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Uplink-Downlink Duality of Interference Alignment in Cellular Relay Networks

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***Abstract*—This paper focuses on signal space interference alignment for the uplink and downlink transmissions in a cellular relay network with multiple cells where a single base station serves multiple mobile stations in each cell and several amplify-and-forward relays are deployed. We show that the interference alignment problems in the uplink and downlink transmissions are a pair of formally dual problems. Exploiting the reciprocity of the channels, a two-step procedure first nullifying the inter-cell interferences following the idea of relay-aided interference alignment and then designing a zero-forcing filter for each base station to suppress the intra-cell interferences is proposed to obtain the dual interference alignment solutions. Furthermore, the dual solutions also achieve the same sum rate in both the uplink and the reciprocal downlink transmissions with a sum transmit power constraint even if the power consumed by the relays and the relay retransmitted noises are considered.**

I. Introduction

In the past few years, interference alignment (IA) has been developed as an efficient interference management technique to achieve a near optimal performance at high signal-to-noise-ratios (SNRs). Signal space IA can be performed using multiple time extensions [1], [2], multiple antennas at the communication nodes [3], [4], or amplify-and-forward (AF) relays [5]–[7]. Among the above three types, relay-aided IA requires the deployment of several simple (AF) relays and few time extensions due to the relaying protocol, while the numbers of antennas at the communication nodes remain small even in large networks. It is therefore of benefit to future cellular systems. The authors of [8] tackled the problem of IA in cellular networks for the first time. In literature, a variety of relay-aided IA schemes have been considered in cellular networks as well. The authors of [9] considered a full-duplex relay combined with a one-way relaying protocol. The IA solution and the achievable degrees of freedom (DoF) are investigated. Besides, two-way relaying has also been considered for cellular networks, [10], [11].

In the present paper, we focus on applying a two-hop relay-aided IA scheme for cellular networks considering a one-way relaying protocol. Exploiting the reciprocity of the channel, we investigate the duality of the considered IA schemes for both the uplink (UL) and the downlink (DL) transmissions. Specifically, we address two problems. The first problem is how to design the filters at the base stations (BSs), the mobile stations (MSs), and the relays to achieve IA in both the UL and the DL. To this end, we propose a two-step procedure, i.e., first to find an inter-cell IA solution for nullifying the inter-cell interferences and then to design a zero-forcing (ZF) filter for each BS to suppress the intra-cell interferences. We will show that the IA solutions for the UL and the DL transmissions are formally dual, i.e., given an IA solution for the UL/DL, a solution for the reciprocal DL/UL can be simply found. The second problem is whether these dual IA solutions also achieve the same performance in the UL and the DL. It is already known that the achievable DoF in both the UL and the DL are the same [8], [12]. In this work, we also compare the achieved sum rates and the consumed sum transmit powers of the dual IA solutions. We will derive a relation between the power allocation schemes for the UL and the DL transmissions such that the same sum rate can be achieved with a sum transmit power constraint.

We will first introduce the system model in Section II. In Section III, the two-step IA procedure is presented. Then we will investigate the achieved sum rates in the UL and the DL in Section IV. Finally, we show the simulation results and conclude this paper.

II. System Model

A cellular network consisting of K cells is considered. Each cell includes a single BS with N_{B} antennas and M single-antenna MSs, where $N_{\mathrm{B}} \geq M$ holds. R amplify-and-forward relays being equipped with N_{R} antennas each are deployed in the network to assist the communications between the BSs and the MSs. The relays are assumed to operate in half duplex mode. A two-hop transmission scheme which exploits the direct channels between the BSs and MSs is applied for both the UL and the DL transmissions. Furthermore, the wireless channels are modeled as single-tap channels and are assumed to remain constant throughout the duration of the transmission. The channel coefficients are independently drawn from continuous distributions. The details of the UL and DL transmissions will be introduced in the following part of this section.

In the UL, each MS transmits a single data stream intended for the BS in its own cell to all the relays and the BSs in the first transmission phase. The signals received by each relay will be linearly processed by the relay. Then in the second transmission phase, the relays and the MSs transmit to the BSs. Let the $N_{\mathrm{B}} \times M$ matrix $\mathbf{H}_{\mathrm{BM}}^{(i,j)}$, the $RN_{\mathrm{R}} \times M$ matrix $\mathbf{H}_{\mathrm{RM}}^{(j)}$, and the $N_{\mathrm{B}} \times RN_{\mathrm{R}}$ matrix $\mathbf{H}_{\mathrm{BR}}^{(i)}$ describe the channels from the MSs in the j-th cell to the i-th BS, the channels from the MSs in the j-th cell to the relays, and the channels from the relays to the i-th BS, respectively. The scenario of the UL transmission is shown in Fig. 1.

