\mathbf{H}_j^{\mathrm{sr}}$, $\mathbf{H}_k^{\mathrm{rd}}$ are assumed to be of full rank N. Furthermore, $\mathbf{H}_j^{\mathrm{sr}}$ for $j = 1, 2, \ldots, 2K$ are independent of each other and hence, their concatenation yields a matrix with rank $\min(2KN, QR)$. Here, the function min() finds the minimum of the two arguments within the brackets. Similarly, the concatenation of $\mathbf{H}_k^{\mathrm{rd}}$ for $k = 1, 2, \ldots, 2K$ yields a matrix with rank $\min(QR, 2KN)$. The relay processing matrix $\mathbf{G}$ can at most be of full rank QR. The rank of product of two or more matrices is less than or equal to the minimum of the rank of each of the matrices involved in the product [Str03]. Therefore,

$$\mathrm{rank}(\widetilde{\mathbf{H}}) \leq \min\left(2KN, QR\right). \tag{2.20}$$

Since it is assumed that the number N of antennas at the nodes is not a limiting factor, (2.20) results in

$$\mathrm{rank}(\widetilde{\mathbf{H}}) \leq QR. \tag{2.21}$$

Kd effective data streams need to be transmitted through the keyhole channel $\widetilde{\mathbf{H}}$ in total. At each of the receivers, the signals from the communication partners should be linearly independent of the signals from other node pairs. Therefore, all the Kd effective data streams should be spatially separable at the output of the effective channel $\widetilde{\mathbf{H}}$. The Kd effective data streams are spatially separable if and only if

$$\mathrm{rank}(\widetilde{\mathbf{H}}) \geq Kd \tag{2.22}$$

holds. From (2.21) and (2.22), we get

$$QR \geq Kd. \tag{2.23}$$

In the above paragraph it has been shown that assuming the nodes have enough antennas, (2.23) is a necessary condition for IA to be feasible in a multi-pair two-way relay network. For a given number of relay antennas satisfying (2.23), the number N of antennas required at the nodes is derived in Chapters 3 and 4 for single and multiple relays scenarios, respectively.

2.6 Achievable sum rate

In this section, the sum of the data rate achieved by all the node pairs is derived. Note that as described in Section 2.2, it is assumed that the data vector $\mathbf{d}_j$ is a zero mean circularly symmetric Gaussian vector [NM93]. The sum rate expression derived in the following is the maximum data rate that is achievable in the network for a given channel realization and the transmit, relay and receive filters obtained using a given

IA algorithm. Let $\mathbf{a}_{jk}$ and $\mathbf{e}_k$ denote the useful signal and the interference signals, respectively, at node k. Then

$$\mathbf{a}_{jk} = \sum_{q=1}^{Q} \mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{jq}^{\mathrm{sr}} \mathbf{V}_j \mathbf{d}_j, \tag{2.24}$$

$$\mathbf{e}_k = \sum_{\substack{i=1\\ i\neq j,k}}^{2K} \sum_{q=1}^{Q} \mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i \mathbf{d}_i. \tag{2.25}$$

The achievable data rate of a MIMO receiver in the presence of interference signals is given by [GPP07]

$$R_{\mathrm{mimo}} = \log_2 \left(\det \left(\mathbf{I} + \mathbf{SINR}\right)\right) \tag{2.26}$$

where **SINR** denotes the ratio of the useful signal covariance matrix to the interference signals plus noise covariance matrix. Two-way relaying involves two time slots and, hence, for the considered multi-pair two-way relay network a factor 0.5 need to be multiplied to the achievable data rate expression. Therefore, the achievable data rate of receiver k is given by

$$R_{jk} = \frac{1}{2} \log_2 \left(\det \left(\mathbf{I} + \frac{\mathbb{E}\left\{\mathbf{a}_{jk}\mathbf{a}_{jk}^{\mathrm{H}}\right\}}{\mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\} + \mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}}\right)\right) \tag{2.27}$$

where

$$\mathbb{E}\left\{\mathbf{a}_{jk}\mathbf{a}_{jk}^{\mathrm{H}}\right\} = \sum_{q=1}^{Q} \sum_{\overline{q}=1}^{Q} \left(\mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{jq}^{\mathrm{sr}} \mathbf{V}_j\right) \left(\mathbf{H}_{\overline{q}k}^{\mathrm{rd}} \mathbf{G}_{\overline{q}} \mathbf{H}_{j\overline{q}}^{\mathrm{sr}} \mathbf{V}_j\right)^{\mathrm{H}}, \tag{2.28}$$

$$\mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\} = \sum_{\substack{i=1\\ i\neq j,k}}^{2K} \sum_{q=1}^{Q} \sum_{\overline{q}=1}^{Q} \left(\mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i\right) \left(\mathbf{H}_{\overline{q}k}^{\mathrm{rd}} \mathbf{G}_{\overline{q}} \mathbf{H}_{i\overline{q}}^{\mathrm{sr}} \mathbf{V}_i\right)^{\mathrm{H}}, \tag{2.29}$$

$$\mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\} = \sum_{q=1}^{Q} \sigma_{1q}^2 \left(\mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q\right) \left(\mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q\right)^{\mathrm{H}} + \sigma_{2k}^2 \mathbf{I}. \tag{2.30}$$

R_{jk} is the rate at which node j can transmit to node k with arbitrarily low bit error probability. The achievable sum rate of the system is defined as the sum of the data rate achievable by all the $2K$ nodes in the system is given by

$$R_{\mathrm{sum}} = \sum_{k=1}^{2K} R_{jk}. \tag{2.31}$$

In this thesis, for simplicity, the term achievable sum rate is expressed simply by the term sum rate.

2.7 Degrees of freedom

In this section, the term Degrees of freedom (DoF) is introduced. In this thesis, DoF is defined as the number of data streams that can be transmitted simultaneously without interference at the receivers per time slot and per subcarrier. Recently, DoF has been utilized as a measure of the capacity of the network at high SNR [CJ08]. DoF is given by the slope of the sum rate curve at high SNR given by

$$d_{\mathrm{DoF}} = \lim_{SNR \to \infty} \frac{R_{\mathrm{sum}}}{\log_2(SNR)} \tag{2.32}$$

[CJ08]. IA focuses at high SNR. Furthermore, since, at high SNR, noise power, amplitude of channel coefficients, and power allocation have almost negligible influence on the sum rate, DoF is an important measure to evaluate the performance of an interference limited system.

Chapter 3

Interference alignment with a single relay

3.1 Introduction

In this chapter, IA with a single relay is considered. In this thesis, it is assumed that the relay does not have enough antennas to spatially separate the data streams. However, the relay assists in performing IA at the receivers. This means the transmit, relay and receive filters are designed jointly such that IA is achieved at the receiver. As described in Section 2.5, the two-way relay channel is a multiple key hole channel and it has been shown that the relay needs $R \geq Kd$ antennas so that the useful and the interfernce signals can be separated at the receivers. Based on this condition, two cases are considered in this chapter. For each of the two cases, first an IA scheme is proposed. The term IA scheme implies the steps that need to be performed to achieve IA. The steps shall be sub-problems represented by a set of equations which are yet to be solved. Secondly, the properness condition is derived based on the steps involved in the IA scheme. Thirdly, an algorithm is proposed to solve the steps involved in the IA scheme. The algorithm shall provide a closed form solution or an iterative solution. In this thesis, the term method is used to imply the combination of the IA scheme and the algorithm.

In Section 3.2, the case where the relay has the minimum number $R = Kd$ of antennas is considered. First an IA scheme is proposed. It will be shown that the problem of IA can be decoupled into three linear problems. Then the number of antennas required at the nodes to perform IA is derived in terms of a properness condition. Finally, a closed form solution to solve the three linear problems and, hence, to achieve IA is proposed. The sum rate performance of the proposed IA algorithm is compared with a reference method based on [LDLG11].

In Section 3.3, the case where the relay has additional antennas is considered. The scheme proposed for $R = Kd$ can be directly extended to $R \geq Kd$ by switching off the additional $R - Kd$ antennas, but this does not utilize the additional antennas available at the relay. In section 3.3, first, a new method to utilize these additional antennas to increase the degrees of freedom in the system or to reduce the number of antennas required at the nodes is proposed. Here, IA is decoupled into two bilinear problems and one linear problem. Then, the properness condition is derived. The properness condition gives the relation between the number R of antennas available at the relay and the number N of antennas required at the nodes to perform IA for a given scenario. An iterative algorithm to achieve the IA solution is proposed. Additionally, it is shown that a closed form solution is possible in certain cases. The condition for the applicability of the closed form solution is derived and the closed form solution is given. Finally, the sum rate performance of the proposed IA algorithm is compared with the reference method based on [LDLG11].

3.2 Relay with minimum number of antennas

3.2.1 Introduction

In this section, the case of the relay having the minimum number $R = Kd$ of antennas given by (2.23) derived in Section 2.5 is considered. First, a new three step IA scheme is proposed in Section 3.2.2. In Section 3.2.3, the properness condition for IA is derived. In Section 3.2.4, a closed form solution IA is proposed. Finally, the performance of the proposed algorithm is investigated in Section 3.2.5. The contents of this section have been published by us in [GWK11] and [GAK+13].

3.2.2 IA Scheme

3.2.2.1 MAC phase: Signal alignment

In this section, the MAC phase of the bi-directional communication is described. In the following, a new concept called signal alignment is developed by considering the fact that the relay has the minimum required number of antennas to achieve IA. It is shown that in order to achieve IA at the receivers, signal alignment is a necessary step.

In the MAC phase, each of the $2K$ nodes transmits d data streams to the relay. The number of antennas at the relay is limited to $R = Kd$. Hence, these $2Kd$ data streams cannot be spatially separated in an R-dimensional relay space. However, our objective is that the relay aids in performing IA. In order to achieve IA, all the interference signals have to be within the ISS and the useful signals have to be within the USS. The signal space of any given receiver k is of dimension N. In order to satisfy the IA conditions given by (2.15) and (2.16), a USS of size d needs to be reserved for the useful signals and all the interferences should be within the ISS of size $N - d$. The self interference is assumed to be known and can be perfectly cancelled at the receiver. Hence, there are $2(K-1)d$ interfering data streams at the receiver.

The channel matrix between the relay and receiver k is a linear map if $R \leq N$ and is a linear projection if $R > N$. Hence, the ISS at receiver k corresponds to a subspace of maximum dimension $R - d = Kd - d = (K-1)d$ at the relay. All the $2(K-1)d$ interfering streams should be within the $(K-1)d$ dimensional subspace at the relay and the d dimensional useful signal subspace should be disjoint from the $(K-1)d$ dimensional subspace containing the interference signals at the relay. The same conditions hold for each of the receivers $k = 1, 2 \cdots 2K$. Since every node needs a d dimensional USS disjoint from the subspace spanned by all the $2(K-1)d$ interfering streams and the relay space is of dimension $R = Kd$, the only possibility to satisfy this condition for all the $2K$ node pairs simultaneously is to align the subspace spanned by the d data streams from node k with the corresponding subspace of its communication partner j. That is, the useful signals from each of the nodes should span the same d dimensional subspace as the useful signals of its communication partner at the relay. We term this signal alignment (SA). Let (j, k) denote the communication partner nodes j and k. Then SA of node pair (j, k) is expressed by

$$\operatorname{span}\left(\mathbf{H}_j^{\mathrm{sr}}\mathbf{V}_j\right) = \operatorname{span}\left(\mathbf{H}_k^{\mathrm{sr}}\mathbf{V}_k\right) \tag{3.1}$$

where span (.) denotes the subspace spanned by the columns of the matrix within the brackets. After performing SA, there are Kd effective data streams at the relay.

3.2.2.2 BC phase: Channel alignment + Transceive zero forcing

In this section, the BC phase of the bi-directional communication is described. First, a new concept called channel alignment is developed as a necessary step in achieving IA at the receivers. Then, it will be shown that channel alignment and transceive ZF will result in IA at the receivers.

In the following, the concept of channel alignment is introduced. After SA in the MAC phase, there are Kd effective data streams available at the relay. In the BC phase, the relay filter coefficients have to be chosen such that IA is achieved at the receivers. After achieving IA, the receive filters can be chosen to nullify the interferences and obtain the useful signals. In other words, the relay and receive filters have to be chosen such that (2.15) and (2.16) are satisfied. In order to obtain a necessary condition on the design of the receive filters, assume that an IA solution exists and the receive filters of the solution of (2.15) and (2.16) are known. Since, the receive filters are known, consider the effective channels from the relay to the receiver including the receive filter given by $\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_j^{\mathrm{rd}}$ for $j = 1, 2, \ldots, 2K$. There are $2Kd$ effective channel directions. Now, the objective is to find the relay filter that satisfies (2.15) and (2.16). In other words, the relay filter has to be chosen such that the interferences are nullified at the receivers after the receive filters. That is, the relay has to perform transmit ZF on the $2Kd$ effective channel directions.

There are Kd effective data streams at the relay. Now, d effective data streams corresponding to a given node pair have to be transmitted orthogonal to the subspace spanned by the $2(K-1)d$ channel directions corresponding to the other node pairs. Since the relay has only Kd antennas, the subspace spanned by the $2(K-1)d$ channel directions corresponding to the undesired receivers should be of maximum dimension $(K-1)d$. This condition needs to hold for each of the K node pairs. Furthermore, the subspace spanned by the effective channel directions of each node pair should be disjoint from each other so that ZF the effective channel directions of one node pair do not zero force the effective channel directions of other node pairs. Therefore, the only possibility for the $2(K-1)d$ channel directions to span a $(K-1)d$ dimensional subspace is that each effective channel directions of one node align with the effective channel directions of its communication partner. We term this channel alignment (CA). Therefore, in order to achieve IA, it is necessary that the receive filters are chosen to perform CA. CA of node pair (j, k) is given by

$$\mathrm{span}\left(\mathbf{H}_j^{\mathrm{rdH}}\mathbf{U}_j\right) = \mathrm{span}\left(\mathbf{H}_k^{\mathrm{rdH}}\mathbf{U}_k\right). \tag{3.2}$$

Note that in the above equation, the Hermitian of the effective channel is considered. This is due to the fact that the span operation is performed along the columns of the matrix within the brackets and the effective channel directions at the relay are given by the rows of the effective channels $\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_j^{\mathrm{rd}}$ and $\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rd}}$.

After SA and CA, there are Kd effective data streams and Kd effective channels. The relay with $R = Kd$ antennas can perform transceive ZF. Hence, (2.15) and (2.16) are satisfied.

In this following, it is shown that CA performed by the nodes and transceive ZF performed by the relay will result in IA at the receivers. Note that the relay filter $\mathbf{G}$ is designed based on the effective channels $\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_j^{\mathrm{rd}}$. Hence, at each of the $2K$ nodes, the interferences will be zero after the receive filter $\mathbf{U}_k^{\mathrm{H}}$. However, the interferences are not necessarily zero at the input of the receive filter. If the interferences are non-zero at the input of the receive filter and zero at the output of the receive filter, then they should be in a subspace orthogonal to the subspace spanned by the rows of the receive filter matrix $\mathbf{U}_k^{\mathrm{H}}$. This subspace in which the interference signals occur is the ISS. However, the useful signals are not nullified at the output of the receive filter and hence, the useful signals should be within a subspace disjoint from the ISS which is USS.

3.2.3 Properness condition

In this section, the number of antennas required at the nodes to perform IA is derived in terms of the properness condition. A system is defined to be proper if the number of variables is larger than or equal to the number of equations. Otherwise, the system is said to be improper [NSGS09], [YGJK09]. The properness condition is in general neither a necessary condition nor a sufficient condition, but proper systems are likely to be feasible, that is, for a proper system it is likely that a solution exists [YGJK09]. However, for the case with the relay having the minimum number $R = Kd$ antennas, IA is decoupled into three linear problems namely, SA, CA, and transceive ZF. For linear problems, the properness condition is also a necessary and sufficient condition [Str03]. Hence, the properness condition derived in this section is a necessary and sufficient condition.

In the proposed scheme, IA is achieved through SA, CA, and transceive ZF. SA and CA are performed at the nodes and transceive ZF is performed at the relay. In the current Section 3.2, the number of antennas at the relay is fixed to $R = Kd$, hence, given SA and CA, the relay can always perform transceive ZF. So, properness is defined by the number N of antennas available at the nodes and the number of equations involved in SA and CA. It can be seen from (3.1) and (3.2), that both SA and CA are identical problems. The term identical implies that both SA and CA involve equations of same type with the MAC channels and the transmit filters replaced with the Hermitian of the corresponding BC channels and the receive filters, respectively. Furthermore, the number of equations and variables involved in both SA and CA are the same. Hence, in the following only the properness condition for the SA is considered. Any system proper with respect to (3.1) is also proper with respect to (3.2).

In this paragraph, the number M_{v} of variables in the system is counted. The variables in the system are due to the multiple antennas at the nodes and at the relay. In the following, first, the number of variables due to N antennas at the nodes are counted. Then the number of variables due to the R antennas in the system are counted. The transmit filter matrix is of size $N \times d$. Hence, Nd variables are available. However, in order to spatially separate all the d data streams at the receiver, the columns of the transmit filter matrix should be linearly independent of each other. d^2 variables are required to be fixed so that the columns of the transmit filter matrix are linearly independent [YGJK09]. Hence, each transmit filter matrix has $Nd - d^2 = (N-d)d$ variables that can be used for SA. In other words, choosing a d-dimensional subspace in an N-dimensional transmit space of the nodes results in $(N-d)d$ variables. There are $2K$ nodes in the system. This leads to $2K(N-d)d$ variables available at the $2K$ nodes. The relay space is R dimensional. Node pairs can perform SA in any of the d dimensional subspaces in the R-dimensional relay space. This gives $(R-d)d$ variables. There are K pairs and hence, the total number of variables in the system is given by

$$M_{\mathrm{v}} = 2K(N-d)d + K(R-d)d. \tag{3.3}$$

In this paragraph, the number M_{c} of constraints involved in SA is counted. We have already counted the number of variables involved in chosing the d-dimensional alignment subspace in the R-dimensional relay space. This means, the subspace in which SA takes place is fixed. The signals transmitted from the nodes of the communication pairs should lie within their corresponding d-dimensional alignment subspace. Consider a single data stream from one of the two nodes of the pair. This data stream should be within the d-dimensional alignment subspace defined for this node, but within the d-dimensional subspace it can be in any direction. This introduces $R-d$ constraints. There are d data streams from each of the $2K$ nodes. Hence, the total number of constraints in the system is given by

$$M_{\mathrm{c}} = 2K(R-d)d. \tag{3.4}$$

For a system to be proper, the number M_{v} of variables has to be larger than or equal to the number M_{c} of constraints. This results in

$$N \geq \frac{(K+1)}{2}d. \tag{3.5}$$

Note that the properness condition in (3.5) for the considered multi-pair two-way relay network is the same as the feasibility condition for IA in a K node pairs interference channel without a relay [YGJK09, NSGS09, BCT11]. In both with and without relay, when each of the $2K$ nodes have $N \geq \frac{(K+1)}{2}d$ antennas, Kd data streams are transmitted per time slot and hence, Kd DoF are achieved. This implies that for a given number N of antennas at each of the $2K$ nodes, the number of DoF achieved in the considered multi-pair two-way relay newtork is the same as the number of DoF achieved in the K node pair interference channel without relay. Furthermore, the introduction of a relay aids in decoupling the trilinear IA problem into three linear sub-problems.

3.2.4 Algorithms

3.2.4.1 Signal alignment algorithm

In this section, an algorithm to achieve SA at the relay is described. The node pair (j, k) is considered and the closed form solution for the transmit filters $\mathbf{V}_j$ and $\mathbf{V}_k$ are obtained. The signal transmitted from node j will be within an N-dimensional subspace at the relay. This N-dimensional subspace is the subspace spanned by the columns of the channel matrix $\mathbf{H}_j^{\mathrm{sr}}$. Let us call this the channel space of node j. The columns of the matrix product $\mathbf{H}_j^{\mathrm{rd}}\mathbf{V}_j$ span a d-dimensional subspace in the N-dimensional channel space of node j. Similarly, the columns of the matrix product $\mathbf{H}_k^{\mathrm{rd}}\mathbf{V}_k$ span a d-dimensional subspace in the N-dimensional channel space of node k. In order to satisfy the SA condition (3.1), the d-dimensional subspaces of the channel subspaces of each node j and k should overlap, i.e., the channel spaces of nodes j and k should have of at least a d-dimensional intersection subspace. If the system is proper, then SA is feasible and hence, such an intersection exists. Assuming that there exists an intersection subspace of dimension at least d between these two subspaces, the columns of the matrices $\mathbf{H}_j^{\mathrm{sr}}\mathbf{V}_j$ and $\mathbf{H}_k^{\mathrm{sr}}\mathbf{V}_k$ each form a basis for a common d-dimensional subspace of the intersection subspace. Then without loss of generality, the following will be satisfied:

$$\begin{bmatrix}\mathbf{H}_j^{\mathrm{sr}} & -\mathbf{H}_k^{\mathrm{sr}}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix} = \mathbf{0}. \tag{3.6}$$

Let $\mathbf{H}_{jk}^{\mathrm{sr}'} = \begin{bmatrix}\mathbf{H}_j^{\mathrm{sr}} & -\mathbf{H}_k^{\mathrm{sr}}\end{bmatrix}$. Then,

$$\mathrm{span}\left\{\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix}\right\} \subseteq \mathrm{null}\left(\mathbf{H}_{jk}^{\mathrm{sr}'}\right) \tag{3.7}$$

where null(.) denotes the null space of the matrix within the brackets. Let the columns of the matrix $\begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix}$ denote a basis for the null space of the matrix $\mathbf{H}_{jk}^{\mathrm{sr}'}$. Then

$$\mathrm{span}\left\{\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix}\right\} \subseteq \mathrm{span}\left\{\begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix}\right\}. \tag{3.8}$$

If the intersection subspace is of dimension larger than d, then there are multiple solutions for (3.1). $\mathbf{V}_j$ and $\mathbf{V}_k$ can be chosen as a basis for any d-dimensional subspace of span$\{\mathbf{A}_j\}$ and span$\{\mathbf{A}_k\}$, respectively. Let $\mathbf{W}_j^{(\mathrm{v})}$ be a matrix with d columns and rank d. Here, the alphabet v in $\mathbf{W}_j^{(\mathrm{v})}$ implies that the matrix varible $\mathbf{W}_j^{(\mathrm{v})}$ is related to the design of the transmit filters. Then the transmit filters are given by

$$\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix} = \begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix} \mathbf{W}_j^{(\mathrm{v})}. \tag{3.9}$$

There are several possibilities to chose such a matrix $\mathbf{W}_j^{(\mathrm{v})}$ of rank d. One can chose a $\mathbf{W}_j^{(\mathrm{v})}$ that maximizes a given objective, for instance, the SNR.

Remark: As shown in (3.7), the design of the transmit filters of the node pair (j, k) involves only the channel matrices from node j and k to the relay. Therefore, the SA subspace of node pair (j, k) at the relay depends only on the channel matrices $\mathbf{H}_j^{\mathrm{sr}}$ and $\mathbf{H}_k^{\mathrm{sr}}$ of the node pair (j, k). Furthermore, the channel matrices $\mathbf{H}_i^{\mathrm{sr}}$ for $i = j, k$ are assumed to be independent of that of the nodes $i \neq j, k$. Hence, the alignment subspace of each node pair will be almost surely disjoint of the alignment subspace of the other node pairs and after SA, there will be Kd effective data streams at the relay.

3.2.4.2 Channel alignment algorithm

It can be seen from (3.1) and (3.2), that both SA and CA are identical problems with the MAC channels and the transmit filters replaced with the corresponding BC channels and the receive filters. Hence, the algorithm described in Section 3.2.4.1 can used to solve (3.2) and hence, perform CA.

3.2.4.3 Transceive zero forcing

In this section, the relay filter is designed. After SA and CA, there are Kd effective data streams at the relay and Kd effective channels between the relay and the receivers. The relay performs receive ZF to spatially separate the Kd effective data streams and performs transceive ZF to spatially orthogonalize the signals transmitted through the Kd effective channels. Let $\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}$ and $\mathbf{G}_{\mathrm{tx}}$ denote the receive and transmit ZF matrices, respectively. Then $\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}$ and $\mathbf{G}_{\mathrm{tx}}$ are given by

$$\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}} = \begin{bmatrix} \mathbf{H}_1^{\mathrm{sr}}\mathbf{V}_1 & \dots & \mathbf{H}_K^{\mathrm{sr}}\mathbf{V}_K \end{bmatrix}^{-1} \tag{3.10}$$

$$\mathbf{G}_{\mathrm{tx}} = \begin{bmatrix} \mathbf{U}_1^{\mathrm{H}}\mathbf{H}_1^{\mathrm{rd}} \\ \vdots \\ \mathbf{U}_K^{\mathrm{H}}\mathbf{H}_K^{\mathrm{rd}} \end{bmatrix}^{-1} . \tag{3.11}$$

The matrices on the right hand side of (3.10) and (3.11) are square matrices of size $Kd \times Kd$. Since the subspace of the effective data streams corresponding to a given node pair is disjoint from the subspaces of the effective data streams of all the other node pairs, the matrix on the right hand side of (3.10) is almost surely full rank Kd

and hence, the inverse exists. Similarly, the matrix on the right hand side of (3.11) is also of full rank Kd and hence, the inverse exists. The relay filter is given by

$$\mathbf{G} = \alpha_{\mathrm{r}} \mathbf{G}_{\mathrm{tx}} \mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}. \tag{3.12}$$

Here, α_{r} is a scalar used to satisfy the relay power constraint given by (2.8). α_{r} is given by

$$\alpha_{\mathrm{r}} = \sqrt{\frac{P_{\mathrm{relay}}}{\mathrm{Trace}\left(\mathbf{G}_{\mathrm{tx}}\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}} \sum_{q=1}^{Q} \left(\sum_{i=1}^{2K} \mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i \mathbf{V}_i^{\mathrm{H}} \mathbf{H}_{iq}^{\mathrm{srH}} + \sigma_{1q}^2 \mathbf{I}\right) (\mathbf{G}_{\mathrm{tx}}\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}})^{\mathrm{H}}\right)}}. \tag{3.13}$$

Note that the scaling factor α_{r} does not affect the IA conditions.

Remark: Although, in this thesis, global CSI is assumed at all the nodes and at the relay, from (3.1) and (3.2), it can be seen that for the design of the transmit and the receive filters, each of the $2K$ nodes need to know only the channel between the node and the relay and the channel between its communication partner and the relay called pair-wise channel knowledge in the MAC and the BC phases. However, relay needs global CSI to perform transceive ZF.

3.2.5 Performance analysis

3.2.5.1 Introduction

In this section, the performance of the proposed IA algorithm is analysed and compared with a reference algorithm. The achievable DoF and the sum rate introduced in Sections 2.7 and 2.6, respectively, are used to evaluate the performance of these algorithms. In the following, first the reference algorithm is briefly described in Section 3.2.5.2. Following this, different numbers N and R of node and relay antennas, respectively, considered for the analysis are introduced and the DoF achieved by the proposed IA algorithm and the reference algorithm are investigated in Section 3.2.5.3. Then the assumptions regarding the simulation scenario are introduced in Section 3.2.5.4. Finally, the sum rate achieved by these algorithms at different SNRs are obtained through MATLAB simulations in Section 3.2.5.5.

3.2.5.2 Reference algorithm

In this section, the reference algorithm which is based on the ZF algorithm described in [LDLG11] is briefly explained. In contrast to the ZF algorithm proposed in Section

3.2.4.3 which spatially separates Kd effective data streams and Kd effective channel directions to achieve IA, in [LDLG11], the $2Kd$ data streams and $2Kd$ channel directions are spatially separated such that interference signals do not arrive at the receivers. In [LDLG11], the nodes have a single antenna each. The multiple antennas at the relays are used to spatially separate the data streams [LDLG11]. Hence, the relay requires $R \geq (2K-1)d$ antennas [LDLG11]. For a given number R of antennas, the number K of node pairs that can be served simultaneously in the reference algorithm is less than that in the proposed IA algorithm. Furthermore, in the scenarios considered in this thesis, multiple antennas are available at the nodes. To make a fair comparison, in the reference algorithm described below, the multiple antennas at the nodes are used to improve the SNR of the received signal. This is achieved by choosing the transmit and the receive filters such that the transmit and the receive directions, respectively, of d data streams of each of the K nodes correspond to the directions of the d largest singular values of the channel between each node and the relay. In the following, the design of the transmit, receive and relay filters is described in detail.

First, in the following, the design of the transmit filters of each of the $2K$ nodes is described. The transmit filters are designed such that for a fixed transmit power, the signal power of the received signal is maximized. Consider node i for $i = 1, 2, \dots, 2K$. The channel matrix between node i and the relay is given by $\mathbf{H}_i^{\mathrm{sr}}$. The objective is to maximize the received signal power at the relay as follows:

$$\begin{aligned} \underset{\mathbf{V}_i}{\text{maximize}} \quad & \mathrm{Tr}\left(\mathbf{V}_i^{\mathrm{H}} \mathbf{H}_i^{\mathrm{srH}} \mathbf{H}_i^{\mathrm{sr}} \mathbf{V}_i\right) \\ \text{subject to} \quad & \mathrm{Tr}\left(\mathbf{V}_i \mathbf{V}_i^{\mathrm{H}}\right) \leq P_{\mathrm{node}}. \end{aligned} \tag{3.14}$$

The optimum $\mathbf{V}_i$ is given by

$$\mathbf{V}_i = \alpha_{\mathrm{n}} \lambda_{\max,d}\left(\mathbf{H}_i^{\mathrm{srH}} \mathbf{H}_i^{\mathrm{sr}}\right) \tag{3.15}$$

[Str03] where $\lambda_{\max,d}(.)$ represents the matrix containing as its columns the eigenvectors corresponding to the first d largest eigenvalues of the matrix within the brackets and $\alpha_{\mathrm{n}} = \sqrt{\frac{P_{\mathrm{node}}}{d}}$.

Similar to the transmit filters, the receive filters are designed to maximize the received signal power and are given by

$$\mathbf{U}_i = \lambda_{\max,d}\left(\mathbf{H}_i^{\mathrm{rd}} \mathbf{H}_i^{\mathrm{rdH}}\right) \tag{3.16}$$

[Str03].

In the following, the design of the relay filter is described according to [LDLG11]. $R \geq (2K-1)d$ antennas are required at the relay to spatially separate the data streams from different node pairs [LDLG11]. Since the objective of IA is to maximize the DoF, for a fair comparison, in the reference algorithm maximum multiplexing gain is obtained by fully utilizing all the R spatial dimensions available at the relay. In the following, as many node pairs are accommodated as to satisfy the condition $R \geq (2K-1)d$ with equality sign. Hence, in the following the case $R = (2K-1)d$ is considered.

The relay filter is denoted as the product of the following three matrices

$$\mathbf{G} = \mathbf{G}_{\mathrm{tx}}\mathbf{G}_{\mathrm{p}}\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}} \tag{3.17}$$

where $\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}$ and $\mathbf{G}_{\mathrm{tx}}$ denote the receive and transmit ZF matrices, respectively [LDLG11] and $\mathbf{G}_{\mathrm{p}}$ is the power allocation matrix. In this thesis, equal power allocation for all the data streams of all the nodes is assumed i.e., $\mathbf{G}_{\mathrm{p}} = \alpha_{\mathrm{r}}\mathbf{I}$. Here, α_{r} is a scaling factor to satify the relay power constraint. Let $\mathbf{H}_j^{\mathrm{sr,int}}$ denote the matrix consisting of all the receive signatures $\mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i$ except the one corresponding to the node j and its communication partner k given by

$$\mathbf{H}_j^{\mathrm{sr,int}} = \left[\begin{array}{ccccc} \mathbf{H}_1^{\mathrm{sr}}\mathbf{V}_1 & \ldots & \mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i & \ldots & \mathbf{H}_{2K}^{\mathrm{sr}}\mathbf{V}_{2K} \end{array}\right]_{i \neq j,k} \tag{3.18}$$

[LDLG11]. Let the columns of the matrix $\mathbf{G}_{\mathrm{rx}}^{j}$ form a basis for a d dimensional subspace in null $\left(\mathbf{H}_j^{\mathrm{sr,int}}\right)$ for $j = 1, \ldots, K$. Then the receive ZF matrix is given by

$$\mathbf{G}_{\mathrm{rx}} = \left[\begin{array}{ccc} \mathbf{G}_{\mathrm{rx}}^{1} & \ldots & \mathbf{G}_{\mathrm{rx}}^{K} \end{array}\right] \tag{3.19}$$

[LDLG11]. Similarly, the transmit ZF matrix $\mathbf{G}_{\mathrm{tx}}$ can be obtained from the following equations

$$\mathbf{H}_j^{\mathrm{rd,int}} = \left[\begin{array}{ccccc} \mathbf{H}_1^{\mathrm{rdH}}\mathbf{U}_1 & \ldots & \mathbf{H}_i^{\mathrm{rdH}}\mathbf{U}_i & \ldots & \mathbf{H}_{2K}^{\mathrm{rdH}}\mathbf{U}_{2K} \end{array}\right]_{i \neq j,k} \tag{3.20}$$

and

$$\mathbf{G}_{\mathrm{tx}} = \left[\begin{array}{ccc} \mathbf{G}_{\mathrm{tx}}^{1} & \ldots & \mathbf{G}_{\mathrm{tx}}^{K} \end{array}\right] \tag{3.21}$$

where the columns of the matrix $\mathbf{G}_{\mathrm{tx}}^{j}$ form a basis for a d dimensional subspace of null $\left(\mathbf{H}_j^{\mathrm{rd,int}}\right)$ [LDLG11]. As this reference algorithm is based on the ZF objective function, in the following, the reference algorithm is denoted by the term ZF.

Remark: Note that for the special case when the relay has more than the minimum required number of antennas to serve K node pairs but does not have sufficient antennas to serve $K+1$ node pairs, only K node pairs are served. In this case, multiple ZF solutions are available and for performance evaluation, one solution is arbitrarily chosen.

3.2.5.3 Degrees of freedom analysis

In this section, the DoF achieved by the proposed IA algorithm are compared with the reference ZF algorithm. As introduced in Section 2.7, in this thesis, DoF is defined as the total number of interference free data streams that can be transmitted in one channel use, i.e., during one time slot using one subcarrier by all the $2K$ nodes in the system. In two-way relaying, two time slots are necessary to perform the bi-directional communication and there are $2K$ nodes each transmitting d data streams. Hence, the DoF are given by Kd. Depending on the number of node pairs that can be simultaneously served interference free, the DoF achieved by each algorithm differ.

Three different scenarios shown in Table 3.1 are considered for investigation, namely, A1, A2, and A3. Recollect from Section 3.2.3, that for a given number N of antennas at each of the $2K$ nodes, the number of DoF achieved in the considered multi-pair two-way relay network is the same as the number of DoF achieved in the K node pairs interference channel which is equal to Kd. However, in contrast to K node pair interference channel without a relay where closed form solution is known only for the three node pairs case, in the multi-pair two-way relay network a single relay with $R = Kd$ antennas aids to obtain closed form solution for any number of node pairs. In order to emphasize this fact, in scenarios A1, A2, and A3, the parameters N and R are chosen such that three and more node pairs are served simultaneously.

In all the three scenarios, the proposed IA algorithm serves more node pairs than the ZF algorithm. Note that the DoF achieved by each algorithm is given by Kd and hence, more DoF are achieved by the proposed IA algorithm than the ZF algorithm. This increase in DoF is due to the fact that in the proposed IA algorithm, in addition to the antennas at the relays, the node antennas are also utilized in achieving a multiplexing gain. However, in the reference scheme only the relay antennas are utilized in achieving a multiplexing gain.

Compared to scenario A2, in scenario A3, more node pairs are served and hence, more variables are available from the node antennas. However, from Table 3.1 it is observed that in both scenarios, the same number of DoF are achieved. The reason is as follows: Due to the multiple key-hole nature of the multi-pair two-way relay network, the DoF are upper bounded by R [TY14]. In both the scenarios A2 and A3, $R = 10$ and the proposed IA algorithm achieves the upper bound. Therefore, in comparison with A2 the additional variables in A3 do not increase the DoF.

Table 3.1. Scenarios considered for $R = Kd$ and DoF achieved

Scenarios	R	d	N	K		DoF	
				ZF	IA	ZF	IA
A1	3	1	2	2	3	2	3
A2	10	2	6	3	5	6	10
A3	10	1	6	5	10	5	10

3.2.5.4 Assumptions

In this section, the assumptions regarding the simulation set up are described. The algorithms proposed in this thesis are valid with the general assumptions described in 2.2. However, in the following further assumptions are made, especially about the channel model, number K of node pairs and power allocation to simplify the system which helps to understand the influence of different parameters on the system performance.

