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An Interference Alignment Scheme using Partial CSI for Large Partially Connected Relay Networks

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Abstract—In this paper, we consider large partially connected relay networks made up of multiple subnetworks. An interference alignment scheme using only partial channel state information is proposed. In particular, every node and the relay only knows the intra-subnetwork channel of the corresponding subnetwork, the network topology, and the side information from other subnetworks. The proposed scheme requires no additional relay antennas as compared to interference alignment using full channel state information. Furthermore, the proposed scheme can be parallelized to reduce the delay in large networks.

I. INTRODUCTION

Applying interference alignment (IA) in large wireless interference networks has always been an attractive but challenging research topic. Since IA is able to achieve 1/2 degrees of freedom (DoF) per user [1], the limited frequency resource can be more efficiently utilized simply through adding more users. However, most of the conventional IA schemes rely on full channel state information (CSI), i.e., each node has to know the CSI of all the links in the network. As the size of the network increases, acquisition of full CSI becomes more difficult or even impossible. Firstly, the length of the pilot signals required for channel estimation is proportional to the network size. Secondly, the CSI estimated at each receiver shall be fed back to all the other nodes in the network, which requires a high-capacity, low-latency backhaul network.

Recently, IA schemes without full CSI have been investigated. The authors of [2] and [3] first proposed the scheme of blind IA and applied it to cellular networks. Blind IA requires no CSI at all to achieve the maximum DoF. However, a super symbol whose length grows exponentially with the number of receive antennas or with the maximum number of users per cell shall be constructed to this end. In [4] and the references therein, an IA scheme exploiting the partial connectivity of large cellular networks is proposed. It requires that the neighboring cellular users are able to share decoded messages for interference cancellation. From a practical point of view, passing the decoded messages from one user to another accumulates errors. The authors of [5] proposed an IA scheme using local CSI for partially connected two-way relaying networks. With the help of relays, achieving IA requires only few antennas at each node even in large networks. However besides [5], there are few results for relay-aided IA without full CSI.

In this paper, we consider large partially connected relay networks employing the one-way relaying protocol. Suppose several source-destination node pairs and an amplify-and-forward relay are located close by. The subnetwork they form is assumed to be fully connected. The entire network is made up of multiple such subnetworks. Different subnetworks are connected only via the direct links between some of the source and destination nodes in them, which will be referred to as the inter-subnetwork links in this paper. A toy example with two subnetworks is shown in Fig. 1. The IA solutions in such partially connected networks with full CSI have been addressed in our preliminary work [6]. Now we proposed an IA scheme for such networks using partial CSI. Firstly, we assume that every node or relay knows the channel realization of the intra-subnetwork links of the corresponding subnetwork. This requires to exchange CSI within each individual subnetwork only and shall be practicable if the subnetwork size is relatively small. Secondly, every node or relay is assumed to know the network topology, i.e., the existence of every inter-subnetwork link. Finally, we assume that each subnetwork obtains some side information from the other subnetworks. The side information contains a few complex valued numbers, which will be explained in the following discussions of the paper. Using the partial CSI introduced above, the individual subnetworks are able to separately design the filters at the nodes and relays such that both the intra- and inter-subnetwork interference can be nullified and IA can be achieved in the entire network.

In the next Section, the transmission scheme and the interference-nulling conditions are introduced. A toy example is used to explain our key ideas in Section III. More detailed discussions on the proposed IA scheme using partial CSI are in Section IV and V. Finally, we show the performances of the proposed scheme in Section VI and conclude this work.
II. INTERFERENCE-NULLING SOLUTIONS