Let $\mathbf{d}_{\mathrm{UL}}^{(j)} = \left(d_{\mathrm{UL}}^{(j,1)}, \ldots, d_{\mathrm{UL}}^{(j,M)}\right)^{\mathrm{T}}$ denote the independent

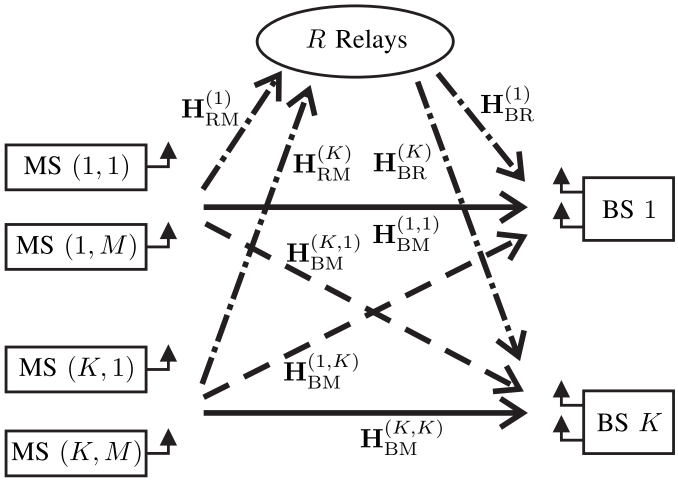

Fig. 1. UL transmission in a multicell network with multiple relays assisting the transmission

data symbols being transmitted by the MSs of the j-th cell. Define the $M \times M$ matrices $\mathbf{V}_1^{(j)}$ and $\mathbf{V}_2^{(j)}$ to be the transmit filter matrices at the MSs in the j-th cell for the first and the second transmission phase, respectively. We assume that the MSs do no cooperate. Therefore, $\mathbf{V}_1^{(j)}$ and $\mathbf{V}_2^{(j)}$ are both diagonal matrices. Their diagonal entries are the transmit filter coefficients of the MSs for the corresponding two phases. The signals received by the i-th BS in the first phase and the signals received by the relays can then be written as

$$\mathbf{r}_{\mathrm{B},1}^{(i)} = \sum_{j=1}^{K} \mathbf{H}_{\mathrm{BM}}^{(i,j)} \mathbf{V}_1^{(j)} \mathbf{d}_{\mathrm{UL}}^{(j)} + \mathbf{n}_{\mathrm{B},1}^{(i)} \tag{1}$$

and

$$\mathbf{r}_{\mathrm{R}} = \sum_{j=1}^{K} \mathbf{H}_{\mathrm{RM}}^{(j)} \mathbf{V}_1^{(j)} \mathbf{d}_{\mathrm{UL}}^{(j)} + \mathbf{n}_{\mathrm{R}}, \tag{2}$$

respectively, where $\mathbf{n}_{\mathrm{B},1}^{(i)}$ denotes the noise received by the BS in the first phase and $\mathbf{n}_{\mathrm{R}}$ denotes the noise received by the relays, both being independently identically distributed (i.i.d.) Gaussian noises with the variance σ^2. Define the $RN_{\mathrm{R}} \times RN_{\mathrm{R}}$ matrix $\mathbf{G}$ to be the processing matrix of the relays. We assume that the relays do not exchange their received signals among each other. Therefore, the matrix $\mathbf{G}$ has a block diagonal structure with the r-th $N_{\mathrm{R}} \times N_{\mathrm{R}}$ diagonal block being the processing matrix of the r-th relay. The signals received by the i-th BS in the second phase can be written as

$$\mathbf{r}_{\mathrm{B},2}^{(i)} = \mathbf{H}_{\mathrm{BR}}^{(i)} \mathbf{G} \mathbf{r}_{\mathrm{R}} + \sum_{j=1}^{K} \mathbf{H}_{\mathrm{BM}}^{(i,j)} \mathbf{V}_2^{(j)} \mathbf{d}_{\mathrm{UL}}^{(j)} + \mathbf{n}_{\mathrm{B},2}^{(i)}, \tag{3}$$

where $\mathbf{n}_{\mathrm{B},2}^{(i)}$ represents the i.i.d. Gaussian noise with the variance σ^2 received by the BS in the second phase. Afterwards, each BS linearly combines the signals it received in the two phases and estimates the data symbols as

$$\begin{aligned}\hat{\mathbf{d}}_{\mathrm{UL}}^{(i)} &= \mathbf{U}_1^{(i)*\mathrm{T}} \mathbf{r}_{\mathrm{B},1}^{(i)} + \mathbf{U}_2^{(i)*\mathrm{T}} \mathbf{r}_{\mathrm{B},2}^{(i)} \\ &= \sum_{j=1}^{K} \Big(\mathbf{U}_1^{(i)*\mathrm{T}} \mathbf{H}_{\mathrm{BM}}^{(i,j)} \mathbf{V}_1^{(j)} + \mathbf{U}_2^{(i)*\mathrm{T}} \mathbf{H}_{\mathrm{BM}}^{(i,j)} \mathbf{V}_2^{(j)} \\ &\quad + \mathbf{U}_2^{(i)*\mathrm{T}} \mathbf{H}_{\mathrm{BR}}^{(i)} \mathbf{G} \mathbf{H}_{\mathrm{RM}}^{(j)} \mathbf{V}_1^{(j)} \Big) \mathbf{d}_{\mathrm{UL}}^{(j)} + \tilde{\mathbf{n}}_{\mathrm{UL}}^{(i)},\end{aligned} \tag{4}$$

where the $N_{\mathrm{B}} \times M$ matrices $\mathbf{U}_1^{(i)}$ and $\mathbf{U}_2^{(i)}$ are the receive filter matrices in the two phases and $\tilde{\mathbf{n}}_{\mathrm{UL}}^{(i)}$ represents the effective noise.

