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# Throughput Maximization in Two-Hop Energy Harvesting Communications

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**Abstract**—Two-hop energy harvesting communications are considered. The scenario consists of a source node which wants to send data to a destination node through a half-duplex amplify-and-forward relay station. The source node and the relay station harvest energy from the environment several times and use it to transmit the data. Our goal is to find the optimal power allocation that maximizes the throughput at the destination node. We show that the use of a half-duplex amplify-and-forward relay station leads to a non-convex optimization problem. Therefore, to find the optimal power allocation we propose to reformulate the problem as the difference between two concave functions (D.C. programming). Moreover, a branch-and-bound algorithm is tailored to fit the energy harvesting constraints. We show that the feasible region has to be adapted to facilitate the branching process. Additionally, we reduce the complexity in the calculation of the bounds by relaxing the problem into a convex problem with a linear objective function. Numerical results compare the performance in different energy harvesting scenarios.

## I. INTRODUCTION

Wireless communication devices such as smart phones, tablets and laptops have a wide range of capabilities that allow popular applications and services like social networks, web browsing, video streaming and localization [1], [2]. While these applications and services increase the user satisfaction, they lead to a challenge in terms of avoiding the fast depletion of the device battery. A promising approach to overcome the problem of extending the battery life is energy harvesting (EH). The idea behind EH is that the devices can recharge their battery in an environmentally friendly way using renewable resources like sunlight or wind [3]. In EH, the devices collect energy from the environment and use it afterwards for transmitting data. This requires a change in the transmission strategies due to the time varying availability of energy [4].

The design of optimal transmission strategies for EH scenarios has recently attracted a lot of attention [3]–[9]. According to the available knowledge on the EH process, the strategies can be classified into offline and online strategies. In the offline case [4], complete knowledge of the EH process is assumed. This includes the time when the energy arrives and the amount of energy that can be collected. On the contrary, in the online case [5], a stochastic model for the EH process is assumed. In this paper, we focus on offline approaches in order to find an upper bound of the achievable rates.

Point to point communications with a single EH transmitter are considered in [4]. For this scenario, the authors find transmission policies to maximize the amount of data transmitted in a fixed period of time. Additionally, it is shown that the maximization of the data transfer within a deadline

is equivalent to the minimization of the completion time for the transmission of a given amount of data. Similarly, in [5], a point-to-point scenario is considered. However, in this case a fading channel is assumed between the source and destination nodes, and the corresponding offline and online optimization problems for the maximization of the amount of data transmitted within a deadline are addressed.

Two-hop EH communications are considered in [6]–[9]. In [6] it is assumed that only the source node harvests energy while the relay station has just a single energy arrival. The authors study the impact of a finite buffer at the relay station for the storage of data. In [7], two-hop communications with full-duplex and half-duplex relay stations are studied. In the full-duplex case, EH is assumed for the source node and the relay station. However, in the half-duplex case, a simplified scenario is assumed where only a single energy arrival is considered at the source node. This scenario is extended in [8], where two energy arrivals at the source node and the relay station are considered. The authors derive transmission policies to maximize the data transmitted from the source node to the destination node within a deadline. Finally, in [9], a convex problem is formulated to find offline policies for parallel relays in the two-hop EH scenario.

In this paper, we consider a two-hop communications scenario where the source node and the relay station harvest energy several times. The goal is to find the optimal power allocation which leads to the maximum throughput at the destination node. We assume a half-duplex amplify-and-forward relay station. In contrast to the afore mentioned approaches, we show that the consideration of an amplify-and-forward relay station results in a non-convex expression at the destination node. Consequently, to find the optimal power allocation, we propose to rewrite the optimization problem as the difference between two concave functions. This reformulation fits in a class of global optimization techniques known as difference of convex functions (D.C.) programming problems [10], [11]. Inspired by the work of [10], [11], a branch-and-bound algorithm is tailored to fit the EH constraints. We show that in order to facilitate the branching process, the feasible region has to be adapted. Furthermore, we reduce the complexity in the calculation of the lower and upper bounds by relaxing the D.C. programming problem into a convex problem with a linear objective function.