- The channel between each of the $2K$ nodes and the relay is assumed to be an i.i.d. frequency-flat Rayleigh fading MIMO channel [GPP07]. In this case, the channel matrices are almost surely of full rank. The channel matrices are normalized such that the average received signal power is the same as the average transmit signal power when the transmit filters are arbitrarily chosen. For simulation, a reciprocal channel is assumed, i.e., the MAC and the BC phase channel matrices are transposes of each other.
- It is assumed that there are many node pairs in the system. As shown in Table 3.1, depending on the number N of antennas at the nodes and the number R of antennas at the relay, the number K of node pairs that are simultaneously served using a single subcarrier is given by the properness condition in case of the proposed algorithm and by the feasibility condition in case of the ZF algorithm. Among all the available node pairs, K node pairs that are simultaneously served are chosen arbitrarily.
- Equal power allocation for all the data streams of all the nodes is assumed.
- Since a statistical channel model is considered, the channel amplitude varies for different realizations. However, the average amplitude of the channel remains constant. Hence, for sum rate analysis, 10^4 channel realizations are generated. For each realization the filters are designed and the corresponding sum rate is calculated. The average of all the 10^4 sum rate values is plotted for investigation

of the performance. Through simulations it has been observed that typically 10^4 channel realizations are sufficient to obtain a smooth sum rate curve.

3.2.5.5 Sum rate analysis

In this section, the sum rate performance of the proposed IA algorithm is compared with the reference ZF algorithm. Let $P_{\text{node}} = P$ denote the maximum power of each of the $2K$ nodes. Let $P_{\text{relay}} = KP$ denote the maximum power available at the relay. In the broadcast phase, the relay transmit the signal received from all the $2K$ nodes and hence, the relay is assumed to have K times more power than each of the $2K$ nodes. In the following, the nodes and the relays perform transmission with their corresponding maximum available transmit power. The noise power at each node and at the relay is assumed to be the same and is denoted by $\sigma_k^2 = \sigma_{\text{relay}}^2 = \sigma^2$.

Figure 3.1 shows the sum rate performance as a function of P/σ^2 for scenario A1 and scenario A2. The black dashed curves and the red solid curves show the sum rates achieved by the reference ZF algorithm and the proposed IA algorithm, respectively. It can be seen that for both scenarios A1 and A2, the proposed IA algorithm performs better than the reference ZF algorithm at high SNR. The IA algorithm utilizes the antennas at the relay and at the nodes to perform IA at the receivers and hence, achieves higher DoF than the reference ZF algorithm. Hence, it outperforms the reference ZF algorithm at high SNR. However, it does not care about the useful signal power. In the reference ZF algorithm, the multiple antennas at the nodes are used to maximize the received signal power and hence, the reference ZF algorithm performs better at low SNR. It should be noted that due to the fact that the proposed IA algorithm achieves higher DoF, the slope of the sum rate curve of the proposed IA algorithm is higher than the reference ZF algorithm. This can be observed at high SNR. Consider the sum rate curve of the proposed IA algorithm in scenario A2. The sum rate values at 40 dB and 50 dB are 117.8 and 84.73, respectively. The difference in the sum rate between these two points is 33.07. In the x-axis, this corresponds to a difference of 10 dB. In terms of bits per channel use, 10 dB corresponds to $\log_2(10) = 3.32$. Therefore, the slope is given by $33.07/3.32 = 9.96 \approx 10$. Similarly, it can be shown that the slope of the sum rate curve of the reference ZF algorithm in scenario A2 is $5.99 \approx 6$. These slopes correspond to the DoF given in Table 3.1. Similarly, the slopes of the sum rate curves for scenario A1 can be shown to be 3 and 2 for the proposed IA algorithm and the reference ZF algorithm, respectively.

For the simulation results shown in Figure 3.1, the relay power has been assumed to be $P_{\text{relay}} = KP$. After SA, there are Kd effective data streams that are transmitted by the

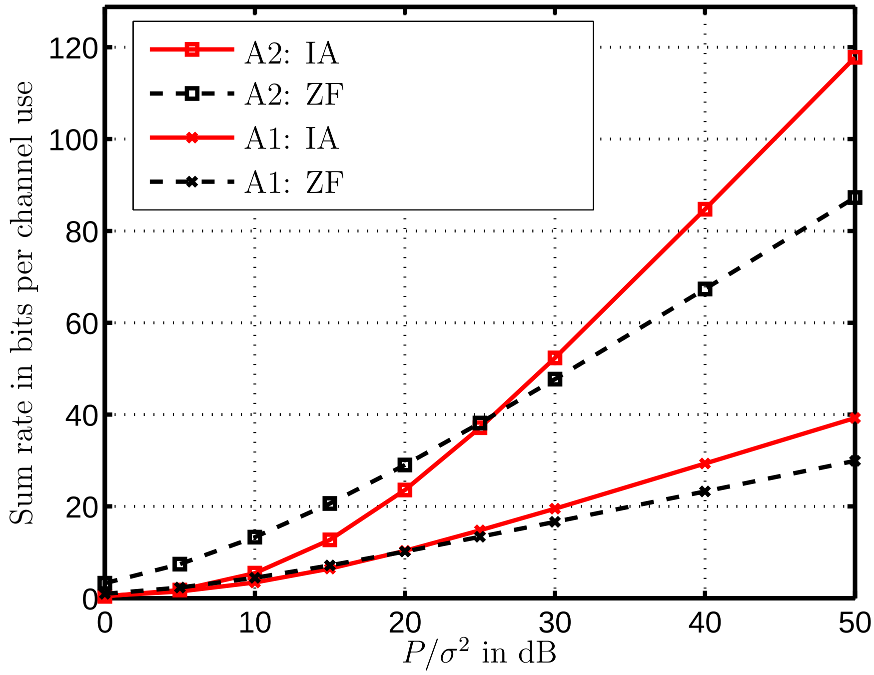

Figure 3.1. Sum rate performance of the proposed IA algorithm and the reference ZF algorithm versus P/σ^2 for scenarios A1 and A2

relay in the BC phase and hence, K times the power used by each node for transmitting d data streams has been chosen. Varying the relay power does not change the achieved DoF. This is shown in Figure 3.2. The slope of the sum rate curves remains the same, but depending on the relay power, the sum rate curve shifts upward or downward. Let us define the crossing point as the value of P/σ^2 after which the sum rate of the IA algorithm is higher than that of the reference ZF algorithm. From Figure 3.2, it can be seen that with the increase in the relay power, the crossing point occurs earlier. This implies that the additional sum rate achieved due to the additional relay power is larger in case of the the proposed IA algorithm than that of the reference ZF algorithm. This is due to the fact that the sum rate curve of the the proposed IA algorithm is higher than that of the reference ZF algorithm.

Figure 3.3 depicts the sum rate curves for the proposed IA algorithm for scenario A2 and scenario A3. In both these scenarios, 10 DoF are achieved and hence, both the sum rate curves have the same slope. However, the absolute value of the sum rate for scenario A3 is higher than that for scenario A2. This difference in performance is due to the additional total power available in the system, i.e., in scenario A2, there are 5 node pairs and hence, a total of $10P$ and $5P$ are used to transmit 20 data streams in the MAC and the BC phases, respectively. However, in scenario A3, there are 10

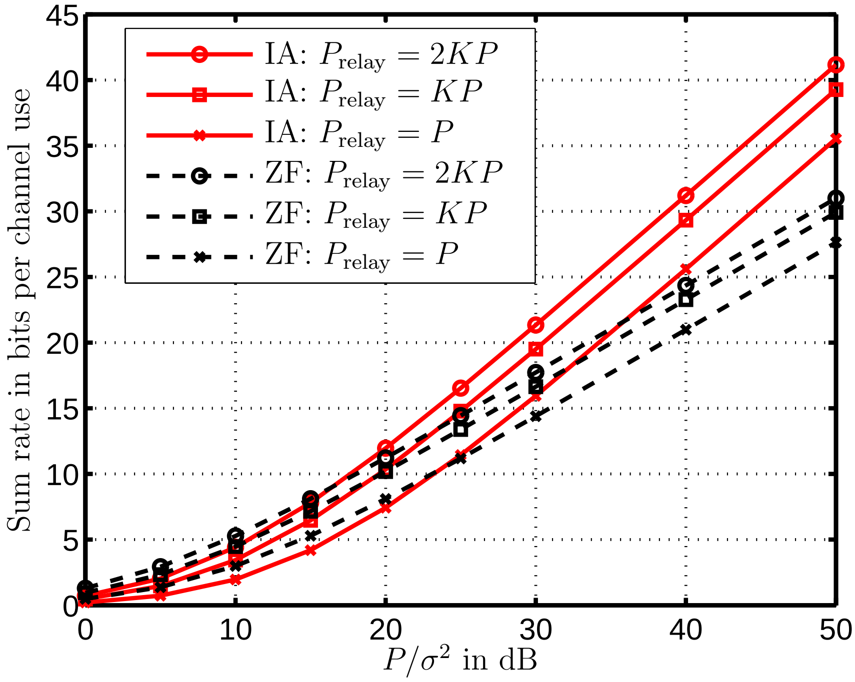

Figure 3.2. Influence of the relay power P_{relay} on the sum rate performance versus P/σ^2 for the proposed IA algorithm and the reference ZF algorithm in Scenario A1

node pairs and hence, a total of $20P$ and 10 are used to transmit 20 data streams in the MAC and the BC phases, respectively. It can be observed in Figure 3.3 that the doubling of the power results in a shift of 3 dB between the sum rate curves. This is further verified in Figure 3.4, where for scenario A3 each node is allocated only $P/2$ and the relay is allocated only $KP/2$, so that the total power becomes the same as that of scenario A2. It can be seen from Figure 3.4 that in both the scenarios, the same sum rate is achieved.

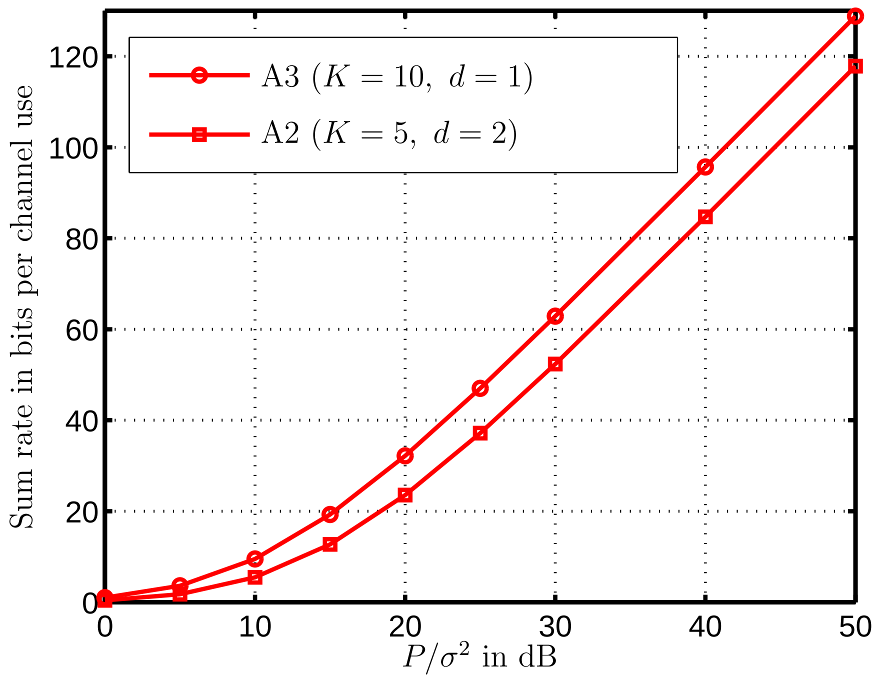

Figure 3.3. Sum rate for the proposed IA algorithm for different number of node pairs and data streams

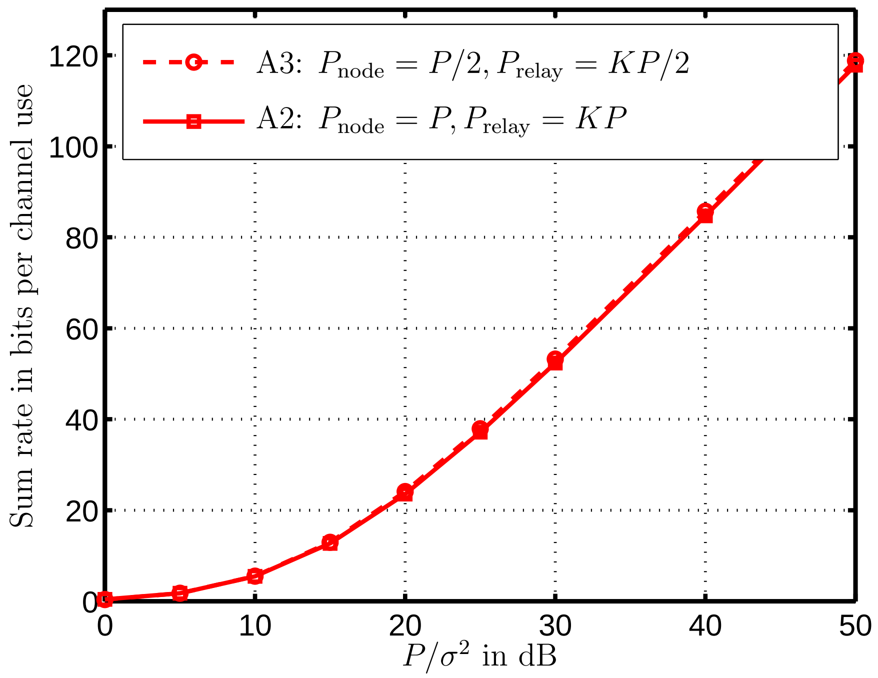

Figure 3.4. Sum rate for the proposed IA algorithm for different number of node pairs and data streams with the same total power in scenario A2 and in scenario A3

3.3 Relay with additional antennas

3.3.1 Introduction

In this section, the case $R \geq Kd$ is considered. In contrast to the case $R = Kd$, for the case $R \geq Kd$ the relay has $R - Kd$ additional antennas. The additional antennas at the relay provide more variables in the system. Hence, from one perspective, for a fixed number N of antennas at the nodes, IA can be achieved for more node pairs or data streams in the system than for the case with $R = Kd$. From another perspective, for a fixed number of node pairs or data streams in the system, for $R \geq Kd$, the number of antennas required at the nodes to perform IA can be reduced. In Section 3.3.3, it will be shown using the properness condition that these two problems are virtually the same and the algorithms proposed in Section 3.3.4 can achieve any of these two objectives. In this section, we propose a novel IA algorithm which is a generalization of the SA and CA based algorithm described in Section 3.2. In the following, first we describe the new IA scheme in Section 3.3.2. Secondly, we derive the properness condition in Section 3.3.3. Thirdly, an iterative algorithm to achieve IA is given in Section 3.3.4. Additionally, a closed form solution is possible in certain cases. The condition for the applicability of the closed form solution is also derived in Section 3.3.4. Finally, the performance is evaluated in terms of the sum rate in Section 3.3.5. The contents of this section have been published by the author of this thesis in [GLA+13] and [GAK+13].

3.3.2 Scheme

3.3.2.1 MAC phase: Partial signal alignment

In this section, the MAC phase of the bi-directional communication is considered. The concept of SA developed in Section 3.2 is generalized into a new concept called partial signal alignment (PSA). In the following, first the concept of PSA is introduced. Then the equations representing PSA are introduced. Finally, an example illustrating the concept of PSA is described.

In this paragraph, the concept of PSA is introduced. In contrast to Section 3.2, in the current section, the relay space is of dimension $R \geq Kd$. For SA, a Kd-dimensional subspace is sufficient at the relay. The additional $(R - Kd)$ dimensional subspace provides additional variables in the system. In this thesis, we utilize these additional variables to reduce the required number N of antennas at the nodes to perform SA.

The basic idea is as follows: For the case $R = Kd$, as derived in Section 3.5, $N \geq \frac{(K+1)d}{2}$ antennas are necessary at the nodes to perform SA in a given Kd dimensional relay space. However, for the case $R \geq Kd$, a Kd dimensional subspace is chosen such that SA is feasible with a smaller number of antennas than that in the case $R = Kd$. We term this subspace relay receive useful subspace $RUSS$. In the other $R - Kd$ dimensional subspace, the signals do not need to be aligned, and it is termed relay receive interference subspace $RISS$. To this extent, in the MAC phase, the relay receive space is divided into two orthogonal subspaces, namely, the $RUSS$ of dimension Kd and the $RISS$ of dimension $R - Kd$. $RUSS$ has to be chosen such that it is possible that each node aligns its signals with that of its communication partner in the $RUSS$. In the $RISS$, the SA does not need to be feasible. The process of performing SA only within $RUSS$ is termed PSA.

In the following, the equations representing PSA are introduced. Since in PSA the signals are pair-wise aligned only within the $RUSS$, the signals in $RISS$ consist of inter-pair interferences and are nullified at the relay. Let $\mathbf{T}$ denote the projection matrix that projects the received signal at the relay to $RUSS$. By this projection operation, the signal components in $RISS$ are nullified. The SA of each pair (j, k), within the $RUSS$ is represented by

$$\text{span}\left(\mathbf{T}^{\text{H}}\mathbf{H}_j^{\text{sr}}\mathbf{V}_j\right) = \text{span}\left(\mathbf{T}^{\text{H}}\mathbf{H}_k^{\text{sr}}\mathbf{V}_k\right). \tag{3.22}$$

Let $RUSS_j = \text{span}\left(\mathbf{T}^{\text{H}}\mathbf{H}_j^{\text{sr}}\mathbf{V}_j\right)$ denote the d-dimensional alignment subspace of the pair (j, k) within the $RUSS$. In addition to the condition given by (3.22), in order to be able to separate the useful signals from the interference signals at each of the $2K$ receivers, the alignment subspace $RUSS_j$ of the node pair (j, k) should be disjoint from that of all the other node pairs. This is given by

$$RUSS_j \bigcap \left(\bigcup_{\substack{j'=1 \\ j' \neq j}}^{K} RUSS_{j'} \right) = 0. \tag{3.23}$$

For illustration, let us consider the example $N = 1, d = 1, R = 3$ and $K = 2$. In this example, the relay space is a 3-dimensional space. Let $S_{\text{r}j}$ denote the subspace spanned by the signals received from node j. In the considered example, each node transmits $d = 1$ data stream and hence, the signals from node j spans an one-dimensional subspace $S_{\text{r}j}$. Let S_j denote the subspace spanned by the signals from the node pair (j, k). In Figure 3.5, the subspaces S_1 and S_2 are shown by the blue colored planes. The grey colored plane denotes the $RUSS$ and the line perpendicular to $RISS$ denotes the $RISS$. It can be observed that the signals received from the node pairs do not

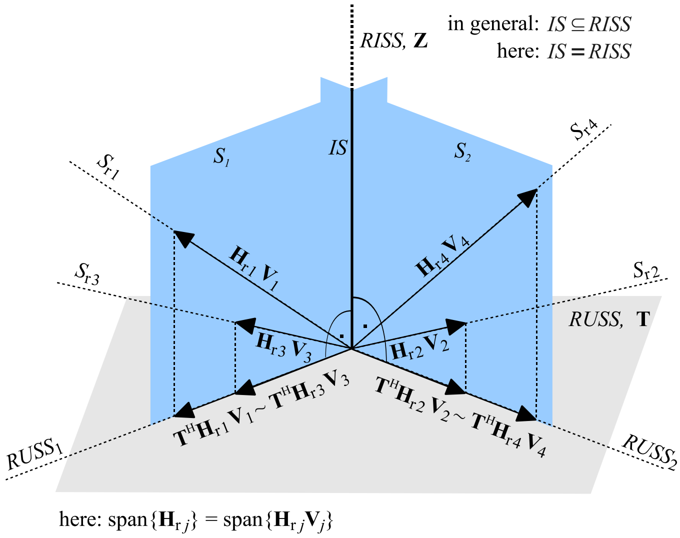

Figure 3.5. Illustration of PSA for $N = 1$, $d = 1$, $R = 3$, and $K = 2$

align pair-wise in the 3-dimensional relay space. However, projection of the received signals on to $RUSS$ results in SA.

3.3.2.2 BC phase: Partial channel alignment + transceive zero forcing

In this section, the BC phase of the two-relaying is considered. After PSA followed by nullifying the interferences in the $RISS$, there are only Kd effective data streams corresponding to the pair-wise aligned signals of the K node pairs. In the following, first the concept of CA is generalized into a new concept called partial channel alignment (PCA). Then the equation representing PCA are introduced. Finally, the transceive ZF performed by the relay is described.

In this paragraph, the concept of PCA is introduced. Similar to PSA in the MAC phase, now, the transmit signal space of the relay is divided into two orthogonal subspaces: relay transmit useful subspace $TUSS$ and relay transmit interference subspace $TISS$. The Kd-dimensional subspace $TUSS$ is chosen such that CA is possible in the $TUSS$. However, the components within the $TISS$ do not need to align pair-wise.

In this paragraph, the equations representing PCA are introduced. Since, the effective channels are not aligned in $TISS$, the signals from the relay are transmitted orthogonal to $TISS$. Let $\mathbf{Q}$ denote the projection matrix that projects the transmit signal at the relay to $TUSS$. Then PCA in $TUSS$ is given by

$$\mathrm{span}\left(\left(\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_j^{\mathrm{rd}}\mathbf{Q}\right)^{\mathrm{H}}\right) = \mathrm{span}\left(\left(\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rd}}\mathbf{Q}\right)^{\mathrm{H}}\right) \tag{3.24}$$

Let $TUSS_j = \text{span}\left(\left(\mathbf{U}_j^{\text{H}}\mathbf{H}_j^{\text{rd}}\mathbf{Q}\right)^{\text{H}}\right)$ denote the alignment subspace of node pair (j, k). In order to guarantee that a ZF filter can be designed for each of the node pairs, the alignment subspace $TUSS_j$ of each of the node pair should be disjoint from the union of the useful subspaces of all the other pairs. This is given by

$$TUSS_j \bigcap \left(\bigcup_{\substack{j'=1 \\ j' \neq j}}^{K} TUSS_{j'} \right) = 0. \tag{3.25}$$

In the following, transceive ZF is explained. After PCA followed by a projection onto the subspace orthogonal to the $TISS$, the effective channel of each node spans the same subspace as the effective channel of its communication partner. Hence, ZF the effective channel of one node by the relay forces also the one of its communication partner to zero. In the BC phase, there are Kd effective data streams and there are Kd effective channels. As the $RUSS$ and the $TUSS$ are of dimension Kd, transceive ZF can be performed at the relay. Let $\mathbf{G}_{\text{s}}$ denote the transceive ZF matrix at the relay. Then the relay processing matrix $\mathbf{G}$ is given by

$$\mathbf{G} = \mathbf{Q}\mathbf{G}_{\text{s}}\mathbf{T}^{\text{H}}. \tag{3.26}$$

After PSA, PCA, and transceive ZF, there will be no unknown interference at the receivers and the d data streams of the desired signal will be linearly independent of each other, i.e., (2.15) and (2.16) are satisfied.

3.3.3 Properness condition

In this section, the properness condition for the proposed IA scheme is derived. It can be seen from (3.22), (3.23), (3.24) and (3.25), that both PSA and PCA are identical problems with the MAC channels, the projection matrix $\mathbf{T}$, and the transmit filters replaced with the corresponding BC channels, the projection matrix $\mathbf{Q}$, and the receive filters. Both are bilinear problems with the same number of variables and equations. Hence, for the consideration of the properness condition, only PSA will be considered.

Note that properness is not a sufficient condition for the feasibility of the system and there exist proper systems that are not feasible [WGJ14]. However, the intuition is that proper systems are likely to be feasible [YGJK09]. In [YGJK09], Bernstein's Theorem is used to verify if the proper systems are feasible by calculating the mixed volume of the polynomials. However, the constraint of (3.22) for the PSA problem is a system of polynomial equations with correlated coefficients and hence, applying Bernstein's Theorem, the mixed volume of the polynomials gives only an upper bound on the number of solutions [YGJK09]. In Section 3.3.5, through simulation results on the leakage interference at the receivers it will be shown that in our considered multi-user two-way relay networks, typically, proper systems are also feasible.

In the following, we count the number M_{v} of variables in the system. There are two kinds of variables in the system. M_{vn} denotes the number of variables corresponding to the antennas at the nodes and M_{vr} denotes the number of variables corresponding to the antennas at the relay.

In this paragraph, we count the number M_{vn} of variables corresponding to the antennas at the nodes. Each node has N antennas and transmits d data streams. The transmit filter matrix at each node is of size $N \times d$. Hence, Nd variables are available in each transmit filter matrix. In order to be able to decode the d data streams, it is necessary that d columns of the transmit filter matrix are linearly independent of each other. d^2 variables are required to make the columns of the transmit filter matrix linearly independent. Hence, $(N-d)d$ variables remain in each of the transmit filters. In other words, choosing a d-dimensional subspace in an N-dimensional space results in $(N-d)d$ variables. There are $2K$ nodes in the system, which leads to

$$M_{\mathrm{vn}} = 2K(N-d)d. \tag{3.27}$$

In this paragraph, we count the number M_{vr} of variables corresponding to the antennas at the relay. In the R-dimensional relay receive space, a Kd-dimensional subspace has to be reserved for the useful signals. Hence, $R - Kd$ dimensions are left for the $RISS$. Choosing a subspace of dimension $R - Kd$ in an R-dimensional space results in $(R - Kd)(R - (R - Kd)) = (R - Kd)Kd$ variables. Given the $RISS$, the $RUSS$ is uniquely defined. Each of the communication pairs can choose its d-dimensional useful signal subspace within this Kd-dimensional $RUSS$. Choosing a d-dimensional subspace in a Kd-dimensional subspace results in $(Kd - d)d$ variables. There are K pairs, hence, $K(Kd-d)d$ free variables in choosing the subspaces $RUSS_j$ within $RUSS$. The $RUSS_j$ of the node pair (j, k) should be disjoint of the subspace spanned by the union of the useful subspaces of all the other node pairs. It will be described in Section 3.3.4 that the choice of $RUSS_j$ depends on the channel matrices corresponding to the pair (j, k). As the channel matrices of the pair (j, k) are assumed to be independent of those of all the other pairs, the probability of two pairs choosing the same $RUSS_j$ is zero. Therefore, M_{vr} is given by

$$M_{\mathrm{vr}} = (R - Kd)Kd + K(K-1)d^2. \tag{3.28}$$

With this, the total number M_{v} of variables is given by

$$M_{\mathrm{v}} = M_{\mathrm{vn}} + M_{\mathrm{vr}} = 2K(N-d)d + (R-Kd)Kd + K(K-1)d^2. \tag{3.29}$$

In this paragraph, the number M_{c} of constraints involved in PSA is counted. The constraints consider that the data streams transmitted by each node pair shall be within its useful subspace at the relay or within the common interference subspace, but not in the useful subspace of the other pairs as described in Section 3.3.2. Now, consider one of the d data streams transmitted by node j. $RISS$ is of dimension $R-Kd$ and $RUSS_j$ is of dimension d. Hence, the considered data stream from node j should be within the $(R-Kd)+d$-dimensional subspace formed by $RISS$ and $RUSS_j$. This introduces $R-((R-Kd)+d) = (K-1)d$ constraints in the systems. There are d data streams per node and $2K$ nodes in the system. Hence, the number of constraints in the system is given by

$$M_{\mathrm{c}} = 2K(K-1)d^2. \tag{3.30}$$

In the following, the properness condition is obtained. For a proper system, the number of variables should be greater than or equal to the number of constraints in the system i.e., $M_{\mathrm{v}} \geq M_c$. This leads to

$$2(N-d) + R + d \geq 2Kd. \tag{3.31}$$

Remark: The properness condition in (3.31) implies that in a proper system, when two antennas are added to the relay, one antenna can be removed from each of the $2K$ nodes and the system will still be proper. The feasibility conditions derived in [LDLG11] and in Section 3.2 are special cases of (3.31). In case of $N = d$, (3.31) becomes $R \geq (2K-1)d$, which is the feasibility condition for pair-aware transceive ZF [LDLG11]. In case of $R = Kd$, (3.31) becomes $N \geq \frac{(K+1)d}{2}$, which is the feasibility condition derived in Section 3.5 for SA.

3.3.4 Algorithms

3.3.4.1 Partial signal alignment algorithms

3.3.4.1.1 Introduction In this section, an iterative algorithm and a closed form solution to obtain PSA are proposed. In order to achieve PSA, the matrices $\mathbf{T}$, $\mathbf{V}_j$, and $\mathbf{V}_k$ have to be chosen such that (3.22) is satisfied. First, $\mathbf{T}$ is chosen such that SA given by (3.22) is feasible. Finding $\mathbf{T}$ is a bilinear problem. An iterative algorithm is proposed to find the solution. If the relays and the nodes have certain numbers of antennas higher than the minimum required numbers of antennas given by (3.31), then the problem of finding $\mathbf{T}$ can be reformulated into a linear problem and, hence, a closed form solution is possible. Once $\mathbf{T}$ is known, the precoding matrices $\mathbf{V}_j$ and $\mathbf{V}_k$ can be calculated in closed form.

In Section 3.3.4.1.2, the problem of PSA is reformulated so that the calculation of $\mathbf{T}$ becomes a problem of finding the subspace $RISS$ intersecting with several subspaces. An iterative algorithm and a closed form solution to find the $RISS$ are proposed in Sections 3.3.4.1.3 and 3.3.4.1.4, respectively, where the condition for the applicability of the closed form solution is also derived. After finding the $RISS$ and hence $\mathbf{T}$, the precoding matrices of all nodes are obtained in Section 3.3.4.1.5.

3.3.4.1.2 Reformulation of partial signal alignment In this section, we reformulate the PSA problem (3.22). Note that $RISS$ uniquely determines $RUSS$ and, hence, uniquely determines $\mathbf{T}$. Therefore, our objective is to choose $RISS$ in such a way that SA given by (3.22) is feasible. For the communication partners (j, k), let $S_{\mathrm{r}j}$ and $S_{\mathrm{r}k}$ represent two subspaces spanned by the columns of the channel matrices $\mathbf{H}_j^{\mathrm{sr}}$ and $\mathbf{H}_k^{\mathrm{sr}}$, respectively. Let $S_j = S_{\mathrm{r}j} \cup S_{\mathrm{r}k}$. In our example in Figure 3.5, we have $K = 2$ node pairs with $N = 1$ and their corresponding subspaces S_1 and S_2 are shown by the two vertical planes. In general, S_j for $j = 1, 2, \ldots, K$ are $2N$-dimensional subspaces and the signals of the node pair (j, k) span a $2d$-dimensional subspace in S_j. The $RUSS_j$ corresponding to this node pair is of dimension d. Hence, to make sure that the signals from this node pair do not interfere with the signals from the other node pairs, d dimensions of the subspace corresponding to the signal received from this node pair should be within the $RUSS_j$ and the other d dimensions should be within the $RISS$. This means that the $RISS$ has to be chosen such that the intersection subspace between S_j and $RISS$ is at least d-dimensional. This needs to hold for each pair (j, k). In Figure 3.5, the dimension of $RISS$ is $R - Kd = d = 1$ and, hence, $RISS$ is directly obtained as the intersection subspace between S_1 and S_2. However, in general $RISS$ is of dimension $R - Kd \geq d$ and the subspaces S_j for $j = 1, 2, \ldots, K$ do not need to intersect with each other. Only $RISS$ needs to intersect with each of the K subspaces S_j. Thus, the problem of determining $\mathbf{T}$ that makes (3.22) feasible is reformulated as the problem of finding $RISS$ that has at least a d-dimensional intersection subspace with each of the subspaces S_j for $j = 1, 2, \ldots, K$.

3.3.4.1.3 Relay Receive Interference Subspace: Iterative algorithm In this section, an iterative algorithm to find RISS is described. First, some terminologies are introduced in the following. Then the basic idea of the iterative algorithm is described. Following this, the iterative algorithm is described. Finally, the convergence of the iterative algorithm is discussed.

Let $RISS_j$ denote the d-dimensional intersection subspace between $RISS$ and S_j for $j = 1, 2, \ldots, K$. Thus, $RISS_j$ is a d-dimensional subspace of both $RISS$ and S_j, i.e., $RISS_j \subseteq RISS$ and $RISS_j \subseteq S_j$. Furthermore, we term the square of the Frobenius norm of the projection of the orthonormal basis vectors of a subspace A on a subspace B as similarity measure of (A, B). This similarity measure of (A, B) is inversely related to the minimum principal angle between the subspaces A and B. Assume that the dimension of A is smaller than or equal to that of B. $A \subseteq B$, if and only if the similarity measure takes its maximum value, which is equal to the dimension of A [Str03]. In this case, the minimum principal angle between A and B is zero.

In the following, the basic idea of the iterative algorithm for finding $RISS$ is explained. The subspace $RISS_j$ is chosen as a subspace of S_j. Our objective is to iteratively find $RISS$ and $RISS_j \subseteq S_j$ for $j = 1, 2 \ldots, K$ such that at the end of the iterations $RISS_j$ is a subspace of both S_j and $RISS$. This means at the end of the iterations the minimum principal angle between $RISS$ and $RISS_j$ should be zero and hence, the similarity measure $(RISS_j, RISS)$ should be maximum. Since, $RISS_j$ is of dimension d, the maximum value of the similarity measure is d. Initially, we arbitrarily choose an $(R - Kd)$-dimensional subspace $RISS^{(0)}$. Then, in iteration step m, first we find a number K of d-dimensional subspaces $RISS_j^{(m)} \subseteq S_j$, for $j = 1, 2, \ldots, K$, such that the K similarity measures of $(RISS_j^{(m)}, RISS^{(m-1)})$ for $j = 1, 2, \ldots, K$ are maximized. Since, the determination of $RISS_j$ for each of the K similarity measures is independent of each other, maximizing each of the K similarity measures is equivalent to maximizing the sum of the K similarity measures. Note that $RISS_j^{(m)}$ for $j = 1, 2, \ldots, K$ is chosen as a d-dimensional subspace of S_j, but in general is not yet a d-dimensional subspace of $RISS^{(m-1)}$, so all or some of the K similarity measures will be smaller than d before convergence of the iterative algorithm. Secondly, in iteration step m, we find a new $(R - Kd)$-dimensional subspace $RISS^{(m)}$ such that the sum of the K similarity measures of $(RISS_j^{(m)}, RISS^{(m)})$ for $j = 1, 2, \ldots, K$ is maximized. These two operations are repeated iteratively. As in each iteration, the sum of the K similarity measures is maximized, the subspaces $RISS_j^{(m)}$ $j = 1, 2, \ldots, K$ and $RISS^{(m)}$ will move in such directions that for increasing m, all $RISS_j^{(m)}$, $j = 1, 2, \ldots, K$ will finally be d-dimensional subspaces of $RISS^{(m)}$ and the sum of the similarity measures will converge to the value Kd.

In the following, the mathematical description of the iterative algorithm is given. Let $\mathbf{S}_j$ denote a unitary matrix of size $R \times 2N$ whose columns are a basis of S_j. Since

$RISS_j^{(m)} \subseteq S_j$, there exists a unitary matrix $\mathbf{X}_j^{(m)}$ of size $2N \times d$ and rank d, such that the columns of the product $\mathbf{S}_j\mathbf{X}_j^{(m)}$ give a basis of $RISS_j^{(m)}$. Let the columns of the unitary matrix $\mathbf{Z}^{(m)}$ of size $R \times (R - Kd)$ denote a basis of $RISS^{(m)}$. Initially, $\mathbf{Z}^{(0)}$ is chosen to be an arbitrary $(R - Kd)$-dimensional subspace. In the m^{th} iteration step, the sum of the squares of the Frobenius norms of the projection of $\mathbf{S}_j\mathbf{X}_j^{(m)}$ on $\mathbf{Z}^{(m-1)}$ is denoted by

$$p^{(m)} = \sum_{j=1}^{K} \text{trace}\left(\mathbf{X}_j^{(m)\text{H}}\mathbf{S}_j^{\text{H}}\mathbf{Z}^{(m-1)}\mathbf{Z}^{(m-1)\text{H}}\mathbf{S}_j\mathbf{X}_j^{(m)}\right). \quad (3.32)$$

For $\mathbf{Z}^{(m-1)}$ determined in the $(m-1)^{\text{th}}$ iteration step, $\mathbf{X}_j^{(m)}$ which maximizes (3.32) is given by

$$\mathbf{X}_j^{(m)} = \lambda_{\text{max},d}\left(\mathbf{S}_j^{\text{H}}\mathbf{Z}^{(m-1)}\mathbf{Z}^{(m-1)\text{H}}\mathbf{S}_j\right), \quad (3.33)$$

where $\lambda_{\text{max},d}(.)$ represents the matrix containing as its columns the eigenvectors corresponding to the first d largest eigenvalues of the matrix within the brackets [Str03]. Using the identity trace($\mathbf{AB}$) = trace($\mathbf{BA}$), the sum of the K similarity measures of $(RISS_j^{(m)}, RISS^{(m)})$ for $j = 1, 2, \ldots, K$ can be written as

$$p^{(m)} = \text{trace}\left(\mathbf{Z}^{(m)\text{H}}\sum_{j=1}^{K}\left(\mathbf{S}_j\mathbf{X}_j^{(m)}\mathbf{X}_j^{(m)\text{H}}\mathbf{S}_j^{\text{H}}\right)\mathbf{Z}^{(m)}\right). \quad (3.34)$$

Next, for given $\mathbf{X}_j^{(m)}$, the $\mathbf{Z}^{(m)}$ that maximizes (3.34) is given by

$$\mathbf{Z}^{(m)} = \lambda_{\text{max},(R-Kd)}\left(\sum_{j=1}^{K}\mathbf{S}_j\mathbf{X}_j^{(m)}\mathbf{X}_j^{(m)\text{H}}\mathbf{S}_j^{\text{H}}\right) \quad (3.35)$$

[Str03]. (3.33) and (3.35) are repeated iteratively until convergence. Finally, the span of the matrix $\mathbf{Z}$ gives $RISS$. $RISS$ uniquely determines the matrix $\mathbf{T}$ and the columns of the matrix $\mathbf{T}$ are given by a basis for the null space of the subspace spanned by the columns of the matrix $\mathbf{Z}$.