Now we introduce the sum power constraint for the UL transmission. Define

$$p_{\mathrm{UL}}^{(j,m)} = \mathrm{E}\left\{ \left| d_{\mathrm{UL}}^{(j,m)} \right|^2 \right\} \tag{5}$$

to be the average power allocated to the data stream transmitted by the m-th MS in the j-th cell. Without loss of generality, let the transmit filters in the UL be normalized as

$$\mathbf{V}_1^{(j)*\mathrm{T}} \mathbf{V}_1^{(j)} + \mathbf{V}_2^{(j)*\mathrm{T}} \mathbf{V}_2^{(j)} = \mathbf{I}, \ \forall j \in \{1, \ldots, K\}. \tag{6}$$

The sum transmit power of the m-th MS in the j-th cell in the two transmission phases therefore reads $p_{\mathrm{UL}}^{(j,m)}$. Given the sum transmit power P_{sum} of all the MSs and the relays, the sum power constraint for the UL transmission can be written as

$$\begin{aligned}P_{\mathrm{sum}} \geq &\sum_{j=1}^{K} \sum_{m=1}^{M} p_{\mathrm{UL}}^{(j,m)} + \sum_{j=1}^{K} \mathrm{E}\left\{ \left\| \mathbf{G} \mathbf{H}_{\mathrm{RM}}^{(j)} \mathbf{V}_1^{(j)} \mathbf{d}_{\mathrm{UL}}^{(j)} \right\|^2 \right\} \\ &+ \sigma^2 \mathrm{tr}\left(\mathbf{G} \mathbf{G}^{*\mathrm{T}} \right).\end{aligned} \tag{7}$$

In (7), the first term on the right hand side is the sum transmit power of the MSs, the second term is the signal power retransmitted by all the relays, and the third term is the noise power retransmitted by the relays.

The two-hop transmission scheme applied for the DL transmission is dual to the one for the UL transmission introduced above. In the first phase, each BS transmits M data streams intended for the MSs in its own cell to all the relays and the MSs. After linear processing at the relays, the relays and the BSs transmit to the MSs in the second phase. Moreover, we will exploit the reciprocity of the channels between the nodes. Thus, the corresponding reciprocal DL channels are described by $\mathbf{H}_{\mathrm{BM}}^{(i,j)*\mathrm{T}}$, $\mathbf{H}_{\mathrm{RM}}^{(j)*\mathrm{T}}$ and $\mathbf{H}_{\mathrm{BR}}^{(i)*\mathrm{T}}$. The noise signals received by the relays and the MSs are again assumed to be white Gaussian noise with the common variance σ^2. Furthermore, we assume that the average power allocated to the data stream $d_{\mathrm{DL}}^{(i,m)}$ is $p_{\mathrm{DL}}^{(j,m)}$ and the sum available transmit power for the DL transmission is P_{sum}, too.

III. Two-Step Interference Alignment Procedure

In this section, we will design the transmit and receive filters at the BSs and the MSs as well as the relay processing matrix to achieve interference free transmissions in both the UL and the DL. The UL transmission will be considered first. Then we show that a dual solution can be applied for the DL transmission if the channels are reciprocal.

Suppose $\left(\mathbf{U}_1^{(i)*\mathrm{T}}, \mathbf{U}_2^{(i)*\mathrm{T}} \right)$ is a receive filter at a BS which along with the transmit filters at the MSs and the relay

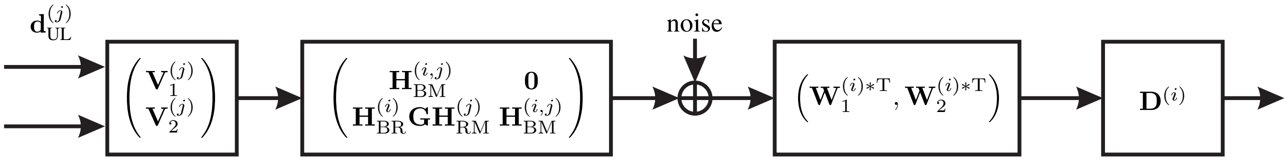

Fig. 2. Equivalent block diagram of the UL transmission from the MSs in the j-th cell to the i-th BS

processing matrix achieves interference-free reception at the BS in the UL transmission. Alternatively, one may use a concatenation of two filters $\left(\mathbf{W}_1^{(i)*\text{T}}, \mathbf{W}_2^{(i)*\text{T}}\right)$ and $\mathbf{D}^{(i)}$ at the BS, which are chosen such that

$$\mathbf{D}^{(i)}\left(\mathbf{W}_1^{(i)*\text{T}}, \mathbf{W}_2^{(i)*\text{T}}\right) = \left(\mathbf{U}_1^{(i)*\text{T}}, \mathbf{U}_2^{(i)*\text{T}}\right) \tag{8}$$