The paper is organized as follows. In Section II, the system model is explained. The formulation of the power allocation for throughput maximization problem as a D.C. program-

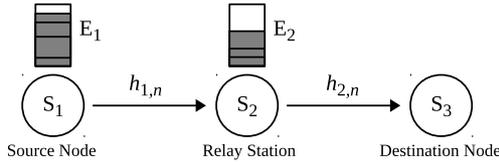


Fig. 1: Two-hop communication scenario with energy harvesting source node and relay station.

ming problem is presented in Section III. In Section IV, the branch-and-bound algorithm for EH two-hop communications is explained. Numerical performance results for different EH scenarios are presented in Section V and Section VI concludes the paper.

## II. SYSTEM MODEL

In this paper, a two-hop EH communications scenario is considered. As depicted in Fig. 1, the scenario consists of three single-antenna half-duplex nodes. The term  $S_k$ ,  $k \in \{1, 2, 3\}$ , is used to label the nodes. In our scenario, the source node  $S_1$  wants to transmit data to the destination node  $S_3$ . It is assumed that the link between these two nodes is weak. Therefore, the nodes cannot communicate directly. To enable the communication,  $S_2$  acts as an amplify-and-forward relay station and it forwards the data from  $S_1$  to  $S_3$ . For simplicity, it is assumed that  $S_1$  has always data available to transmit to  $S_3$ . Moreover,  $S_2$  does not have any own data to transmit to the other nodes.

$S_1$  and  $S_2$  are able to harvest energy from the environment and use it for transmitting data. Consequently, the power available for transmission at  $S_1$  and  $S_2$  depends on their corresponding EH processes. As in [3]–[9], the energy is harvested in a discrete manner, i.e., an amount of energy  $E_{i,n}$ ,  $i \in \{1, 2\}$ , is received by  $S_i$  at a specific time instant  $t_n$ , where  $n = 1, 2, \dots, N$  is the index of the EH time instants and  $N$  is the total number of EH time instants. These EH time instants are not necessarily equal for  $S_1$  and  $S_2$ . However, to simplify the notation and keep one common index, for the case where only  $S_i$  harvests energy at the time instant  $t_n$ , the harvested energy of  $S_j$ ,  $j \neq i$ , is set to zero, i.e.,  $E_{j,n} = 0$ .

We focus on the ideal case where the nodes have all the knowledge about the EH process in advance. This means an offline approach is considered where the time instants  $t_n, \forall n$ , and the amounts of harvested energy  $E_{i,n}, \forall n$ , are known by the nodes at the beginning of the EH process. Although this assumption cannot be completely fulfilled in reality, it allows us to calculate an upper bound of the performance [3]. It is assumed that at  $t_0$ , the nodes have not yet harvested any energy. Furthermore, it is assumed that the nodes are equipped with batteries which can store an unlimited amount of energy and that the energy is only available for transmission after it has been harvested. The received noise at  $S_2$  and  $S_3$  is assumed to be independent and identically distributed (i.i.d.) zero mean additive white Gaussian noise (AWGN) with variance  $\sigma_2^2 = \sigma_3^2 = 1$ . Additionally, the channel coefficients  $h_{i,n} \in \mathbb{C}$  are assumed to be perfectly known at  $S_2$  and  $S_3$ .