The considered network topology has been introduced in Section I. In particular, the entire network is made up of $Q$ subnetworks being partially connected to each other by the inter-subnetwork links. The $q$-th subnetwork includes the $q$-th relay, which has $N_q$ antennas, and $K_q$ single-antenna source-destination nodes pairs. To avoid the discussion of some trivial cases where relays are not required for IA at all, we assume that each subnetwork is fully connected and has at least three node pairs. A two-hop transmission scheme is exploited. In the first time slot, each source node transmits a single data symbol to the connected relay and destination nodes. In the second time slot, each relay forwards the linearly processed signals to the connected destination nodes, while each source node transmits again to the connected destination nodes. If IA is feasible, $1/2$ DoF per node pair are achievable using this transmission scheme. The channel is assumed to be constant throughout the communication. The channel between the $k$-th source node and the $j$-th destination is denoted by the scalar $h_{DS}^{(j,k)}$. The channel between the $k$-th source node and the $q$-th relay is denoted by the $N_q \times 1$ vector $h_{DS}^{(q,k)}$. Finally, the channel between the $q$-th relay and the $j$-th destination node is denoted by the $1 \times N_q$ vector $h_{DR}^{(j,q)}$. For the absent links, the channel coefficients are set to zero. For the present ones, the channel coefficients are assumed to be independently drawn from a continuous distribution and be non-zero with probability one. Furthermore, the transmit filter at the $k$-th source node and the receive filter at the $j$-th destination node are denoted by $(v_1^{(k)},v_2^{(k)})^T$ and $(u_1^{(j)},u_2^{(j)})^T$, respectively. The processing matrix at the $q$-th relay is denoted by the $N_q \times N_q$ matrix $G^{(q)}$.

IA requires that both the intra- and inter-subnetwork interference must be nullified. On the one hand, if the $k$-th source node and the $j$-th destination node both belong to the $q$-th subnetwork and $j \neq k$ holds, the intra-subnetwork interference-nulling (IN) condition can be formulated as

$$
\begin{pmatrix}
v_1^{(k)} \\
v_2^{(k)}
\end{pmatrix}
=\begin{pmatrix}
\frac{1}{v_1^{(k)}} \\
\frac{1}{v_2^{(k)}}
\end{pmatrix}
\begin{pmatrix}
h_{DS}^{(j,k)} \\
G^{(q)}h_{DS}^{(q,k)}
\end{pmatrix}
\begin{pmatrix}
v_1^{(j)} \\
v_2^{(j)}
\end{pmatrix}
= 0.
$$

(1)

Here we introduce the filter coefficients $v^{(k)}=v_2^{(k)}/v_1^{(k)}$ and $u^{(j)*}=u_1^{(j)}/u_2^{(j)}$, which specify the one-dimensional transmit signal subspace at a source node and the one-dimensional receive signal subspace at a destination node, respectively. Using $u^{(k)}$ and $u^{(j)*}$, (1) can be linearized as

$$
h_{DS}^{(j,q)}G^{(q)}h_{DS}^{(q,k)} + h_{DS}^{(j,k)}(u^{(k)} + u^{(j)*}) = 0,
$$

(2)

where the new filter coefficients $v^{(k)}$, $u^{(j)*}$, and the elements of $G^{(q)}$ are the unknowns. We refer to the solution space $\mathbb{W}_q$ of all the $K_q(K_q-1)$ equations of (2) for the $q$-th subnetwork as the intra-subnetwork IN solution space of the $q$-th subnetwork. It is of dimension $N_q^2 + 2K_q - K_q(K_q-1)$. The feasibility conditions for relay-aided IA in fully connected networks require that the dimension of $\mathbb{W}_q$ shall be at least two [7]. On the other hand, if the $k$-th source node and the $j$-th destination node belong to different subnetworks and if they are connected by an inter-subnetwork link, no relay is able to participate in nulling the inter-subnetwork interference.

III. A TOY EXAMPLE

We first consider the simple network shown in Fig. 1 as a toy example. Assuming that full CSI is available, if IA is feasible with $N_q$ antennas at the $q$-th relay, the tuple $(N_1^2,\ldots,N_Q^2)$ is referred to as a proper relay antenna configuration. According to [6], the proper relay antenna configurations of this toy example are illustrated in Fig. 2. The two Pareto points, which represent the minimum required numbers of relay antennas in this network, are $(N_1^2,N_2^2)=(2,4)$ and $(N_1^2,N_2^2)=(3,3)$.