In this paragraph, the convergence of the proposed iterative algorithm is investigated. As $\mathbf{S}_j\mathbf{X}_j$ and $\mathbf{Z}$ are unitary matrices, the sum of the K similarity measures is upper bounded by Kd. In each iteration step, the sum of the K similarity measures is maximized and, hence, the algorithm converges. However, due to the non-concave nature of the problem, convergence to the global maximum cannot be guaranteed. From the simulations, it is observed that iteratively optimizing $\mathbf{X}_j^{(m)}$ and $\mathbf{Z}^{(m)}$ using (3.33) and (3.35), the value of the objective function typically converges to Kd and, hence, the algorithm converges to $RISS$ and $RISS_j$.

3.3.4.1.4 Relay Receive Interference Subspace: Closed form solution In this section, two closed form solutions to find the $RISS$ are proposed and the corresponding conditions for the applicability of the closed form solutions are derived. Finding $RISS$ is a bilinear problem. However, by dividing $RISS$ into smaller subspaces and identifying these subspaces independently, one can linearize the problem. As the subspaces of $RISS$ are determined independently, this approach will need more variables than that required to design $RISS$ as a single subspace by the iterative algorithm proposed in Section 3.3.4.1.3.

The basic idea of the closed form solution is as follows: $RISS$ is of dimension $R-Kd \geq d$. For simplicity of the following notation, assume $R-Kd$ is an integer multiple of d, say $R-Kd = nd$, $n \in \mathbb{N}$. Then $RISS$ can be represented as a union of n subspaces $RISS_{\tau}^{(d)}$ for $\tau = 1, \ldots, n$ each of dimension d as follows

$$RISS = \bigcup_{\tau=1}^{n} RISS_{\tau}^{(d)}, \tag{3.36}$$

or the $RISS$ can be represented as a union of d subspaces $RISS_{l}^{(n)}$ for $l = 1, \ldots, d$, each of dimension n as

$$RISS = \bigcup_{l=1}^{d} RISS_{l}^{(n)}. \tag{3.37}$$

In the following, we show that if we design either the n subspace or the d subspaces independently, then we can find a closed form solution. We consider (3.36) in the method 1 and (3.37) in the method 2. The method 1 is more intuitive, but as we will show in the following the method 1 is more constrained than the method 2.

Method 1: In this method 1, $RISS$ is constructed as the union of n subspaces each of dimension d. In the following, first an example is considered for illustration of the basic idea. Then the generalized solution is given. Finally the condition for the applicability of the closed form solution is derived.

For illustration, consider the example in Figure 3.5 with $N = 1, d = 1, R = 3$, and $K = 2$. In this example, $RISS$ is of dimension $R - Kd = d = 1$ and $RISS$ should have a $d = 1$ dimensional intersection with each of the $2N = 2$ dimensional subspaces S_1 and S_2. Hence, $RISS$ can be found directly by determining the intersection of S_1 and S_2.

In the following, the closed form solution is given. For simplicity of the following notation, assume K is an integer multiple of n, say $K = K_0 n, K_0 \in \mathbb{N}$. Our task is to find a $R - Kd = nd$ dimensional $RISS$ that has at least a d-dimensional intersection subspace with each of the K subspaces S_j for $j = 1, 2, \ldots, K$. In the following, we propose one possible approach to achieve this. First we split up the K subspaces S_j for $j = 1, 2, \ldots, K$ into n disjoint groups with $K_0 = \frac{K}{n}$ subspaces in each group. The n disjoint groups can be formed arbitrarily. $RISS_\tau^{(d)}$ is determined as a d-dimensional subspace of the intersection subspace of the K_0 subspaces of group τ for $\tau = 1, 2, \ldots, n$. This is given by

$$RISS_\tau^{(d)} \subseteq S_{(\tau-1)K_0+1} \cap S_{(\tau-1)K_0+2} \cap \cdots \cap S_{(\tau)K_0}. \tag{3.38}$$

Let $\mathbf{RISS}_\tau^{(d)}$ denote a matrix whose columns are a basis of $RISS_\tau^{(d)}$. If a solution for (3.38) is feasible, then there exist matrices $\mathbf{Y}_j$ for $j = 1, 2, \ldots, K$ such that the following

$$\mathbf{RISS}_\tau^{(d)} = \mathbf{S}_{(\tau-1)K_0+1}\mathbf{Y}_{(\tau-1)K_0+1} = \mathbf{S}_{(\tau-1)K_0+2}\mathbf{Y}_{(\tau-1)K_0+2} \quad \ldots \quad \mathbf{S}_{(\tau)K_0}\mathbf{Y}_{(\tau)K_0} \tag{3.39}$$

holds. (3.39) can be written as

$$\underbrace{\begin{pmatrix} \mathbf{S}_{(\tau-1)K_0+1} & \mathbf{S}_{(\tau-1)K_0+2} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & & & & & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{S}_{(\tau-1)K_0+(K_0-1)} & \mathbf{S}_{(\tau)K_0} \end{pmatrix}}_{\mathbf{S}} \underbrace{\begin{pmatrix} \mathbf{Y}_{(\tau-1)K_0+1} \\ \mathbf{Y}_{(\tau-1)K_0+2} \\ \vdots \\ \mathbf{Y}_{(\tau)K_0} \end{pmatrix}}_{\mathbf{Y}} = \mathbf{0}. \tag{3.40}$$

(3.40) is a system of linear homogeneous equations whose solution is given by

$$\mathbf{Y} = \mathrm{NULL}^d(\mathbf{S}) \tag{3.41}$$

where $\mathrm{NULL}^d(.)$ returns a matrix with its columns being a basis for an arbitrary d-dimensional subspace of the null space of the matrix within the brackets. Substituting (3.41) in (3.39), $\mathbf{RISS}_\tau^{(d)}$ can be obtained. The $RISS$ can be found as the union of all $RISS_\tau^{(d)}$ for $\tau = 1, 2, \ldots n$, i.e.,

$$RISS = \bigcup_{\tau=1}^{n} RISS_\tau^{(d)}. \tag{3.42}$$

Since, $RISS_\tau^{(d)}$ is determined as the d-dimensional subspace of the intersection subspace of K_0 subspaces, the resulting $RISS$ has a d-dimensional intersection subspace with each of the K subspaces.

In the following, the condition for the applicability of the closed form solution is derived. The matrix $\mathbf{S}$ is of size $(K_0 - 1)R \times 2NK_0$. Since the elements of the matrix $\mathbf{S}$ are dependent on the coeffcients of the channel matrices, the elements are independent of each other. Therefore, the matrix $\mathbf{S}$ is almost surely full rank. The dimension of the null space of $\mathbf{S}$ is given by $2NK_0-(K_0-1)R$. For a d-dimensional intersection subspace to exist, the null space should be at least of dimension d. Therefore, the closed form solution derived is applicable if the following condition

$$2NK_0 - (K_0 - 1)R \geq d \tag{3.43}$$

holds. For the $RISS$ to have a d-dimensional intersection subspace with each of the K subspaces, (3.43) needs to be true for all the n groups. Hence, multiplying (3.43) on both sides with n leads to the condition

$$2NK - (K - n)R \geq nd. \tag{3.44}$$

Remark: If K is not an integer multiple of n, then the arbitrarily chosen disjoint groups will have a different number of subspaces and not the same number K_0.

Method 2: In this method 2, $RISS$ is constructed as the union of d subspaces, each of dimension n. First, we choose each $RISS_l$ for $l = 1, \dots, d$ such that it has a one dimensional intersection subspace with each of the K subspaces S_{jk}. Then the resulting $RISS$ will have a d dimensional intersection subspace with each of the S_{jk}. The condition that each $RISS_l$ needs to have a one dimensional intersection subspace with each S_{jk} is more strict than the condition that $RISS$ needs to have d dimensional intersection with each S_{jk}. Hence, more variables are needed to obtain a closed form solution than for the iterative solution. In this section, first the closed form solution is introduced. Then, the condition for the applicability of the closed form solution is derived.

Let $\mathbf{s}_{jk}^l$ denote the basis vector of the one-dimensional intersection subspace between S_{jk} and $RISS_l$. Without loss of generality, we assume that $\mathbf{s}_{jk}^l$ is within the subspace spanned by l^{th} data streams of the node pair (j, k). This can be expressed as

$$\mathbf{s}_{jk}^l \in \text{span}\left\{\begin{bmatrix}\mathbf{H}_j^{\text{sr}}\mathbf{v}_j^l & \mathbf{H}_k^{\text{sr}}\mathbf{v}_k^l\end{bmatrix}\right\} \tag{3.45}$$

where $\mathbf{v}_j^l$ is the l^{th} column vector of the transmit filter $\mathbf{V}_j$. Since $\mathbf{v}_j^l$ and $\mathbf{v}_k^l$ are variables in the system, (3.45) can be written as

$$\mathbf{s}_{jk}^l = \mathbf{H}_j^{\text{sr}}\mathbf{v}_j^l + \mathbf{H}_k^{\text{sr}}\mathbf{v}_k^l. \tag{3.46}$$

Let $\chi_l = \{\mathbf{s}_{jk}^l \mid j = 1, 2, \dots, K,\ k = j + K\}$ denote the set of one dimensional intersections of $RISS_l$ with S_{jk} for $j = 1, 2, \dots, K,\ k = j + K$. Now $RISS_l$ can be constructed as

$$RISS_l = \text{span}\left\{\chi_l\right\}. \tag{3.47}$$

However, $RISS_l$ can be of maximum dimension n. Hence, any $n+1$ vectors in χ_l should be within an n dimensional subspace. This means that any $n+1$ vectors in the set χ_l should be linearly dependent of each other. At least $Kd-n$ sets of $n+1$ vectors are necessary to make sure that all the vectors within χ are in a subspace of size n. This is given by

$$\underbrace{\begin{pmatrix} \mathbf{H}_1^{\mathrm{sr}} & \mathbf{H}_{(K+1)}^{\mathrm{sr}} & \cdots & \mathbf{H}_{(n+1)}^{\mathrm{sr}} & \mathbf{H}_{(K+n+1)}^{\mathrm{sr}} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & & & & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{H}_{(K-n)}^{\mathrm{sr}} & \mathbf{H}_{(2K-n)}^{\mathrm{sr}} & \cdots & \mathbf{H}_K^{\mathrm{sr}} & \mathbf{H}_{(2K)}^{\mathrm{sr}} \end{pmatrix}}_{\mathbf{H}} \begin{pmatrix} \mathbf{v}_1^l \\ \mathbf{v}_{K+1}^l \\ \vdots \\ \mathbf{v}_K^l \\ \mathbf{v}_{2K}^l \end{pmatrix} = \mathbf{0}. \tag{3.48}$$

Note that in (3.48), the matrix $\mathbf{H}$ is independent of the data stream index l. Hence, the equations corresponding to $l = 1, 2, \ldots, d$ can be concatenated as

$$\underbrace{\begin{pmatrix} \mathbf{H}_1^{\mathrm{sr}} & \mathbf{H}_{(K+1)}^{\mathrm{sr}} & \cdots & \mathbf{H}_{(n+1)}^{\mathrm{sr}} & \mathbf{H}_{(K+n+1)}^{\mathrm{sr}} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & & & & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{H}_{(K-n)}^{\mathrm{sr}} & \mathbf{H}_{(2K-n)}^{\mathrm{sr}} & \cdots & \mathbf{H}_K^{\mathrm{sr}} & \mathbf{H}_{(2K)}^{\mathrm{sr}} \end{pmatrix}}_{\mathbf{H}} \begin{pmatrix} \mathbf{V}_1 \\ \mathbf{V}_{K+1} \\ \vdots \\ \mathbf{V}_K \\ \mathbf{V}_{2K} \end{pmatrix} = \mathbf{0}. \tag{3.49}$$

Let

$$\begin{pmatrix} \mathbf{A}_1^{\mathrm{T}} & \mathbf{A}_{K+j}^{\mathrm{T}} & \cdots & \mathbf{A}_K^{\mathrm{T}} & \mathbf{A}_{2K}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}} = \mathrm{NULL}\left(\mathbf{H}\right) \tag{3.50}$$

where NULL (.) returns a matrix with its columns being a basis for the null space of the matrix within the brackets. Let $\mathbf{W}_j^{(\mathrm{v})}$ be an arbitrary matrix with d columns and rank d. Then the transmit filters are given by

$$\mathbf{V}_j = \mathbf{A}_j \mathbf{W}^{(\mathrm{v})}. \tag{3.51}$$

There are several possibilities to choose such a matrix $\mathbf{W}_j^{(\mathrm{v})}$ of rank d. One can choose a $\mathbf{W}_j^{(\mathrm{v})}$ that maximizes a given objective, for instance the SNR. Once $\mathbf{V}_j$ is determined, $RISS_l$ and, hence, $RISS$ can be obtained from (3.47) and (3.37), respectively.

In the following, the condition for the applicability of the closed form solution is derived. (3.48) is a system of homogenous linear equations with at least one non-trivial solution if the number of variables is greater than the number of equations given by

$$2KN \geq (K-n)R + 1. \tag{3.52}$$

We have such a constraint for each of the subspaces $RISS_l$ for $l = 1, \ldots, d$ as given in (3.49). Hence, the solution space of (3.48) should be of dimension of at least d. This results in the following condition:

$$2KN \geq (K-n)R + d. \tag{3.53}$$

Remark 1: In the closed form solution described above, $RISS$ is obtained as a union of d subspaces $RISS_l$ for $l = 1, \ldots, d$ and $RISS_l$ is obtained as the span of χ_l which is constrained to be in an n dimensional subspace. In general, $RISS$ can be obtained as the subspace spanned by all the Kd vectors $\mathbf{s}_{jk}^l$ for $j = 1, \ldots, K$ and $k = j + K$ and $l = 1, \ldots, d$ which are constrained to be in an $R - Kd$ dimensional subspace. However, in this case, one additionally needs to make sure that the resulting transmit filters are of rank d.

Remark 2: It should be noted that if $R - Kd$ is not an integer multiple of d, then in the above closed form solution, $R - Kd - nd$ dimensions of $RISS$ are not utilized. Hence, for some cases it might be better to reduce d and increase K so that $RISS$ is fully utilized. In this case, more degrees of freedom can be achieved compared to the case where $R - Kd$ is not an integer multiple of d.

Remark 3: Note that (3.53) and (3.44) are more strict than (3.31) and hence, the closed form solution is possible only if the nodes and/or the relays have more antennas than required by the properness condition (3.31). Also, from (3.53) and (3.44), we see that method 1 is more constrained than method 2.

Remark 4: For both the methods, note that for the case $R - Kd = d$, there is only a single group, i.e., $n = 1$. In this case, both (3.44) and the properness condition of (3.31) yield the same result, $2N + d \geq R$. Therefore, for the case $R - Kd = d$, the closed form solution is possible whenever the system is proper. In this case, the properness condition is also a sufficient condition.

3.3.4.1.5 Transmit filters In this section, the transmit filters $\mathbf{V}_j$ and $\mathbf{V}_{j+K}$ are obtained in closed form. In Sections 3.3.4.1.3 and 3.3.4.1.4, the $RISS$ has been determined using the iterative algorithm and in a closed form, respectively. Note that in Section 3.3.4.1, while determining the $RISS$ in method 2 for the closed form solution, the transmit filters are already given by (3.51). Hence, the method described in the following is only needed for the $RISS$ determined through the iterative algorithm or the closed form solution from method 1.

Consider the node pair (j, k). The $RISS$ calculated in the subsections 3.3.4.1.3 and 3.3.4.1.4 has an at least d-dimensional intersection subspace with the subspace S_j. Let $RISS_j^{(d)}$ denote an arbitrary d-dimensional subspace in the intersection subspace $RISS_j$. Recollect from Section 3.3.4.1.2 that $S_j = S_{\mathrm{r}j} \cup S_{\mathrm{r}k}$. In order to guarantee that the signal from a node pair (j, k) is either within the $RUSS_j$ or within the $RISS_j$, we need to choose a d-dimensional subspace from $S_{\mathrm{r}j}$ and a d-dimensional subspace from $S_{\mathrm{r}k}$ such that the corresponding $2d$-dimensional subspace in S_j includes the d-dimensional intersection subspace $RISS_j^{(d)}$.

Let the columns of the matrix $\mathbf{Z}_j$ of size $R \times d$ span the intersection subspace $RISS_j^{(d)} \subseteq (RISS \cap S_j)$. The columns of the matrices $\mathbf{H}_j^{\mathrm{sr}}\mathbf{V}_j$ and $\mathbf{H}_k^{\mathrm{sr}}\mathbf{V}_k$ span the d-dimensional subspaces chosen from $S_{\mathrm{r}j}$ and $S_{\mathrm{r}k}$, respectively. Then the subspace spanned by the columns of $\left[\mathbf{H}_j^{\mathrm{sr}}\mathbf{V}_j \;\; \mathbf{H}_k^{\mathrm{sr}}\mathbf{V}_k\right]$ has to contain $\mathbf{Z}_j$. This can be written as

$$\mathbf{Z}_j = \mathbf{H}_j^{\mathrm{sr}}\mathbf{V}_j + \mathbf{H}_k^{\mathrm{sr}}\mathbf{V}_k. \tag{3.54}$$

Since by design, ${RISS_j}^{(d)} \subseteq (S_{\mathrm{r}j} \cup S_{\mathrm{r}k})$, the above equation has a unique solution for a given ${RISS_j}^{(d)}$ and is given by

$$\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix} = \begin{bmatrix}\mathbf{H}_j^{\mathrm{sr}} & -\mathbf{H}_k^{\mathrm{sr}}\end{bmatrix}^{-1} \mathbf{Z}_j. \tag{3.55}$$

Chosing a different $RISS_j^{(d)} \subseteq RISS_j$ will result in another solution. Let the columns of the matrix $\widetilde{\mathbf{Z}}_j$ denote a basis for the subspace $RISS_j$. Let $\mathbf{W}_j^{(\mathrm{v})}$ be an arbitrary matrix with d columns and rank d. Then $\mathbf{Z}_j = \widetilde{\mathbf{Z}}_j\mathbf{W}_j^{(\mathrm{v})}$. The transmit filters are given by

$$\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix} = \begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix} \mathbf{W}_j^{(\mathrm{v})} \tag{3.56}$$

where

$$\begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix} = \begin{bmatrix}\mathbf{H}_j^{\mathrm{sr}} & -\mathbf{H}_k^{\mathrm{sr}}\end{bmatrix}^{-1} \widetilde{\mathbf{Z}}_j. \tag{3.57}$$

There are several possibilities to choose such a matrix $\mathbf{W}_j^{(\mathrm{v})}$ of rank d. One can choose a $\mathbf{W}_j^{(\mathrm{v})}$ that maximizes a given objective, for instance the SNR.

3.3.4.2 Partial channel alignment algorithms

Since, PSA and PCA are identical problems, the algorithms proposed for PSA can be directly applied for PCA by replacing the matrix $\mathbf{T}$ by the matrix $\mathbf{Q}$, the transmit filters by the receive filters and the MAC channel matrices by the corresponding BC channel matrices.

3.3.4.3 Transceive zero forcing

In this section, the relay filter is designed. The transceive ZF performed by the relay for the case $R \geq Kd$ is same as that for the case $R = Kd$, except that the transceive ZF is performed in a Kd-dimensional subspace at the relay and the projection matrices $\mathbf{T}$ and $\mathbf{Q}$ need to be taken into account. We explain this briefly for completeness. After PSA and PCA, in a Kd dimensional $RUSS$ and $TUSS$, there are Kd effective data streams and Kd effective channels. The relay performs receive ZF to spatially separate the Kd effective data streams and performs transmit ZF to spatially orthogonolize the signals transmitted through the Kd effective channels. Let $\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}$ denote the receive ZF matrix. Then $\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}$ is given by

$$\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}} = \begin{bmatrix} \mathbf{T}^{\mathrm{H}}\mathbf{H}_1^{\mathrm{sr}}\mathbf{V}_1 & \dots & \mathbf{T}^{\mathrm{H}}\mathbf{H}_K^{\mathrm{sr}}\mathbf{V}_K \end{bmatrix}^{-1}. \tag{3.58}$$

Let $\mathbf{G}_{\mathrm{tx}}$ denote the transmit ZF matrix. Then $\mathbf{G}_{\mathrm{tx}}$ is given by

$$\mathbf{G}_{\mathrm{tx}} = \begin{bmatrix} \mathbf{U}_1^{\mathrm{H}}\mathbf{H}_1^{\mathrm{rd}}\mathbf{Q} \\ \vdots \\ \mathbf{U}_K^{\mathrm{H}}\mathbf{H}_K^{\mathrm{rd}}\mathbf{Q} \end{bmatrix}^{-1}. \tag{3.59}$$

The relay filter is given by

$$\mathbf{G} = \alpha_{\mathrm{r}}\mathbf{Q}\mathbf{G}_{\mathrm{tx}}\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}. \tag{3.60}$$

Here, α_{r} is a scalar used to satisfy the relay power constraint given by (2.8). α_{r} is given by (3.13). Note that the scaling factor α_{r} does not affect the IA conditions.

3.3.5 Performance analysis

3.3.5.1 Introduction

In this section, the performances of the proposed IA algorithms are compared with those of the reference ZF algorithm and the convergence of the iterative algorithm is investigated. The achievable DoF and the sum rate are used to evaluate the performance of the algorithms. The reference algorithm introduced in Section 3.2.5.2 is used also in this section. In the following, first different simulation scenarios are introduced and the DoF achieved by the proposed IA scheme and the reference scheme in each of the scenarios are investigated in Section 3.3.5.2. Following this, the sum rates achieved by these schemes are obtained through MATLAB simulations in Section 3.3.5.3. The same assumptions as in Section 3.2.5.4 about the channel model, number of node pairs, power allocation and channel realizations are assumed in this section. Finally, the convergence of the iterative algorithm is investigated in Section 3.3.5.4

3.3.5.2 Degrees of freedom analysis

In this section, the DoF of the proposed IA algorithm are investigated and compared with the DoF of the reference algorithm. In the following, first using the properness condition (3.31), it is shown that the proposed IA algorithm based on PSA and PCA is a generalization of the IA algorithm based on SA and CA proposed in Section 3.2. Secondly, the scenarios considered for the investigation of the DoF are introduced and the DoF are investigated. Furthermore, among the three proposed algorithms the one which can be used to determine the IA solution is also identified for each of the scenarios.

The properness condition for the case $R \geq Kd$ considered in this section is given by (3.31). For the special case of $R = Kd$, (3.31) becomes $2N \leq Kd + d$, which is the properness condition derived in Section 3.2.3. Hence, the proposed IA algorithm based on PSA and PCA is the generalization of the IA algorithm based on SA and CA proposed in Section 3.2.

Table 3.2 shows the scenarios considered for DoF investigations. The scenarios are chosen such that the following facts become clear:

- The additional antennas at the relays can be used to reduce the minimum required number N of antennas at the nodes for a fixed number of DoF.
- For a given scenario, if only the iterative algorithm is applicable, then by reducing the number of simultaneously served node pairs by one, closed form solution shall be obtained.
- The condition given by (3.44) derived for the applicability of the closed form solution of method 1 is more constrained than the condition given by (3.53) for the applicability of the closed form solution of method 2.
- For a given number of DoF, reducing d and K shall result in multiple IA solutions.

In addition to the numbers N, R, and d, the Table 3.2 also shows the number of simultaneously served node pairs and the DoF achieved by the proposed IA algorithms and the reference algorithm. From the Table 3.2, it can be observed, that the DoF achieved by the proposed IA scheme in all the above scenarios are larger than that achieved by the reference ZF algorithm. Also, note that the feasibility conditions derived in [LDLG11] is a special case of the properness condition given by (3.31). For

Table 3.2. Scenarios considered for $R \geq Kd$ and DoF achieved

Scenarios	R	d	N	K		DoF	
				ZF	IA	ZF	IA
B1	5	1	2	3	4	3	4
B2	12	2	5	3	5	6	10
B3	9	1	3	5	7	5	7
B4	9	1	3	5	6	5	6
B5	9	1	2	5	6	5	6
B6	9	1	2	5	5	5	5
B7	12	1	5	3	10	6	10

$N = d$, the properness condition given by (3.31) becomes $R \geq (2K-1)d$, which is the feasibility condition for pair-aware transceive ZF [LDLG11]. Hence, the DoF of the reference algorithm is a special case of that of the proposed IA algorithm.

In this paragraph, the minimum number N of antennas required at the nodes is investigated. Consider the scenario B1 in Table 3.2. Here, $K = 4$. In order to perform SA and CA in an arbitrarily chosen 4-dimensional relay subspace, $N \geq \frac{(K+1)d}{2} = 2.5$ antennas are necessary. However, using the PSA based IA algorithm, only $N = 2$ antennas are necessary. Similarly, in scenario B2, $N = 6$ antennas are necessary to perform SA in an arbitrarily chosen 10-dimensional relay subspace, but for PSA only $N = 5$ antennas are necessary. In both scenarios B1 and B2, the reduction in the minimum required number N of node antennas is due to the fact that the additional antennas at the relay is used to reduce the required number N of node antennas. In general, it can be seen from the properness condition (3.31), that for every two additional antennas at the relay, the required number N of antennas at each of the nodes is reduced by one. Note that scenarios B1 and B2 satisfy (3.44) and hence, closed form solutions can be obtained using Method 1.

In this paragraph, it is shown that for a proper system in which only the iterative solution is applicable, closed form solution shall be obtained by reducing the number of simultaneously served node pairs by one. Consider the scenario B3 given in Table 3.2. This scenario is proper, however, neither condition (3.44) nor condition (3.53) is satisfied. Hence, neither Method 1 or Method 2 to obtain the closed form solution can be applied. For the same number of node and relay antennas as in scenarion B3, in scenario B4 given in Table 3.2 only $K = 6$ node pairs are served. In this case, both the conditions (3.44) and (3.53) are satisfied and hence, both Method 1 and Method 2 can be applied to obtain a closed form solution.

In this paragraph, two scenarios, namely, scenario B5 and scenario B6 are considered to show that the applicability condition (3.44) derived for method 1 is more constrained than (3.53) derived for method 2. Both the scenarios B5 and B6 are proper systems. In B5, only the iterative algorithm could be applied because the conditions (3.44) and (3.53) are not satisfied. However, reducing the number of node pairs to $K = 5$, condition (3.53) is satisfied, but still, condition (3.44) is not satisifed and hence, only Method 2 can be applied to find the closed form solution.

In this paragraph, it is shown that reducing d and increasing K shall result in multiple IA solutions. Consider the scenarios B2 and B7 in Table 3.2. Both the scenarios are proper and a closed form solution can be found using Method 1 or Method 2. In both the scenarios, the relay has $R = 12$ antennas and each of the $2K$ the nodes have $N = 5$ antennas. In scenario B2, there are $K = 5$ nodes and each node transmits $d = 2$ data streams. The properness condition is satisfied with equality sign i.e., a single closed form solution is obtained using the proposed algorithm. In scenario B7, there are $K = 10$ nodes and each node transmits $d = 1$ data streams. The properness condition (3.31) is satisfied with an inequality sign, i.e., multi-dimensional solution space is obtained in closed form using the proposed algorithms. The multiple-dimensional solution space is due to the additional variables obtained through the antennas at the additional 5 node pairs.

3.3.5.3 Sum rate analysis

In this section, the sum rate performances of the proposed IA algorithms are compared with that of the reference algorithm for the scenarios B1, B2, B3, and B4. Let $P_{\text{node}} = P$ denote the power of each of the $2K$ nodes. Let $P_{\text{relay}} = KP$ denote the power available at the relay. The noise power at each node and at the relay is assumed to be the same and is denoted by $\sigma_k^2 = \sigma_{\text{relay}}^2 = \sigma^2$. Let IA_PSA_Itr denote the iterative IA algorithm proposed in Section 3.3.4.1.3. Let IA_PSA_M1 and IA_PSA_M2 denote the closed form solution using method 1 and method 2, respectively, proposed in Section 3.3.4.1.4.

In the following, first the sum rate performances of the IA_PSA_M1 algorithm are compared with the reference ZF algorithm in scenario B1 and scenario B2. Then, the sum rate performance of the IA_PSA_Itr algorithm and that of the IA_PSA_M2 algorithm are compared with the reference ZF algorithm in scenarios B3 and B4, respectively. Finally, the influence of the number of iterations on the sum rate performance of the IA_PSA_Itr algorithm is investigated.

In this paragraph, the sum rate performances of the IA_PSA_M1 algorithm are compared with the reference ZF algorithm in scenario B1 and scenario B2. Figure 3.6 shows the sum rate performance as a function of P/σ^2 for scenario B1 and scenario B2. The black dashed curve and the red solid curve show the sum rate achieved by the reference algorithm and the proposed IA algorithm, respectively. It can be seen that at high SNR, the proposed IA algorithm achieves higher sum rate due to the higher DoF. At low SNR, the reference algorithm has better sum rate. The performance gap at the low SNR region is due to the following: In general, projection of a signal to a subspace results in the reduction in the signal amplitude because the component of the signal orthogonal to the subspace will be nullified. In the reference ZF algorithm, at the relay, the transceive ZF performed in the R-dimensional relay space results in a reduction in the signal amplitude and hence, reduction in the useful signal power. However, for the proposed IA algorithm, in addition to the transceive ZF performed in the Kd-dimensional relay useful subspace, the reduction in amplitude also takes place due to the projection to the $RUSS$ and to the $TUSS$ during the PSA and the PCA, respectively. Furthermore, the multiple antennas at the nodes are used to improve the received signal power in the reference algorithm, while for the proposed IA algorithm, they are used to achieve more number of DoF compared to the reference algorithm.

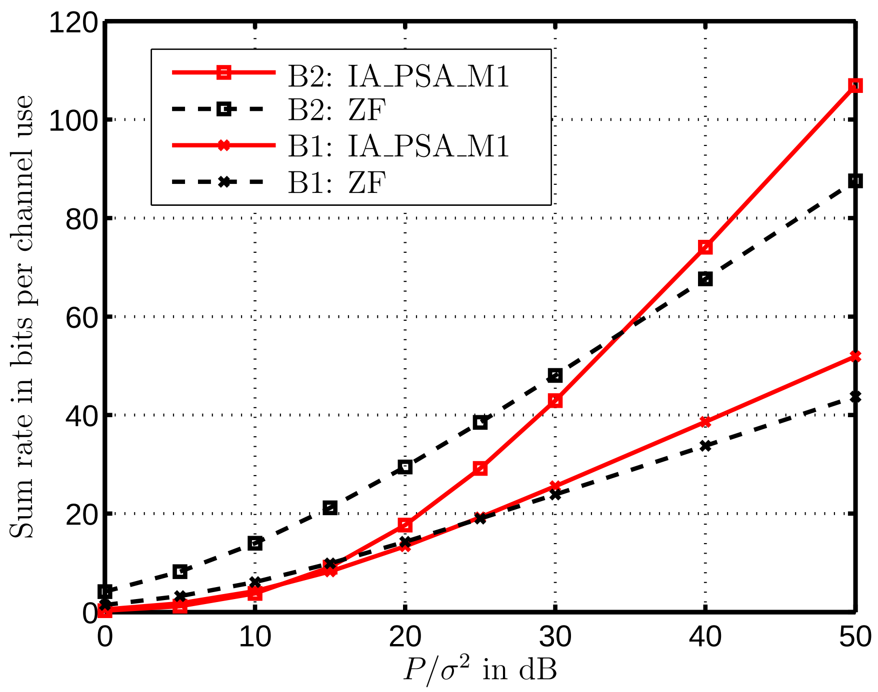

Figure 3.6. Sum rate performance of the IA_PSA_M1 algorithm and the reference ZF algorithm versus P/σ^2 for scenarios B1 and B2

In this paragraph, the sum rate performance of the IA_PSA_Itr algorithm and that of the IA_PSA_M2 algorithm are compared with the reference ZF algorithm in scenarios B3 and B4, respectively. Note that scenarios B1 and B2 satisfy (3.44) and hence, closed form solutions are obtained using Method 1. However, for scenario B3, only iterative algorithm can be applied. 2000 iterations are considered for the simulations. Figure 3.7 shows the sum rate of the proposed IA algorithm and of the reference algorithm. In scenario B4, one node pair less than that in scenario B3 is served simultaneously. Serving one node pair lesser results in less constraints which make it possible to obtain a closed form solution. However, as seen in Figure 3.7, the slope of the curve and the sum rate performance in scenario B4 are less than that in scenario B3. Furthermore, similar to scenarios B1 and B3, also in scenarios B3 and B4, it can be seen that at high SNR, the proposed IA algorithm achieves higher sum rate due to the higher DoF. At low SNR, the reference algorithm has better sum rate performance.

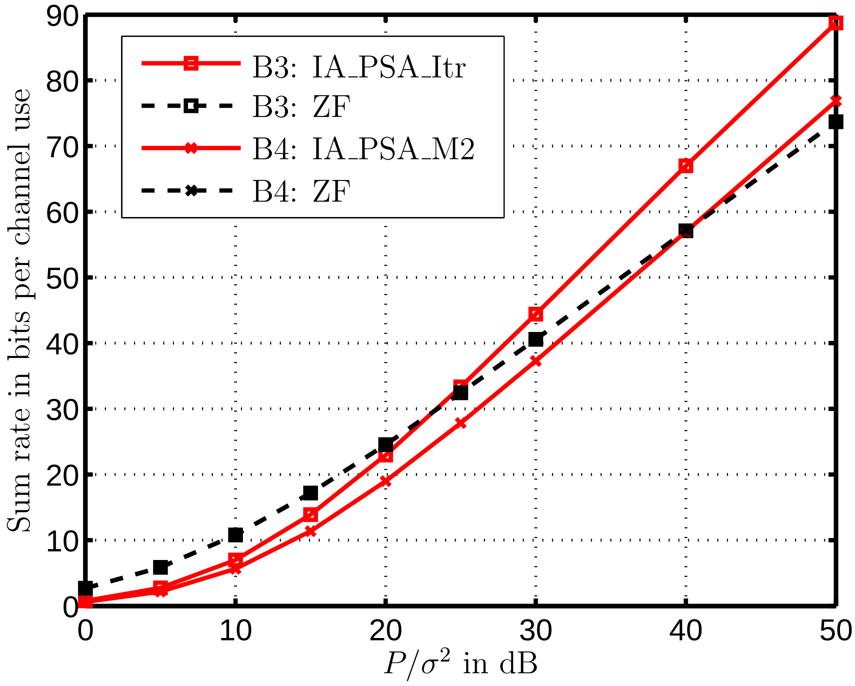

Figure 3.7. Sum rate performance of the IA_PSA_Itr, the IA_PSA_M2 and the reference ZF algorithms versus P/σ^2 for scenarios B3 and B4

In this paragraph, the influence of the number of iterations on the sum rate performance of the IA_PSA_Itr algorithm is investigated. In the following, first a measure of accuracy of the alignment of the signals within the $RUSS$ is introduced. Then sum rate performance of IA_PSA_Itr algorithm for different number of iterations is investigated. Recollect that in the iterative algorithm, the similarity measure between the $RISS_j$ and $RISS$ is maximized and the similarity measure is upper bounded by d. Let leakage measure is defined as the difference between d and the similarity measure, this means, the leakage measure is reduced in every iteration in the iterative algorithm. The value of leakage measure at the end of the iterations gives a measure of accuracy of the alignment of the signals within the $RUSS$ and is dependent on the number of iterations considered in the algorithm. Figure 3.8 shows the sum rate performance of IA_PSA_Itr algorithm for different number of iterations. In general, the leakage measure reduces with the number of iterations. At high SNR, the noise is almost zero and the sum rate is defined by the interference. Therefore at high SNR, increasing the number of iterations reduces the leakage measure which reduces the interferences and hence, increases the sum rate. However, after certain number of iterations, the leakage measure is almost zero and hence, the sum rate does not change. In contrast to high SNR, at low SNR, the noise plays a significant role compared to the interference signals. Hence, as soon as the leakage measure is below the noise level, increasing the number of iterations does not change the sum rate. Hence, in Figure 3.8, at low SNR, the sum rate performance is almost the same for different numbers of iterations.