In the following, the system equations for the transmission from  $S_1$  to  $S_3$  are presented in the equivalent baseband. As shown in [4], [5], for each link, a constant transmission power in the time interval  $\tau_n = t_{n+1} - t_n$  between two consecutive EH time instants  $t_n$  and  $t_{n+1}$  is optimum. It is assumed that during the time interval  $\tau_n$ , the communication is performed in two hops of equal duration. Moreover,  $h_{i,n}$  is assumed to be constant during this time interval. First,  $S_1$  transmits the signal  $x_{1,n}$  with  $\mathbb{E}[|x_{i,n}|^2] = p_{1,n}$  to  $S_2$ .  $p_{1,n}$  is the transmit power of  $S_1$ . Let  $y_{2,n}$  be the received signal at  $S_2$ . Then, the received power at  $S_2$  is given by

$$\mathbb{E}[|y_{2,n}|^2] = p_{1,n}|h_{1,n}|^2 + 1. \quad (1)$$

Afterwards,  $S_2$  amplifies and retransmits the received signal to  $S_3$ . The amplification factor  $\alpha_n \in \mathbb{C}$  has to fulfill the power constraint at  $S_2$  given by

$$|\alpha_n|^2(p_{1,n}|h_{1,n}|^2 + 1) \leq p_{2,n}. \quad (2)$$

Let  $y_{3,n}$  be the received signal at  $S_3$ . Then the received signal power at  $S_3$  is

$$\mathbb{E}[|y_{3,n}|^2] = |h_{2,n}|^2|\alpha_n|^2(p_{1,n}|h_{1,n}|^2 + 1) + 1. \quad (3)$$

From (2) and (3), the signal to noise ratio (SNR) at  $S_3$  in the time interval  $\tau_n$  is written as

$$\gamma_n = \frac{|h_{1,n}|^2|h_{2,n}|^2p_{1,n}p_{2,n}}{|h_{1,n}|^2p_{1,n} + |h_{2,n}|^2p_{2,n} + 1}. \quad (4)$$

The throughput at  $S_3$  in the time interval  $\tau_n$  is given by

$$B_n = \frac{\tau_n}{2} \log_2(1 + \gamma_n), \quad (5)$$

where the factor  $1/2$  comes from the two-hop nature of the communication. Our goal is to find the optimal power allocation in order to maximize the throughput at  $S_3$  given the EH processes of  $S_1$  and  $S_2$ . The total throughput at  $S_3$  is the sum of the throughput in each interval and is given by

$$B = \sum_{n=1}^N B_n. \quad (6)$$

It is assumed that the energy consumption of the nodes is exclusively due to the transmission of data. Therefore, since the power can be allocated only after it has been harvested, the following causality condition must be fulfilled by any feasible power allocation solution in the time interval  $\tau_n$

$$\sum_{l=1}^n \tau_l p_{i,l} \leq \sum_{l=1}^n E_{i,l}. \quad (7)$$

## III. OPTIMIZATION PROBLEM FORMULATION

In this section, the power allocation problem for throughput maximization is formulated. To simplify the notation, let the vector  $\mathbf{p} \in \mathbb{R}^{2N \times 1}$  contain the power allocation of nodes  $S_1$  and  $S_2$  such that  $\mathbf{p} = [p_{1,1}, \dots, p_{1,N}, p_{2,1}, \dots, p_{2,N}]^T$ . Moreover, let the vector  $\mathbf{e} \in \mathbb{R}^{2N \times 1}$  contain the cumulative energy values of  $S_1$  and  $S_2$ , i.e.,  $\mathbf{e} = [E_{1,1}, E_{1,1} + E_{1,2}, \dots, E_{1,1} + \dots + E_{1,N}, E_{2,1}, E_{2,1} + E_{2,2}, \dots, E_{2,1} + \dots + E_{2,N}]^T$ .

Additionally, let the matrix  $\mathbf{T} \in \mathbb{R}^{2N \times 2N}$  be defined as

$$\mathbf{T} = \begin{pmatrix} \tau_1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \tau_1 & \tau_2 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tau_1 & \tau_2 & \cdots & \tau_N & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \tau_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \tau_1 & \tau_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \tau_1 & \tau_2 & \cdots & \tau_N \end{pmatrix}. \quad (8)$$