If full CSI is not available, an intuitive scheme is to fix the transmit and receive signal subspaces at the nodes being connected by the inter-subnetwork links to fulfill the inter-subnetwork IN conditions of (4) first. To this end, one can simply choose $u^{(1)*}=u^{(2)*}=1$ and $v^{(4)}=v^{(5)}=v^{(6)}=-1$, for instance. Note that this requires only the knowledge of the network topology. Given $u^{(1)*}$, $u^{(2)*}$, $v^{(4)}$, $v^{(5)}$, and $v^{(6)}$, the intra-subnetwork IN conditions in each subnetwork can be solved separately using the corresponding intra-subnetwork CSI. However, since some nodes are no longer able to assist in intra-subnetwork IN, both relays shall provide more variables, thus need more antennas. In this particular example, the required numbers of relay antennas are $N_1^2 \geq 3$ and $N_2^2 \geq 4$. In larger networks with more nodes being connected by inter-subnetwork links, more antennas will be required at every relay. For this reason, we look forward to a new scheme which uses only partial CSI but requires no additional relay antennas as compared to using full CSI.

The proposed scheme starts by solving the intra-subnetwork IN conditions of subnetwork 1. To ensure that its intra-subnetwork IN solution space $\mathbb{W}_1$ is at least two dimensional, $N_1^2 \geq 2$ is required. Subnetwork 1 then randomly selects a solution from $\mathbb{W}_1$ and forwards the filter coefficients $u^{(1)*}$ and $u^{(2)*}$ to subnetwork 2. Being aware
of \(u^{(1)*}\) and \(u^{(2)*}\), subnetwork 2 will then select a solution from its intra-subnetwork IN solution space \(\mathcal{W}_2\) such that the selected solution also fulfills the inter-subnetwork IN conditions \(v^{(4)} = -u^{(1)*}\) and \(v^{(5)} = u^{(0)} = -u^{(2)*}\). If such a solution exists, all the interferences in the entire network can be nullified. Obviously, this requires \(\mathcal{W}_2\) having at least three dimensions. Therefore, \(N_2^2 \geq 4\) must hold. Hence, the Pareto point of proper relay antenna configurations \((N_1^2, N_2^2) = (2, 4)\) is achievable. The key idea is to forward the filter coefficients \(u^{(1)*}\) and \(u^{(2)*}\) from subnetwork 1 to subnetwork 2, which will be referred to as side information in this paper. The side information can also be regarded as a compressed version of the intra-subnetwork CSI of other subnetworks. In this toy example, the side information contains only two complex valued numbers.

Alternatively, subnetwork 2 could also solve its intra-subnetwork IN conditions first. Note that the only way to align the inter-subnetwork interferences caused by source 5 and 6 at destination 2 is to choose the transmit signal subspaces at source 5 and at source 6 to be parallel, i.e., \(v^{(5)} = v^{(6)}\) must be fulfilled. Therefore, subnetwork 2 has to select a solution from its intra-subnetwork IN solution space \(\mathcal{W}_2\) under the additional constraint \(v^{(5)} = v^{(6)}\). Based on the selected solution, \(v^{(4)}\) and any one of \(v^{(5)}\) or \(v^{(6)}\) can be forwarded to subnetwork 1 as the side information. Using the alternative, the other Pareto point \((N_1^2, N_2^2) = (3, 3)\) is achievable and the side information contains two complex valued numbers as well.

### IV. Extension to Large Networks

In this Section, we will describe the proposed IA scheme using partial CSI in detail. We first introduce a few useful terms and preliminary results, which can be found in [6].

**Definition 1 (Set of Subnetworks):** The subnetworks being indexed by the elements of \(S \subseteq \{1, \ldots, Q\}\) form a set of subnetworks denoted by \(S\).