holds, to achieve the same performance as the one which could be achieved by using $\left(\mathbf{U}_1^{(i)*\text{T}}, \mathbf{U}_2^{(i)*\text{T}}\right)$. Specifically, the $M \times 2N_{\text{B}}$ matrix $\left(\mathbf{W}_1^{(i)*\text{T}}, \mathbf{W}_2^{(i)*\text{T}}\right)$ has the same row space as $\left(\mathbf{U}_1^{(i)*\text{T}}, \mathbf{U}_2^{(i)*\text{T}}\right)$ and serves the purpose of inter-cell IA. The $M \times M$ matrix $\mathbf{D}^{(i)}$ is an invertible matrix which is designed to suppress the intra-cell interferences. In other words, $\mathbf{D}^{(i)}$ is a ZF filter. A block diagram of the UL transmission is illustrated in Fig. 2.

We define the effective IA channel to be

$$\begin{aligned}\mathbf{H}_{\text{IA}}^{(i,j)} = &\mathbf{W}_1^{(i)*\text{T}}\mathbf{H}_{\text{BM}}^{(i,j)}\mathbf{V}_1^{(j)} + \mathbf{W}_2^{(i)*\text{T}}\mathbf{H}_{\text{BM}}^{(i,j)}\mathbf{V}_2^{(j)} \\ &+ \mathbf{W}_2^{(i)*\text{T}}\mathbf{H}_{\text{BR}}^{(i)}\mathbf{G}\mathbf{H}_{\text{RM}}^{(j)}\mathbf{V}_1^{(j)}.\end{aligned} \tag{9}$$

Then the UL inter-cell IA problem can be formulated as solving the matrix equations

$$\mathbf{H}_{\text{IA}}^{(i,j)} = \mathbf{0}, \quad \forall\, i,j = \{1,\ldots,K\},\ i \neq j. \tag{10}$$

The interference leakage minimization algorithm which minimizes the inter-cell interference leakage

$$L = \sum_{i=1}^{K}\sum_{\substack{j=1 \\ j\neq i}}^{K} \operatorname{tr}\left(\mathbf{H}_{\text{IA}}^{(i,j)}\mathbf{H}_{\text{IA}}^{(i,j)*\text{T}}\right) \tag{11}$$

can be used to approach a numerical solution of (10). The interference leakage minimization algorithm was originally proposed in [3] for MIMO IA without relays. We may directly extend it to the considered cellular relay network for inter-cell IA. However, the interference leakage minimization algorithm is computationally expensive and can not yield an exact solution. Therefore, we introduce a linear IA algorithm for the special case $M = N_{\text{B}}$ where the number of MSs in each cell equals the number of antennas at a BS. The linear IA algorithm is able to solve the inter-cell IA problem in closed form for this case.

If $N_{\text{B}} = M$ holds, the matrices $\mathbf{W}_1^{(i)}$ and $\mathbf{W}_2^{(i)}$ are both square matrices. Based on the intuition that all the variables which could be provided by the transmit and receive filters at the MSs and the BSs shall be utilized, we assume that $\mathbf{W}_2^{(i)}$ and $\mathbf{V}_1^{(j)}$ are both of full rank and, therefore, invertible. Hence (10) can be linearized as

$$\mathbf{W}^{(i)*\text{T}}\mathbf{H}_{\text{BM}}^{(i,j)} + \mathbf{H}_{\text{BM}}^{(i,j)}\mathbf{V}^{(j)} + \mathbf{H}_{\text{BR}}^{(i)}\mathbf{G}\mathbf{H}_{\text{RM}}^{(j)} = \mathbf{0}, \forall i \neq j, \tag{12}$$

where the elements of the matrix

$$\mathbf{W}^{(i)*\text{T}} = \left(\mathbf{W}_1^{(i)}\left(\mathbf{W}_2^{(i)}\right)^{-1}\right)^{*\text{T}}, \tag{13}$$

the non-zero elements of the matrix

$$\mathbf{V}^{(j)} = \mathbf{V}_2^{(j)}\left(\mathbf{V}_1^{(j)}\right)^{-1}, \tag{14}$$

and the non-zero elements of the relay processing matrix $\mathbf{G}$ are chosen as the new variables. The linearized UL inter-cell IA problem of (12) can be readily solved for the new variables. Once a solution is obtained, (13) and (14) can be used to reconstruct the receive IA filters at the BSs and the transmit filters at the MSs. Based on the UL inter-cell IA solution, designing the ZF filter $\mathbf{D}^{(i)}$ at each BS to suppress the intra-cell interferences is straightforward. Since $\mathbf{H}_{\text{IA}}^{(i,i)}$ is an $M \times M$ square matrix here, the ZF filter at the i-th BS is simply

$$\mathbf{D}^{(i)} = \left(\mathbf{H}_{\text{IA}}^{(i,i)}\right)^{-1}. \tag{15}$$