Using (5), (6) and (7) and the notation described above, the optimization problem can be written as

$$\mathbf{p}^{\text{opt}} = \underset{\mathbf{p}}{\text{argmax}} B(\mathbf{p}) \quad (9a)$$

$$\text{subject to } \mathbf{T}\mathbf{p} \leq \mathbf{e} \quad (9b)$$

$$\mathbf{p} \geq \mathbf{0}_{2N \times 1}, \quad (9c)$$

where the inequalities hold element-wise and  $\mathbf{0}_{2N \times 1}$  is a  $2N \times 1$  vector of zeros. The constraints given by (9b) and (9c) correspond to an affine set. However, the objective function is non-convex with respect to the optimization variables. The result is that (9) is a non-convex optimization problem and a closed-form solution cannot be obtained. Nevertheless, using basic properties of logarithms, the objective function can be rewritten as the difference of two concave functions. Consequently, the optimization problem of (9) is reformulated as a D.C. programming problem. Applying the quotient and product properties of logarithms, (9a) is rewritten as

$$B(\mathbf{p}) = f(\mathbf{p}) - g(\mathbf{p}), \quad (10)$$

where  $f(\mathbf{p})$  and  $g(\mathbf{p})$  are two concave functions given by

$$f(\mathbf{p}) = \frac{1}{2} \sum_{n=1}^N \tau_n [\log_2(|h_{1,n}|^2 p_{1,n} + 1) + \log_2(|h_{2,n}|^2 p_{2,n} + 1)], \quad (11)$$

$$g(\mathbf{p}) = \frac{1}{2} \sum_{n=1}^N \tau_n \log_2(|h_{1,n}|^2 p_{1,n} + |h_{2,n}|^2 p_{2,n} + 1). \quad (12)$$

Using (11) and (12), problem (9) is reformulated as

$$\mathbf{p}^{\text{opt}} = \underset{\mathbf{p}}{\text{argmax}} [f(\mathbf{p}) - g(\mathbf{p})] \quad (13a)$$

$$\text{subject to } \mathbf{T}\mathbf{p} \leq \mathbf{e}, \quad (13b)$$

$$\mathbf{p} \geq \mathbf{0}_{2N \times 1}. \quad (13c)$$

#### IV. BRANCH-AND-BOUND ALGORITHM FOR EH TWO-HOP COMMUNICATIONS

In this section, the branch-and-bound algorithm for D.C. programming problems presented in [10] is tailored to consider the power allocation problem in an EH two-hop communications scenario. In general, branch-and-bound is an iterative algorithm which works as follows. A recurrent partitioning of the feasible region is performed. In each iteration one partition is considered and the corresponding lower and an

upper bounds of the objective function are calculated. Based on these bounds, decision rules are applied to decide if the partition should be further divided. The algorithm stops when there are no more partitions to examine.

##### A. Partitioning of the feasible region

According to [10], to facilitate the branching an initial simplex is constructed from the feasible region. An  $m$ -simplex is a polytope which is the convex hull of its  $m+1$  affinely independent vertices [10]. Depending on the decision rules, this initial simplex is partitioned using bisection in each iteration. The use of bisection ensures that the resulting partitions are simplices as well. However, the feasible region described by (13b) and (13c) does not fulfill the definition of a simplex because the available power in each time interval depends on the previous power allocations. In the considered scenario, two nodes harvest energy independently in  $N$  time instants. Therefore, for each node,  $N$  power values are calculated. This means that the dimension of the problem is  $2N$  and the feasible region is a  $2N$ -dimensional polytope. Consequently, to construct a simplex, non-feasible power values must be considered in addition to the feasible region.

The initial simplex must include the complete feasible region. Hence, we propose to create the initial simplex based on the maximum power values that can be allocated to the nodes. If a node saves all the harvested energy and transmits only during the last interval, the maximum power that can be allocated to it is calculated using (13b) as  $\frac{1}{\tau_N} \sum_{n=1}^N E_{i,n}$ . A simplex whose vertices are given by the sum of the maximum power values of all the EH nodes, is guaranteed to include the complete feasible region. In other words, the  $2N+1$  vertices  $\mathbf{v}_l, l=0, \dots, 2N$  of the initial simplex are calculated as

$$\mathbf{v}_l = \begin{cases} \mathbf{0}_{2N \times 1} & l=0, \\ [v_{l,1}, v_{l,2}, \dots, v_{l,2N}]^T & l=1, \dots, 2N, \end{cases} \quad (14)$$

where  $v_{l,j}, j=1, \dots, 2N$  are the elements in  $\mathbf{v}_l$  which are calculated as

$$v_{l,j} = \begin{cases} \frac{1}{\tau_N} \sum_{n=1}^N (E_{1,n} + E_{2,n}) & j=l, \\ 0 & j \neq l. \end{cases} \quad (15)$$