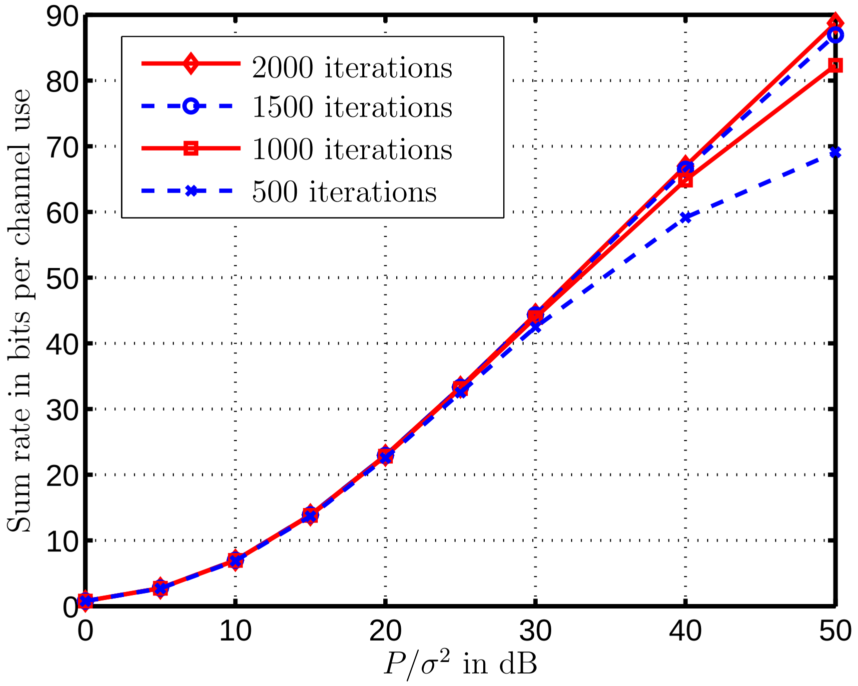

Figure 3.8. Sum rate comparison with different number of iterations for scenario B3

3.3.5.4 Convergence analysis

In this section, the convergence of the iterative algorithm is investigated. Since the optimization problem is non-convex, convergence to a global optimum cannot be guaranteed. However, after each iteration the leakage measure is minimized and it is lower bounded by zero and hence, the proposed iterative algorithm IA_PSA_Itr is guaranteed to converge at least to a local minimum. In the following, first the convergence of IA_PSA_Itr is investigated in scenario B3. Then it is shown that increasing the number of variables in the system either by increasing R or by increasing N, the IA_PSA_Itr converges faster.

Figure 3.9 shows the leakage measure defined in Section 3.3.5.3 versus the iteration number for 10 arbitrarily chosen channel realizations. It can be seen that the leakage measure is reduced after each iteration and hence, converges at least to a local optimum.

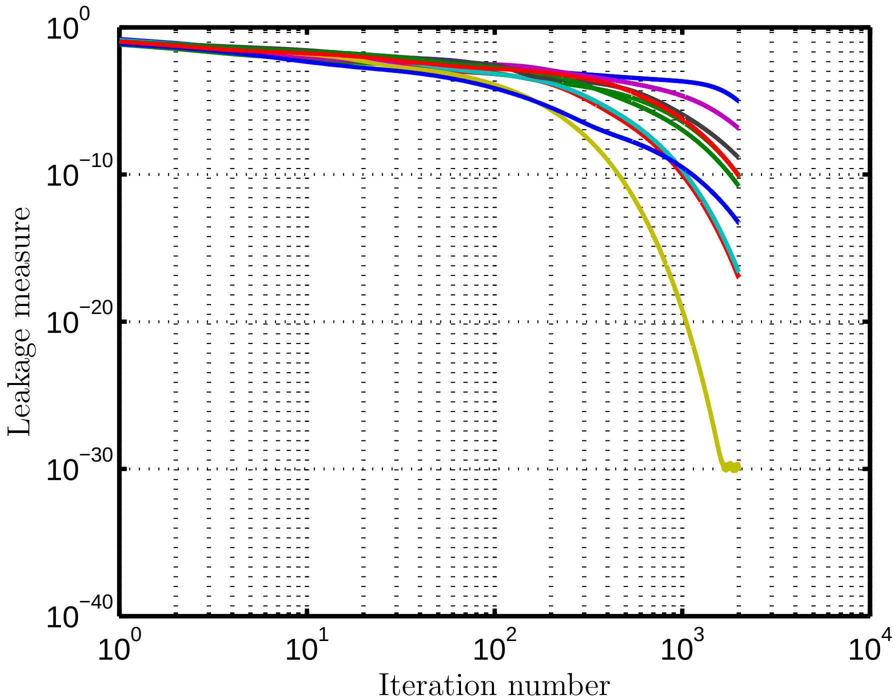

Figure 3.9. Leakage measure of IA_PSA_Itr versus number of iterations in B3

In the following, the convergence of the IA_PSA_Itr is investigated by varying R or N. Since the channel amplitude of different realizations are different, in order to have fair comparison between scenarios with different number R of relay antennas or number N of node antennas, in the following, the mean of the leakage measure of all the 10^4 channel realizations is considered as the measure of accuracy of SA. Figure 3.10 shows the mean of the leakage measure versus the iteration number for the IA_PSA_Itr for different number R of antennas at the relay in a scenario with $N = 3, d = 1$, and $K = 7$. Note that for $R = 9$, the considered scenario becomes the scenario B3. Reducing from $R = 9$ to $R = 8$, the system becomes improper and hence, the leakage measure saturates to 0.05. If the number of antennas at the relay is increased to $R = 10$, then there are multiple IA feasible and hence, the algorithm converges faster than that of the case $R = 9$.

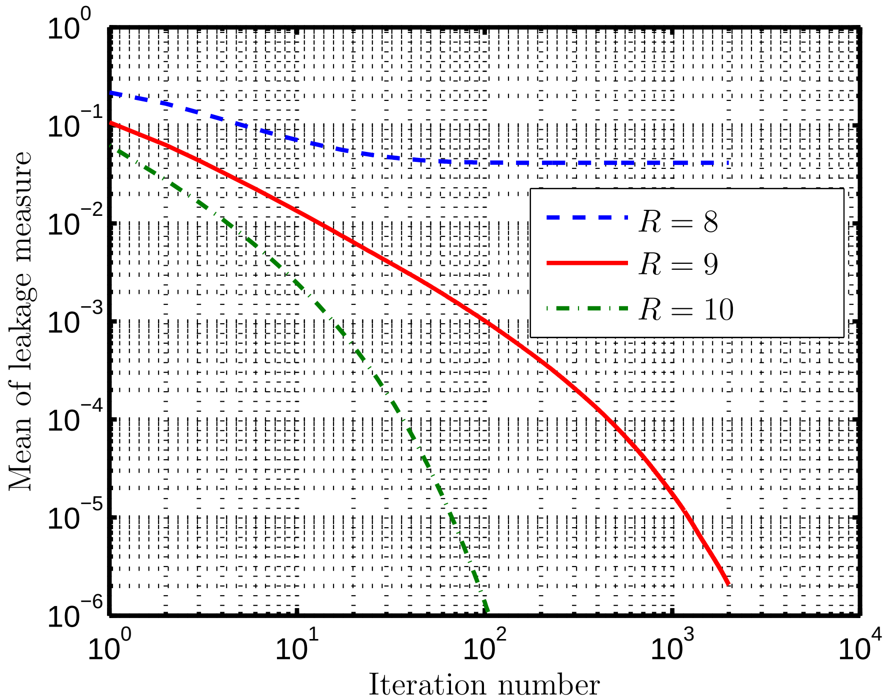

Figure 3.10. Mean of the leakage measure versus the iteration number for the IA_PSA_Itr for different number R of antennas at the relay

In Figure 3.11, similar convergence behavior can be observed by varying N in a scenario with $R = 9, d = 1$, and $K = 7$. Note that for $N = 3$, the considered scenario becomes the scenario B3. Furthermore, for $N = 4$, the leakage measure is zero after one or two iterations and hence, is not visible in Figure 3.11.

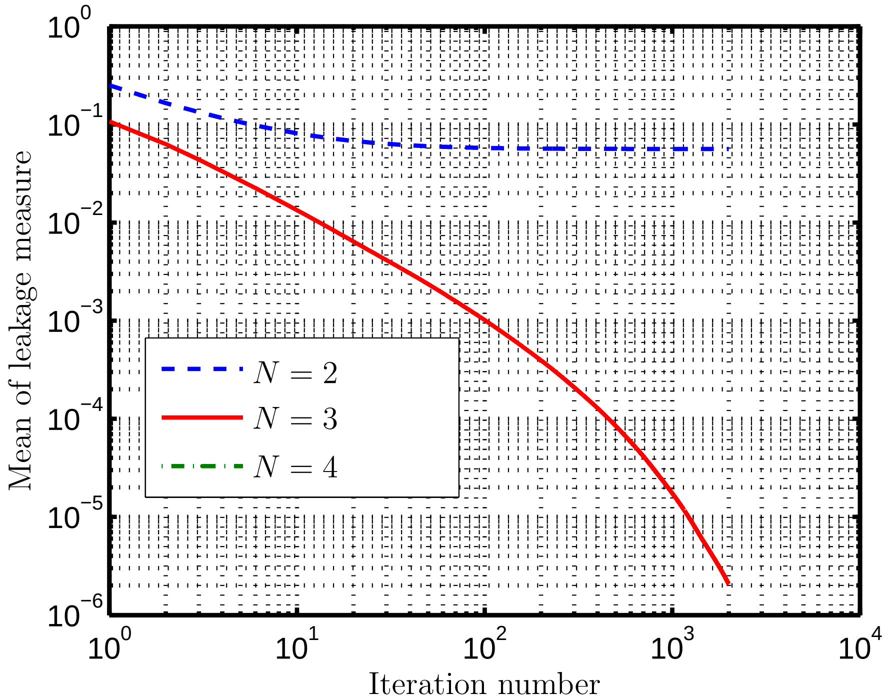

Figure 3.11. mean of the leakage measure versus the iteration number for the IA_-PSA_Itr for different number N of antennas at the nodes

3.4 Summary

In this chapter, IA aided by a single relay in a multi-pair two-way relay network has been considered. Based on the number of antennas at the relay, two different cases have been considered, namely, the relay with the minimum number $R = Kd$ of antennas and the relay with additional number $R \geq Kd$ antennas. For both cases, new schemes to perform IA has been proposed and closed form solutions and / or iterative algorithms have been developed to design the transmit, the relay and the receive filters. The properness conditions have also been derived. The following summarizes the main contributions and conclusions of this chapter.

- For the case $R = Kd$, it has been shown in this chapter that SA is a necessary condition in the MAC phase. After SA there are Kd effective data streams at the relay. In the BC phase, it has been shown in this chapter that CA is a necessary condition. Thereby, IA is decoupled into three linearly problems namely, SA, CA, and transceive ZF. CA is an identical problem as SA. Algorithms developed to perform SA can be directly applied to perform CA. In order to perform SA or CA, the nodes need at least $N \geq \frac{(K+1)d}{2}$ antennas. It has been revealed that, for the case $R = Kd$, same DoF as IA using spatial dimension in a K-user interference channel without relays is achieved. However, in contrast to K-user interference channel, now a closed form solution is available. In addition, the nodes need only pair-wise channel knowledge and the relay needs global channel knowledge to perform IA.

- For the case $R \geq Kd$, SA and CA are generalized as PSA and PCA. In this case, SA and CA takes place in a chosen Kd-dimensional relay subspace. This subspace is chosen such that the achieved DoF are increased and / or the number N of antennas required at the nodes is as small as possible. Increasing R, the dimension of the relay space increases and hence, there are more possibilities to choose a Kd-dimensional relay subspace. Therefore, the number of antennas required at the nodes to perform PSA and PCA reduces. This can be seen directly from the properness condition $2Kd \leq 2N + R - d$. For the special case $R = Kd$, the properness condition derived for PSA becomes the properness condition derived for SA. PSA and PCA are identical problems. PSA is a bilinear problem. An iterative algorithm is proposed to solve the bilinear problem. Two methods to decouple the problem into smaller linear subproblems have been proposed and closed form solutions have been obtained. Method 1 is more intuitive than method 2 but method 1 is more constrained than method 2. In both methods, the decoupling into linear subproblems is suboptimal in the sense that now more variables are needed than for solving PSA without decoupling. The conditions for the applicability of the closed form solutions have also been derived.

In both cases, the proposed algorithms achieve higher DoF and hence, higher multiplexing gain than the reference algorithm. Furthermore, the proposed algorithms achieve higher sum rate than the reference algorithm especially at high SNR.

Chapter 4

Interference alignment with multiple relays

4.1 Introduction

In this chapter, IA in a multi-pair two-way relay network with multiple relays is considered. As introduced in Section 2.5, the two-way relay channel is a multiple keyhole channel. It has been derived in Section 2.5 that in order to make sure that the useful and the interference signals are linearly independent of each other at the receiver, the total number of antennas at the relays has to be larger than Kd. There are Q relays each with R antennas. Hence, $QR \geq Kd$ is necessary to perform IA. Two cases are considered in this chapter.

In Section 4.2, the case where the relays have the minimum number $QR = Kd$ of antennas is addressed. In this case, the relay space is Kd-dimensional. First a scheme to achieve IA is proposed. Recollect from Chapter 3 that the term IA scheme implies the steps that need to be performed to achieve IA. The steps shall be sub-problems represented by a set of equations which are yet to be solved. In Section 4.2, by generalizing the concept of SA and CA proposed in Chapter 3 for multiple relays, it will be shown that IA can be decoupled into two linear problems and one bilinear problem. Then the properness condition is derived. Finally, an iterative algorithm to achieve IA is proposed. In comparison to the single relay scenario considered in Chapter 3, in the current chapter for multiple relays it is to be noted that the relays jointly design their signal processing matrices, but do not share the received signals. Therefore, the relay processing matrix $\mathbf{G}$ is a block diagonal matrix. The block diagonal structure brings additional challenges in performing IA.

In Section 4.3, the case of additional relays and / or relay antennas $QR \geq Kd$ is considered. For the single relay scenario considered in the previous chapter, the additional antennas at the relays are used to increase the DoF by performing IA in three independent steps, PSA, PCA and ZF. In Section 4.3, first it will be shown that the generalization of the PSA and the PCA based IA scheme developed for a single relay to multiple relays increases the complexity of the IA equations given by (2.15) and (2.16). Hence, a new scheme to design the transmit, relay and receive filters iteratively is proposed in this chapter. Then the properness condition is derived. Finally an iterative algortihm to achieve IA is proposed.

In Section 4.4, the DoF of the proposed schemes and the sum rates achieved by the proposed schemes are shown and compared with a reference method based on [RW07].

4.2 Relays with minimum number of antennas

4.2.1 Introduction

In this section, the case of relays having the minimum total number $QR = Kd$ of antennas derived in Section 2.5 is considered. In the following, first the proposed scheme to achieve IA is described in Section 4.2.2. Following this, in Section 4.2.3, the properness condition is derived. An algorithm to obtain the IA solution is proposed in Section 4.2.4.

4.2.2 Scheme

4.2.2.1 MAC phase: Signal alignment

In this section, the MAC phase of the bi-directional communication is described. In the following, it is shown that similar to the single relay scenario with the minimum number $R = Kd$ of antennas at the relay, also for the multiple relay scenario with the minimum number $QR = Kd$ of antennas at the relays, SA is a necessary condition. Then, the SA equation is introduced.

In the MAC phase, $2Kd$ data streams are received at the Q relays. The total number of antennas at the relays is $QR = Kd$. Even in the case that the relays share their signals, the $2Kd$ data streams cannot be spatially separated with QR antennas at the relays. However, our objective is that the relays aid in performing IA. From Section 3.2.2.1, it is known that for a single relay with $R = Kd$, SA at the relay is a necessary condition to achieve IA at the receiver. Furthermore, as described in Section 2.3, the linear processing at the relays can be represented by a single block diagonal matrix $\mathbf{G}$. In comparison to a single relay with $R = Kd$ antennas, now for the multiple relays with $QR = Kd$ antennas, the relay processing matrix $\mathbf{G}$ is block diagonal. The block-diagonal structure adds additional constraint in the system. Including an additional constraint in the system does not influence the necessary condition to achieve IA. However, it may influence the sufficient condition for the existence of IA. Since, SA is a necessary condition for the single relay case, SA is also a necessary condition for the multiple relay case, when the relays have the minimum number of antennas. Therefore, the signals from the nodes should be pair-wise aligned at the relays, i.e., the d-dimensional subspace spanned by the d data streams transmitted by node j should align with the corresponding d-dimensional subspace spanned by the d data streams of its communication partner k in the $QR = Kd$ dimensional relay space. The SA is given by

$$\mathrm{span}\left(\mathbf{H}_j^{\mathrm{sr}}\mathbf{V}_j\right) = \mathrm{span}\left(\mathbf{H}_k^{\mathrm{sr}}\mathbf{V}_k\right). \tag{4.1}$$

Note that the channel matrix $\mathbf{H}_j^{\mathrm{sr}}$ corresponds to the channel between node j and all Q relays. The signals from the nodes are pair-wise aligned in the QR-dimensional relay space.

4.2.2.2 BC phase: Channel alignment + Cooperative zero forcing

In this section, the BC phase of the bi-directional communication is described. First, it will be shown that CA is a necessary condition and the CA equation is given. Then, a new concept called cooperative zero forcing is proposed to achieve IA at the receivers.

In the following, the necessity of CA and the CA equation are given. After SA, there are Kd effective data streams at the relay. In Section 3.2.2.2, it has been shown that for a single relay with $R = Kd$ antennas, CA is a necessary condition for performing IA at the receivers. In comparison to the single relay scenario, for the multiple relays with the minimum number $QR = Kd$ of antennas, the relay processing matrix $\mathbf{G}$ is constrained to be block-diagonal. The addition of a constraint to a system does not influence the necessary condition for achieving IA. Hence, CA is also a necessary condition for the multiple relay scenario with the relays having the minimum number $QR = Kd$ of antennas. The receive filters are chosen such that the effective channels $\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_j^{\mathrm{rd}}$ and $\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rd}}$ for each node pair (j, k) align at the relays. The CA is given by

$$\mathrm{span}\left(\mathbf{H}_j^{\mathrm{rdH}}\mathbf{U}_j\right) = \mathrm{span}\left(\mathbf{H}_k^{\mathrm{rdH}}\mathbf{U}_k\right). \tag{4.2}$$

In the following, the concept of cooperative zero forcing is proposed. After SA and CA, there are Kd effective data streams and Kd effective channels. In the case of a single relay with $R = Kd$ antennas, the effective data streams and the effective channels can be spatially separated by performing transceive ZF. However, for the case of multiple relays with $QR = Kd$, as shown in Section 2.3, the relays do not share the signals received in the MAC phase and the relay processing matrix $\mathbf{G}$ is a block diagonal matrix. Hence, transceive ZF cannot be performed. Fortunately, the relay processing matrix with the block diagonal structure still has a maximum rank Kd and hence, there is a set of receive signatures and transmit signatures for which the relays can perform transceive ZF. This joint design of transmit, relay, and receive filters is called cooperative zero forcing (CZF). The basic idea of CZF is that the nodes have to jointly choose their SA and CA directions such that the relays will be able to perform transceive ZF with the block-diagonal relay processing matrix.

4.2.3 Properness condition

In this section, the properness condition is derived by counting the number of variables and number of equations. In order to achieve IA, three steps need to be performed: SA, CA, and CZF. The variables are given by the antennas at the nodes and at the relays. The constraints are determined by the fact that the relays have to be able to perform transceive ZF with a block-diagonal $\mathbf{G}$ matrix.

In the following, the number of variables is counted. The nodes need to perform SA and CA in the first step. The SA and CA solution spaces have to be multi-dimensional so that the nodes can perform CZF together with the relays. Hence, the dimension of the solution space provides the variables. From Section 3.2.3, we know that if the nodes have N antennas each, the dimension of the SA solution space of performing SA in a Kd dimensional relay space is given by $2N - Kd$. In this solution space, a d-dimensional subspace is chosen as the SA subspace for aligning d data streams of the considered node pair. This results in $d(2N - Kd - d)$ variables. There are K node pairs, and, hence, $Kd(2N - Kd - d)$ variables. Similarly, there are $Kd(2N - Kd - d)$ variables in chosing a CA solution. Hence, $2Kd(2N - Kd - d)$ variables are provided by the nodes after SA and CA. The relays have R antennas each and hence, QR^2 variables. Therefore, there are $2Kd(2N - Kd - d) + QR^2$ variables in total.

In the following, the number of equations is counted. The transceive ZF of $QR = Kd$ effective data streams and $QR = Kd$ effective channels involves Q^2R^2 equations. For the system to be proper, the number of variables should be larger than the number of equations. This results in in the properness condition

$$4N + R \geq 3Kd + 2d. \tag{4.3}$$

Note that for the special case $Q = 1$, the number R of antennas at the relays is given by $R = Kd$. In this case, (4.3) results in the same equation as (3.5) which is the properness condition for the single relay scenario.

4.2.4 Algorithm

4.2.4.1 Introduction

In this section, an algorithm is proposed to achieve IA in the three steps SA, CA and CZF. First the transmit filters are designed to achieve SA. We assume that the nodes have enough antennas such that multiple SA solutions are possible and hence, the solution space of SA is determined. Secondly, the solution space of CA is determined. Then one specific solution for SA and CA is jointly chosen such that the relays can together perform transceive ZF with a block-diagonal relay processing matrix $\mathbf{G}$.

4.2.4.2 Signal alignment algorithm

In this section, the solution space for SA is determined and an auxiliary matrix variable is introduced to choose one of the many possible SA solution in the determined solution space. The SA is independent of the fact whether there is a single relay which forms a Kd-dimensional relay space or if there are $Q > 1$ relays that together form a Kd-dimensional relay space. Hence, the algorithm in Section 3.2.4.1 can be used to find the SA solution.

Consider the pair (j, k). Using the algorithm proposed in Section 3.2.4.1, the precoding matrices $\mathbf{V}_j$ and $\mathbf{V}_k$ that solve (4.1) is given by

$$\begin{bmatrix} \mathbf{V}_j \\ \mathbf{V}_k \end{bmatrix} = \begin{bmatrix} \mathbf{A}_j \\ \mathbf{A}_k \end{bmatrix} \mathbf{X}_j^{(\mathrm{v})}, \tag{4.4}$$

where $\mathbf{X}_j^{(\mathrm{v})}$ is an auxiliary matrix variable to choose a d dimensional subspace in the SA solution space. Recollect from Section 3.2.4.1 that the columns of the matrix $\begin{bmatrix} \mathbf{A}_j \\ \mathbf{A}_k \end{bmatrix}$ denote a basis for the null space of the matrix $\mathbf{H}_{jk}^{\mathrm{sr}'}$, where $\mathbf{H}_{jk}^{\mathrm{sr}'} = \begin{bmatrix} \mathbf{H}_j^{\mathrm{sr}} & -\mathbf{H}_k^{\mathrm{sr}} \end{bmatrix}$.

4.2.4.3 Channel alignment algorithm

In this section, the solution space for CA is determined and an auxiliary variable is introduced to choose one of the many possible CA solutions. SA and CA are identical problems. Hence, the solution space of the CA equation given in (4.2) can be determined in the same way as that of SA. Let the columns of the matrix $\begin{bmatrix} \mathbf{B}_j \\ \mathbf{B}_k \end{bmatrix}$ span the solution space for the CA equation given in (4.2). The columns of the matrix $\begin{bmatrix} \mathbf{U}_j \\ \mathbf{U}_k \end{bmatrix}$ are chosen as a basis for a d dimensional subspace of the CA solution space $\begin{bmatrix} \mathbf{B}_j \\ \mathbf{B}_k \end{bmatrix}$ as follows:

$$\begin{bmatrix} \mathbf{U}_j \\ \mathbf{U}_k \end{bmatrix} = \begin{bmatrix} \mathbf{B}_j \\ \mathbf{B}_k \end{bmatrix} \mathbf{X}_j^{(\mathrm{u})}, \tag{4.5}$$

where $\mathbf{X}_j^{(\mathrm{u})}$ is an auxiliary matrix variable to choose a d dimensional subspace in the CA solution space.

4.2.4.4 Cooperative zero forcing algorithm

In this section, an algorithm to perform CZF is proposed, i.e., an algorithm to find one specific solution for SA and CA such that transceive ZF is possible with a block-diagonal $\mathbf{G}$ matrix. In the following, first CZF is represented by a set of bilinear equations. Then an iterative algorithm to solve the set of bilinear equations is proposed. Finally, the convergence of the proposed iterative algorithm is considered.

For any arbitrary matrices $\mathbf{X}_j^{(\mathrm{v})}$ and $\mathbf{X}_j^{(\mathrm{u})}$ of rank d, (4.4) and (4.5) result in SA and CA, respectively, at the relay. After SA and CA, there are Kd effective signals and Kd effective channels. In order to spatially separate these signals and channels, the following transceive ZF equation needs to be satisfied:

$$\mathbf{I} = \begin{bmatrix} \mathbf{U}_1^{\mathrm{H}} \mathbf{H}_1^{\mathrm{rd}} \\ \vdots \\ \mathbf{U}_K^{\mathrm{H}} \mathbf{H}_K^{\mathrm{rd}} \end{bmatrix} \mathbf{G} \begin{bmatrix} \mathbf{H}_j^{\mathrm{sr}} \mathbf{V}_1 & \dots & \mathbf{H}_K^{\mathrm{sr}} \mathbf{V}_K \end{bmatrix}. \tag{4.6}$$

The above equation can be rewritten as

$$\mathbf{G}^{-1} = \begin{bmatrix} \mathbf{H}_j^{\mathrm{sr}} \mathbf{V}_1 & \dots & \mathbf{H}_K^{\mathrm{sr}} \mathbf{V}_K \end{bmatrix} \begin{bmatrix} \mathbf{U}_1^{\mathrm{H}} \mathbf{H}_1^{\mathrm{rd}} \\ \vdots \\ \mathbf{U}_K^{\mathrm{H}} \mathbf{H}_K^{\mathrm{rd}} \end{bmatrix}. \tag{4.7}$$

Now inserting (4.4) and (4.5) in (4.7) we get

$$\mathbf{G}^{-1} = \begin{bmatrix} \mathbf{H}_1^{\mathrm{sr}} \mathbf{A}_1 \mathbf{X}_1^{(\mathrm{v})} & \dots & \mathbf{H}_K^{\mathrm{sr}} \mathbf{A}_K \mathbf{X}_K^{(\mathrm{v})} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1^{(\mathrm{u})\mathrm{H}} \mathbf{B}_1^{\mathrm{H}} \mathbf{H}_1^{\mathrm{rd}} \\ \vdots \\ \mathbf{X}_K^{(\mathrm{u})\mathrm{H}} \mathbf{B}_K^{\mathrm{H}} \mathbf{H}_K^{\mathrm{rd}} \end{bmatrix}. \tag{4.8}$$

$\mathbf{G}$ is a block-diagonal matrix. The inverse of a block-diagonal matrix is also a block-diagonal matrix. Therefore, the left hand side of (4.8) is a block-diagonal matrix. In order to satisfy (4.8), the variables $\mathbf{X}_j^{(\mathrm{v})}$ and $\mathbf{X}_j^{(\mathrm{u})}$ need to be chosen such that the product of the two matrices on the right hand side of equation (4.8) results in a block-diagonal matrix. This can be achieved by equating the off-block-diagonal elements of the product of the two matrices on the right hand side of equation (4.8) to zero. First let us express (4.8) in terms of the channels from the nodes to individual relays. Then (4.8) becomes

$$\begin{pmatrix} \mathbf{G}_1 & \cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \ldots & \mathbf{G}_Q \end{pmatrix}^{-1} = \underbrace{\begin{bmatrix} \tilde{\mathbf{H}}^{\mathrm{sr}}_{11}\mathbf{X}^{(\mathrm{v})}_1 & \ldots & \tilde{\mathbf{H}}^{\mathrm{sr}}_{K1}\mathbf{X}^{(\mathrm{v})}_K \\ \vdots & & \vdots \\ \tilde{\mathbf{H}}^{\mathrm{sr}}_{1Q}\mathbf{X}^{(\mathrm{v})}_1 & \ldots & \tilde{\mathbf{H}}^{\mathrm{sr}}_{KQ}\mathbf{X}^{(\mathrm{v})}_K \end{bmatrix}}_{\mathbf{H}^{\mathrm{sr}}_{\mathrm{eff}}} \underbrace{\begin{bmatrix} \mathbf{X}^{(\mathrm{u})\mathrm{H}}_1\tilde{\mathbf{H}}^{\mathrm{rd}}_{11} & \cdots & \mathbf{X}^{(\mathrm{u})\mathrm{H}}_1\tilde{\mathbf{H}}^{\mathrm{rd}}_{Q1} \\ \vdots & & \vdots \\ \mathbf{X}^{(\mathrm{u})\mathrm{H}}_K\tilde{\mathbf{H}}^{\mathrm{rd}}_{1K} & \cdots & \mathbf{X}^{(\mathrm{u})\mathrm{H}}_K\tilde{\mathbf{H}}^{\mathrm{rd}}_{QK} \end{bmatrix}}_{\mathbf{H}^{\mathrm{rd}}_{\mathrm{eff}}} \tag{4.9}$$

with $\tilde{\mathbf{H}}^{\mathrm{sr}}_{jq} = \mathbf{H}^{\mathrm{sr}}_{jq}\mathbf{A}_j$ and $\tilde{\mathbf{H}}^{\mathrm{rd}}_{jq} = \mathbf{B}^{\mathrm{H}}_j\mathbf{H}^{\mathrm{rd}}_{qj}$. Equating the off-block-diagonal elements of both sides of (4.9) we get

$$\sum_{k=1}^{K} \tilde{\mathbf{H}}^{\mathrm{sr}}_{kq}\mathbf{X}^{(\mathrm{v})}_k\mathbf{X}^{(\mathrm{u})\mathrm{H}}_k\tilde{\mathbf{H}}^{\mathrm{rd}}_{\overline{q}k} = \mathbf{0} \tag{4.10}$$

for $q, \overline{q} = 1, \ldots, Q$ and $q \neq \overline{q}$. (4.10) is a set of bilinear equations.

In the following, we propose an iterative algorithm to solve (4.10). First arbitrarily fix $\mathbf{X}^{(\mathrm{u})\mathrm{H}}_k$, then the resulting equations are a set of linear equations. Vectorizing both sides of (4.10) results in

$$\sum_{k=1}^{K} \left(\left(\mathbf{X}^{(\mathrm{u})\mathrm{H}}_k\tilde{\mathbf{H}}^{\mathrm{rd}}_{\overline{q}k} \right)^{\mathrm{T}} \otimes \tilde{\mathbf{H}}^{\mathrm{sr}}_{kq} \right) \mathrm{vec}\left(\mathbf{X}^{(\mathrm{v})}_k \right) = \mathrm{vec}\left(\mathbf{0} \right) \tag{4.11}$$

for $q, \overline{q} = 1, \ldots, Q$ and $q \neq \overline{q}$, where $\otimes$ denotes the Kronecker product operator and $\mathrm{vec}\left(.\right)$ denotes the vectorization of the matrix within the brackets. Let

$$\mathbf{H}_{q\overline{q}} = \begin{bmatrix} \left(\mathbf{X}^{(\mathrm{u})\mathrm{H}}_1\tilde{\mathbf{H}}^{\mathrm{rd}}_{\overline{q}1} \right)^{\mathrm{T}} \otimes \tilde{\mathbf{H}}^{\mathrm{sr}}_{1q} & \cdots & \left(\mathbf{X}^{(\mathrm{u})\mathrm{H}}_K\tilde{\mathbf{H}}^{\mathrm{rd}}_{\overline{q}K} \right)^{\mathrm{T}} \otimes \tilde{\mathbf{H}}^{\mathrm{sr}}_{Kq} \end{bmatrix} \tag{4.12}$$

and

$$\mathbf{X}^{(\mathrm{v})} = \begin{bmatrix} \mathrm{vec}\left(\mathbf{X}^{(\mathrm{v})}_1 \right)^{\mathrm{T}} \cdots \mathrm{vec}\left(\mathbf{X}^{(\mathrm{v})}_K \right)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \tag{4.13}$$

Then (4.11) can be written as

$$\underbrace{\begin{bmatrix} \mathbf{H}_{12} \\ \vdots \\ \mathbf{H}_{q\overline{q}} \\ \vdots \\ \mathbf{H}_{QQ-1} \end{bmatrix}_{q\neq\overline{q}}}_{\mathbf{H}_{\mathbf{X}^{(\mathrm{v})}}} \mathbf{X}^{(\mathrm{v})} = \mathrm{vec}\left(\mathbf{0} \right). \tag{4.14}$$

A least squares solution for (4.14) is obtained as

$$\mathbf{X}^{(\mathrm{v})} = \lambda_{\min}^{d}\left(\mathbf{H}_{\mathbf{X}^{(\mathrm{v})}}\right) \tag{4.15}$$

where the operator $\lambda_{\min}^{d}(.)$ obtains d eigenvectors corresponding to the minimum eigenvalue of the matrix within the brackets. In the second step, $\mathbf{X}^{(\mathrm{v})}$ determined in the previous step is fixed and a least squares solution for (4.10) is obtained by solving for $\mathbf{X}^{(\mathrm{u})}$. This is similar to determining $\mathbf{X}^{(\mathrm{v})}$ in the first step. These two steps are repeated till the product $\mathbf{H}_{\mathrm{eff}}^{\mathrm{sr}}\mathbf{H}_{\mathrm{eff}}^{\mathrm{rd}}$ results in a block diagonal matrix.

As (4.10) is a bilinear equation and minimizing the sum of squared errors (SEs) of (4.10) is a non-convex problem, the convergence to a block-diagonal matrix is not guaranteed. However, the sum of SEs is minimized in each iteration step. Hence, the algorithm is guaranteed to converge at least to a local minimum. Simulation results show that typically the algorithm finds the global minimum, i.e., the sum of SEs becomes zero.

After determining $\mathbf{X}^{(\mathrm{v})}$ and $\mathbf{X}^{(\mathrm{u})}$, the block diagonal matrix $\mathbf{G}$ that spatially separates all the effective signals and effective channels is given by

$$\mathbf{G} = \alpha_{\mathrm{r}}\left(\mathbf{H}_{\mathrm{eff}}^{\mathrm{sr}}\mathbf{H}_{\mathrm{eff}}^{\mathrm{rd}}\right)^{-1} \tag{4.16}$$

where α_{r} is a scalar used to satisfy the relay power constraint given by (2.8). α_{r} is given by (3.13). Note that the scaling factor α_{r} does not affect the IA conditions. In case the iterative algorithm converges to a local minimum, the resulting matrix $\mathbf{G}$ will not be block-diagonal. In this case, the off-block-diagonal elements of the matrix $\mathbf{G}$ are set to zero and there will be a leakage inter-pair interference in the system.

4.3 Relays with additional antennas

4.3.1 Introduction

In this section, the case where the relays have additional antennas compared to the minimum required number is considered. First in Section 4.3.2, the suitability of the SA and the CA based IA scheme for the considered scenario is investigated. Then, an iterative IA scheme is proposed in Section 4.3.3. In Section 4.3.4, the properness condition is derived. An iterative algorithm to design the transmit, the relay, and the receive filters is described in Section 4.3.5.

4.3.2 Suitability of SA and CA based scheme

In this section, the suitability of the SA and CA based scheme for $Q > 1$ with the total number QR of antennas at the relays larger than Kd is considered. In Chapter 3, it is shown that for the case $Q = 1$ and $R > Kd$, the additional antennas at the relays can be used to reduce the required number N of antennas at the nodes or to increase the achievable number of DoF. This is achieved by performing PSA and PCA. The PSA and PCA based schemes can be extended for the case $Q > 1$ and $QR > Kd$. However, from Section 4.2, we know that the nodes need to cooperate with the relays to perform CZF and hence, PSA and PCA have to be done jointly. Note that PSA and PCA are bilinear problems. Therefore, jointly performing PSA and PCA results in a quad-linear problem. Our original problem of IA is only trilinear, and hence, introducing SA and CA constraints do not simplify the problem. Hence, the SA and the CA based IA schemes are not suitable for $Q > 1$ with the total number QR of antennas at the relays larger than Kd.

4.3.3 Proposed scheme

In this section, the proposed IA scheme for the general case $QR \geq Kd$ is introduced. First we recollect that our objective is to align all the interferences within an $N - d$ dimensional ISS and to ensure that the useful signals fully occupy a d dimensional USS which is linearly independent from the ISS. This means the conditions (2.15) and (2.16) have to be fullfilled. Without loss of generality, (2.15) and (2.16) can be rewritten as

$$\mathbf{U}_k^{\mathrm{H}} \sum_{q=1}^{Q} \mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i = \begin{cases} \mathbf{0} & \text{if } i \neq j, k \\ \mathbf{I} & \text{if } i = j \end{cases} \tag{4.17}$$

for $i = 1, \ldots, 2K$ and $k = 1, \ldots, 2K$. Our objective is to design $\mathbf{G}_q$ for $q = 1, \ldots, Q$ and $\mathbf{V}_i$ for $i = 1, \ldots, 2K$ such that all the interference signals are aligned at the receive nodes within the ISS and the useful signals are within the USS and to design $\mathbf{U}_k$ for $k = 1, \ldots, 2K$ to zero force the interference signals in order to satisfy (4.17). The basic idea of the proposed iterative scheme is as follows. The condition of (4.17) is a set of trilinear equations in $\mathbf{U}_k$, $\mathbf{G}_q$ and $\mathbf{V}_i$. Fixing two of the three matrices results in a set of linear equations. Hence, we successively optimize $\mathbf{U}_k$, $\mathbf{G}_q$ and $\mathbf{V}_i$ to satisfy (4.17).