However, not all solutions in the solution subspace of (12) guarantee that each matrix $\mathbf{H}_{\text{IA}}^{(i,i)}$, which describes the useful link in the i-th cell, is of full rank. For instance, we may choose $\mathbf{G}$ to be a zero matrix, then choose $\mathbf{W}^{(i)*\text{T}}$ and $\mathbf{V}^{(j)}$ to be $\mathbf{I}_{M\times M}$ and $-\mathbf{I}_{M\times M}$, respectively. Obviously, with this solution neither the inter-cell interferences nor the useful signals can be transmitted to the BSs. Note that the above solution spans a one-dimensional subspace in the solution space of (12). The solution space of (12) therefore shall be at least of dimension two to allow the existence of other solutions. Thus the number of relays and the number of antennas at each relay must satisfy

$$RN_{\text{R}}^2 \geq K(K-2)M^2 - KM + 2. \tag{16}$$

The condition of (16) is derived by comparing the number of variables and the number of equations of (12). It is a necessary condition for obtaining a valid solution of the inter-cell IA problem as reasoned above. In fact, it is also a sufficient condition in the almost sure sense. This means if (16) holds, a randomly selected solution of (12) almost surely guarantees that each matrix $\mathbf{H}_{\text{IA}}^{(i,i)}$ is of full rank. However, as the feasibility conditions for IA are not the main focus of this paper, the proof of the sufficiency of (16) is omitted here. More details about the validity of the linear IA solutions can be found in our paper [13].

For the DL transmission, a similar two-step procedure can be applied to achieve interference-free transmission. In other words, we may first design a transmit IA filter at each BS along with the receive filters at the MSs and the relay processing matrix to align the inter-cell interferences, then design a ZF filter for each BS such that the MSs in the cell receive no intra-cell interferences.

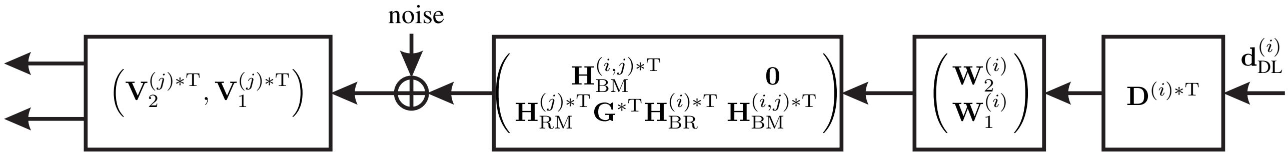

Fig. 3. Equivalent block diagram of the dual DL transmission from the i-th BS to the MSs in the j-th cell

If an UL inter-cell IA solution is already found, it is easy to find a solution of the dual DL inter-cell IA problem by exploiting the reciprocity of the channels. More specifically, we choose $\mathbf{W}_2^{(i)}$ and $\mathbf{W}_1^{(i)}$ to be the IA filter matrices at the i-th BS for the first and the second phase of the DL transmission, respectively. We choose $\mathbf{V}_2^{(j)*\mathrm{T}}$ and $\mathbf{V}_1^{(j)*\mathrm{T}}$ to be the receive filter matrices at the MSs in the j-th cell for the first and the second phase of the DL transmission, recpectively. Furthermore, we choose $\mathbf{G}^{*\mathrm{T}}$ to be the relay processing matrix. In this way, one can verify that the DL inter-cell IA problem is also solved since

$$\begin{aligned}\mathbf{H}_{\mathrm{IA}}^{(i,j)*\mathrm{T}} &= \mathbf{V}_2^{(j)*\mathrm{T}}\mathbf{H}_{\mathrm{BM}}^{(i,j)*\mathrm{T}}\mathbf{W}_2^{(i)} + \mathbf{V}_1^{(j)*\mathrm{T}}\mathbf{H}_{\mathrm{BM}}^{(i,j)*\mathrm{T}}\mathbf{W}_1^{(i)} \\ &\quad + \mathbf{V}_1^{(j)*\mathrm{T}}\mathbf{H}_{\mathrm{RM}}^{(j)*\mathrm{T}}\mathbf{G}^{*\mathrm{T}}\mathbf{H}_{\mathrm{BR}}^{(i)*\mathrm{T}}\mathbf{W}_2^{(i)} \\ &= \mathbf{0}, \qquad \forall\, i,j=\{1,\ldots,K\},\ i\neq j \end{aligned} \tag{17}$$

holds. Furthermore, the ZF filter

$$\left(\mathbf{H}_{\mathrm{IA}}^{(i,i)*\mathrm{T}}\right)^{-1} = \mathbf{D}^{(i)*\mathrm{T}} \tag{18}$$

shall be used at the i-th BS to suppress the intra-cell interferences. The DL transmission from the i-th BS to the MSs in the j-th cell, which is dual to the UL transmission shown in Fig. 2, is illustrated in Fig. 3.