To illustrate the feasible region, let us consider the simplest case of  $N=1$ . From the constraint of (13b), the maximum power values for  $S_1$  and  $S_2$  are given by  $\frac{E_{1,1}}{\tau_1}$  and  $\frac{E_{2,1}}{\tau_1}$ , respectively. Similarly, from (13c), the minimum power value is zero for both nodes. The resulting feasible region corresponds to a rectangle as shown in Fig. 2. The required initial simplex is calculated using (14) and (15). The result is the triangle shown in Fig. 2 which contains the complete feasible region.

##### B. Lower and upper bounds

In this section, the calculation of the lower and upper bounds of the objective function is presented. As mentioned, the branch-and-bound algorithm works in an iterative fashion. In each iteration, a partition of the initial simplex is considered and the corresponding lower and upper bounds are calculated. Decision rules are applied to these bounds to decide if the considered partition should be further divided.

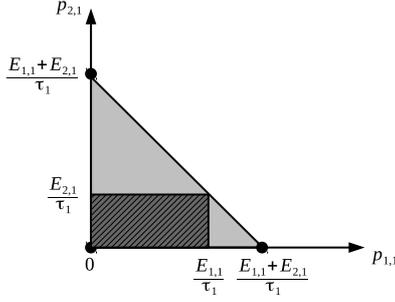


Fig. 2: Example of the feasible region and the initial simplex in a scenario where  $N = 1$ .

In [10], [11] the bounds are calculated by relaxing the D.C. problem into a linear problem. However, in this approach the number of constraints increases linearly with the number of iterations. Therefore, to reduce the complexity in the calculation of the bounds, we propose to linearize only the objective function. As a result, (13) is relaxed into a convex problem. As described in [10], to linearize the objective function, an artificial variable  $\xi$  is included in (13). Moreover, a property of simplices is used to rewrite the power variables as a function of the vertices of the considered simplex. It is known that any point in a simplex can be uniquely represented as a weighted sum of the vertices [10]. Consequently, any vector  $\mathbf{p}$  in the considered simplex can be written as

$$\mathbf{p} = \sum_{l=0}^{2N} w_l \mathbf{v}_l, \quad (16)$$

where  $w_l, l = 0, \dots, 2N$  are the weighting factors which satisfy  $\sum_{l=0}^{2N} w_l = 1$ . Having in mind that  $g(\mathbf{p})$  is a concave function and using (16),  $g(\mathbf{p})$  is lower bounded by

$$\sum_{l=0}^{2N} w_l g(\mathbf{v}_l) \leq g\left(\sum_{l=0}^{2N} w_l \mathbf{v}_l\right), \quad (17)$$

where the equality is met at the vertices. To include the variable  $\xi$ , the constraint

$$\xi - f\left(\sum_{l=0}^{2N} w_l \mathbf{v}_l\right) \leq 0, \quad (18)$$

has to be fulfilled, where (16) is used. Using (16), (17) and (18) the problem in (13) is relaxed into a convex problem:

$$(\xi^{\text{opt}}, w_0^{\text{opt}}, \dots, w_{2N}^{\text{opt}}) = \underset{\xi, w_0, \dots, w_{2N}}{\text{argmax}} \left( \xi - \sum_{l=0}^{2N} w_l g(\mathbf{v}_l) \right) \quad (19a)$$

$$\text{subject to } \xi - f\left(\sum_{l=0}^{2N} w_l \mathbf{v}_l\right) \leq 0, \quad (19b)$$