4.3.4 Properness condition

In this section, the properness condition for IA is derived. The variables in the system are the transmit, the relay and the receive filter matrices. Each relay filter matrix is of dimension $R \times R$ and the transmit and the receive filter matrices are of dimension $d \times N$. Hence, there are $4KNd + QR^2$ variables. From (4.17), there are $2K(2K-1)d^2$ equations. Therefore, for the system to be proper, the condition

$$4KNd + QR^2 \geq 2K(2K-1)d^2 \tag{4.18}$$

needs to hold. It has to be noted that in contrast to [YGJK09], where the number of variables corresponding to each transmit and receive filter matrix is counted as $Nd - d^2$ to make sure that the columns of the receive filter matrix span a d dimensional subspace, in this section, the number of variables corresponding to each transmit and receive filter is counted as Nd. This is due to the fact that in (4.17), the dimension of the subspace spanned by the columns of the receive filter matrix is explicitly made to be equal to d by equating the effective channel matrices corresponding to the useful signal to the identity matrix.

4.3.5 Algorithm

4.3.5.1 Design of receive filters

In this section, we arbitrarily fix the relay filters $\mathbf{G}_q$ and the transmit filters $\mathbf{V}_i$ and find the optimum receive filters that minimizes the sum of SEs between the left hand side and the right hand side of (4.17). After fixing the relay and the transmit filters, (4.17) is a set of linear equations. Vectorizing (4.17) and using the identity $\mathrm{vec}(\mathbf{YXZ}) = \left(\mathbf{Z}^{\mathrm{T}} \otimes \mathbf{Y}\right)\mathrm{vec}(\mathbf{X})$ results in

$$\mathbf{H}_{ik}^{\mathrm{eff}}\mathrm{vec}\left(\mathbf{U}_k^{\mathrm{H}}\right) = \begin{cases} \mathrm{vec}(\mathbf{0}) & \text{if } i \neq j,k \\ \mathrm{vec}(\mathbf{I}) & \text{if } i = j \end{cases} \tag{4.19}$$

where $\mathbf{H}_{ik}^{\mathrm{eff}} = \left(\left(\sum_{q=1}^{Q} \mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i\right)^{\mathrm{T}} \otimes \mathbf{I}\right)$. With

$$\begin{aligned} \mathbf{c} &= \left[\mathrm{vec}(\mathbf{I})^{\mathrm{T}}\ \mathrm{vec}(\mathbf{0})^{\mathrm{T}}\ \ldots\ \mathrm{vec}(\mathbf{0})^{\mathrm{T}}\right]^{\mathrm{T}}, \\ \mathbf{u}_k &= \mathrm{vec}\left(\mathbf{U}_k^{\mathrm{H}}\right), \\ \mathbf{H}_k &= \left[\mathbf{H}_{jk}^{\mathrm{effT}}\ \mathbf{H}_{1k}^{\mathrm{effT}}\ \cdots\ \mathbf{H}_{ik}^{\mathrm{effT}}\ \cdots\ \mathbf{H}_{2k}^{\mathrm{effT}}\right]_{i \neq j,k}^{\mathrm{T}}. \end{aligned} \tag{4.20}$$

(4.19) can be expressed as $\mathbf{H}_k\mathbf{u}_k = \mathbf{c}$. Then the least squares solution for the receive filter is obtained as

$$\mathbf{u}_k = \mathbf{H}_k^{\dagger}\mathbf{c} \tag{4.21}$$

for $k = 1, \ldots, 2K$, where $\mathbf{H}_k^{\dagger}$ is the pseudo inverse of $\mathbf{H}_k$.

4.3.5.2 Design of relay filters

In this section, we fix $\mathbf{U}_k$ and $\mathbf{V}_i$ and find the optimum relay filter that minimizes the sum of SEs of (4.17). Vectorizing (4.17), we get

$$\sum_{q=1}^{Q} \left(\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i\right)^{\mathrm{T}} \otimes \left(\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{qk}^{\mathrm{rd}}\right) \mathrm{vec}\left(\mathbf{G}_q\right) = \begin{cases} \mathrm{vec}\left(\mathbf{0}\right) & \text{if } i \neq j, k \\ \mathrm{vec}\left(\mathbf{I}\right) & \text{if } i = j \end{cases} \tag{4.22}$$

for $i, k = 1, \ldots, 2K$. Let $\mathbf{D}_{i,q,k} = \left(\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i\right)^{\mathrm{T}} \otimes \left(\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{qk}^{\mathrm{rd}}\right)$ and $\mathbf{g}_q = \mathrm{vec}\left(\mathbf{G}_q\right)$. Then, (4.22) can be written as

$$\underbrace{\begin{pmatrix} \mathbf{D}_{j,1,k} & \cdots & \mathbf{D}_{j,Q,k} \\ \mathbf{D}_{1,1,k} & \cdots & \mathbf{D}_{1,Q,k} \\ \vdots & & \vdots \\ \mathbf{D}_{i,1,k} & \cdots & \mathbf{D}_{i,Q,k} \\ \vdots & & \vdots \\ \mathbf{D}_{2K,1,k} & \cdots & \mathbf{D}_{2K,Q,k} \end{pmatrix}_{i \neq j,k}}_{\mathbf{D}_k} \begin{pmatrix} \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_Q \end{pmatrix} = \begin{pmatrix} \mathrm{vec}\left(\mathbf{I}\right) \\ \mathrm{vec}\left(\mathbf{0}\right) \\ \vdots \\ \mathrm{vec}\left(\mathbf{0}\right) \end{pmatrix} \tag{4.23}$$

for $k = 1, \ldots, 2K$. Furthermore with $\mathbf{D} = \left[\mathbf{D}_1^{\mathrm{T}} \ \cdots \ \mathbf{D}_{2K}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\mathbf{g} = \left[\mathbf{g}_1^{\mathrm{T}} \ \ldots \ \mathbf{g}_Q^{\mathrm{T}}\right]^{\mathrm{T}}$, $\mathbf{b}_k = \left[\mathrm{vec}\left(\mathbf{I}\right)^{\mathrm{T}} \ \mathrm{vec}\left(\mathbf{0}\right)^{\mathrm{T}} \ \ldots \ \mathrm{vec}\left(\mathbf{0}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, and $\mathbf{b} = \left[\mathbf{b}_1^{\mathrm{T}} \ \mathbf{b}_2^{\mathrm{T}} \ \ldots \ \mathbf{b}_{2K}^{\mathrm{T}}\right]^{\mathrm{T}}$, (4.23) can be written as

$$\mathbf{D}\mathbf{g} = \mathbf{b}. \tag{4.24}$$

In this case, the least squares solution for (4.24) is given by

$$\mathbf{g} = \alpha_{\mathrm{g}}\mathbf{D}^{\dagger}\mathbf{b} \tag{4.25}$$

where $\alpha_{\mathrm{v}} = \frac{1}{\sqrt{\mathrm{trace}(\mathbf{g}\mathbf{g}^{\mathrm{H}})}}$ is a scalar used to normalize the Frobenius norm of the vector $\mathbf{g}$ to 1. This normalization is done to prevent the coefficients of the relay filters from converging to very small values at the end of the iterations. Thus $\mathbf{G}_q$ for $q = 1, \ldots, Q$ is obtained for fixed receive filters and transmit filters.

4.3.5.3 Design of transmit filters

In this section, we fix $\mathbf{U}_k$ and $\mathbf{G}_q$ and find the optimum transmit filter that minimizes the sum of SEs of (4.17). For simplicity of the notation, we relabel the indices of (4.17) as follows:

$$\mathbf{U}_{\overline{k}}^{\mathrm{H}} \sum_{q=1}^{Q} \mathbf{H}_{q\overline{k}}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{jq}^{\mathrm{sr}}\mathbf{V}_j = \begin{cases} \mathbf{0} & \text{if } \overline{k} \neq j, k \ , \\ \mathbf{I} & \text{if } \overline{k} = k \ . \end{cases} \tag{4.26}$$

Let $\mathbf{H}_{\text{eff}}^{j\bar{k}} = \mathbf{U}_{\bar{k}}^{\text{H}} \sum_{q=1}^{Q} \mathbf{H}_{q\bar{k}}^{\text{rd}} \mathbf{G}_q \mathbf{H}_{jq}^{\text{sr}}$. Then (4.26) can be written as

$$\mathbf{H}_{\text{eff}}^{j\bar{k}} \mathbf{V}_j = \begin{cases} \mathbf{0} & \text{if } \bar{k} \neq j,k \\ \mathbf{I} & \text{if } \bar{k} = k \end{cases} \tag{4.27}$$

Let

$$\mathbf{D}_j = \left[\mathbf{H}_{\text{eff}}^{jk\text{T}} \; \mathbf{H}_{\text{eff}}^{j1\text{T}} \cdots \; \mathbf{H}_{\text{eff}}^{j\bar{k}\text{T}} \; \cdots \mathbf{H}_{\text{eff}}^{j(2)K\text{T}} \right]_{\bar{k} \neq j,k}^{\text{T}} \tag{4.28}$$

and

$$\mathbf{b}_j = \left[\text{vec}\,(\mathbf{I})^{\text{T}} \; \text{vec}\,(\mathbf{0})^{\text{T}} \; \ldots \; \text{vec}\,(\mathbf{0})^{\text{T}} \right]^{\text{T}}. \tag{4.29}$$

Then (4.27) can be expressed as

$$\mathbf{D}_j \mathbf{V}_j = \mathbf{b}_j. \tag{4.30}$$

The least squares solution for (4.30) is given by

$$\mathbf{V}_j = \alpha_{\text{v}} \mathbf{D}_j^{\dagger} \mathbf{b}_j \tag{4.31}$$

where $\alpha_{\text{v}} = \frac{1}{\sqrt{\text{trace}\left(\mathbf{V}_j \mathbf{V}_j^{\text{H}}\right)}}$ is a scalar used to normalize the Frobenius norm of the matrix $\mathbf{V}_j$ to 1. This normalization is done to avoid IA solutions in which the transmit filter coefficients become very small.

Iteratively optimizing the receive filters $\mathbf{U}_k^{\text{H}}$, for $k = 1, \ldots, 2K$, the relay filters $\mathbf{G}_q$ for $q = 1, \ldots, Q$ and the tranmit filters $\mathbf{V}_j$ for $j = 1, \ldots, 2K$ using (4.21), (4.25) and (4.31), respectively, a solution for (4.17) that minimizes the sum of SEs can be found. As in each steps of the iterations, the sum of SEs is reduced and the sum of SEs is lower bounded by zero, the algorithm converges to a local optimum. Convergence to a global optimum cannot be guaranteed due to the non-convex nature of the problem.

4.4 Performance analysis

4.4.1 Introduction

In this section, the DoF and the sum rate performance of the two schemes proposed in this chapter are investigated. First, the reference scheme used for comparison is introduced in Section 4.4.2. Secondly, the DoF achieved by the proposed and the reference schemes are investigated in Section 4.4.3. Following this, in Section 4.4.4, the assumptions regarding the simulation setup are introduced. Finally, the sum rate achieved and the convergence of the proposed iterative algorithms are evaluated in Section 4.4.5 and 4.4.6, respectively.

4.4.2 Reference algorithm

A reference scheme to compare the performance of the proposed IA algorithms is introduced in this section. The reference scheme is based on the orthogonalize and forward scheme from [RW07]. In [RW07], single antenna nodes and single antenna relays have been considered and a reciprocal channel is assumed. The relay coefficients are chosen such that at the receivers, the inter-pair interference is completely suppressed [RW07]. Furthermore, in [RW07], the useful links are not considered in the design of the relay filters.

In this thesis, multiple antennas are considered at the nodes and at the relays, and hence, the following extensions are made to [RW07]: First, the transmit and receive filters at the nodes are designed to maximize the received signal power as in Section 3.2.5.2. Secondly, the inter-pair interference is completely suppressed by appropriately choosing the relay filter coefficients. For this purpose, the orthogonalize and forward scheme from [RW07] is generalized to the case of multiple antennas at the relays and for non-reciprocal channels.

Since the transmit and the receive filters are designed similar to the reference scheme described in Section 3.2.5.2, they are assumed to be fixed in the following. Now the objective is to find the relay filters that satisfy the following equations:

$$\mathbf{U}_k^{\mathrm{H}} \sum_{q=1}^{Q} \mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i = \mathbf{0} \text{ if } i \neq j, k \tag{4.32}$$

for $i = 1, \ldots, 2K$ and $k = 1, \ldots, 2K$. Vectorizing (4.32) we get

$$\sum_{q=1}^{Q} \left(\mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i\right)^{\mathrm{T}} \otimes \left(\mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{qk}^{\mathrm{rd}}\right) \operatorname{vec}\left(\mathbf{G}_q\right) = \operatorname{vec}\left(\mathbf{0}\right) \text{ if } i \neq j, k \tag{4.33}$$

for $i, k = 1, \ldots, 2K$. Let $\mathbf{D}_{i,q,k} = \left(\mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i\right)^{\mathrm{T}} \otimes \left(\mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{qk}^{\mathrm{rd}}\right)$ and $\mathbf{g}_q = \operatorname{vec}\left(\mathbf{G}_q\right)$. Then (4.33) can be written as

$$\underbrace{\begin{pmatrix} \mathbf{D}_{1,1,k} & \cdots & \mathbf{D}_{1,Q,k} \\ \vdots & & \vdots \\ \mathbf{D}_{i,1,k} & \cdots & \mathbf{D}_{i,Q,k} \\ \vdots & & \vdots \\ \mathbf{D}_{2K,1,k} & \cdots & \mathbf{D}_{2K,Q,k} \end{pmatrix}_{i \neq j,k}}_{\mathbf{D}_k} \begin{pmatrix} \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_Q \end{pmatrix} = \begin{pmatrix} \operatorname{vec}\left(\mathbf{0}\right) \\ \vdots \\ \operatorname{vec}\left(\mathbf{0}\right) \end{pmatrix} \tag{4.34}$$

for $k = 1, \ldots, 2K$. Furthermore with $\mathbf{D} = \left[\mathbf{D}_1^{\mathrm{T}} \; \cdots \; \mathbf{D}_{2K}^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\mathbf{g} = \left[\mathbf{g}_1^{\mathrm{T}} \; \ldots \; \mathbf{g}_Q^{\mathrm{T}}\right]^{\mathrm{T}}$, (4.34) can be written as

$$\mathbf{D}\mathbf{g} = \mathbf{0}. \tag{4.35}$$

The solution for (4.35) is given by

$$\mathbf{g} = \mathrm{NULL}^{(1)}\left(\mathbf{D}\right) \tag{4.36}$$

where $\mathrm{NULL}^{(1)}(.)$ gives a basis for an arbitrary one-dimensional subspace in the null space of the matrix within the brackets. The size of the matrix $\mathbf{D}$ is $2K(2K-1)d^2 \times QR^2$. For a non-trivial null space to exist, the condition

$$QR^2 > 2K(2K-2)d^2 \tag{4.37}$$

needs to hold.

4.4.3 Degrees of freedom analysis

In this section, the achievable DoF of the proposed IA schemes and of the reference scheme are investigated. For simplicity, let us denote the cooperative ZF based IA scheme proposed in Section 4.2 by IA_CZF and the iterative IA scheme proposed in Section 4.3 by IA_Iterative. Since the reference scheme is a generalization of the orthogonalize and forward scheme proposed in [RW07], let us call it generalized orthogonalize and forward (GOF) in the following.

In the following, the scenarios considered for the DoF analysis are introduced. Table 4.1 shows five different scenarios considered for investigation. The scenarios are chosen such that the cases $Q = 1$, $Q > 1$, $QR = Kd$, $QR > Kd$, $d = 1$, and $d > 1$ are considered. A single relay is considered in scenario A1 which is the same scenario as introduced in Chapter 3. Multiple relays are considered in scenarios C1 - C4. $QR = Kd$ is considered in scenarios C1 - C3 and $QR > Kd$ is considered in scenario C4. Scenario C2 considers $d = 2$ and in all the other scenarios, $d = 1$ is considered. The algorithms that can be used to obtain IA in these five scenarios and the number of DoF achieved in these scenarios are described in the following paragraphs.

In A1, a single relay with $R = 3$ antennas is considered. In this case, both (4.3) and (4.18) are satisfied and hence, IA_CZF and IA_Iterative can be used to find IA solutions. Three DoF are achieved, which is the same as that achieved by the IA algorithm proposed in Chapter 3 for a single relay. The reference GOF algorithm achieves only two DoF.

Table 4.1. Scenarios considered for $Q \geq 1$ and DoF achieved

Scenarios	Q	R	d	N	K		DoF	
					GOF	IA	GOF	IA
A1	1	3	1	2	2	3	2	3
C1	2	2	1	3	1	4	1	4
C2	2	3	2	5	1	3	2	6
C3	2	3	1	5	2	6	2	6
C4	2	4	1	3	3	5	3	5

In scenario C1, both (4.3) and (4.18) are satisfied with equality sign and hence, both IA_CZF and IA_Iterative can be used to find an IA solution. The proposed algorithms serve $K = 4$ node pairs simultaneously and achieve 4 DoF. However, the reference GOF algorithm can serve only $K = 1$ node pair simultaneously and hence, achieves 1 DoF. Note that in the reference algorithm, for the case of a single node pair, the nodes can transmit $N = 3$ data streams each and hence, achieve 3 DoF. However, this difference between the DoF achieved by transmitting N data streams per node and that achieved by transmitting d data streams per node becomes negligible when $K \geq 2$. Hence, in the following, for simplicity, the same number of data streams is considered in IA_CZF, IA_Iterative, and GOF. Note that in either case, the proposed algorithms achieve more DoF than the reference GOF algorithm.

In scenario C2, $d = 2$ data streams per node are considered. For the same numbers N and R, in scenario C3, only $d = 1$ data stream per node but double the number of node pairs is considered. In both scenarios, both (4.3) and (4.18) are satisfied and $Kd = 6$ DoF are achieved. The reference GOF algorithm achieves only 2 DoF in both the scenarios C2 and C3.

In scenario C4, $QR > Kd$ and hence, only the IA_Iterative scheme can be applied. The condition (4.18) is satisfied and hence, the system is proper. The IA_Iterative algorithm achieves 5 DoF while the GOF algorithm achieves only 3 DoF.

In all the scenarios mentioned in Table 4.1, the proposed IA schemes achieve more DoF than the reference GOF scheme. This is due to the fact that in GOF, only the relays are used to suppress the interferences at the receivers, but in the proposed IA schemes, the transmit, the relay, and the receive filters are jointly designed to achieve interference-free communication.

4.4.4 Assumptions

In this section, the same assumptions as in Section 3.2.5.4 regarding the channel model, number of node pairs, power allocation and channel realizations are assumed, except for the reciprocity of the MAC and BC channels. In Chapter 3, the properness condition derived for both the proposed and the reference schemes are independent of the reciprocity of the channel. Hence, for simplicity channel reciprocity is assumed between the MAC and the BC channels in the simulation setup in Chapter 3. However, in contrast to this, in the current chapter for the reference GOF algorithm, depending on whether the MAC and BC channels are reciprocal to each other or not, the number of equations involved in (4.32) changes and hence, the properness condition derived for GOF also changes. At the same time, for the proposed IA schemes, the properness conditions derived are independent of the reciprocity of the channel. Hence, no assumption on channel reciprocity is made.

4.4.5 Sum rate analysis

In this section, the sum rate performance of the proposed IA algorithms are compared with the reference GOF algorithm. Let $P_{\text{node}} = P$ denote the power of each of the $2K$ nodes. Let $P_{\text{relay}} = KP$ denote the power available at the relay. The noise power at each node and at the relay is assumed to be the same and is denoted by $\sigma_k^2 = \sigma_{\text{relay}}^2 = \sigma^2$.

In the following, first the sum rate performance of the IA_CZF algorithm is compared with that of the reference GOF algorithm. Then for a fixed number of DoF, the influence of d, K, and the maximum transmit power at the nodes and at the relays on the sum rate is investigated. Since IA_Iterative can also be used to find an IA solution in a scenario with $QR = Kd$, the sum rate performances of the proposed IA_CZF algorithm and the proposed IA_Iterative algorithm are compared. Finally, the sum rate performance of the proposed IA_Iterative algorithm is compared with the reference GOF algorithm.

In this paragraph, the sum rate performance of the IA_CZF algorithm is compared with that of the reference GOF algorithm in scenario C1 and scenario C2. Figure 4.1 shows the sum rate in bits per channel use as a function of P/σ^2. The solid red curves and the dotted black curves correspond to the sum rate achieved by IA_-CZF and GOF, respectively. The slope of each of the curves correspond to the DoF achieved by the corresponding scheme. IA_CZF achieves more DoF than GOF and hence, achieves higher sum rate at high SNR regime. At low SNR, GOF has better performance because the antennas at the nodes are used to improve the received useful signal power. In contrast to the fact that IA_CZF achieves more DoF in scenario C2 than in scenario C1, for $P/\sigma^2 < 50$ dB, the absolute value of the sum rate achieved by IA_CZF in scenario C2 is smaller than that in scenario C1. This is mainly due to two factors: First, in scenario C1, the total power available to transmit all the $2Kd$ data streams in the MAC and the BC phases are $8P$ and $4P$, respectively. However, in scenario C2, the total power available to transmit all the $2Kd$ data streams in the MAC and BC phases are $6P$ and $3P$, respectively. Secondly, in scenario C2, the fact that the two data streams transmitted from a transmitter to a receiver need to be linearly independent of each other adds additional constraints in the system compared to the case $d = 1$.

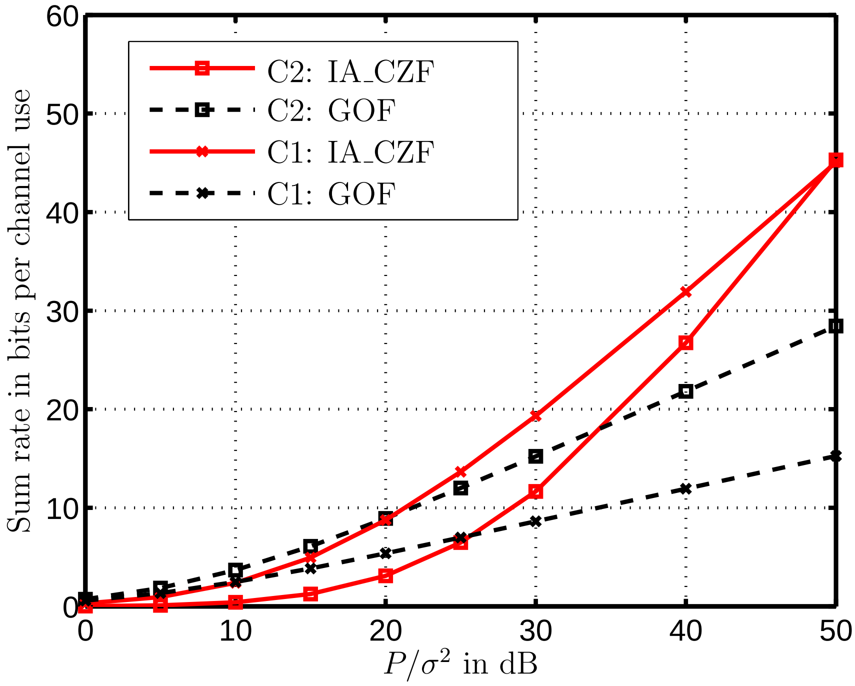

Figure 4.1. Sum rate performance of the IA_CZF, the IA_Iterative and the reference GOF algorithms versus P/σ^2 for scenarios C1 and C2

In order to investigate separately the two factors mentioned above, consider the scenario C3. In contrast to C2, in C3, the number K of node pairs is doubled and the number d of data streams per node is halfed so that the DoF remain the same in both scenarios. Furthermore, the maximum transmit power available at each node is halfed, i.e., $P_{\text{node}} = \frac{P}{2}$, so that the total power available in C2 and C3 is the same. In this case, each node transmits only $d = 1$ data stream and hence, there is no additional constraint in C3 to ensure linear independency among the data streams. The sum rate performances of C2 and C3 are shown in Figure 4.2, using the solid red curve and the solid blue curve, respectively. There is a sum rate gain of approximately 7.5 bits per channel use at medium and high SNR which is due to the fact that in comparison to C2, in C3 there is no additional constraint to ensure linear independency among the data streams. Furthermore, if we double the transmit power of each node and each relay in C3, then the achieved sum rate increases further. This is shown by the solid green curve in Figure 4.2.

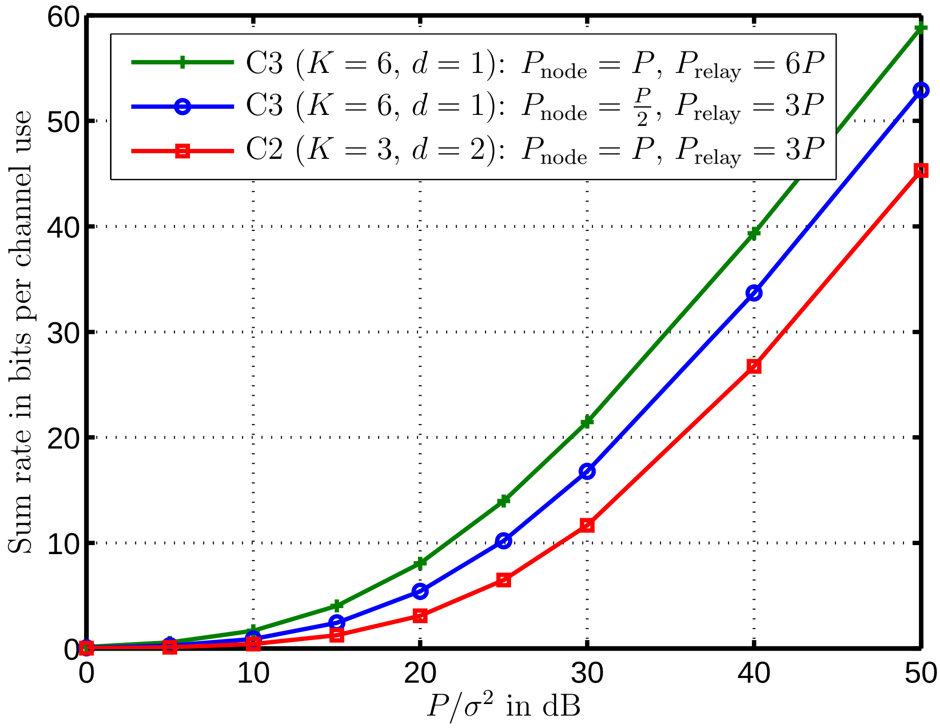

Figure 4.2. Influence of d, K, and the maximum transmit power at the nodes and at the relays on the sum rate in scenario C2 and in scenario C3 for a fixed number of DoF

In this paragraph, the sum rate performance of the proposed IA_CZF algorithm and the proposed IA_Iterative algorithm are compared for the case $QR = Kd$. It is to be noted that the IA_Iterative algorithm can also be applied to scenarios C1, C2, and C3 to obtain the IA solution. However, in these scenarios, $QR = Kd$ and hence, SA and CA are necessary conditions. IA_CZF takes SA and CA into consideration while designing the transmit, relay and receive filters. Hence, IA_CZF should have better results than IA_Iterative. In order to verify this, consider the sum rate achieved by IA_CZF and IA_Iterative in scenario C1. In Figure 4.3, the solid red curve and the solid blue curve correspond to the sum rates achieved by IA_CZF and IA_Iterative, respectively. At low SNR, the noise is the dominating factor and hence, both schemes have similar performance. At high SNR, interference is the dominating factor and hence, IA_CZF has better performance than IA_Iterative.

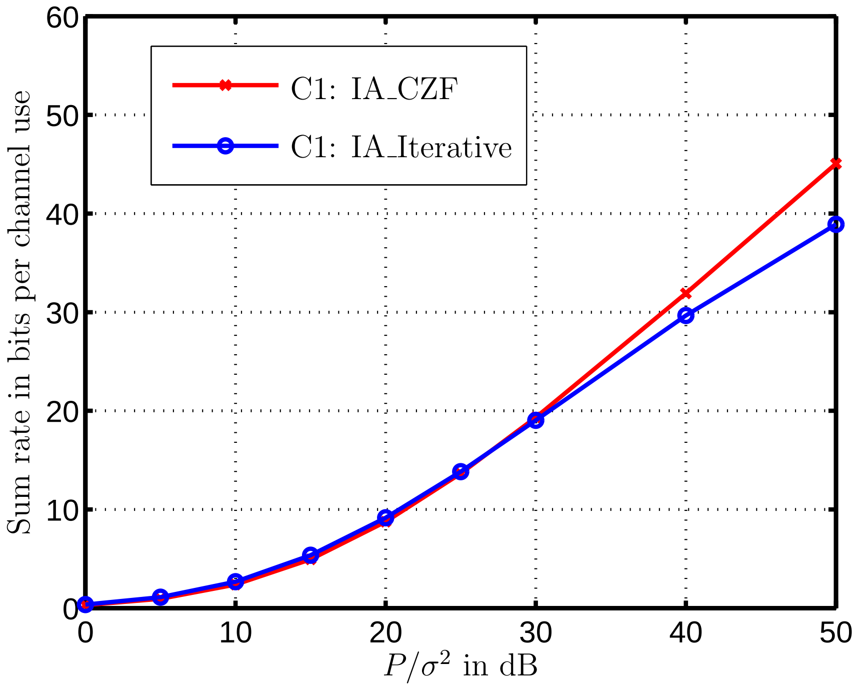

Figure 4.3. Sum rate performance of the IA_CZF and the IA_Iterative algorithms versus P/σ^2 for scenario C1

In this paragraph, the sum rate performance of the proposed IA_Iterative algorithm is compared with the reference GOF algorithm. In C4, only IA_Iterative can be applied. Figure 4.4 shows the sum rate performance of IA_Iterative and GOF in C4. It can be observed IA_Iterative performs better than the reference GOF. However, at high SNR, the slope of the sum rate curve corresponding to IA_Iterative reduces. This is due to the residual sum of SEs at high SNR.

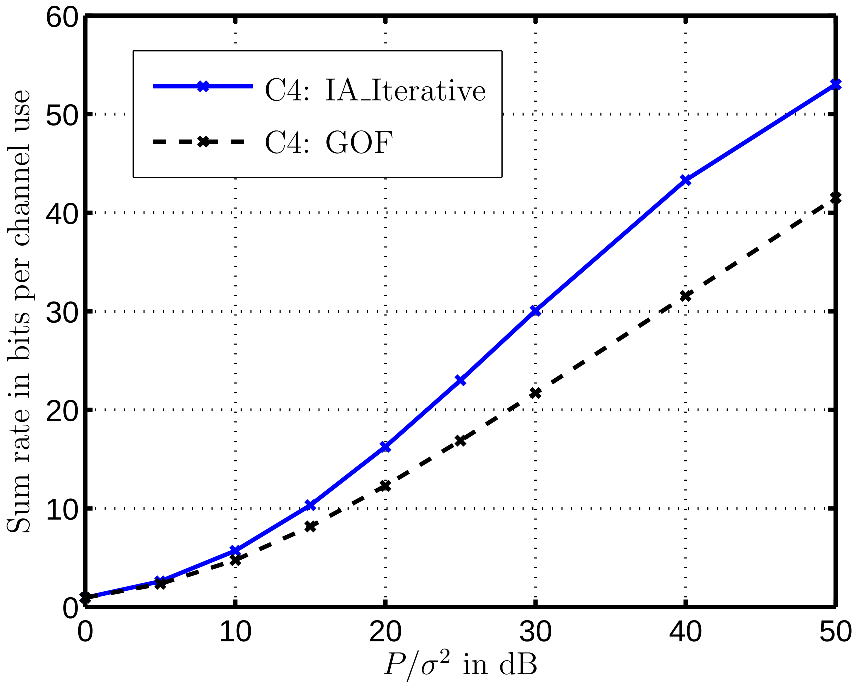

Figure 4.4. Sum rate performance of the IA_CZF algorithm and the reference GOF algorithm versus P/σ^2 for scenario C4

4.4.6 Convergence analysis

In this section, the convergence of both proposed IA algorithms is investigated. Recollect from Sections 4.2.4.4 and 4.3.5, that in both proposed iterative IA algorithms, the variables are iteratively designed to minimize the sum of SEs between the left hand side and right hand side of a set of equations. In each step, the sum of SEs is reduced and is lower bounded by zero. Hence, the sum of SEs is guaranteed to converge. However, convergence to a global minimum is not guaranteed due to the non-convex nature of the problem. In the following, first the convergence of the IA_CZF algorithm is investigated for scenario C1. Then the convergence of the IA_Iterative algorithm is investigated for scenario C4.

Figure 4.5 shows the sum of SEs versus the iteration number for IA_CZF in C1. The sum of SEs values are plotted for ten arbitrarily chosen channel realizations. It can been seen that the sum of SEs reduces during each iteration and is almost zero after 10 iterations. Since for each of the channel realizations, the least square error values reduces after each iteration, the average of the sum of SEs for several channel realizations will also reduce after each iterations. This can be verified in Figure 4.6 where the average of the sum of SEs of 10^4 channel realizations versus the iteration number is plotted.

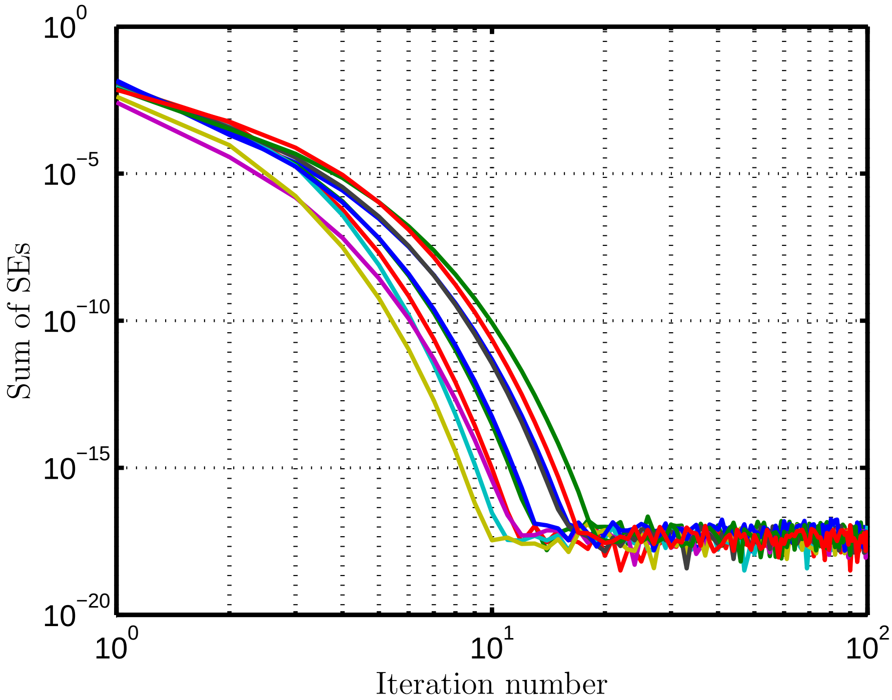

Figure 4.5. Sum of SEs versus the iteration number in scenario C1

Figure 4.7, shows the average of the sum of SEs of 10^4 channel realizations in C4. It can be observed that after 1000 iterations the MSE converges to a value of 0.01. However, at iteration numbers 510 and 660, the sum of SEs increases by a small value. This is due to the fact that the relay and the transmit filter matrices are normalized after each iteration. This normalization is a non-linear step and hence, affects the sum of SEs after each iteration. However, the normalization does not affect the IA solutions, as it modifies only the amplitude of the received signal. In contrast to this, if the relay and receive filters are not normalized after each iteration, the sum of SEs is guaranteed to reduce after each iteration, but the sum of SEs at the end of the iterations shall be quite significant. It can be observed from Figure 4.8 that after 1000 iterations the sum of SEs converges to a value of 0.15. Hence, the normalization of the relay and the transmit filter matrices is necessary.