IV. Achieved Sum Rate

A. Sum Rate with Sum Power Constraint

Employing the duality of the filter designs in the UL and the DL transmissions shown in Section III, we can calculate the sum rate achieved for both the UL and the DL transmissions in the considered cellular relay network. Let $\left(p_{\mathrm{UL}}^{(1,1)},\ldots,p_{\mathrm{UL}}^{(K,M)}\right)$ be the powers allocated to the data streams in the UL satisfying the sum power constraint of (7) with equality, i.e.,

$$\begin{aligned}P_{\mathrm{sum}} &= \sum_{i=1}^{K}\sum_{m=1}^{M}\mathrm{diag}_m\big(\mathbf{I}_{M\times M} \\ &\quad +\mathbf{V}_1^{(i)*\mathrm{T}}\mathbf{H}_{\mathrm{RM}}^{(i)*\mathrm{T}}\mathbf{G}^{*\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{RM}}^{(i)}\mathbf{V}_1^{(i)}\big)p_{\mathrm{UL}}^{(i,m)} \\ &\quad + \sigma^2\mathrm{tr}\left(\mathbf{G}\mathbf{G}^{*\mathrm{T}}\right) \\ &= \sum_{i=1}^{K}\sum_{m=1}^{M}\alpha^{(i,m)}p_{\mathrm{UL}}^{(i,m)} + \sigma^2\mathrm{tr}\left(\mathbf{G}\mathbf{G}^{*\mathrm{T}}\right)\end{aligned} \tag{19}$$

holds, where $\mathrm{diag}_m(\cdot)$ denotes the m-th diagonal element of a matrix. In the second equality of (19), we introduce the coefficients $\alpha^{(i,m)}$ to simplify the notations. Note that the value of $\alpha^{(i,m)}$ depends on the inter-cell IA solution we selected. The physical meaning of $\alpha^{(i,m)}p_{\mathrm{UL}}^{(i,m)}$ is the sum power consumed by the m-th MS in the i-th cell and all the relays for transmitting the data stream $d_{\mathrm{UL}}^{(i,m)}$ in the UL.

Furthermore, the covariance matrix of the effective noise introduced in (4) can be written as

$$\begin{aligned}\mathbf{R}_{\tilde{\mathbf{n}}_{\mathrm{UL}}^{(i)}\tilde{\mathbf{n}}_{\mathrm{UL}}^{(i)}} &= \sigma^2\mathbf{D}^{(i)}\Big(\mathbf{W}_2^{(i)*\mathrm{T}}\mathbf{H}_{\mathrm{BR}}^{(i)}\mathbf{G}\mathbf{G}^{*\mathrm{T}}\mathbf{H}_{\mathrm{BR}}^{(i)*\mathrm{T}}\mathbf{W}_2^{(i)} \\ &\quad + \mathbf{W}_1^{(i)*\mathrm{T}}\mathbf{W}_1^{(i)} + \mathbf{W}_2^{(i)*\mathrm{T}}\mathbf{W}_2^{(i)}\Big)\mathbf{D}^{(i)*\mathrm{T}},\end{aligned} \tag{20}$$

where the matrix $\mathbf{D}^{(i)}$ is given by (15). Therefore, the SNR of the m-th data stream at the i-th BS can be computed as

$$\gamma_{\mathrm{UL}}^{(i,m)} = \frac{\mathrm{diag}_m\big(\mathbf{D}^{(i)}\mathbf{H}_{\mathrm{IA}}^{(i,i)}\big)p_{\mathrm{UL}}^{(i,m)}}{\mathrm{diag}_m\big(\mathbf{R}_{\tilde{\mathbf{n}}_{\mathrm{UL}}^{(i)}\tilde{\mathbf{n}}_{\mathrm{UL}}^{(i)}}\big)} = \frac{p_{\mathrm{UL}}^{(i,m)}}{\beta^{(i,m)}\sigma^2}, \tag{21}$$

where $\mathrm{diag}_m(\cdot)$ denotes the m-th diagonal element of a matrix and the coefficient $\beta^{(i,m)}$ is introduced to simplify the notations.

Similarly, let $\left(p_{\mathrm{DL}}^{(1,1)},\ldots,p_{\mathrm{DL}}^{(K,M)}\right)$ be the powers allocated to the data streams in the DL satisfying the sum power constraint with equality, i.e.,

$$\begin{aligned}P_{\mathrm{sum}} &= \sum_{i=1}^{K}\sum_{m=1}^{M}\mathrm{diag}_m\Big(\mathbf{D}^{(i)}\big(\mathbf{W}_1^{(i)*\mathrm{T}}\mathbf{W}_1^{(i)} + \mathbf{W}_2^{(i)*\mathrm{T}}\mathbf{W}_2^{(i)} \\ &\quad + \mathbf{W}_2^{(i)*\mathrm{T}}\mathbf{H}_{\mathrm{BR}}^{(i)}\mathbf{G}\mathbf{G}^{*\mathrm{T}}\mathbf{H}_{\mathrm{BR}}^{(i)*\mathrm{T}}\mathbf{W}_2^{(i)}\big)\mathbf{D}^{(i)*\mathrm{T}}\Big) \\ &\quad + \sigma^2\mathrm{tr}\left(\mathbf{G}^{*\mathrm{T}}\mathbf{G}\right)\end{aligned} \tag{22}$$

holds. Comparing the expression of (22) with (20), we may further simplify (22) as

$$P_{\mathrm{sum}} = \sum_{i=1}^{K}\sum_{m=1}^{M}\beta^{(i,m)}p_{\mathrm{DL}}^{(i,m)} + \sigma^2\mathrm{tr}\left(\mathbf{G}^{*\mathrm{T}}\mathbf{G}\right), \tag{23}$$

where $\beta^{(i,m)}$ has been introduced in (21). Here, the physical meaning of $\beta^{(i,m)}p_{\mathrm{DL}}^{(i,m)}$ is the sum power consumed by the i-th BS and all the relays for transmitting the data stream $d_{\mathrm{DL}}^{(i,m)}$ in the DL. Furthermore, the coefficient $\beta^{(i,m)}$ yields a direct relation between the signal power transmitted by a BS and the relays in the DL and the variance of the noise received by the same BS in the UL.