$$\mathbf{T} \sum_{l=0}^{2N} w_l \mathbf{v}_l \leq \mathbf{e}, \quad (19c)$$

$$\sum_{l=0}^{2N} w_l = 1, \quad (19d)$$

$$0 \leq w_l \leq 1, \quad l = 0, \dots, 2N. \quad (19e)$$

The new optimization variables are the weighting factors  $w_l$  and  $\xi$ . The solution of (19) leads to the calculation of the upper bound

$$u_b = \xi^{\text{opt}} - \sum_{l=0}^{2N} w_l^{\text{opt}} g(\mathbf{v}_l). \quad (20)$$

However,  $u_b$  is a non-achievable throughput value because it is obtained by linearizing the original objective function, i.e., (19a) is an outer approximation of (13a).

The lower bound  $l_b$  is calculated by applying the throughput function of (13a) to the obtained power vector as

$$l_b = f\left(\sum_{l=0}^{2N} w_l^{\text{opt}} \mathbf{v}_l\right) - g\left(\sum_{l=0}^{2N} w_l^{\text{opt}} \mathbf{v}_l\right). \quad (21)$$

In contrast to  $u_b$ ,  $l_b$  is an achievable throughput value. It has to be noticed that in each iteration of the algorithm, the lower and upper bounds are calculated for the considered simplex. The largest value of  $l_b$  among all the simplices leads to the maximum throughput.

### C. Decision rules

In this section, the decision rules used to decide if the considered simplex should be partitioned, are presented. As the initial simplex includes non-feasible power values, it is possible that simplices obtained during branching lie in a non-feasible region and consequently, lead to non-feasible solutions. These solutions are ignored and the corresponding simplices are not further partitioned. The decision rules presented in the following apply only to feasible solutions of (19).

As  $l_b$  is an achievable throughput value and since our goal is to maximize the throughput, the highest lower bound, termed  $l_b^{\text{best}}$ , leads to the maximum throughput. The value  $l_b^{\text{best}}$  is updated only if for a given simplex, the calculated  $l_b$  is higher than the current  $l_b^{\text{best}}$ . The following decision rules are applied to the considered simplex in each iteration:

- 1) If  $u_b < l_b^{\text{best}}$ , the considered simplex is not further partitioned because the current  $l_b^{\text{best}}$  exceeds the corresponding  $u_b$ . This means that the power vector which leads to the maximum throughput cannot be in the region determined by the considered simplex.
- 2) If  $u_b - l_b^{\text{best}} > \epsilon$ , where  $\epsilon$  is the desired tolerance, the considered simplex is partitioned because it may contain a power allocation that leads to the maximum throughput.
- 3) If  $0 \leq u_b - l_b^{\text{best}} \leq \epsilon$ , the considered simplex contains a local maximum given by  $l_b^{\text{best}}$ . If no other simplex leads to a higher lower bound, then the current  $l_b^{\text{best}}$  is considered as the maximum throughput.

Finally, the branch-and-bound algorithm is summarized as:

- 1: create the initial simplex ▷ Eqs. (14) and (15)
- 2: set  $l_b^{\text{best}} = 0$
- 3: **while** there are simplices to be inspected **do**
- 4:     select a simplex and calculate  $u_b$  ▷ Eqs. (19) and (20)
- 5:     calculate the corresponding  $\mathbf{p}$  ▷ Eq. (16)
- 6:     **if** a feasible solution is found **then**
- 7:         calculate the corresponding  $l_b$  ▷ Eq. (21)

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8:   if  $l_b > l_b^{\text{best}}$  then
9:       update  $l_b^{\text{best}}$  and the corresponding  $\mathbf{p}^{\text{best}}$ 
10:   end if
11:   if  $u_b - l_b^{\text{best}} > \epsilon$  then
12:       the simplex is partitioned using bisection
13:   end if
14: end if
15: end while
16: return  $R_{\text{max}} = l_b^{\text{best}}$  and  $\mathbf{p}_{\text{opt}} = \mathbf{p}^{\text{best}}$ 