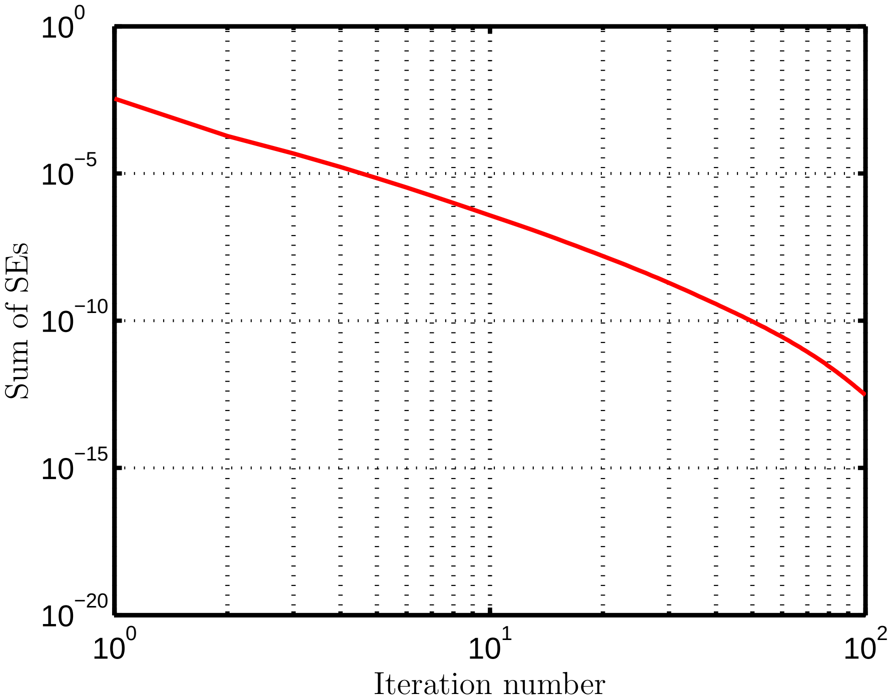

Figure 4.6. Average of the sum of SEs of different channel realizations versus the iteration number in scenario C1

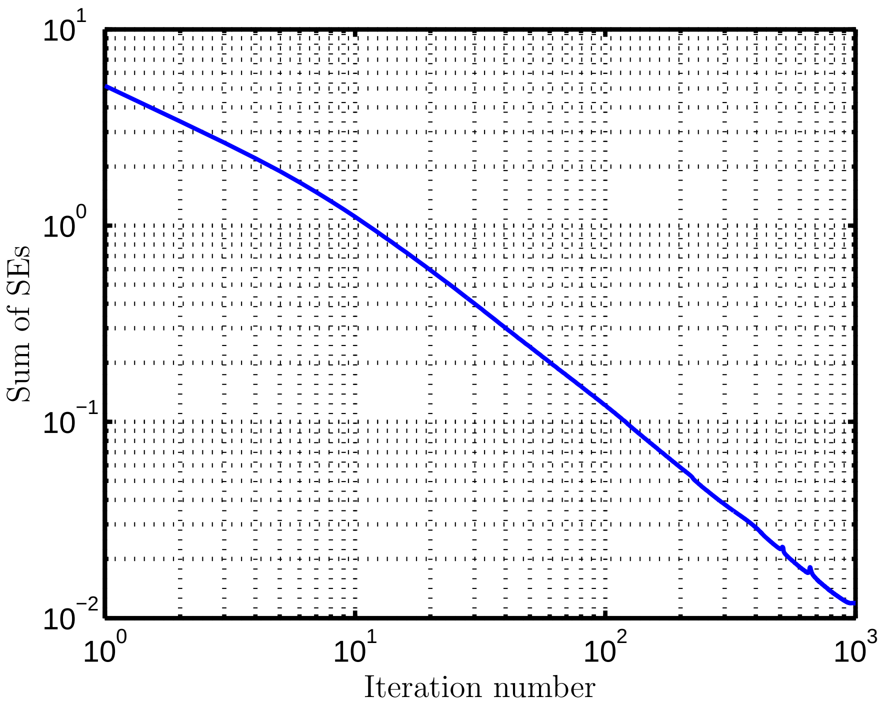

Figure 4.7. Average of the sum of SEs of different channel realizations versus the iteration number in scenario C4 with normalization of the variables

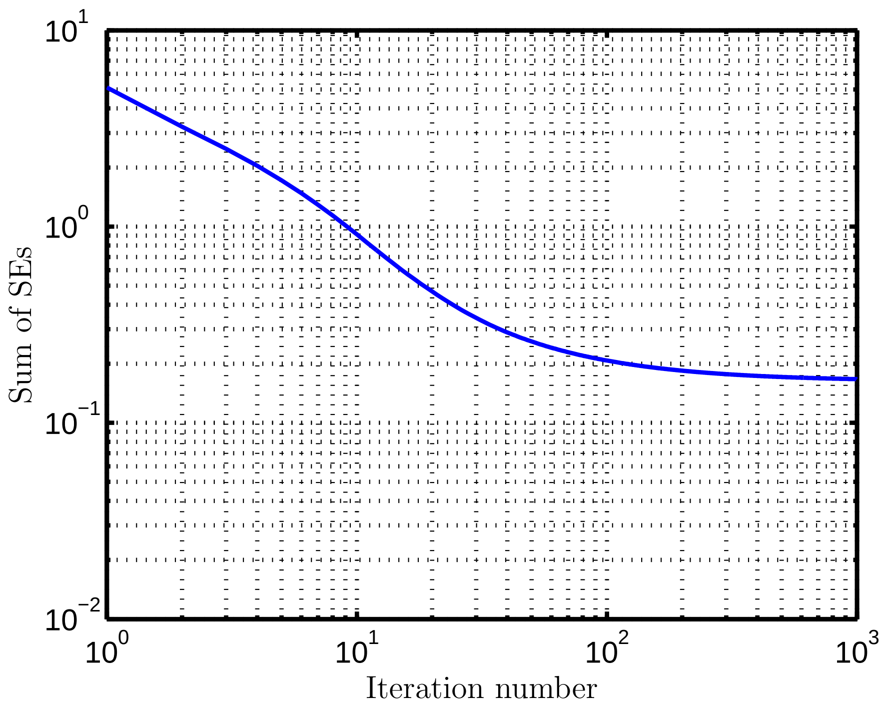

Figure 4.8. Average of the sum of SEs of different channel realizations versus the iteration number in scenario C4 without normalization of the variables

4.5 Summary

In this chapter, IA aided by multiple relays in a multi-pair two-way relay network has been considered. Based on the number of antennas at the relays, two different cases have been considered, namely, the relays with minimum number $QR = Kd$ of antennas and the relays with additional number $QR \geq Kd$ of antennas. In both cases, new schemes and algorithms to perform IA have been proposed. The properness conditions have also been derived. The following summarizes the main contributions and the conclusions of this chapter.

- For the case $QR = Kd$, SA and CA are necessary conditions. Similar to the single relay scenario, for the multiple relay scenario, after SA and CA, there are Kd effective data streams and Kd effective channels. However, for the multiple relay scenario, the relay processing matrix is a block diagonal matrix and hence, the relays alone cannot perform transceive ZF in the $QR = Kd$ dimensional relay space. The nodes need to jointly choose the SA and the CA directions such that the relay can perform transceive ZF with the block diagonal relay processing matrix. This is defined as CZF. SA and CA are linear problems and hence, closed form solutions are obtained. However, CZF is a trilinear problem, which is reformulated into a bilinear problem and an iterative algorithm to solve the bilinear problem has been proposed. The algorithm is guaranteed to converge at least to a local optimum. The nodes need at least $N \geq \frac{3Kd-R+2d}{4}$ antennas to perform SA, CA, and CZF along with the relay.

- For the case $QR \geq Kd$, it has been shown that a SA and CA based scheme results in a quad-linear problem which is more complex than the original trilinear IA problem where the transmit, relay and receive filters are jointly designed. Therefore, in this thesis, these three filters are designed iteratively by fixing two of the three filters and designing the third one. The algorithm is guaranteed to converge at least to a local minimum. The properness condition is given by $4KNd + QR^2 \geq 2K(2K-1)d^2$.

In both cases, the proposed IA algorithms achieve higher DoF and higher multiplexing gain than the reference scheme. Furthermore, through simulations it has been shown that the proposed IA algorithms achieve higher sum rate than the reference algorithm at high SNR.

Chapter 5

Optimization algorithms for low and medium SNR

5.1 Introduction

In this chapter, algorithms to optimize the useful signal power are proposed. IA aims at maximizing the DoF, i.e., increasing the number of data streams being transmitted simultaneously without interference. However, the power of the useful signal at the receiver is not taken into account. The signal power could be reduced due to several factors. First, the precoding matrices at the transmitters are designed for performing IA and the resulting vector directions might correspond to the channel directions with low singular value. Secondly, receive ZF at the relays and at the receivers and the transmit ZF at the relays may result in a reduction of the signal power. Thirdly, there is loss in the signal power when the signal is projected to a subspace during PSA and PCA. All these three factors lead to a reduction in the useful signal power. At high SNR, the noise power is negligible and after IA, the interference signals' power is zero and hence, IA is optimum. However, at low and medium SNR, suppressing only the interference signals is not sufficient as noise plays a significant role in these regions. So other algorithms that take into account the useful signals in addition to the interference signals are required. Two algorithms are developed in this chapter.

The first algorithm is an IA based algorithm. This algorithm is applicable when there are multiple IA solution and these solutions are known in closed form. In this case, the interference signals are completely suppressed through IA and out of all the available IA solutions, the solution that maximizes the SNR is chosen. Finding the IA solution that maximizes the SNR is a non-convex problem. In this thesis, a gradient based iterative algorithm is proposed.

The second algorithm is based on the minimum mean squared error (MMSE). In this algorithm, the transmit, the relay, and the receive filters are designed to minimize the MSE subject to the power constraints at the nodes and at the relays. In this case, the joint design of all the three filters results in a non-convex problem. However, in this thesis, we show that fixing two of these three filters results in a convex problem and an iterative algorithm to minimize the MSE is proposed.

This chapter is organized as follow: The IA based SNR maximization algorithm is introduced in Section 5.2. The MMSE based algorithm is described in Section 5.3. The sum rate performances of these two algorithms are investigated in Section 5.4. Section 5.5 summarizes the contents of this chapter.

5.2 Interference alignment with SNR maximization

In this section, an algorithm which in addition to IA also maximizes the SNR is proposed. This algorithm is applicable when there are multiple IA solutions and these solutions are available in closed form. As described in Chapter 3, a closed form solution for IA is achievable through SA, CA and transceive ZF or through PSA, PSA and transceive ZF.

In order to improve the signal power, in this thesis, out of all possible SA and CA solutions, the one with maximum receive useful signal power is chosen. This is done in two steps. The transmit filters are chosen to satisfy the SA conditions and maximize the SNR of the useful signals received at the relay in the MAC phase. Similarly, the receive filters are chosen to satisfy the CA conditions and maximize the SNR of the useful signals received at the receivers in the BC phase. As SA and CA are identical problems, we will consider only SA, i.e., the design of the transmit filters. In order to take into account the projection performed during the receive ZF denoted by the matrix $\mathbf{G}_{\text{rx}}$ in the Kd-dimensional relay subspace, the SNR of the useful signals is obtained after receive ZF at the relay.

In the following, first the expression for the SNR at the relay is derived. Then, an iterative gradient algorithm to find the SA solution that maximize the SNR is described.

As currently our focus is only on SA, we drop the index (v) in (3.51), which results in

$$\mathbf{V}_j = \mathbf{A}_j \mathbf{W} \tag{5.1}$$

for $j = 1, 2, \ldots, 2K$. Note that $\mathbf{W}$ chooses a basis for a d-dimensional subspace in the solution space given by the columns of the matrix $\mathbf{A}_j$. Each of the columns of $\mathbf{W}$ corresponds to one of the d data streams. In this thesis, we optimize the SA directions of each of the d data streams independently. Let $\mathbf{v}_j^l$ and $\mathbf{w}^l$ denote the l^{th} column of the matrices $\mathbf{V}_j$ and $\mathbf{W}$, respectively. Then the precoding direction of the l^{th} data stream is given by

$$\mathbf{v}_j^l = \mathbf{A}_j \mathbf{w}^l \tag{5.2}$$

for $l = 1, 2, \ldots, d$.

In the following, the expression for the SNR at the relay is derived. Let $\mathbf{g}_j^l$ denote the l^{th} column of the receive ZF matrix corresponding to the j^{th} pair at the relay. Due to the fact that the data streams from the communication partners j and k are aligned at the relay, we have $\mathbf{g}_j^l = \mathbf{g}_k^l$. The SNR of the l^{th} data stream from node j at the relay is given by

$$SNR_j^l = \frac{P_{\text{node}}}{(d\mathbf{v}_j^{l\text{H}}\mathbf{v}_j^l)} \frac{\mathbf{v}_j^{l\text{H}}\mathbf{H}_{\text{r}j}^{\text{H}}\mathbf{T}\mathbf{g}_j^l\mathbf{g}_j^{l\text{H}}\mathbf{T}^{\text{H}}\mathbf{H}_{\text{r}j}\mathbf{v}_j^l}{\text{E}\left[\mathbf{n}_1^{\text{H}}\mathbf{T}\mathbf{g}_j^l\mathbf{g}_j^{l\text{H}}\mathbf{T}^{\text{H}}\mathbf{n}_1\right]}. \tag{5.3}$$

The term $P_{\text{node}}/(d\mathbf{v}_j^{l\text{H}}\mathbf{v}_j^l)$ guarantees that after performing the optimization, the nodes transmit with maximum power P_{node}. Using the fact that $\mathbf{n}_1^{\text{H}}\mathbf{T}\mathbf{g}_j^l$ is a complex scalar, (5.3) can be rewritten as

$$SNR_j^l = \frac{P_{\text{node}}}{(d\mathbf{v}_j^{l\text{H}}\mathbf{v}_j^l)} \frac{\mathbf{v}_j^{l\text{H}}\mathbf{H}_{\text{r}j}^{\text{H}}\mathbf{T}\mathbf{g}_j^l\mathbf{g}_j^{l\text{H}}\mathbf{T}^{\text{H}}\mathbf{H}_{\text{r}j}\mathbf{v}_j^l}{\text{E}\left[\mathbf{g}_j^{l\text{H}}\mathbf{T}^{\text{H}}\mathbf{n}_1\mathbf{n}_1^{\text{H}}\mathbf{T}\mathbf{g}_j^l\right]}. \tag{5.4}$$

Without loss of generality, we assume the projection matrix to be an orthonormal matrix, i.e., $\mathbf{T}^{\text{H}}\mathbf{T} = \mathbf{I}$ and the columns of the receive ZF matrix to be normalized, i.e., $\mathbf{g}_j^{l\text{H}}\mathbf{g}_j = 1$. This results in

$$SNR_j^l = \frac{P_{\text{node}}}{d\sigma_1^2} \frac{\mathbf{v}_j^{l\text{H}}\mathbf{H}_{\text{r}j}^{\text{H}}\mathbf{T}\mathbf{g}_j^l\mathbf{g}_j^{l\text{H}}\mathbf{T}^{\text{H}}\mathbf{H}_{\text{r}j}\mathbf{v}_j^l}{\mathbf{v}_j^{l\text{H}}\mathbf{v}_j^l} \tag{5.5}$$

The optimization problem is to maximize the sum of the SNRs subject to the partial SA conditions of (5.2). Using (5.2), the optimization problem becomes

$$\max_{\mathbf{w}^1,\ldots,\mathbf{w}^d} \sum_{j=1}^{2K}\sum_{l=1}^{d} \frac{P_{\text{node}}}{d\sigma_1^2} \frac{\mathbf{w}^{l\text{H}}\mathbf{A}_j^{\text{H}}\mathbf{H}_{\text{r}j}^{\text{H}}\mathbf{T}\mathbf{g}_j^l\mathbf{g}_j^{l\text{H}}\mathbf{T}^{\text{H}}\mathbf{H}_{\text{r}j}\mathbf{A}_j\mathbf{w}^l}{\mathbf{w}^{l\text{H}}\mathbf{A}_j^{\text{H}}\mathbf{A}_j\mathbf{w}^l}. \tag{5.6}$$

In the following, the gradient based algorithm is described. The objective function in (5.6) is non-convex [BV04] in $\mathbf{w}^l$ for $l = 1, \ldots, d$ and a local solution is obtained iteratively using the gradient approach [BV04]. The initial value of $\mathbf{w}^l$ is chosen arbitrarily. Let $SSNR$ denote the sum of all the SNRs. The gradient of the objective function is calculated and the variable $\mathbf{w}^l$ is updated iteratively using the following relation:

$$\mathbf{w}^l \rightarrow \mathbf{w}^l + \alpha\frac{\partial SSNR}{\partial \mathbf{w}^{l*}} \tag{5.7}$$

where the parameter α controls the step size. The derivative can be obtained by calculating the derivative with respect to each element of $\mathbf{w}^{l*}$ [Dat05, Fel04]. The detailed derivation of the derivative is given in Appendix A.1. After each iteration, $\mathbf{w}^l$ and $\frac{\partial SSNR}{\partial \mathbf{w}^{l*}}$ are normalized to one so that the norm of $\mathbf{w}^l$ does not grow to a large value during the iterations. The algorithm is described in Algorithm 1.

Algorithm 1 Iterative gradient algorithm

1. Initialize α, $\mathbf{w}^{l*}$ for $l = 1, \dots, d$
2. $i = 0$
3. Calculate $\mathbf{T}$, $\mathbf{g}_j^l$ for $j = 1, \dots, 2K$ and $l = 1, \dots, d$
4. Calculate the derivative $\frac{\partial SSNR}{\partial \mathbf{w}^{l*}}$ for $l = 1, \dots, d$
5. Normalize $\frac{\partial SSNR}{\partial \mathbf{w}^{l*}}$ for $l = 1, \dots, d$
6. Update $\mathbf{w}^{l*}$,
$$\mathbf{w}^l \to \mathbf{w}^l + \alpha \frac{\partial SSNR}{\partial \mathbf{w}^{l*}}$$
7. Normalize $\mathbf{w}^{l*}$ for $l = 1, \dots, d$
8. $i = i + 1$
9. Repeat *Steps 3 to 8*, till $i =$ total number of iterations

---

Note that receive ZF is performed at the relay to spatially separate all the Kd effective data streams. This ensures that the d data streams from any node j are linearly independent of each other and, hence, the matrix $\mathbf{W}$ and the precoding matrices $\mathbf{V}_j$ are of full rank d.

5.3 Minimization of mean squared error

5.3.1 Introduction

In this section, an iterative algorithm to minimize the MSE subject to transmit power constraints at the nodes and at the relays is described. In this algorithm, IA is not guaranteed, however IA may implicitly take place especially at high SNR.

In the following, first the motivation for considering MMSE as the objective function is described. Then the optimization problem is formulated. Finally, an overview of the proposed iterative MMSE algorithm is described.

The motivation for considering MMSE is as follows: In an interference limited wireless communication system, increasing the sum rate makes it possible to increase the number of node pairs served using a given limited bandwidth. IA is a technique to achieve this at high SNR. At low and medium SNR, one shall perform IA with SNR maximization. However, this may not be optimal as for the optimum solution that maximizes the system throughput, IA may not be necessary. In addition, for the method described in Section 5.2, all the IA solutions need to be known in closed form. To overcome these two challenges a MMSE algorithm is considered. In MMSE algorithm, the mean of the squared error between the transmitted data symbols and the estimated data symbols is considered for the design of the transmit, the relay and the receive filters. Furthermore, the MMSE objective function is convex with respect to each of the three filters to be designed. Hence, the MSE is considered as the objective in this section.

In the following, first the optimization problem is introduced. From the system model described in Chapter 2, recollect that the estimated data symbols are given by

$$\hat{\mathbf{d}}_j = \mathbf{U}_k^{\mathrm{H}} \left(\widetilde{\mathbf{A}}_{jk}\mathbf{d}_j + \mathbf{e}_k + \widetilde{\mathbf{n}}_k \right) \tag{5.8}$$

for $j = 1, \ldots, 2K$, with

$$\widetilde{\mathbf{A}}_{jk} = \sum_{q=1}^{Q} \mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{jq}^{\mathrm{sr}} \mathbf{V}_j \tag{5.9}$$

and $\mathbf{e}_k$ is given by (2.25). The MSE of $\hat{\mathbf{d}}_j$ at receiver k is given by

$$MSE_k = \mathbb{E}\left\{ \|\hat{\mathbf{d}}_j - \mathbf{d}_j\|^2 \right\}. \tag{5.10}$$

(5.10) can be expressed as

$$\begin{aligned} MSE_k =& \mathrm{Tr}\left(\left(\mathbf{U}_k^{\mathrm{H}} \widetilde{\mathbf{A}}_{jk} - \mathbf{I} \right) \mathbf{R}_{\mathbf{d}_j} \left(\widetilde{\mathbf{A}}_{jk}^{\mathrm{H}} \mathbf{U}_k - \mathbf{I} \right) \right) + \\ & \mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}} \mathbb{E}\left\{ \mathbf{e}_k \mathbf{e}_k^{\mathrm{H}} \right\} \mathbf{U}_k \right) + \mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}} \mathbb{E}\left\{ \widetilde{\mathbf{n}}_k \widetilde{\mathbf{n}}_k^{\mathrm{H}} \right\} \mathbf{U}_k \right). \end{aligned} \tag{5.11}$$

Now the objective is to design the transmit filters, the receive filters and the filters at the relays such that the MSE is minimized subject to the power constraints at the nodes and at the relays. This is given by

$$\begin{aligned} \underset{\mathbf{V}_j, \mathbf{U}_k, \mathbf{G}_q}{\text{minimize}} \quad & \overline{MSE} = \sum_{k=1}^{2K} MSE_k \\ \text{subject to} \quad & \mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q \mathbf{G}^{\mathrm{H}} \right) \leq P_{\text{relay}} \\ & \mathrm{Tr}\left(\mathbf{V}_j \mathbf{R}_{\mathbf{d}_j} \mathbf{V}_j^{\mathrm{H}} \right) \leq P_{\text{node}} \;\; \text{for} \;\; j = 1, \ldots, 2K \end{aligned} \tag{5.12}$$

where

$$\mathbf{R}_Q = \mathbb{E}\left\{ \mathbf{r}\mathbf{r}^{\mathrm{H}} \right\} = \sum_{i=1}^{2K} \left(\mathbf{H}_i^{\mathrm{sr}} \mathbf{V}_i \right) \mathbf{R}_{\mathbf{d}_i} \left(\mathbf{H}_i^{\mathrm{sr}} \mathbf{V}_i \right)^{\mathrm{H}} + \mathbf{R}_{n1q}. \tag{5.13}$$

The optimization problem in (5.12) is non-convex [BV04].

In the following, we propose an iterative scheme to obtain a local minimum. The algorithm consist of three steps. First we arbitrarily fix the relay and the transmit filters and derive the optimum receive filters that minimize the MSE at the receivers. In the second step, we fix the receive and the transmit filters and using the Lagrange multiplier method, we derive the optimum relay filters that minimize the MSE subject to the power constraint at the relays. In the third step, we fix the receive and the relay filters and using the Lagrange multiplier method, we derive the optimum transmit filters. The receive filters, relay filters and the transmit filters are iteratively optimized until the algorithm converges to a local optimum. As explained in Chapter 1, a similar iterative MMSE algorithm has been proposed in [MXF+10] for one-way relaying.

5.3.2 Design of receive filters

In this subsection, for fixed transmit and relay filters, the optimum receive filters are derived in closed form. First, we initialize the transmit and relay filters arbitrarily. As the MSE_k involves only the receive filter $\mathbf{U}_k$ at receiver k, the receive filters can be optimized independently. For fixed transmit and relay filters, the optimization problem described in (5.12) is an unconstrained quadratic optimization problem. The optimum $\mathbf{U}_k$ is given by

$$\frac{\partial MSE_k}{\partial \mathbf{U}_k^*} \stackrel{!}{=} \mathbf{0}. \tag{5.14}$$

Substituting (5.12) in (5.14), the optimum $\mathbf{U}_k$ that minimizes $\overline{MSE}$ is given by

$$\mathbf{U}_k = \left[\widetilde{A}_{jk}\mathbf{R}_{\mathbf{d}_j}\widetilde{A}_{jk}^{\mathrm{H}} + \mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\} + \mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}\right]^{-1}\widetilde{A}_{jk}\mathbf{R}_{\mathbf{d}_j}, \tag{5.15}$$

where

$$\mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\} = \sum_{\substack{i=1\\ i\neq j,k}}^{2K}\left(\mathbf{H}_k^{\mathrm{rd}}\mathbf{G}\mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i\right)\mathbf{R}_{\mathbf{d}_i}\left(\mathbf{H}_k^{\mathrm{rd}}\mathbf{G}\mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i\right)^{\mathrm{H}} \tag{5.16}$$

$$\mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\} = \left(\mathbf{H}_k^{\mathrm{rd}}\mathbf{G}\right)\mathbf{R}_{n1}\left(\mathbf{H}_k^{\mathrm{rd}}\mathbf{G}\right)^{\mathrm{H}} + \mathbf{R}_{n2k}. \tag{5.17}$$

and

$$\mathbf{R}_{\widetilde{\mathbf{n}}_k} = \mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}. \tag{5.18}$$

5.3.3 Design of relay filters

In this subsection, for fixed receive and transmit filters, using the Lagrange multiplier method, the optimum relay filters subject to the power constraint at the relays are derived. For fixed receive and transmit filters, the optimization problem of (5.12) is a quadratically constrained quadratic minimization problem. This is a convex problem whose optimum can be obtained using the Lagrange multiplier method.

In the following, first starting from the Lagrangian function, the Karush-Kuhn-Tucker (KKT) conditions are obtained. Then the closed form solution for the relay filters is obtained as a function of the Langrange multiplier. Finally, the Lagrange multiplier is determined.

The Lagrangian function is given by

$$L(\mathbf{G},\lambda) = \overline{MSE} + \lambda\left(\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{relay}}\right) \tag{5.19}$$

where λ denotes the Lagrange multiplier. Substituting $\overline{MSE}$ in (5.19) using (5.12) and (5.11) results in

$$\begin{aligned} L(\mathbf{G},\lambda) =& \mathrm{Tr}\left[\mathbf{G}^{\mathrm{H}}\sum_{k=1}^{2K}\left(\mathbf{F}_k^{(2)}\mathbf{G}\mathbf{F}_k^{(1)} - \mathbf{F}_{jk}^{(3)} + \mathbf{F}_k^{(2)}\mathbf{G}\mathbf{R}_{\mathrm{n}1}\right)\right] \\ &+ \mathrm{Tr}\left[\mathbf{G}^{\mathrm{H}}\lambda\mathbf{G}\mathbf{R}_Q\right] + \mathrm{Tr}\left[\mathbf{C}\right], \end{aligned} \tag{5.20}$$

where

$$\mathbf{F}_k^{(1)} = \sum_{i=1,\ i\neq k}^{2K}\mathbf{H}_{i\mathrm{r}}\mathbf{V}_i\mathbf{R}_{\mathbf{dd}i}\mathbf{V}_i^{\mathrm{H}}\mathbf{H}_{i\mathrm{r}}^{\mathrm{H}}, \quad \mathbf{F}_k^{(2)} = \mathbf{H}_{\mathrm{r}k}^{\mathrm{H}}\mathbf{U}_k\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{\mathrm{r}k},$$

$$\mathbf{F}_{jk}^{(3)} = \mathbf{H}_{\mathrm{r}k}^{\mathrm{H}}\mathbf{U}_k\mathbf{R}_{\mathbf{dd}j}\mathbf{V}_j^{\mathrm{H}}\mathbf{H}_{j\mathrm{r}}^{\mathrm{H}}, \quad \mathbf{R}_{\mathrm{n}1} = \mathbb{E}\left\{\mathbf{n}_1\mathbf{n}_1^{\mathrm{H}}\right\}, \tag{5.21}$$

and $\mathbf{C}$ consists of the terms independent of $\mathbf{G}^*$. The optimum $\mathbf{G}$ and λ satisfy the KKT conditions given by

$$\frac{\partial L(\mathbf{G},\lambda)}{\partial\mathbf{G}^*} = 0, \tag{5.22}$$

$$\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) \le P_{\mathrm{relay}}, \tag{5.23}$$

$$\lambda\left(\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{relay}}\right) = 0, \tag{5.24}$$

$$\lambda \ge 0. \tag{5.25}$$

In the following, the closed form solution for the relay filters is derived as a function of λ. Since $\mathbf{G}$ is a block diagonal matrix, in (5.22), the partial derivative is taken only with respect to the block diagonal elements of $\mathbf{G}$. Let $\mathbf{B}$ denote a block diagonal matrix of same size and same block diagonal structure as $\mathbf{G}$, with all the block diagonal elements being equal to one. Then (5.22) implies

$$\mathbf{B}\circ\left(\sum_{k=1}^{2K}\left(\mathbf{F}_k^{(2)}\mathbf{G}\mathbf{F}_k^{(1)} - \mathbf{F}_{jk}^{(3)} + \mathbf{F}_{2k}^{(2)}\mathbf{G}\mathbf{R}_{\mathrm{n}1}\right) + \lambda\mathbf{G}\mathbf{R}_Q\right) = \mathbf{0}, \tag{5.26}$$

where $\mathbf{A} \circ \mathbf{B}$ denotes the Hadamard product of $\mathbf{A}$ and $\mathbf{B}$. With $\mathbf{F}_k^{(4)} = \mathbf{F}_k^{(1)} + \mathbf{R}_{\mathrm{n}1}$ and $\mathbf{F} = \sum_{k=1}^{2K} \mathbf{F}_{jk}^{(3)}$, (5.26) can be written as

$$\mathbf{B} \circ \left(\sum_{k=1}^{2K} \mathbf{F}_k^{(2)} \mathbf{G} \mathbf{F}_k^{(4)} + \lambda \mathbf{G} \mathbf{R}_Q \right) = \mathbf{B} \circ \mathbf{F}. \tag{5.27}$$

In (5.27), $\mathbf{F}_k^{(2)}$, $\mathbf{F}_k^{(4)}$, $\mathbf{R}_Q$, and $\mathbf{F}$ are matrices of dimension $QR \times QR$. Each of these matrices is composed of Q^2 block matrices of dimension $R \times R$ each. Let $\mathbf{F}_k^{(2)l,q}$, $\mathbf{F}_k^{(4)l,q}$, $\mathbf{R}_Q^{l,q}$ and $\mathbf{F}^{l,q}$ denote the $(l,q)^{\text{th}}$ block of the matrices $\mathbf{F}_k^{(2)}$, $\mathbf{F}_k^{(4)}$, $\mathbf{R}_Q$, and $\mathbf{F}$, respectively. Then (5.27) becomes

$$\sum_{q=1}^{Q} \sum_{k=1}^{2K} \mathbf{F}_k^{(2)l,q} \mathbf{G}_q \mathbf{F}_k^{(4)q,l} + \lambda \mathbf{G}_l \mathbf{R}_Q^{l,l} = \mathbf{F}^{l,l} \tag{5.28}$$

for $l = 1, \ldots, Q$. With $\mathbf{X}_q^l = \sum_{k=1}^{2K} \mathbf{F}_k^{(4)q,l\mathrm{T}} \otimes \mathbf{F}_k^{(2)l,q}$ and $\mathbf{Y}^l = \mathbf{R}_Q^{l,l\mathrm{T}} \otimes \mathbf{I}$, vectorizing (5.28) we get

$$\sum_{q=1}^{Q} \mathbf{X}_q^l \mathrm{vec}\left(\mathbf{G}_q\right) + \lambda \mathbf{Y}^l \mathrm{vec}\left(\mathbf{G}_l\right) = \mathrm{vec}\left(\mathbf{F}^{l,l}\right). \tag{5.29}$$

Let $\mathbf{X}$ denote a block matrix whose $(l,q)^{\text{th}}$ block is $\mathbf{X}_q^l$ and $\mathbf{Y}$ denote a block diagonal matrix whose $(l,l)^{\text{th}}$ block is $\mathbf{Y}^l$. Also, let $\mathbf{f} = \left[\mathrm{vec}\left(\mathbf{F}^{1,1}\right)^{\mathrm{T}} \ \ldots \ \mathrm{vec}\left(\mathbf{F}^{Q,Q}\right)^{\mathrm{T}} \right]^{\mathrm{T}}$ and $\mathbf{g} = \left[\mathrm{vec}\left(\mathbf{G}_1\right)^{\mathrm{T}} \ \ldots \ \mathrm{vec}\left(\mathbf{G}_Q\right)^{\mathrm{T}} \right]^{\mathrm{T}}$. Then (5.29) becomes

$$\left(\mathbf{X} + \lambda \mathbf{Y}\right) \mathbf{g} = \mathbf{f}. \tag{5.30}$$

From the above equation, we get

$$\mathbf{g} = \left(\mathbf{X} + \lambda \mathbf{Y}\right)^{-1} \mathbf{f}. \tag{5.31}$$

Now we have the optimum $\mathbf{G}$ in closed form. However, λ needs to be determined such that (5.23), (5.24) and (5.25) are satisfied [MXF+10]. From (5.25), $\lambda \geq 0$. First set $\lambda = 0$. If (5.23) is satisfied, then the optimum λ is equal to zero. If (5.23) is not satisfied, then $\lambda > 0$. Hence, in order to satisfy (5.23) and (5.24), the condition

$$\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{relay}} = 0. \tag{5.32}$$

needs to hold. In the following, we describe a method to find the optimum λ. This method is a similar to that of the method described in [MXF+10] for one-way relaying. From (5.31), we know that $\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{relay}}$ is a decreasing function of λ. Using the fact that $\mathbf{X}$ is a positive semidefinite matrix and setting $\mathbf{X} = \mathbf{0}$, it can be proven that λ is bounded by

$$0 \leq \lambda_{\mathrm{opt}} \leq \lambda_{\mathrm{up}}^{(\mathrm{G})} = \sqrt{\frac{\mathbf{f}^{\mathrm{H}} \mathbf{Y}^{-1} \mathbf{f}}{P_{\mathrm{relay}}}}. \tag{5.33}$$

The detailed proof is given in Appendix A.2. Given $\lambda_{\mathrm{up}}^{(\mathrm{G})}$, (5.32) can be solved using the bisection algorithm.

5.3.4 Design of transmit filters

In this subsection, for fixed receive and relay filters, using the Lagrange multiplier method, the transmit filters are designed. For fixed receive and relay filters, the optimization problem in (5.12) becomes a quadratic minimization problem with quadratic constraint and is convex [BV04]. Note that the transmit filters are involved in both the node power constraint and the relay power constraint.

In the following, first starting from the Lagrangian function, the KKT conditions are obtained. Then the closed form solution for the transmit filters is obtained as a function of the Langrange multipliers. Finally, the Lagrange multipliers are determined using a new quadsection algorithm proposed in the following.