Moreover, it is also not difficult to find that the SNR of the estimated data stream at the m-th MS in the i-th cell is

$$\gamma_{\mathrm{DL}}^{(i,m)} = \frac{p_{\mathrm{DL}}^{(i,m)}}{\alpha^{(i,m)}\sigma^2}. \tag{24}$$

Therefore, the variance of the noise received by a MS in the DL and the signal power transmitted by the same MS and the relays in the UL are connected by the coefficient $\alpha^{(i,m)}$.

Comparing the equations (19), (21), (23) and (24), the relation between the achievable sum rates in the UL and DL transmissions with a sum transmit power constraint is almost clear. Given any power allocation scheme $\left(p_{\text{UL}}^{(1,1)}, \ldots, p_{\text{UL}}^{(K,M)}\right)$ in the UL satisfying the sum power constraint with equality as shown in (19), then allocating the powers as

$$p_{\text{DL}}^{(i,m)} = \frac{\alpha^{(i,m)}}{\beta^{(i,m)}} p_{\text{UL}}^{(i,m)}, \quad \forall\, i,\ m \tag{25}$$

to the data streams in the DL also satisfies the sum power constraint with equality as shown in (23). A direct result of (25) is that

$$\gamma_{\text{DL}}^{(i,m)} = \frac{p_{\text{DL}}^{(i,m)}}{\alpha^{(i,m)}\sigma^2} = \frac{p_{\text{UL}}^{(i,m)}}{\beta^{(i,m)}\sigma^2} = \gamma_{\text{UL}}^{(i,m)} \tag{26}$$

holds for any i and m. Therefore, the same sum rate can be achieved in both the UL and the DL transmissions.

B. Optimal Power Allocation

By (25) and (26), we can prove that if a power allocation scheme for the UL transmission is optimal with respect to the given UL inter-cell IA solution, the power allocation scheme for the DL transmission given by (25) is also optimal with respect to the dual DL inter-cell IA solution. This can be proved by contradiction. In this section, we will give another proof by deriving the optimal power allocation schemes for both the UL and DL transmissions.

For the UL transmission, the sum rate maximizing power allocation scheme is the solution of the optimization problem

$$\underset{p_{\text{UL}}^{(i,m)},\ \forall\, i,m}{\arg\max} \left\{ \sum_{i=1}^{K} \sum_{m=1}^{M} \text{ld}\left(1 + \frac{p_{\text{UL}}^{(i,m)}}{\beta^{(i,m)}\sigma^2}\right) \right\} \tag{27}$$

subject to the sum power constraint

$$\sum_{i=1}^{K} \sum_{m=1}^{M} \alpha^{(i,m)} p_{\text{UL}}^{(i,m)} + \sigma^2 \text{tr}\left(\mathbf{G}\mathbf{G}^{*\text{T}}\right) \leq P_{\text{sum}}. \tag{28}$$

Using the method of Lagrange multipliers, a water-filling-like solution can be obtained as

$$p_{\text{UL,opt}}^{(i,m)} = \max\left\{0,\ \frac{P_{\text{W}}}{\alpha^{(i,m)}} - \beta^{(i,m)}\sigma^2\right\}, \tag{29}$$

where P_{W} is chosen such that the sum power constraint of (28) is satisfied with equality. Substituting (29) into (25) yields

$$p_{\text{DL,opt}}^{(i,m)} = \frac{\alpha^{(i,m)}}{\beta^{(i,m)}} p_{\text{UL,opt}}^{(i,m)} = \max\left\{0,\ \frac{P_{\text{W}}}{\beta^{(i,m)}} - \alpha^{(i,m)}\sigma^2\right\}. \tag{30}$$

We may easily verify that (30) is the solution of the DL sum rate maximization problem

$$\underset{p_{\text{DL}}^{(i,m)},\ \forall\, i,m}{\arg\max} \left\{ \sum_{i=1}^{K} \sum_{m=1}^{M} \text{ld}\left(1 + \frac{p_{\text{DL}}^{(i,m)}}{\alpha^{(i,m)}\sigma^2}\right) \right\} \tag{31}$$

subject to a total power constraint

$$\sum_{i=1}^{K} \sum_{m=1}^{M} \beta^{(i,m)} p_{\text{DL}}^{(i,m)} + \sigma^2 \text{tr}\left(\mathbf{G}^{*\text{T}}\mathbf{G}\right) \leq P_{\text{sum}}, \tag{32}$$

because (32) is satisfied with equality as well. This proves our statement.