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## V. PERFORMANCE RESULTS

In this section, numerical results for the evaluation of the proposed algorithm are presented. It is assumed that the amount of harvested energy  $E_{i,n}$  for each node  $S_i$  in time instant  $t_n$  is taken from a uniform distribution with maximum value  $E^{\text{max}}$  and each realization is assumed to be known non-causally. Moreover, the time intervals  $\tau_n$  are assumed to be equal and the magnitude of the channel gains are assumed to be one, i.e.,  $|h_{i,n}| = 1, \forall i, n$ .

Fig. 3 shows the average throughput versus the maximum amount of harvested energy  $E^{\text{max}}$  when  $N = 3$  EH time instants are considered. In the figure, four cases are compared:

- **Equal Energy:** The EH processes of  $S_1$  and  $S_2$  are exactly the same. Consequently, the amount of harvested energy in each time instant is equal for both nodes.
- **Equal Mean:** The EH process of each node is a uniformly distributed random variable with mean  $\mu_1 = \mu_2 = \frac{E^{\text{max}}}{2}$ .
- **Double Mean - Relay Station:** The EH process of each node is a uniformly distributed random variable. In this case,  $\mu_1 = \frac{E^{\text{max}}}{4}$  and  $\mu_2 = \frac{E^{\text{max}}}{2}$ .
- **Double Mean - Source Node:** The EH process of each node is a uniformly distributed random variable. In this case,  $\mu_1 = \frac{E^{\text{max}}}{2}$  and  $\mu_2 = \frac{E^{\text{max}}}{4}$ .

The results show that the maximum throughput is achieved when the EH processes of the two nodes are equal. The reason is that the two-hop communication channel can be seen as a single overall channel whose capacity in each time interval depends on  $P_{1,n}$  and  $P_{2,n}$  simultaneously. Therefore, the throughput is maximized when the available energies at the nodes are equal. When only the mean value of the two EH process is equal, the throughput is reduced compared to the initial case because in each realization, one of the nodes is limited compared to the other. The maximum reduction is observed in the cases where the means are not equal. However, the throughput achieved when  $\mu_2 = 2\mu_1$  is on average equal to the throughput achieved when  $\mu_1 = 2\mu_2$ . This means that the reduction in the throughput due to energy limitation does not depend on which node is limited, but on the difference between the maximum energy values of  $S_1$  and  $S_2$ .

## VI. CONCLUSIONS

We have investigated a two-hop EH communications scenario with an amplify-and-forward relay station. The source node and the relay station harvest energy several times and use it for the transmission of data. We show that the resulting power allocation problem for throughput maximization is a

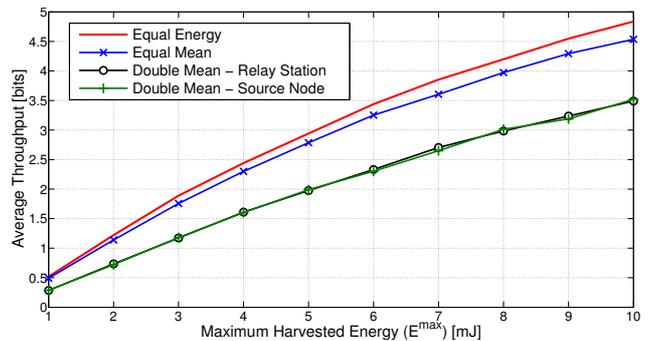


Fig. 3: Average throughput vs. maximum harvested energy for an EH two-hop scenario and  $N = 3$  EH time instants.

non-convex problem for which a closed-form solution cannot be found. To overcome this, we reformulate the problem as a D.C. programming problem and tailor a branch-and-bound algorithm to find a numerical solution. The results show that the maximum throughput is achieved when the available energies at the source node and at the relay station are equal. Additionally, it is shown that the throughput does not depend on which node is energy limited, but on the difference between the EH processes of the nodes.

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