**Definition 2 (External Constraints):** A path consisting of present inter-subnetwork links results in an external constraint between the end nodes of the path, which only depends on the types of the end nodes of the path. Specifically, if such a path exists between the \(k\)-th source node and the \(j\)-th destination node, the external constraint \(v^{(k)} + u^{(j)*} = 0\) follows. If such a path exists between two source nodes or between two destination nodes, the corresponding external constraint is \(v^{(k)} = u^{(j)}\) or \(v^{(k)*} = u^{(j)*}\), respectively.

Note that if all the external constraints are satisfied, all the inter-subnetwork IN conditions are also fulfilled, and vice versa. The external constraints between nodes belonging to the set of subnetworks \(S\) can be represented by the edges of a graph \(G_S\). It has been shown in [6] that the rank of the incidence matrix of the graph, which is simply denoted by \(\text{rank}(G_S)\), represents the number of linearly independent external constraints for \(S\).

**Proposition 1:** Suppose \(S\) is formed by two disjoint sets of subnetworks \(S_1\) and \(S_2\), i.e., \(S = S_1 \cup S_2\) and \(S_1 \cap S_2 = \emptyset\) hold, then \(\text{rank}(G_S) \geq \text{rank}(G_{S_1}) + \text{rank}(G_{S_2})\) follows.

**Proposition 2:** IA is feasible, almost surely, in the considered partially connected networks if and only if the numbers of relay antennas satisfy the inequality

\[
\sum_{q \in S} N_q^2 \geq \sum_{q \in S} K_q(K_q - 3) + \text{rank}(G_{S_q}) + 2
\]

for any non-empty set of subnetworks \(S \subseteq \{1, \ldots, Q\}\).

If a network can be divided into two sets of subnetworks which are not connected to each other, they can be considered separately. Therefore, we exclude this case from the following discussions. The proposed scheme has \(Q\) steps. Roughly speaking, in each step, one of the \(Q\) subnetworks will select a solution from its intra-subnetwork IN solution space and forwards some of its filter coefficients as the side information to other subnetworks. The answers to the following three questions will explain the details of this procedure. To facilitate the description, let the subnetworks be indexed such that the \(q\)-th subnetwork will select its IN solution in the \(q\)-th step.

**Q1. Which subnetwork will select the IN solution in the \(q\)-th step?** The 1st subnetwork can be arbitrarily chosen. The \(q\)-th subnetwork shall be connected to at least one of the subnetworks \(1, \ldots, q - 1\) by a path consisting of inter-subnetwork links, which results in an external constraint for the set of subnetworks \(\{1, \ldots, q\}\).

**Q2. Which filter coefficients shall be forwarded to other subnetworks as the side information?** One subnetwork does not need to forward all the filter coefficients of its member nodes to all the other subnetworks. Instead, if the \(q\)-th and the \(r\)-th subnetworks, with \(r \in \{q + 1, \ldots, Q\}\), are connected by a path consisting of inter-subnetwork links, the \(q\)-th subnetwork shall forward the filter coefficient of the end node of the path to the \(r\)-th subnetwork.

**Q3. How does a subnetwork select its IN solution?** The \(q\)-th subnetwork shall first solve its intra-subnetwork IN conditions under the external constraints for itself, i.e., the external constraints between two nodes in the \(q\)-th subnetwork. The obtained solution space \(\mathcal{W}_q\) is a subspace of the intra-subnetwork IN solution space of the \(q\)-th subnetwork \(\mathcal{W}_q\). Having the side information from the subnetworks \(1, \ldots, q - 1\), the \(q\)-th subnetwork will select an IN solution from the solution space \(\mathcal{W}_q\) such that all the external constraints between the \(q\)-th subnetwork and the set of subnetworks \(\{1, \ldots, q - 1\}\) are satisfied. Specially, the 1st subnetwork can randomly choose an IN solution from the solution space \(\mathcal{W}_1\).