V. Simulation Results

For the simulations, we consider a cellular relay network consisting of three cells. A single BS equipped with two antennas and two single-antenna MSs are included in each cell. Two relays being equipped with two antennas each are deployed in the network to help achieving interference-free communications. The feasibility condition for inter-cell IA (16) is satisfied with equality. The proposed two-hop transmission scheme can be applied for both the UL and the DL transmissions.

We assume that the wireless channels between the nodes are i.i.d. Rayleigh channels with unit average channel gains. The performance is measured by the average sum rate per transmission phase as a function of the total transmit power to noise ratio P_{sum}/σ^2. The simulation results are based on an arbitrary UL inter-cell IA solution obtained by either the proposed linear IA algorithm or the interference leakage minimization algorithm and its dual solution of the DL inter-cell IA problem. Depending on the inter-cell IA solution we select, the ZF filters $\mathbf{D}^{(i)}$ at each BS are designed to suppress the intra-cell interferences. Both the optimal water-filling-like power allocation and the uniform power allocation are considered.

The results based on linear IA solutions are shown in Fig. 4. As discussed in Section IV, the sum rate achieved in the UL and in the DL shall be the same if the optimal power allocations are applied. The performance for both the UL and the DL transmissions are shown by the solid curve with plus signs. The dashed curve with squares shows the performance in the UL transmission if all the data streams have the same average power. This is nearly optimal at high SNRs. However, a constant gap exists between the sum rate achieved by the optimal power allocation and the one achieved by the uniform power allocation even at very high SNRs. This is because at very high SNRs,

$$p_{\text{UL,opt}}^{(i,m)} \approx \frac{P_{\text{sum}}}{\alpha^{(i,m)}KM} \tag{33}$$

holds for all the MSs. The values of the coefficient $\alpha^{(i,m)}$ depend on the IA solution we select and are likely to be different from each other. The dashed curve with circles shows the performance in the DL transmission if all the data streams transmitted by the BSs have equal average powers. In this case, the achieved sum rate is lower than ones achieved in the other two cases. This implies that the values of the coefficients $\beta^{(i,m)}$ usually differ a lot from each other, which makes the uniform power allocation in the DL transmission to be far away from the optimum. Fig. 5 shows the performances based on the interference leakage minimization solutions. Similar results as in Fig. 4 can be observed. However, the interference leakage minimization algorithm yields inferior average performances as compared to the linear IA algorithm. The exact reason for this is still unclear. Nevertheless, if we take a closer look at the simulation results, it can be observed that the interference leakage minimization algorithm usually yields a solution with relatively large relay gains. Therefore, a large amount of power is wasted on retransmitting the relay noise.

VI. Conclusion

In this paper, a two-hop relay-aided IA scheme is applied in cellular relay networks. We investigate the duality between the

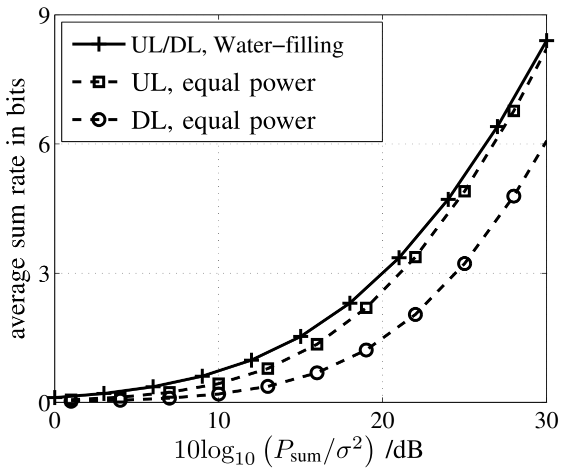

Fig. 4. Average sum rate per transmission phase based on linear IA solutions

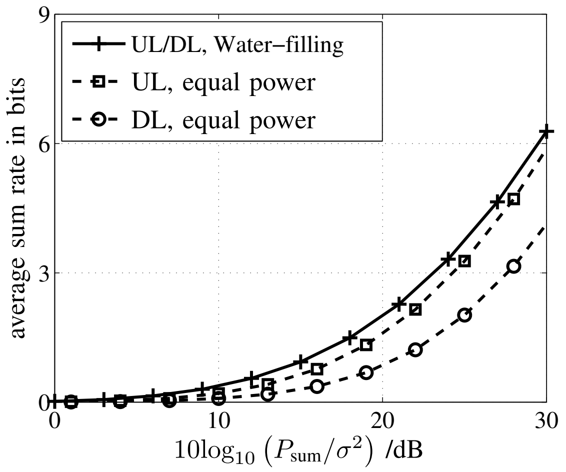

Fig. 5. Average sum rate per transmission phase based on interference leakage minimization solutions

IA schemes for the UL and DL transmissions. We show that IA for achieving interference-free transmission in the network can be decomposed into inter-cell IA and intra-cell interference ZF. The filters for inter-cell IA and the ZF filters can be designed sequentially. A pair of dual IA solutions can be obtained for the UL and DL. We also show a relation between the sum power consumed by the network for transmitting a data stream from a MS to the corresponding BS in the UL and the variance of the effective noise received by the same MS in the DL, and vice versa. Based on these results, we conclude that the sum rates achieved by IA with a sum transmit power constraint are the same in both UL and DL.

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