The Lagrangian function is given by

$$L\left(\mathbf{V}_{\bar{j}},\lambda,\mu\right)=\overline{MSE}+\lambda\left(\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_{Q}\mathbf{G}^{\mathrm{H}}\right)-P_{\mathrm{relay}}\right)+\mu\left(\mathrm{Tr}\left(\mathbf{V}_{\bar{j}}\mathbf{R}_{d_{\bar{j}}}\mathbf{G}_{\bar{j}}^{\mathrm{H}}\right)-P_{\mathrm{node}}\right) \tag{5.34}$$

where λ and μ are Lagrange multipliers. Substituting (5.11) in (5.34) results in

$$\begin{aligned}L\left(\mathbf{V}_{\bar{j}},\lambda,\mu\right)=\mathrm{Tr}\left(\left(\mathbf{U}_k^{\mathrm{H}}\widetilde{\mathbf{A}}_{jk}-\mathbf{I}\right)\mathbf{R}_{\mathbf{d}_j}\left(\widetilde{\mathbf{A}}_{jk}^{\mathrm{H}}\mathbf{U}_k-\mathbf{I}\right)\right)+\mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}}\mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\}\mathbf{U}_k\right)+\\ \mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}}\mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}\mathbf{U}_k\right)+\lambda\left(\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right)-P_{\mathrm{relay}}\right)+\mu\left(\mathrm{Tr}\left(\mathbf{V}_{\bar{j}}\mathbf{R}_{d_{\bar{j}}}\mathbf{G}_{\bar{j}}^{\mathrm{H}}\right)-P_{\mathrm{node}}\right).\end{aligned} \tag{5.35}$$

In the above equation, substituting for $\widetilde{\mathbf{A}}_{jk}$, $\mathbf{e}_k$, $\widetilde{\mathbf{n}}_k$, and using the property Tr($\mathbf{XYZ}$) = Tr($\mathbf{ZXY}$) we get

$$\begin{aligned}L\left(\mathbf{V}_{\bar{j}},\lambda,\mu\right) &= \mathrm{Tr}\left(\sum_{k=1}^{2K}\sum_{i=1,i\neq k}^{2K}\left(\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rd}}\mathbf{G}\mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i\right)^{\mathrm{H}}\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rd}}\mathbf{G}\mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i\mathbf{R}_{d_i}\right)\\ &- \mathrm{Tr}\left(\sum_{k=1}^{2K}\mathbf{V}_j^{\mathrm{H}}\mathbf{H}_j^{\mathrm{sr}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rd}}\mathbf{U}_k\mathbf{R}_{d_{\bar{j}}}\right)+\lambda\mathrm{Tr}\left(\sum_{i=1}^{2K}\left(\mathbf{G}\mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i\right)^{\mathrm{H}}\mathbf{G}\mathbf{H}_i^{\mathrm{sr}}\mathbf{V}_i\mathbf{R}_{d_{\bar{j}}}\right)\\ &+ \mu\mathrm{Tr}\left(\mathbf{V}_{\bar{j}}^{\mathrm{H}}\mathbf{V}_{\bar{j}}\mathbf{R}_{d_{\bar{j}}}\right)+\mathbf{C}_{\mathrm{const}}\end{aligned} \tag{5.36}$$

where $\mathbf{C}_{\mathrm{const}}$ consists of the terms indpendent of the transmit filters. Let $\bar{k}$ be the communication partner of node $\bar{j}$. Then the partial derivative of $L\left(\mathbf{V}_{\bar{j}},\lambda,\mu\right)$ with respect to $\mathbf{V}_{\bar{j}}^*$ is given by

$$\begin{aligned}\frac{\partial L\left(\mathbf{V}_{\bar{j}},\lambda,\mu\right)}{\partial\mathbf{V}_{\bar{j}}^*} &= \sum_{k=1,k\neq\bar{j}}^{2K}\mathbf{H}_{\bar{j}}^{\mathrm{srH}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rdH}}\mathbf{U}_k\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{rd}}\mathbf{G}\mathbf{H}_{\bar{j}}^{\mathrm{sr}}\mathbf{V}_{\bar{j}}\mathbf{R}_{d_{\bar{j}}}-\mathbf{H}_{\bar{j}}^{\mathrm{sr}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_{\bar{k}}^{\mathrm{rd}}\mathbf{U}_{\bar{k}}\mathbf{R}_{d_{\bar{j}}}\\ &+ \lambda\mathbf{H}_{\bar{j}}^{\mathrm{srH}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{H}_{\bar{j}}^{\mathrm{sr}}\mathbf{V}_{\bar{j}}\mathbf{R}_{d_{\bar{j}}}+\mu\mathbf{V}_{\bar{j}}\mathbf{R}_{d_{\bar{j}}}.\end{aligned} \tag{5.37}$$

Let

$$\mathbf{X}_{(v)} = \sum_{k=1,k\neq\overline{j}}^{2K} \mathbf{H}_{\overline{j}}^{\mathrm{srH}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_{k}^{\mathrm{rdH}}\mathbf{U}_{k}\mathbf{U}_{k}^{\mathrm{H}}\mathbf{H}_{k}^{\mathrm{rd}}\mathbf{G}\mathbf{H}_{\overline{j}}^{\mathrm{sr}}, \tag{5.38}$$

$$\mathbf{Y}_{(v)} = \mathbf{H}_{\overline{j}}^{\mathrm{srH}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{H}_{\overline{j}}^{\mathrm{sr}}, \tag{5.39}$$

$$\mathbf{F}_{(v)} = \mathbf{H}_{\overline{j}}^{\mathrm{sr}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_{\overline{k}}^{\mathrm{rd}}\mathbf{U}_{\overline{k}}. \tag{5.40}$$

Then (5.37) becomes

$$\frac{\partial L\left(\mathbf{V}_{\overline{j}},\lambda,\mu\right)}{\partial \mathbf{V}_{\overline{j}}^{*}} = \left(\mathbf{X}_{(v)} + \lambda\mathbf{Y}_{(v)} + \mu\mathbf{I}\right)\mathbf{V}_{\overline{j}}\mathbf{R}_{d_{\overline{j}}} - \mathbf{F}_{(v)}\mathbf{R}_{d_{\overline{j}}}. \tag{5.41}$$

The optimum $\mathbf{V}_{\overline{j}}$, λ, and μ for node $\overline{j}$ satisfy the KKT conditions given by

$$\frac{\partial L\left(\mathbf{V}_{\overline{j}},\lambda,\mu\right)}{\partial \mathbf{V}_{\overline{j}}^{*}} = \mathbf{0}, \tag{5.42}$$

$$\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_{Q}\mathbf{G}^{\mathrm{H}}\right) \leq P_{\mathrm{relay}}, \tag{5.43}$$

$$\lambda\left(\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_{Q}\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{relay}}\right) = 0, \tag{5.44}$$

$$\lambda \geq 0, \tag{5.45}$$

$$\mathrm{Tr}\left(\mathbf{V}_{\overline{j}}\mathbf{R}_{d_{\overline{j}}}\mathbf{V}_{\overline{j}}^{\mathrm{H}}\right) \leq P_{\mathrm{node}}, \tag{5.46}$$

$$\mu\left(\mathrm{Tr}\left(\mathbf{V}_{\overline{j}}\mathbf{R}_{d_{\overline{j}}}\mathbf{V}_{\overline{j}}^{\mathrm{H}}\right) - P_{\mathrm{node}}\right) = 0, \tag{5.47}$$

$$\mu \geq 0. \tag{5.48}$$

Using (5.41), the first KKT condition becomes

$$\left(\mathbf{X}_{(v)} + \lambda\mathbf{Y}_{(v)} + \mu\mathbf{I}\right)\mathbf{V}_{\overline{j}}\mathbf{R}_{d_{\overline{j}}} = \mathbf{F}_{(v)}\mathbf{R}_{d_{\overline{j}}}. \tag{5.49}$$

Hence, the closed form solution for $\mathbf{V}_{\overline{j}}$ is given by

$$\mathbf{V}_{\overline{j}} = \left(\mathbf{X}_{(v)} + \lambda\mathbf{Y}_{(v)} + \mu\mathbf{I}\right)^{-1}\mathbf{F}_{(v)}. \tag{5.50}$$

Now we need to find λ and μ that satisfy the conditions given by (5.43) - (5.48).

In the following, an algorithm to find the optimum λ and μ is described. Let λ_{opt} and μ_{opt} denote the optimum λ and μ, respectively. Let $\phi(\lambda,\mu) = \mathrm{Tr}\left(\mathbf{G}\mathbf{R}_{Q}\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{relay}}$ and $\psi(\lambda,\mu) = \mathrm{Tr}\left(\mathbf{V}_{\overline{j}}\mathbf{R}_{d_{\overline{j}}}\mathbf{V}_{\overline{j}}^{\mathrm{H}}\right) - P_{\mathrm{node}}$. Note that the functions $\phi(\lambda,\mu)$ and $\psi(\lambda,\mu)$ are decreasing in each of the variables λ and μ. Since the optimization problem is convex, there is a unique minimum. The optimum λ and μ can be one of the four values as follow:

- Case 1: $\lambda = 0$ and $\mu = 0$. In this case, if the conditions (5.43) and (5.46) are satisfied, then $\lambda_{\text{opt}} = 0$ and $\mu_{\text{opt}} = 0$ satisfy the conditions (5.43) - (5.48).

- Case 2: $\lambda = 0$ and $\mu > 0$. Here μ needs to be determined. Because $\mu \neq 0$, (5.46) need to be satisified with equality sign so that (5.47) can be satisfied, i.e., $\psi(0, \mu) = 0$. The upper bound for μ can be found by setting the positive semi-definite matrix $\mathbf{X}_{(v)} = 0$. Therefore μ_{opt} is bounded by

$$0 \leq \mu_{\text{opt}} \leq \mu_{\text{u}}^{(\text{V})} = \sqrt{\frac{\text{Tr}\left(\mathbf{F}_{(v)}^{\text{H}} \mathbf{R}_{d_{\bar{j}}} \mathbf{F}_{(v)}\right)}{P_{\text{node}}}}. \tag{5.51}$$

The derivation of the upper bound μ_{u} is identical to the derivation of $\lambda_{\text{up}}^{(\text{G})}$ for the relay filter in Appendix A.2. We know that $\psi(\lambda, \mu)$ is a monotonically decreasing function. Through bisection, the point $\mu = \mu_1$ can be found where $\psi(0, \mu_1) = 0$. Now if $\phi(0, \mu_1) \leq 0$, i.e., if (5.43) is satisfed, then $\lambda_{\text{opt}} = 0$ and $\mu_{\text{opt}} = \mu_1$.

- Case 3: $\lambda > 0$ and $\mu = 0$. Here λ needs to be determined. Because $\lambda \neq 0$, (5.43) need to be satisified with equality sign so that (5.44) can be satisfied i.e., $\phi(\lambda, 0) = 0$. The upper bound for λ can be found by setting the positive semi-definite matrix $\mathbf{X}_{(v)} = 0$. Therefore λ_{opt} is bounded by

$$0 \leq \lambda_{\text{opt}} \leq \lambda_{\text{u}}^{(\text{V})} = \sqrt{\frac{\text{Tr}\left(\mathbf{F}_{(v)}^{\text{H}} \left(\mathbf{Y}_{(v)}^{-1}\right)^{\text{H}} \mathbf{R}_{d_{\bar{j}}} \mathbf{F}_{(v)}\right)}{P_{\text{relay}} - \mathbf{Y}_o}} \tag{5.52}$$

where $\mathbf{Y}_o = \text{Tr}\left(\sum_{l=1, l\neq i}^{2K} \mathbf{G}\mathbf{H}_l^{\text{sr}} \mathbf{V}_l \mathbf{R}_{\mathbf{d}_l} \mathbf{V}_l^{\text{H}} \mathbf{H}_l^{\text{srH}} \mathbf{G}^{\text{H}} + \mathbf{G}\text{E}\{\mathbf{n}_1 \mathbf{n}_1^{\text{H}}\}\mathbf{G}^{\text{H}}\right)$. The derivation of the upper bound λ_{u} is identical to the derivation of $\lambda_{\text{up}}^{(\text{G})}$ for the relay filter in Appendix A.2. $\phi(\lambda, \mu)$ is a monotonically decreasing function. Through bisection, the point $\lambda = \lambda_1$ where $\phi(\lambda_1, 0) = 0$ can be found. Now if $\psi(\lambda_1, 0) \leq 0$, i.e., if (5.46) is satisfied, then $\lambda_{\text{opt}} = \lambda_1$ and $\mu_{\text{opt}} = 0$.

- Case 4: $\lambda > 0$ and $\mu > 0$. In this case, both the node and the relay power constraints need to be satisfied with equality sign i.e., $\phi(\lambda, \mu) = 0$ and $\psi(\lambda, \mu) = 0$. In order to find the λ_{opt} and μ_{opt} consider the following axes: Let the x-axis correspond to λ and let the y-axis correspond to μ. Then the amplitude of the functions $\phi(\lambda, \mu)$ and $\psi(\lambda, \mu)$ can be plotted along the z-axis. This will result in two planes. Let us name them as the relay plane and the node plane. Let $\omega = (\lambda, \mu)$. At $\omega_{\text{opt}} = (\lambda_{\text{opt}}, \mu_{\text{opt}})$, the relay and the node plane will meet the $z = 0$ plane. Figure 5.1 shows an example where these planes meet at $\omega_{\text{opt}} = (\lambda_{\text{opt}}, \mu_{\text{opt}})$. For ease of understanding, the $z = 0$ plane is plotted in Figure 5.2. The two curves in Figure 5.2 show the intersection of the relay plane

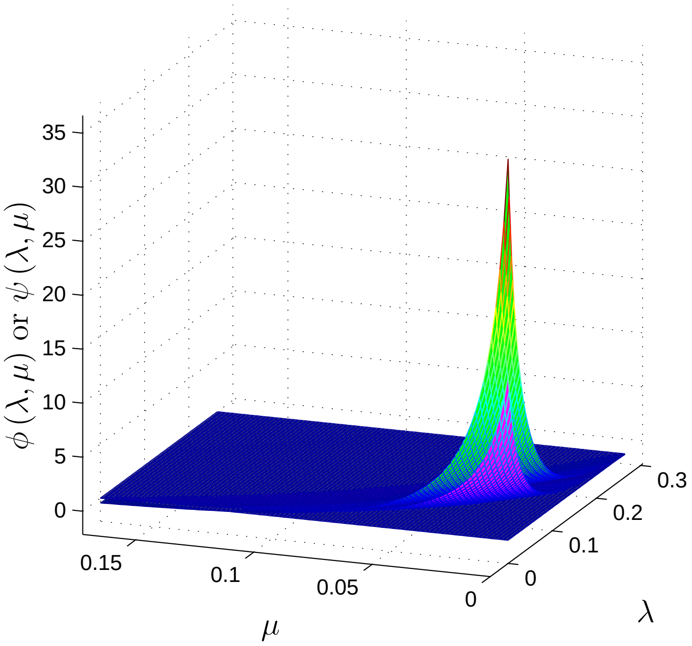

Figure 5.1. Relay plane, node plane and $z = 0$ plane in 3D plot

and the node plane with the $z = 0$ plane, say relay line and node line. For each of the two lines, the area between the origin and the line corresponds to the points ω where the functions $\phi(\lambda, \mu)$ and $\psi(\lambda, \mu)$, respectively, are greater than zero. At the points beyond the lines, these functions are smaller than zero. Along the lines, the functions are zero. Hence, we are interested in finding the point of intersection between these two lines.

In the following, we describe a quadsection algorithm, which is a generalization of bisection algorithm to planes. First, let the intersection of the node line with the $\lambda = 0$ and $\mu = 0$ lines be denoted by μ_1 and λ_2, respectively. Similarly, let the intersection of the relay line with the $\lambda = 0$ and $\mu = 0$ lines be denoted by μ_2 and λ_1, respectively. Note that the points λ_1 and μ_1 have already been calculated in Case 3 and Case 2, respectively, in this section. Similarly, λ_2 and μ_2 can be calculated. Secondly, take four points in the four regions created by the intersection of the relay and the node lines as follow: $\mathbf{x}_1 = (0, 0)$, $\mathbf{x}_2 = (0, (\mu_1 + \mu_2)/2)$, $\mathbf{x}_3 = (\max(\lambda_1, \lambda_2), \max(\mu_1, \mu_2))$, and $\mathbf{x}_4 = ((\lambda_1 + \lambda_2)/2, 0)$. Finally, similar to the bisection algorithm, these points are iteratively updated by taking the mean with their neighbouring points. This is described in Algorithm 2. The proposed quadsection algorithm halves the search space after every iteration. Also, note that out of the four cases described above, the steps involved in case c for $c = 1, 2, 3, 4$ are performed if the optimum is not found in all the previous

Algorithm 2 Quadsection search

while $\delta > \epsilon$ **do**
  $\mathbf{x}_{12} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}$
  if $\text{sign}(\phi(\mathbf{x}_1) * \psi(\mathbf{x}_1)) = \text{sign}(\phi(\mathbf{x}_{12}) * \psi(\mathbf{x}_{12}))$ **then**
    $\mathbf{x}_1 \leftarrow \mathbf{x}_{12}$
  else
    $\mathbf{x}_2 \leftarrow \mathbf{x}_{12}$
  end if $\mathbf{x}_{14} = \frac{\mathbf{x}_1 + \mathbf{x}_4}{2}$
  if $\text{sign}(\phi(\mathbf{x}_1) * \psi(\mathbf{x}_1)) = \text{sign}(\phi(\mathbf{x}_{14}) * \psi(\mathbf{x}_{14}))$ **then**
    $\mathbf{x}_1 \leftarrow \mathbf{x}_{14}$
  else
    $\mathbf{x}_4 \leftarrow \mathbf{x}_{14}$
  end if $\mathbf{x}_{32} = \frac{\mathbf{x}_3 + \mathbf{x}_2}{2}$
  if $\text{sign}(\phi(\mathbf{x}_3) * \psi(\mathbf{x}_3)) = \text{sign}(\phi(\mathbf{x}_{32}) * \psi(\mathbf{x}_{32}))$ **then**
    $\mathbf{x}_3 \leftarrow \mathbf{x}_{32}$
  else
    $\mathbf{x}_2 \leftarrow \mathbf{x}_{32}$
  end if $\mathbf{x}_{34} = \frac{\mathbf{x}_3 + \mathbf{x}_4}{2}$
  if $\text{sign}(\phi(\mathbf{x}_3) * \psi(\mathbf{x}_3)) = \text{sign}(\phi(\mathbf{x}_{34}) * \psi(\mathbf{x}_{34}))$ **then**
    $\mathbf{x}_3 \leftarrow \mathbf{x}_{34}$
  else
    $\mathbf{x}_4 \leftarrow \mathbf{x}_{34}$
  end if
end while
return $\delta = ||\mathbf{x}_1 - \mathbf{x}_3|| + ||\mathbf{x}_2 - \mathbf{x}_4||$
return $\mathbf{x} = (\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3 + \mathbf{x}_4)/4$

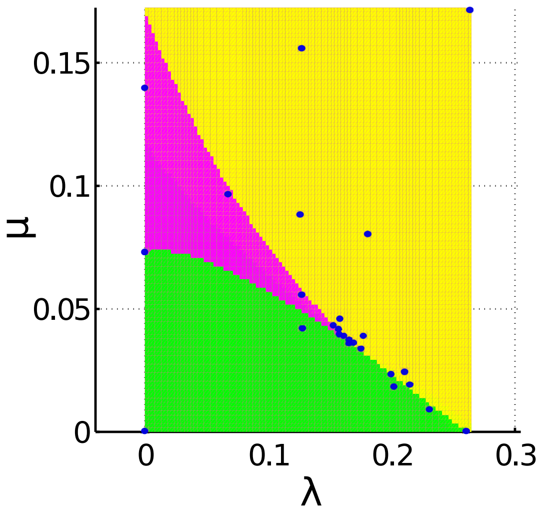

Figure 5.2. Relay plane, node plane and $z = 0$ plane in 2D plot

$c - 1$ cases. At the end of the iterations, the distance between the points $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, and $\mathbf{x}_4$ converges to a very small value and $\mathbf{x} = (\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3 + \mathbf{x}_4)/4$ gives λ_{opt} and μ_{opt}, which satisfies both the relay and the node power constraints. Hence, the optimum transmitter filter is found. The blue dots in Figure 5.2 show the position of $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, and $\mathbf{x}_4$ during the iterations. It can be seen that after a few iterations, $\mathbf{x}_i$ for $i = 1, 2, 3, 4$ converges to the point where the relay, the node and the $z = 0$ planes intersect i.e, where the magenta, green, and yellow planes intersect.

The receive, the relay, and the transmit filters are optimized iteratively either till the MSE does not change significantly or till a specified number of iterations is reached. Since at each iteration step, the MSE is reduced, the algorithm is guaranteed to converge to a minimum, though not necessarily to a global minimum.

5.4 Performance analysis

5.4.1 Introduction

In this section, the sum rate performance and the convergence of the proposed SNR maximization (maxSNR) algorithm and the MMSE algorithm are investigated. First, the sum rate achieved by the maxSNR and MMSE algorithms proposed in this chapter are compared with the IA algorithms proposed in this thesis and the reference algorithms described in Chapter 3 and Chapter 4. Recollect that the maxSNR algorithm can be applied only when the IA solutions are known in closed form. However, the MMSE scheme can be applied to any scenario. Then, the convergence of the proposed algorithms is investigated.

5.4.2 Sum rate analysis

In this section, the sum rate performances of the proposed maxSNR and MMSE algorithms are compared with the proposed IA algorithms and the reference algorithms intoduced in Section 3.2.5.2 and in Section 4.4.2. In this section, the same assumptions as in Section 3.2.5.4 regarding the channel model, number of node pairs, power allocation and channel realizations are made. Let $P_{\text{node}} = P$ denote the power of each of the $2K$ nodes. Let $P_{\text{relay}} = KP$ denote the total power available at the relays. The noise power at each node and at the relays is assumed to be the same and is denoted by $\sigma_k^2 = \sigma_{\text{relay}}^2 = \sigma^2$.

Table 5.1 shows the scenarios considered for the performance analysis. A single relay is considered in B3 and B4. Multiple relays are considered in C1 and C4. Note that these scenarios have been considered either in Chapter 3 or in Chapter 4 for the performance analysis of the IA schemes proposed in the corresponding chapters.

In the following, first the sum rate performances of the proposed algorithms and the reference ZF algorithm are investigated in scenarios B3 and B4 where a single relay is considered. Secondly, the sum rate performances of the proposed algorithms and the reference GOF algorithm are investigated in scenarios C1 and C4.

A single relay is considered in the scenarios B3 and B4. Both scenarios B3 and B4 have the same numbers N and R. However, the number of node pairs simultaneously served in each scenario is different. In scenario B3, $K = 7$ node pairs are simultaneously

Table 5.1. Scenarios considered and DoF achieved

Scenarios	Q	R	d	N	K		DoF	
					ZF / GOF	IA	ZF / GOF	IA
B3	1	9	1	3	5	7	5	7
B4	1	9	1	3	5	6	5	6
C1	2	2	1	3	1	4	1	4
C4	2	4	1	3	3	5	3	5

served. An IA solution can be found using the IA_PSA_Itr algorithm. However, by serving only $K = 6$ node pairs, in scenario B4, a closed form solution is feasible using the IA_PSA_M2 algorithm. In addition, multiple IA solutions are feasible and all these solutions are available in closed form. Hence, the proposed maxSNR algorithm can be used to find an IA solution that maximizes the SNR. Note that the proposed maxSNR algorithm finds only a local maximum. Figure 5.3 shows the sum rate in bits per channel use as a function of P/σ^2. It can be seen that maxSNR achieves higher sum rate than the IA_PSA_Itr, the IA_PSA_M2 and the reference ZF algorithms. Note that since only$K = 6$ nodes pairs are simultaneously served, the slope of maxSNR corresponds to the slope of IA_PSA_M2 in B4. However, the absolute value of the sum rate achieved by the maxSNR algorithm is even better than the IA_PSA_Itr, the IA_PSA_M2 and the reference ZF algorithm in both B3 and B4 at low and medium SNR. Figure 5.4 shows the sum rate achieved by the MMSE algorithm in B3 and B4, in comparison to the maxSNR algorithm. It can be seen from Figure 5.4 that the maxSNR algorithm achieves a higher sum rate than the MMSE algorithm especially at high SNR. This can be explained at follow: In the MMSE algorithm, the MSE is minimized iteratively. Here, the interference and the noise are treated in the same way. The proposed MMSE algorithm guarantees to achieve at least a local minimum and not necessarily the global minimum and hence, at the end of the iterations, there will be a residual error. At high SNR, the noise is almost zero and the interference plays a major role in determining the sum rate. In the maxSNR algorithm, the interference signals are completely removed by performing IA. However, in the MMSE algorithm, there is a residual error which is due to the interference signals. Hence, the maxSNR algorithm achieves a higher sum rate than the MMSE algorithm at high SNR. Furthermore, for MMSE the slope of the sum rate curve is reduced compared to the slope of maxSNR at high SNR. Also, in B3 where one additional node pair is served, the residual interference is further increased compared to B4 and hence, the slope of the sum rate curve is reduced further.

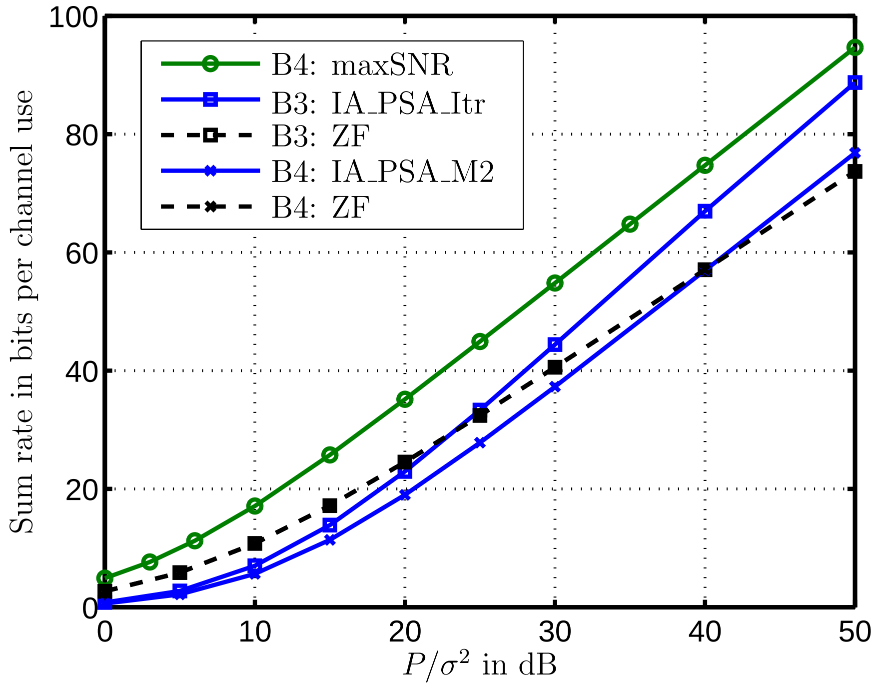

Figure 5.3. Sum rate performance of the maxSNR, the IA_PSA_Itr, the IA_PSA_M2, and the reference ZF algorithms versus P/σ^2 for scenarios B3 and B4

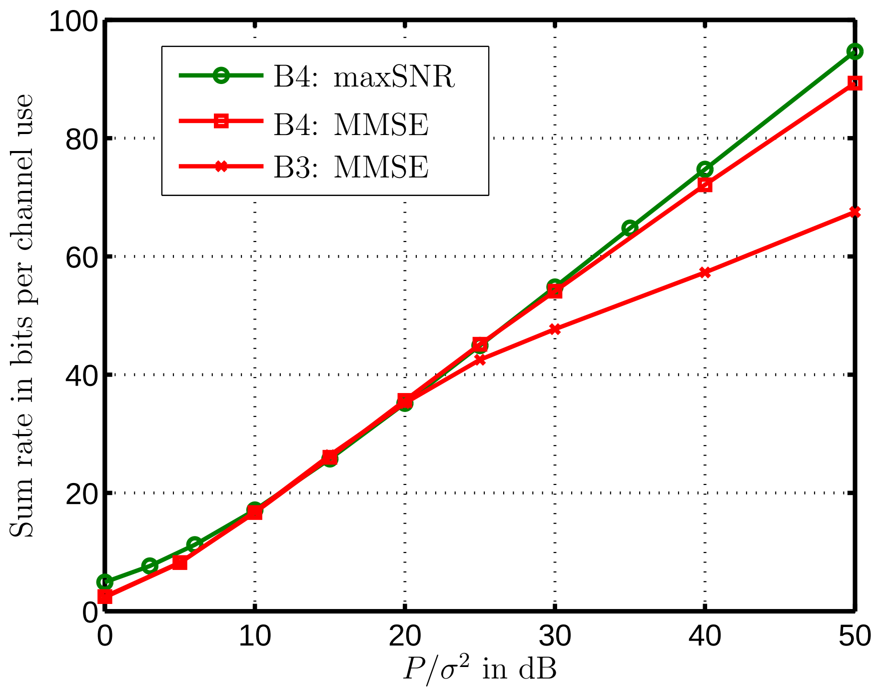

Figure 5.4. Sum rate performance of the maxSNR and the MMSE algorithms versus P/σ^2 for scenarios B3 and B4

Secondly, consider the scenarios C1 and C4. Multiple relays are considered in these scenarios. In scenario C1, $QR = Kd$. The IA_CZF scheme can be used to find the IA solution. As the IA solutions are not known in closed form, the maxSNR algorithm cannot be applied to this scenario. However, the MMSE algorithm can be applied. Figure 5.5 shows the sum rate in bits per channel use as a function of P/σ^2. It can be observed that the MMSE algorithm has better performance than the IA_CZF algorithm and the reference GOF algorithm at low and medium SNR and the IA_CZF algorithm has better performance than the other two algorithms at high SNR. In C4, $QR > Kd$ and, hence, the IA_CZF algorithm cannot be applied and the IA_Iterative algorithm shall be used to find an IA solution. Figure 5.6 shows that similar to scenario C1, also in scenario C4, the MMSE algorithm has better performance at low and medium SNR and the IA_Iterative algorithm has better performance at high SNR.

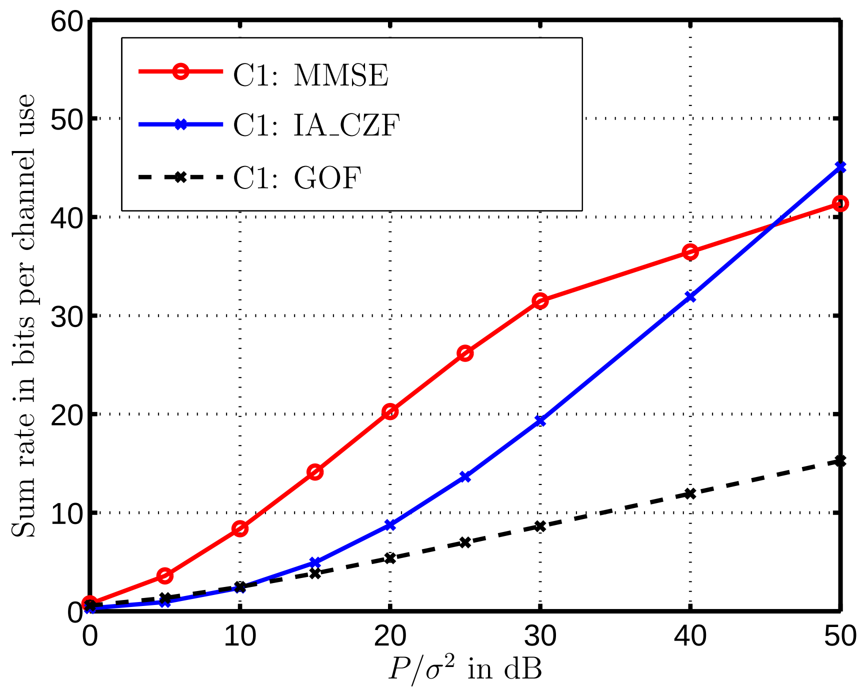

Figure 5.5. Sum rate performance of the MMSE, the IA_CZF, and the reference GOF algorithms versus P/σ^2 for scenario C1

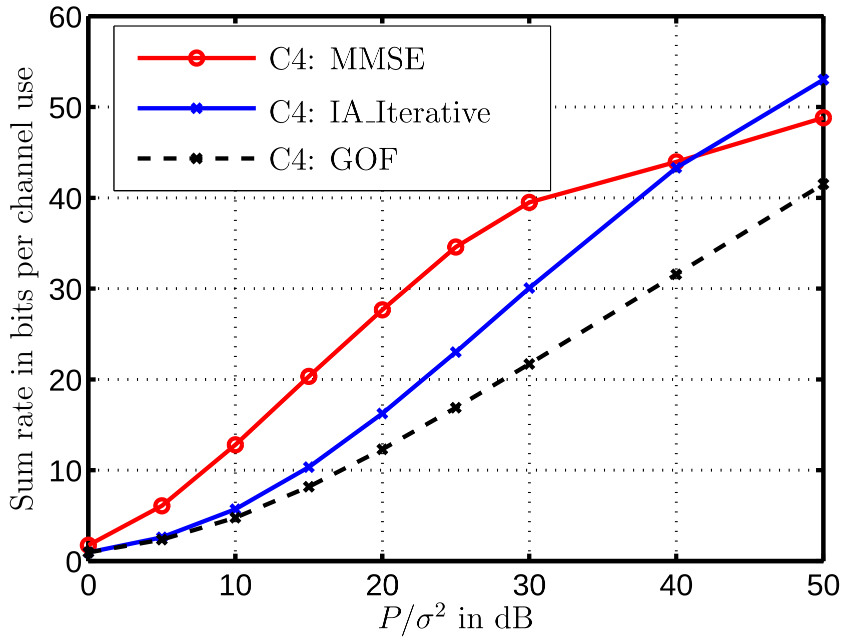

Figure 5.6. Sum rate performance of the MMSE, the IA_Iterative, and the reference GOF algorithms versus P/σ^2 for scenario C4

5.4.3 Convergence analysis

In this section, the convergence of the maxSNR algorithm and the MMSE algorithm are investigated in scenario B4. In the maxSNR algorithm, the objective is to maximize the sum of SNRs at the receivers. This objective function is a non-convex function and hence, convergence to the global optimum cannot be guaranteed. However, at each iteration step, the gradient is computed and the IA solution in the increasing direction of the gradient is chosen. Therefore, at each step, the sum of the SNRs is guaranteed to increase and hence, the algorithm is guaranteed to converge at least to a local maximum. Figure 5.7 shows the sum of SNRs versus the number of iterations for 10 arbitrarily chosen channel realizations. It can be seen that the sum of SNRs converges already after 20 iterations. However, the curves are not smooth. This is due to the normalization of $\mathbf{w}^l$ and $\frac{\partial SSNR}{\partial \mathbf{w}^{l*}}$ after each iteration which introduces a non-linear operation in the maxSNR algorithm.

In the MMSE algorithm, the MSE is reduced during each iteration step. Similar to the sum of SNRs objective function, minimization of the MSE is also a non-convex function and hence, convergence to the global optimum cannot be guaranteed. However, at each step the MSE is reduced and the MSE is lower bounded by zero and hence, it is guaranteed to converge at least to a local minimum. From Figure 5.8, it can be seen that the MSE is monotonically decreasing with increasing number of iterations and it is almost constant after 200 iterations.

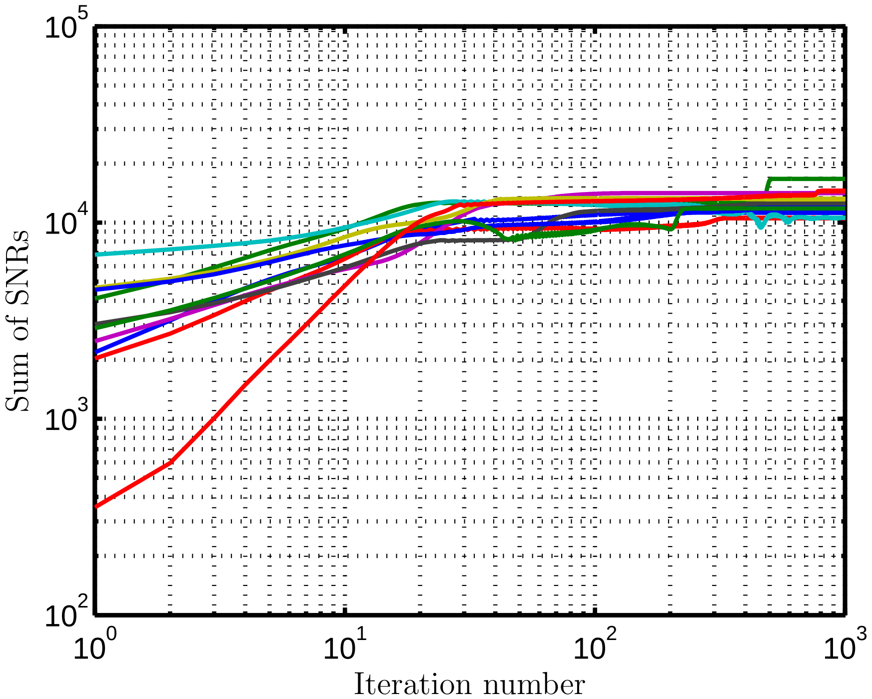

Figure 5.7. Sum of SNRs versus the iteration number in scenario B4 with normalization of the variables

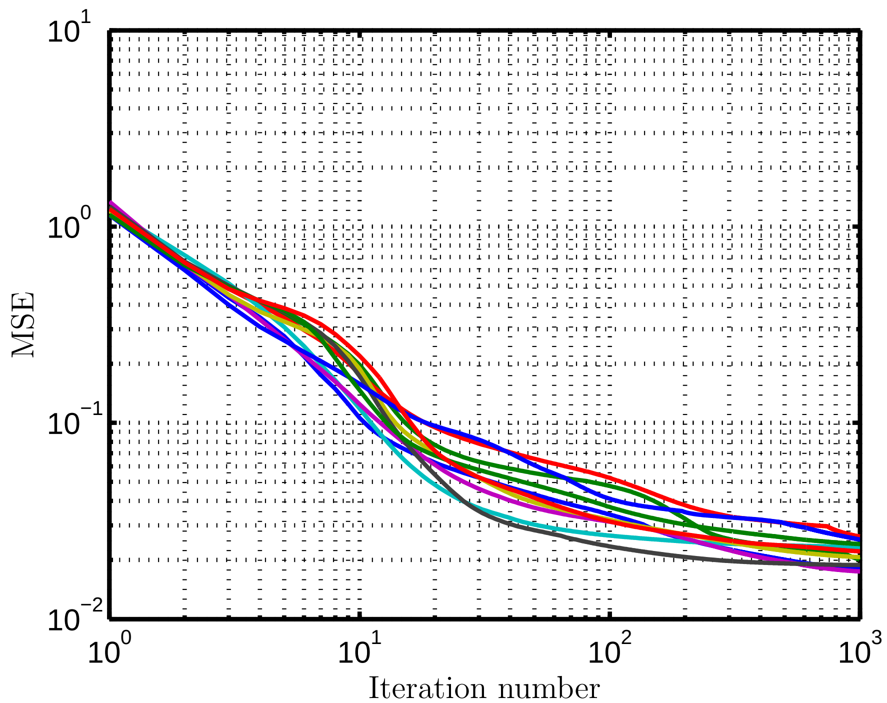

Figure 5.8. MSE versus the iteration number in scenario B4

5.5 Summary

In this chapter, two optimization algorithms to improve the sum rate performance at low and medium SNR in a multi-pair two-way relay network have been proposed. IA is optimum at high SNR and has better sum rate performance than the reference schemes introduced in Chapter 3 and in Chapter 4 at high SNR. For low and medium SNR, two optimization algorithms have been proposed:

- The first algorithm named maxSNR is based on IA and maximization of SNR. This algorithm is applicable whenever multiple IA solutions exists and the solutions are known in closed form. Out of all the IA solutions, maxSNR aims at finding a solution that maximize the sum of SNRs at all the receivers. This is a non-convex optimization problem. A gradient based method to find at least a local maximum has been proposed. By applying the maxSNR algorithm, the proposed IA schemes achieve better sum rate even at low and medium SNR. Furthermore, it has been shown that when there is no closed form solution available in a given scenario with K node pairs, reducing the number of simultaneously served node pairs by one, i.e., serving $K-1$, node pairs results in multiple solutions which can be obtained in closed form. At low and medium SNR, the sum rate achieved by serving $K-1$ node pairs and applying the maxSNR algorithm is even higher than the sum rate achieved by serving K node pairs without maxSNR.