We now derive the required numbers of relay antennas for the proposed scheme. In the \(q\)-th step, the \(q\)-th subnetwork first needs to solve its intra-subnetwork IN conditions under the external constraints for itself. Hence the number of antennas at the \(q\)-th relay must satisfy

\[
N_q^2 \geq K_q(K_q - 3) + \text{rank}(G_{\{q\}}) + 2
\]

such that the solution space \(\mathcal{W}_q\) has at least two dimensions. In addition to this, the \(q\)-th subnetwork, except for the first one, shall also consider the external constraints between itself and the set of subnetworks \(\{1, \ldots, q - 1\}\). Suppose there are \(M_q\) such external constraints, where \(M_q\) can be computed as

\[
M_q = \text{rank}(G_{\{1, \ldots, q\}}) - \text{rank}(G_{\{1, \ldots, q - 1\}}) - \text{rank}(G_{\{q\}})
\]

using Prop. 1. Remember that the filter coefficient at one of the end nodes of each of those external constraints has been
forwarded to the $q$-th subnetwork as the side information in the previous $q - 1$ steps. To satisfy these $M_q$ external constraints, $M_q$ filter coefficients in the $q$-th subnetwork have to be chosen accordingly. Hence, the solution space $\mathbb{W}_q$ must have at least $M_q$ dimensions as well. In other words, the number of antennas at the $q$-th relay shall fulfill both

$$N_q^2 \geq K_q(K_q - 3) + \operatorname{rank} \left( G_{(1,..,q)} \right) - \operatorname{rank} \left( G_{(1,..,q-1)} \right)$$

(8)

and the inequality of (6). Suppose $N_{q,\min}$ is the minimum $N_q$ satisfying both (6) and (8). By setting the numbers of relay antennas to be $N_{1,\min}, \ldots, N_{Q,\min}$, respectively, it is obviously possible to achieve IA in the entire network using the above scheme. Therefore, the tuple $(N_{1,\min}, \ldots, N_{Q,\min})$ is a proper relay antenna configuration. Moreover, if $N_{q,\min}$ satisfies (6) with equality, then further reducing the number of antennas at the $q$-th relay violates the feasibility conditions (5) for $S = \{q\}$. If $N_{q,\min}$ satisfies (8) with equality and if $N_{1,\min}, \ldots, N_{q-1,\min}$ satisfy the feasibility conditions (5) for any $S \subseteq \{1, \ldots, q - 1\}$, then further reducing the number of antennas at the $q$-th relay violates (5) for $S = \{1, \ldots, q\}$. Hence, the tuple $(N_{1,\min}, \ldots, N_{Q,\min})$ is also a Pareto point of the proper relay antenna configurations. In plain words, the proposed IA scheme using partial CSI requires no additional relay antennas as compared to IA using full CSI.

Remark 1: According to the answer to Q1, the choice of the $q$-th subnetwork is not unique. For any possible ordering of the subnetworks, a certain tuple of $(N_{1,\min}, \ldots, N_{Q,\min})$ can be achieved, which is only one of the Pareto points of the proper relay antenna configurations. Note that there may exist proper relay antenna configurations which cannot be achieved by any ordering of the subnetworks.

V. PARALLELIZATION

The scheme proposed in Sec. IV requires that the subnetworks have to select their IN solutions one after another, which causes significant delay if there are lots of subnetworks. Therefore, we propose to reduce the delay by allowing several subnetworks to select their IN solutions simultaneously.

Suppose $q - 1$ subnetworks have selected their IN solutions in the previous steps. The $q$-th and the $r$-th subnetworks are both candidates as the next subnetwork to select the IN solution. They can select their IN solutions simultaneously in the next step if the $r$-th subnetwork does not need any side information from the $q$-th subnetwork and vice versa. In fact, there are only two cases where the parallelization is possible.

Case 1. If there are no external constraints between the $q$-th and the $r$-th subnetwork, no side information needs to be exchanged between them.