- The second algorithm named MMSE is based on the minimization of the MSE. In contrast to the maxSNR algorithm, the MMSE algorithm is applicable also to scenarios, where IA solution is not available in closed form. In the MMSE algorithm, the objective is to design the transmit, the relay and the receive filters to minimize the MSE at the receivers subject to the relay and the node power constraints. This is a non-convex optimization problem. However, fixing two of the three kinds of filters results in a convex problem. For fixed transmit and relay filters, a closed form solution to find the receive filters has been derived. Then for fixed transmit and receive filters, the optimum relay filter that minimizes the MSE has been obtained using the KKT conditions and the bisection algorithm. Finally, for fixed relay and receive filters, the optimum transmit filter is obtained using the KKT conditions and a new quadsection algorithm. Simulation results show that the MMSE algorithm has better sum rate performance than the proposed IA algorithms and than the reference algorithm introduced in Chapter 3 and in Chapter 4 at low and medium SNR.

Both algorithms are guaranteed to converge at least to a local optimum. Since the maxSNR algorithm can be applied only for scenarios where multiple IA solutions are known in closed form, it converges faster than the MMSE algorithm. Furthermore, the maxSNR algorithm guarantees zero interference and hence, at high SNR the maxSNR algorithm achieves better sum rate performance than the MMSE algorithm.

Chapter 6

Conclusions and outlook

6.1 Conclusions

In this thesis, IA in a multi-pair two-way relay network has been investigated. In contrast to the conventional use of relays, where the relays are used for coverage extension, in this thesis, relays are used to aid in the process of IA. IA in a multi-pair two-way relay network is a trilinear problem. In general, closed form solutions are not available for trilinear problems. However, in this thesis, utilizing the fact that the communication takes place in two time slots and that the nodes can cancel their self-interference, several algorithms have been developed to perform IA and closed form and / or iterative solutions have been proposed. The properness conditions have been derived in terms of the number K of node pairs, the number Q of relays, the number N of node antennas, the number R of relay antennas, and the number d of data streams. Furthermore, IA is capacity optimum at high SNR. In order to improve the sum rate performance at low and medium SNR, optimization algorithms to improve the SNR at the receiver and to minimize the MSE of the decoded symbols have been developed. Through simulations, it has been shown that the proposed algorithms achieve higher sum rate than the reference algorithms known from literature.

In Chapter 1, as known from the literature, IA is introduced as the potential technique to handle interferences at high SNR. The challenges involved in performing IA have been listed. It has also been revealed through the current literature that for a unidirectional communication in a 3-user interference channel, introducing a relay with a single antenna and performing one-way relaying can overcome some of the challenges involved in performing IA. For bi-directional communication, we propose to use multiple relays with multiple antennas to manipulate the effective channels between the transmitters and the receivers to aid in the process of IA. Following this, works involving one-way relaying, two-way relaying, one-way relay aided IA, and two-way relay aided IA have been analysed and the open issues related to two-way relay aided IA have been identified. The contributions of this thesis have been introduced.

In Chapter 2, the focus is on the system model and the assumptions considered in this thesis. The two-way relay channel is a multiple key-hole channel and it is shown that the relays need at least $QR \geq Kd$ antennas for an interference free communication to be feasible. This expression helps in investigating different aspects like DoF and required number of node antennas in the considered bi-directional communication in multi-pair two-way relay networks. The sum rate expression is also derived.

In Chapter 3, the focus is on the single relay scenario. The relay needs at least $R \geq Kd$ antennas. The case $R = Kd$ is first investigated. Two new concepts namely, SA and CA are proposed and it is shown that SA and CA are necessary conditions for IA. Using SA and CA, we decoupled the process of IA into three linear steps, namely, SA, CA and ZF. A closed for solution has been obtained. The number of antennas required at the nodes to perform SA and CA is also derived as $N \geq \frac{(K+1)d}{2}$. It has been revealed that for the case $R = Kd$, the same number of DoF as for IA using spatial dimension in a K-user interference channel without a relay is achieved. However, in contrast to the K-user interference channel, now a closed form solution is available. In addition, for the proposed IA algorithm, the nodes need only pair-wise channel knowledge and only the relay needs global channel knowledge to perform IA. Furthermore, for the case $R \geq Kd$, the concepts of SA and CA are generalized to PSA and PCA. In this case, it is possible to choose the Kd-dimensional relay subspace in which SA and CA are performed. In this thesis, the Kd-dimensional subspace is chosen such that, compared to the case $R = Kd$, for the current case either the number of DoF is increased or the required number N of node antennas is decreased. This is directly revealed from the derived properness condition given by $2Kd \geq R + 2N - d$. It is to be noted that for a proper system, by increasing the number of antennas at the relay by 2, the number of antennas at each of the $2K$ nodes can be reduced by 1 and the system still remains proper. Hence, the proposed scheme gives a method to make a trade-off between R and N. An iterative algorithm to find the IA solution has been proposed. Furthermore, for the proper system satisfying the additional condition given by $2KN - (K - n)R \geq d$, a closed form solution has been proposed. Simulation results show that the proposed IA algorithms achieve higher sum rate than the reference ZF algorithm without IA at high SNR.

In Chapter 4, the focus is on the multiple relay scenario. Since the relays do not share their data streams, the relay processing matrix is a block diagonal matrix. This introduces a new challenge in the multiple relay scenario in comparison with the single relay scenario. First for the case $QR = Kd$, a new algorithm is proposed to perform IA in three steps, namely, SA, CA, and CZF. SA and CA, which are performed by the nodes remain the same as in the case of a single relay. However, the ZF performed in the single relay case is now generalized into CZF in which the nodes choose their SA and CA directions such that the relays are able to perform ZF with a block diagonal relay processing matrix. The properness condition is derived as $4N + R \geq 3Kd + 2d$ and an iterative algorithm to achieve IA is proposed. Then the case $QR \geq Kd$ is investigated. It is shown that the SA and CA based scheme is not suitable for this case because the complexity of the problem is increased in comparison to the original trilinear IA problem. An iterative algorithm to solve the IA conditions is proposed. The properness condition is derived as $QR^2 > 2K(2K-2)d^2$. Simulation results show that the proposed IA algorithms achieve higher sum rate than the reference GOF algorithm at high SNR.

In Chapter 5, the focus is on the optimization of the sum rate at low and medium SNR. Simulation results from Chapter 3 and Chapter 4 show that the proposed IA algorithms have better sum rate performance than the reference algorithms at high SNR. In order to improve the sum rate performance at low and medium SNR, two optmization algorithms, namely, the maxSNR algorithm and the MMSE algorithm, have been proposed. The maxSNR algorithm is applicable only when the IA solutions are known in closed form. Furthermore, it has been shown that when there is no closed form solution available in a given scenario with K node pairs, reducing the number of simultaneously served node pairs by one, i.e., serving $K-1$, node pairs results in multiple solutions which can be obtained in closed form. It is revealed that, at low and medium SNR, the sum rate achieved by serving $K-1$ node pairs and applying the maxSNR algorithm is even higher than the sum rate achieved by serving K node pairs without maxSNR. Both proposed algorithms achieve better sum rate performance than the reference algorithm and the proposed IA algorithms at low and medium SNR.

6.2 Outlook

In this thesis, the multi-pair two-way relay network has been investigated. Several assumptions like global CSI at the relays, all communication links being similar strength and the nodes having the same number of antennas have been made. The concepts developed in this thesis can be extended depending on the new challenges introduced by relaxing one or more assumptions. The considered multi-pair two-way relay network is an example of a fully connected ad-hoc network. In addition, it is a special case of several other networks, e.g., of a cellular network with each base station serving a single mobile terminal and with interference links as strong as the useful links. The algorithms and the properness conditions derived in this thesis can be generalized to the new network topologies. Both relaxation of the assumptions made in this thesis and generalization of the considered scenario lead to several research questions as described below:

- In this thesis, all the communication links are assumed to be of similar strength. However, if the communication links are of different strength, it leads to the possibility to choose the node pairs that are simulataneously served such that the maximum sum rate is achieved. This results in a resource allocation problem which need to be solved.

- If some of the interference links are significantly weaker compared to the useful links, then the corresponding nodes shall may be assumed to be disconnected. The resulting network is no more fully connected, but partially connected. A given partially connected network may contain two or more fully connected subnetworks. From one perspective, partially connected networks may be considered as an ad-hoc scenario considered in this thesis, but with some interference links missing. From another perspective, the ad-hoc scenario in this thesis may be considered as the fully connected subnetwork in the larger partially connected network. In both cases, new challenges are introduced by the partial connectivity and lead to several challenges which are yet to be addressed. For instance, due to the limited connectivity, the relays can only provide limited assistance in aligning the interferences coming from other subnetworks.

- In this thesis, global CSI is assumed at the relays. In a large partially connected network, on the one hand the CSI of the node pairs that are not connected to a given subnetwork are not required at the relays within that subnetwork to perform IA. On the other hand, subnetworks may have only local CSI and the CSI of the interfering nodes from neighbouring subnetworks may not be available. In this case, distributed IA algorithms which align interferences based on local CSI need to be developed. The convergence of such distributed algorithms is one of the key challenges that needs to be addressed in a partially connected network.

- From the properness conditions derived in this thesis, it has been revealed that for a given number of relays, relay antennas and node antennas, the number of node pairs that can be served simultaneously without interference is limited. However, in a partially connected network, at any receiver, the number of interference signals is limited by the connectivity. It has to be investigated if it is possible to scale the network without increasing the number of relays, relay antennas, and node antennas. Scalability of the network is an interesting open question in a partially connected network. The requirements on the network topology to enable scalability of the network has to be analysed.

- Pair-wise communication has been considered in this thesis. This can be generalized to a cellular network with each base station communicating to multiple mobile terminals. In this case, on the one hand, each base station requires a larger number of antennas to serve multiple mobile terminals. On the other hand, joint processing of the signals transmitted to multiple mobile terminals is possible at the base station. Both fully connected and partially connected network may be investigated in a cellular network.

- In scenarios like video conferencing or multi-player gaming, the nodes want to share their messages with all the nodes in their group. The multi-pair two-way relay network investigated in this thesis is a multi-group multi-way relay network with each group consisting of only two nodes. The concepts like SA and CA developed for pairs of nodes need to be generalized to groups of nodes. Furthermore, as each node is interested in the message from all the other nodes in its group, more than one BC phase will be necessary. Several new challenges arise when IA is performed jointly in spatial and time dimensions. For instance, a trade-off between the number of BC phases and the number of antennas at the nodes has to be identified. Furthermore, if multiple MAC phases are considered, then the minimum number of antennas required at the relays to perform IA can be derived as a function of the number of MAC phases.

Appendix

A.1 Derivation of gradient of SSNR of (5.7)

In this section, the gradient of $SSNR$ of (5.7) is obtained. From (5.5), SSNR is given by

$$SSNR = \sum_{j=1}^{2K}\sum_{l=1}^{d} SNR_j^l. \tag{A.1}$$

where

$$\mathrm{SNR}_j^l = \frac{P}{d\sigma_1^2}\frac{\mathbf{w}^{l\mathrm{H}}\mathbf{A}_j^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}^{\mathrm{H}}\mathbf{T}\mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}\mathbf{A}_j\mathbf{w}^l}{\mathbf{w}^{l\mathrm{H}}\mathbf{A}_j^{\mathrm{H}}\mathbf{A}_j\mathbf{w}^l} \tag{A.2}$$

Now the derivative is obtained as follows:

$$\frac{\partial \mathrm{SSNR}}{\partial \mathbf{w}^{l*}} = \sum_{j=1}^{2K}\sum_{l=1}^{d}\frac{\partial SNR_j^l}{\partial \mathbf{w}^{l*}}. \tag{A.3}$$

Since the derivative is a linear operator, we focus on the term $\frac{\partial SNR_j^l}{\partial \mathbf{w}^{l*}}$. Let

$$\gamma = \mathbf{w}^{l\mathrm{H}}\mathbf{A}_j^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}^{\mathrm{H}}\mathbf{T}\mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}\mathbf{A}_j\mathbf{w}^l \tag{A.4}$$

and

$$\nu = \mathbf{w}^{l\mathrm{H}}\mathbf{A}_j^{\mathrm{H}}\mathbf{A}_j\mathbf{w}^l, \tag{A.5}$$

then using the quotient rule of differentiation, we have

$$\frac{\partial SNR_j^l}{\partial \mathbf{w}^{l*}} = \frac{P}{d\sigma_1^2}\frac{\nu\frac{\partial\gamma}{\partial \mathbf{w}^{l*}} - \frac{\partial\nu}{\partial \mathbf{w}^{l*}}\gamma}{\nu^2} \tag{A.6}$$

with

$$\frac{\partial \nu}{\partial \mathbf{w}^{l*}} = \mathbf{A}_j^{\mathrm{H}}\mathbf{A}_j\mathbf{w}^l. \tag{A.7}$$

Let $\mathbf{b}_1 = \mathbf{w}^{l\mathrm{H}}\mathbf{A}_j^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}^{\mathrm{H}}\mathbf{T}$ and $\mathbf{b}_2 = \mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}\mathbf{A}_j\mathbf{w}^l$. Then using the product rule of differentiation, the i^{th} element of the row vector $\frac{\partial\gamma}{\partial \mathbf{w}^{l*}}$ is given by

$$\frac{\partial\gamma}{\partial \mathbf{w}_i^{l*}} = \frac{\partial \mathbf{b}_1}{\partial \mathbf{w}_i^{l*}}\mathbf{b}_2 + \mathbf{b}_1\frac{\partial \mathbf{b}_2}{\partial \mathbf{w}_i^{l*}} \tag{A.8}$$

with

$$\frac{\partial \mathbf{b}_1}{\partial \mathbf{w}_i^{l*}} = \frac{\partial \mathbf{b}_3}{\partial \mathbf{w}_i^{l*}}\mathbf{T} + \mathbf{b}_3\frac{\partial \mathbf{T}}{\partial \mathbf{w}_i^{l*}}, \tag{A.9}$$

$$\frac{\partial \mathbf{b}_2}{\partial \mathbf{w}_i^{l*}} = \frac{\partial\left(\mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}\right)}{\partial \mathbf{w}_i^{l*}}\mathbf{b}_4 + \mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}\frac{\partial \mathbf{b}_4}{\partial \mathbf{w}_i^{l*}}, \tag{A.10}$$

$$\frac{\partial \mathbf{b}_3}{\partial \mathbf{w}_i^{l*}} = \left(\mathbf{A}_j^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}^{\mathrm{H}}\right)(i,:), \tag{A.11}$$

and

$$\frac{\partial \mathbf{b}_4}{\partial \mathbf{w}_i^{l*}} = \frac{\partial \mathbf{T}}{\partial \mathbf{w}_i^{l*}}\mathbf{H}_{\mathrm{r}j}\mathbf{A}_j\mathbf{w}^l. \tag{A.12}$$

Now $\frac{\partial \mathbf{T}}{\partial \mathbf{w}_i^{l*}}$ and $\frac{\partial\left(\mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}\right)}{\partial \mathbf{w}_i^{l*}}$ have to be obtained to complete the derivation. First let us derive $\frac{\partial \mathbf{T}}{\partial \mathbf{w}_i^{l*}}$. $\mathbf{T}$ is dependent on $\mathbf{w}^1, \ldots, \mathbf{w}^d$. Let

$$\mathbf{SN}_\tau^l = \mathbf{H}_{\mathrm{r}\tau}\mathbf{A}_\tau + \mathbf{H}_{\mathrm{r}K+\tau}\mathbf{A}_{K+\tau} \tag{A.13}$$

for $\tau = 1, \ldots, n$ and $l = 1, \ldots, d$. $\mathbf{SN}_\tau^l\mathbf{w}_l$ gives the l^{th} intersection of *RISS* with $S_{\tau(K+\tau)}$. Let

$$\mathbf{N}_l = \left[\mathbf{SN}_1^l\mathbf{w}^l \ \cdots \ \mathbf{SN}_n^l\mathbf{w}^l\right]. \tag{A.14}$$

Then

$$\mathbf{N} = [\mathbf{N}_1 \ \cdots \ \mathbf{N}_d] \tag{A.15}$$

represent the *RISS*. The projection matrix $\mathbf{T} = \mathbf{T}^{\mathbf{H}}$ is given by

$$\mathbf{T} = \mathbf{T}^{\mathbf{H}} = \mathbf{I} - \mathbf{N}\left(\mathbf{N}^{\mathbf{H}}\mathbf{N}\right)^{-1}\mathbf{N}^{\mathbf{H}}. \tag{A.16}$$

Let $\mathbf{E} = \mathbf{N}^{\mathbf{H}}\mathbf{N}$, then

$$\frac{\partial \mathbf{T}}{\partial \mathbf{w}_i^{l*}} = -\mathbf{N}\left[\frac{\partial \mathbf{E}^{-1}}{\partial \mathbf{w}_i^{l*}}\mathbf{N}^{\mathbf{H}} + \mathbf{E}^{-1}\frac{\partial \mathbf{N}^{\mathrm{H}}}{\partial \mathbf{w}_i^{l*}}\right] \tag{A.17}$$

with

$$\frac{\partial \mathbf{E}^{-1}}{\partial \mathbf{w}_i^{l*}} = -\mathbf{E}^{-1}\frac{\partial \mathbf{E}}{\partial \mathbf{w}_i^{l*}}\mathbf{E}^{-1} \quad \text{and} \quad \frac{\partial \mathbf{E}}{\partial \mathbf{w}_i^{l*}} = \frac{\partial \mathbf{N}^{\mathrm{H}}}{\partial \mathbf{w}_i^{l*}}\mathbf{N}. \tag{A.18}$$

Now we need to find $\frac{\partial \mathbf{N}^{\mathrm{H}}}{\partial \mathbf{w}_i^{l*}}$. Note that only few terms of the matrix $\mathbf{N}$ are dependent on $\mathbf{w}^l$ and hence

$$\frac{\partial \mathbf{N}^{\mathrm{H}}}{\partial \mathbf{w}_i^{l*}} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \frac{\partial \mathbf{N}_l^{\mathrm{H}}}{\partial \mathbf{w}_i^{l*}} \\ \vdots \\ \mathbf{0} \end{bmatrix} \quad \text{with} \quad \frac{\partial \mathbf{N}_l^{\mathrm{H}}}{\partial \mathbf{w}_i^{l*}} = \begin{bmatrix} \left(\mathbf{SN}_1^{l\mathrm{H}}\right)(i,:) \\ \vdots \\ \left(\mathbf{SN}_n^{l\mathrm{H}}\right)(i,:) \end{bmatrix}. \tag{A.19}$$

Second let us derive $\frac{\partial\left(\mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}\right)}{\partial \mathbf{w}_i^{l*}}$. $\mathbf{g}_j^l$ is dependent on $\mathbf{w}^1, \ldots, \mathbf{w}^d$. Let

$$\mathbf{H}_{\mathrm{w}}^l = \left[\ \mathbf{H}_{\mathrm{r}1}\mathbf{A}_1\mathbf{w}^l \ \cdots \ \mathbf{H}_{\mathrm{r}\bar{j}}\mathbf{A}_{\bar{j}}\mathbf{w}^l \ \cdots \ \mathbf{H}_{\mathrm{r}(2K)}\mathbf{A}_{(2K)}\mathbf{w}^l\ \right]_{\bar{j}\neq j}, \tag{A.20}$$

$$\overline{\mathbf{H}}_{\mathrm{w}}^l = \left[\ \mathbf{H}_{\mathrm{r}j}\mathbf{A}_j\mathbf{w}^1 \ \cdots \ \mathbf{H}_{\mathrm{r}j}\mathbf{A}_j\mathbf{w}^{\bar{l}} \ \cdots \ \mathbf{H}_{\mathrm{r}j}\mathbf{A}_j\mathbf{w}^d\ \right]_{\bar{l}\neq l}, \tag{A.21}$$

and

$$\mathbf{H}_{\mathrm{w}} = \left[\begin{array}{cccc} \mathbf{H}_{\mathrm{w}}^{1} & \cdots & \mathbf{H}_{\mathrm{w}}^{d} & \overline{\mathbf{H}}_{\mathrm{w}}^{l} \end{array} \right]. \tag{A.22}$$

Here, the columns of the matrix $\mathbf{P} = \mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{w}}$ span the subspace orthogonal to $\mathbf{g}_j^l$. Hence, $\mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}$ is given by

$$\mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}} = \mathbf{I} - \mathbf{P}\left(\mathbf{P}^{\mathrm{H}}\mathbf{P}\right)^{-1}\mathbf{P}^{\mathrm{H}}. \tag{A.23}$$

Let $\mathbf{E}_1 = \left(\mathbf{P}^{\mathrm{H}}\mathbf{P}\right)^{-1}\mathbf{P}^{\mathrm{H}}$, then

$$\frac{\partial\left(\mathbf{g}_j^l\mathbf{g}_j^{l\mathrm{H}}\right)}{\partial\mathbf{w}_i^{l*}} = -\left(\frac{\partial\mathbf{P}}{\partial\mathbf{w}_i^{l*}}\mathbf{E}_1 + \mathbf{P}\frac{\partial\mathbf{E}_1}{\partial\mathbf{w}_i^{l*}}\right). \tag{A.24}$$

Let $\mathbf{E}_2 = \mathbf{P^H P}$, then

$$\frac{\partial\mathbf{E}_1}{\partial\mathbf{w}_i^{l*}} = \frac{\partial\mathbf{E}_2^{-1}}{\partial\mathbf{w}_i^{l*}}\mathbf{P^H} + \mathbf{E}_2^{-1}\frac{\partial\mathbf{P}^{\mathrm{H}}}{\partial\mathbf{w}_i^{l*}} \tag{A.25}$$

with

$$\frac{\partial\mathbf{E}_2^{-1}}{\partial\mathbf{w}_i^{l*}} = -\mathbf{E}_2^{-1}\frac{\partial\mathbf{E}_2}{\partial\mathbf{w}_i^{l*}}\mathbf{E}_2^{-1} \quad \text{and} \quad \frac{\partial\mathbf{E}_2}{\partial\mathbf{w}_i^{l*}} = \frac{\partial\mathbf{P}^{\mathrm{H}}}{\partial\mathbf{w}_i^{l*}}\mathbf{P} + \mathbf{P}^{\mathrm{H}}\frac{\partial\mathbf{P}}{\partial\mathbf{w}_i^{l*}}. \tag{A.26}$$

In these expressions, $\frac{\partial\mathbf{P}}{\partial\mathbf{w}_i^{l*}}$ and $\frac{\partial\mathbf{P}^{\mathrm{H}}}{\partial\mathbf{w}_i^{l*}}$ are given by

$$\frac{\partial\mathbf{P}}{\partial\mathbf{w}_i^{l*}} = \frac{\partial\mathbf{T}^{\mathrm{H}}}{\partial\mathbf{w}_i^{l*}}\mathbf{H}_{\mathrm{w}} \tag{A.27}$$

and

$$\frac{\partial\mathbf{P}^{\mathrm{H}}}{\partial\mathbf{w}_i^{l*}} = \frac{\partial\mathbf{H}_{\mathrm{w}}^{\mathrm{H}}}{\partial\mathbf{w}_i^{l*}}\mathbf{T} + \mathbf{H}_{\mathrm{w}}^{\mathrm{H}}\frac{\partial\mathbf{T}}{\partial\mathbf{w}_i^{l*}}. \tag{A.28}$$

Note that only few terms of the matrix $\mathbf{H}_{\mathrm{w}}$ are dependent on $\mathbf{w}^l$ and hence

$$\frac{\partial\mathbf{H}_{\mathrm{w}}^{\mathrm{H}}}{\partial\mathbf{w}_i^{l*}} = \left[\begin{array}{c} \mathbf{0} \\ \vdots \\ \frac{\partial\mathbf{H}_{\mathrm{w}}^{l\mathrm{H}}}{\partial\mathbf{w}_i^{l*}} \\ \vdots \\ \mathbf{0} \end{array}\right] \quad \text{with} \quad \frac{\partial\mathbf{H}_{\mathrm{w}}^{l\mathrm{H}}}{\partial\mathbf{w}_i^{l*}} = \left[\begin{array}{c} \left(\mathbf{A}_1^{\mathrm{H}}\mathbf{H}_{\mathrm{r}1}^{\mathrm{H}}\right)(i,:) \\ \vdots \\ \left(\mathbf{A}_{\bar{j}}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}\bar{j}}^{\mathrm{H}}\right)(i,:) \\ \vdots \\ \left(\mathbf{A}_{(2K)}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}(2K)}^{\mathrm{H}}\right)(i,:) \end{array}\right]_{\bar{j}\neq j}. \tag{A.29}$$

Now we have obtained all the terms required for the calculation of $\frac{\partial\gamma}{\partial\mathbf{w}_i^{l*}}$ for $i = 1, \ldots, L$. Now these elements can be put together in a row vector to form $\frac{\partial\gamma}{\partial\mathbf{w}^{l*}}$. Substituting $\frac{\partial\gamma}{\partial\mathbf{w}^{l*}}$ and (A.7) in (A.6) gives $\frac{\partial\mathrm{SNR}_j^l}{\partial\mathbf{w}^{l*}}$. Furthermore, substitution of $\frac{\partial\mathrm{SNR}_j^l}{\partial\mathbf{w}^{l*}}$ in (A.3) gives $\frac{\partial\mathrm{SSNR}}{\partial\mathbf{w}^{l*}}$ which can be used in (5.7).

A.2 Derivation of upper bound for $\lambda_{\mathbf{up}}^{(\mathbf{G})}$ of (5.33)

In this section, the upper bound $\lambda_{\text{up}}^{(\text{G})}$ of (5.33) for the Langrangian variable corresponding to the relay power constraint is derived. For the optimum λ and $\mathbf{G}$, the KKT conditions given by (5.31) and (5.32) need to be satisfied. $\text{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\text{H}}\right) - P_{\text{relay}}$ is a decreasing function of λ and the matrix $\mathbf{X}$ is a positive semi-definite matrix. Hence, $\left(\text{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\text{H}}\right) - P_{\text{relay}}\right)_{\mathbf{X}=\mathbf{0}}$ gives an upper bound for λ. Substituting $\mathbf{X} = \mathbf{0}$ in (5.31) results in

$$\mathbf{g} = \frac{1}{\lambda_{\text{u}}}\mathbf{Y}^{-1}\mathbf{f}. \tag{A.30}$$

Furthermore, (5.32) can be rewritten as

$$\sum_{q=1}^{Q} \text{Tr}\left(\mathbf{G}_q\mathbf{R}_Q^{q,q}\mathbf{G}_q^{\text{H}}\right) - P_{\text{relay}} = 0. \tag{A.31}$$

Here

$$\begin{aligned}
\text{Tr}\left(\mathbf{G}_q\mathbf{R}_Q^{q,q}\mathbf{G}_q^{\text{H}}\right) &= \text{Tr}\left(\mathbf{G}_q^{\text{H}}\mathbf{G}_q\mathbf{R}_Q^{q,q}\right) \\
&= \text{vec}\left(\mathbf{G}_q\right)^{\text{H}} \text{vec}\left(\mathbf{G}_q\mathbf{R}_Q^{q,q}\right) \\
&= \text{vec}\left(\mathbf{G}_q\right)^{\text{H}} \left(\mathbf{R}_Q^{\text{T}q,q} \otimes \mathbf{I}\right) \text{vec}\left(\mathbf{G}_q\right) \\
&= \text{vec}\left(\mathbf{G}_q\right)^{\text{H}} \left(\mathbf{Y}^q\right) \text{vec}\left(\mathbf{G}_q\right).
\end{aligned} \tag{A.32}$$

Substituting (A.32) in (A.31) results in

$$\mathbf{g}^{\text{H}}\mathbf{Y}\mathbf{g} - P_{\text{relay}} = 0. \tag{A.33}$$

Substituting (A.30) in (A.33) results in

$$\lambda_{\text{up}}^{(\text{G})} = \sqrt{\frac{\mathbf{f}^{\text{H}}\mathbf{Y}^{-1}\mathbf{f}}{P_{\text{relay}}}}. \tag{A.34}$$

(A.34) gives the $\lambda_{\text{up}}^{(\text{G})}$ of (5.33).

List of Acronyms

BC	Broadcast
CA	Channel Alignment
CFS	Closed Form Solution
CSI	Channel State Information
CZF	Cooperative Zero Forcing
DoF	Degrees of Freedom
FDMA	Frequency Division Multiple Access
IA	Interference Alignment
ISS	Interference Subspace
KKT	Karush Kuhn Tucker
LSE	Least Squared Error
MAC	Multiple Access
MIMO	Multiple Input Multiple Output
MMSE	Minimum Mean Squared Error
MSE	Mean Squared Error
PCA	Partial Channel Alignment
PSA	Partial Signal Alignment
SA	Signal Alignment
SE	Squared Error
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access
USS	Useful Subspace
ZF	Zero Forcing

List of Symbols

$\mathbf{a}_{jk}$	Useful signal from node j to node k
(A, B)	Similarity measure between the subspaces A and B
$\mathbf{A}_j$	span $\{\mathbf{A}_j\}$ gives SA solution space of node j
$\widetilde{\mathbf{A}}_{jk}$	Effective useful link from node j to node k including the precoding matrix
$\mathbf{B}$	Block diagonal matrix with all block-diagonal entries equal to one
$\mathbf{B}_j$	span $\{\mathbf{B}_j\}$ gives CA solution space of node j
d	Number of data streams transmitted by a node
$\mathbf{d}_j$	Data symbols transmitted by node j
$\mathbf{e}_k$	Interference signal received at node k
$\mathbb{E}\{.\}$	Expectation operator
$\mathbf{g}$	Vectorized form of the block-diagonal elements of the matrix $\mathbf{G}$
$\mathbf{g}_j^{l\mathrm{H}}$	Row of the relay receive zero forcing matrix corresponding to l^{th} data stream of node j
$\mathbf{G}$	Block-diagonal matrix corresponding to the linear signal processing at Q relays
$\mathbf{G}_q$	Linear signal processing matrix at relay q
$\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}$	Relay receive zero forcing matrix at the relays
$\mathbf{G}_{\mathrm{s}}$	Relay transceive zero forcing matrix at the relays
$\mathbf{G}_{\mathrm{tx}}$	Relay transmit zero forcing matrix at the relays
$\mathbf{H}_j^{\mathrm{sr}}$	MIMO channel matrix between node j and all the Q relays in the MAC phase
$\mathbf{H}_{jq}^{\mathrm{sr}}$	MIMO channel matrix between node j and relay q
$\tilde{\mathbf{H}}_{jq}^{\mathrm{sr}}$	Effective channel of node j in the MAC phase in CZF scheme
$\mathbf{H}_k^{\mathrm{rd}}$	MIMO channel matrix between the relays and node j in the BC phase
$\mathbf{H}_{qk}^{\mathrm{rd}}$	MIMO channel matrix between relay q and node j
$\tilde{\mathbf{H}}_{qk}^{\mathrm{rd}}$	Effective channel of node k in the BC phase in CZF scheme
i	Index $i = 1, \ldots, 2K$
$\mathbf{I}$	Identity matrix
j	Index $j = 1, \ldots, 2K$
(j, k)	Communication partner nodes j and k
k	index k denotes the communication partner of node j
K	Number of node pairs
K_0	Integer number such that $K = K_0 n$

l	Index $l = 1, \ldots, d$
$L(.)$	Lagrangian function
M_c	Number of constraints
MSE_k	Mean squared error at the node k
$\overline{MSE}$	Sum mean squared error in the system
M_v	Number of variables
M_vn	Number of variables corresponding the antennas at the nodes
M_vr	Number of variables corresponding the antennas at the relay
n	Integer number such that $R - Kd = nd$
$\mathbf{n}_{1q}$	Noise at relay q
$\mathbf{n}_{2k}$	Noise at node k
$\widetilde{\mathbf{n}}_k$	Effective noise at node k
N	Number of antennas at each node
P_node	Maximum transmit power of each node
P_relay	Maximum total transmit power the relays
Q	Number of relays
$\mathbf{Q}$	Relay transmit projection matrix
R	Number of antennas at each relay
R_k	Average data rate achievable by node k
$RISS$	Relay receive interference subspace
$RISS^m$	$RUSS$ at the m^th iteration step
$RISS_j$	d-dimensional intersection subspace between $RISS$ and S_j
$RISS_j^m$	$RISS_j$ at the m^th iteration step
$RISS_l^{(n)}$	n-dimensional subspace of $RISS$
$RISS_\tau^{(d)}$	d-dimensional subspace of $RISS$
$RUSS$	Relay receive useful subspace
$RUSS_j$	Relay receive useful subspace of node j
$\mathbf{r}$	Signal received by all the Q relay
$\mathbf{r}_q$	Signal received at relay q
$\mathbf{R}_{\mathbf{d}_j}$	Covariance matrix of the symbols $\mathbf{d}_j$
$\mathbf{R}_{\mathbf{e}_k}$	Covariance matrix of the interference signal received at node k
$\mathbf{R}_{n1}$	Covariance matrix of noise at the relay
$\mathbf{R}_{\widetilde{\mathbf{n}}_k}$	Covariance matrix of the effective noise at node k
$\mathbf{R}_Q$	Covariance matrix of the relay received signal
$\mathrm{sign}(.)$	Sign function

$\mathbf{s}_{jk}^{l}$	l^{th} basis vector of the one dimensional intersection subspace between S_{jk} and $RISS_l$
$\text{span}\{.\}$	Span operator
S_j	Subspace $S_{\text{r}j} \cup S_{\text{r}k}$ of the communication partners (j,k)
$S_{\text{r}j}$	subspace spanned by the columns of the matrix $\mathbf{H}_j^{\text{sr}}$
$\mathbf{S}$	Signal transmitted from Q relays
$\mathbf{S}_j$	Unitary matrix whose columns form a basis for S_j
SNR_j^l	Signal to noise ratio of the l^{th} data stream of node j
SR	Achievable sumrate of the system
$SSNR$	Sum of signal to noise ratios of all the nodes
$\mathbf{T}$	Relay receive projection matrix
$TISS$	Relay transmit interference subspace
$TUSS$	Relay transmit useful subspace
$\mathbf{U}_k^{\text{H}}$	Receive filter matrix at node k
$\mathbf{v}_j^l$	l^{th} column vector of the transmit filter matrix $\mathbf{V}_j$
$\mathbf{V}_j$	Transmit filter matrix at node j
$\mathbf{w}^l$	l^{th} column of the matrix $\mathbf{W}$
$\mathbf{W}$	Same as $\mathbf{W}^{(\text{v})}$ with the index (v) dropped for simplicity
$\mathbf{W}^{(\text{u})}$	An auxillary variable to choose a d-dimensional subspace in CA solution space
$\mathbf{W}^{(\text{v})}$	An auxillary variable to choose a d-dimensional subspace in SA solution space
$\mathbf{X}_j^{(m)}$	Unitary matrix such that columns of $\mathbf{S}_j\mathbf{X}_j^{(m)}$ form a basis for $RISS_j^m$
$\mathbf{X}_j^{(\text{u})}$	An auxillary variable to choose a d-dimensional subspace in CA solution space
$\mathbf{X}_j^{(\text{v})}$	An auxillary variable to choose a d-dimensional subspace in SA solution space
$\mathbf{Z}_j$	Unitary matrix whose columns form a basis of $RISS_j^{(d)}$
$\mathbf{Z}^{(m)}$	Unitary matrix whose columns form a basis of $RISS^m$
χ_l	The set of one dimensional intersections of $RISS_l$ with S_{jk} for $\forall_{j,k}$
λ	Lagrange multiplier corresponding to relay power constraint
$\lambda_{\text{u}}^{(\text{G})}$	Upper bound for λ for the design of $\mathbf{G}$
$\lambda_{\text{u}}^{(\text{V})}$	Upper bound for λ for the design of $\mathbf{V}_j$
μ	Lagrange multiplier corresponding to node power constraint
$\mu_{\text{u}}^{(\text{V})}$	Upper bound for μ for the design of $\mathbf{V}_j$
σ_{1q}^2	Variance of the noise at relay q

τ	Index $\tau = 1, \ldots, n$
$(.)^{\mathrm{H}}$	Conjugate transpose operator
$(.)^{-1}$	Inverse operator
$(.)^{\dagger}$	Pseudo inverse operator
$(.)^{\mathrm{T}}$	Transpose operator

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Erklärung laut §9 der Promotionsordnung

Ich versichere hiermit, dass ich die vorliegende Dissertation allein und nur unter Verwendung der angegebenen Literatur verfasst habe. Die Arbeit hat bisher noch nicht zu Prüfungszwecken gedient.

Darmstadt, 08. Dezember 2014