Case 2. Suppose there is an external constraint between node $j$ in the $q$-th subnetwork and node $k$ in the $r$-th subnetwork. Furthermore, there are also external constraints between each of the two nodes and a common node in the set of subnetworks $\{1, \ldots, q - 1\}$. Since the three external constraints are linearly dependent, the filter coefficients at node $j$ and node $k$ can be determined separately based on the side information from the set of subnetworks $\{1, \ldots, q - 1\}$ and the external constraint between node $j$ and node $k$ is automatically satisfied. In this case, no side information needs to be exchanged between node $j$ and node $k$ either.

VI. SIMULATION RESULTS

We first investigate how the average number of antennas per relay increases with the network size, i.e., the total number of node pairs $\sum_{q=1}^{Q} K_q$ in the network. Two cases are considered. In the first case, the number of subnetworks $Q$ is fixed to 5. In the second case, the subnetwork size $K_q$ is fixed to 5. We assume that every inter-subnetwork link exists with an equal probability $p$, which is chosen to be either 0.9 or 0.1. As a reference, we also consider fully connected networks of the same size, where every relay is also connected to all the node pairs in other subnetworks. It is shown in Fig. 4 that the average number of antennas per relay required by the proposed
The proposed IA scheme mainly depends on the size of the subnetworks and not on their number. The reason is that the number of antennas at a relay is always upper bounded by a number related to the size of the subnetwork it belongs to [6]. Therefore, it is more favorable to construct large networks using many relatively small subnetworks so that every relay just needs few antennas.

In the following, the achieved sum rates of the proposed IA scheme and the IA scheme using full CSI are compared. Three partially connected subnetworks are considered. Each subnetwork includes three node pairs and a single relay with three antennas such that IA is always feasible regardless of the topology of the network. Every inter-subnetwork link exists with an equal probability \( p \). The channel coefficients are independently drawn from the Gaussian distribution with zero mean and unit variance. The performance is measured by the average sum rate per subnetwork in bits/s. A sum power constraint \( 3P \), which is the sum of the power consumed by the relays in the second time slot and the powers consumed by the source nodes in both time slots, is assumed. The sum power can be considered as the total transmitted energy normalized by the duration of a single time slot. The pseudo signal-to-noise ratio (SNR) is then defined to be \( P/\sigma^2 \), where \( \sigma^2 \) is the variance of the white Gaussian noise at the relays and at the destination nodes. If full CSI is available, the IN conditions in the entire network are jointly solved. Based on the selected solution, the transmit powers of all the source nodes and the relays are adapted using the water-filling algorithm to maximize the sum rate under the sum power constraint \( 3P \). If the proposed IA scheme is applied, the subnetworks determine their solutions one after another as described in Sec. IV. However, power optimization over the entire network is no longer valid. Therefore, the source nodes and the relay in each subnetwork adapted their transmit powers to maximize the sum rate of the corresponding subnetwork under the sum power constraint \( P \). As shown in Fig. 5, IA using full CSI outperforms the proposed scheme, which is a natural consequence of the power optimization over the entire network. However, if the network is sparse, the solution space \( \mathbb{W}_q \) of every subnetwork has larger dimensions. Therefore, it is easier for each subnetwork to select a “good” IN solution and the proposed scheme suffers from little performance loss.

VII. CONCLUSION

To conclude, the key idea of the proposed IA scheme is to solve the intra-subnetwork IN conditions in each individual subnetwork sequentially using partial CSI. In our scheme, only a few complex valued filter coefficients need to be exchanged between different subnetworks as a side information. The proposed IA scheme needs no additional relay antennas as compared to IA using full CSI. Furthermore, in large networks with realistic assumptions on the network topology, the proposed scheme can be parallelized to reduce the delay.

ACKNOWLEDGMENT

This work is supported by Deutsche Forschungsgemeinschaft (DFG), grants No. WE2825/11-1 and KL907/5-1.

REFERENCES