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Bi-Directional Differential Beamforming for Multi-Antenna Relaying

Adrian Schad, Samer J. Alabed, Holger Degenhardt, Marius Pesavento

Abstract—**In this work, we propose a differential beamforming (DBF) scheme for bi-directional communication between two single-antenna terminals via a multi-antenna relay station (RS). The proposed scheme utilizes differential phase-shift keying modulation to enable beamforming at the RS without knowledge of the instantaneous channel state information (CSI) at any entity in the network. In our differential scheme, receive and transmit beamforming at the RS is performed based on the implicit CSI contained in the received signals in the preceding time slots. Thus, the DBF scheme is applicable even if the communication channels are time-variant. For time-invariant channels, we show that our DBF scheme is associated with a performance penalty of 3 dB as compared to the ideal amplify-and-forward relaying scheme, which requires perfect CSI. Our simulation results confirm the analytical results for time-invariant channels. For time-variant channels, the simulation demonstrate a high performance gain of the DBF scheme compared to schemes of the literature.**

Index Terms—**Amplify-and-forward relaying, bi-directional relaying, differential phase-shift keying, differential beamforming, relay networks**

I. Introduction

In this work we consider differential beamforming for bi-directional amplify-and-forward (AF) relaying.

In the AF protocol, relays apply complex weights to their received signals to adjust their phases and amplitudes before broadcasting the resulting signals to the respective destinations [1]–[22]. Beamforming techniques for bi-directional AF networks have been developed in [1]–[19], where [1]–[9] and [18] consider a single multi-antenna relay, [10]–[16] and [19] consider multiple single-antenna relays and [17] considers multiple multi-antenna relays. In [2]–[17], it is assumed that the perfect instantaneous channel state information (CSI) is available at one or more nodes to compute the relay weights. Instantaneous CSI can be acquired by the use of training symbols which results in signaling overhead.

To establish relay communication without CSI, differential techniques [19], [22]–[27] and non-differential techniques [28], [29] for single antenna relays have been proposed.

In this work, we develop a beamforming-based differential relaying scheme employing a single multi-antenna relay station (RS), which requires neither the instantaneous CSI nor the second order statistics of the CSI at any node in the network. In the proposed differential beamforming (DBF) scheme, the RS exploits implicit CSI contained in previously received signal vectors to perform receive and transmit beamforming.

In our theoretical analysis, we derive an expression for the approximated signal-to-noise ratio (SNR) achieved by the proposed DBF scheme for time-invariant channels. Our simulations demonstrate that the latter approximation is highly accurate in the medium to high-power regime. Based on this approximation, we propose to distribute the total power among the terminals and the RS such that both terminals achieve the same approximated SNR. The proposed power allocation scheme requires only the knowledge of the noise powers at the RS and the terminals but no CSI. In the simulations, we consider time-variant channels using the model of [30] and compare the bit error rate (BER) of the proposed DBF scheme with state of the art approaches known from literature for different terminal velocities. The simulation results demonstrate that the proposed DBF scheme dramatically outperforms the differential single-antenna relaying scheme of [19] as well as the multi-antenna relaying scheme of [20], which requires the knowledge of the second order statistics of the CSI.

The contribution of this paper are:

- We propose a novel DBF scheme for bi-directional AF relaying that does not require CSI at any node.
- We derive an approximation expression for the SNR at each receiver which is accurate for medium and high transmission powers.
- We derive a simple power allocation scheme that achieves almost optimum performance.
- We test our proposed DBF scheme under realistic high mobility scenarios using numerical simulations and demonstrate that the proposed scheme outperforms existing schemes.

Notation: $\mathrm{E}\{\cdot\}$, $|\cdot|$, $\Re\{\cdot\}$, $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\|\cdot\|$ denote the statistical expectation, absolute value of a complex number, real part of a complex number, complex conjugate, transpose, Hermitian transpose, and the Euclidean norm of a vector, respectively.

II. System Model

We consider a wireless single carrier network with two single-antenna terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ and one multi-antenna AF RS comprising R antennas. We assume frequency flat fading channels and that there is no direct link between the terminals available. We further assume that reciprocity holds for transmissions from the terminals to the RS and vice versa. This assumption has also been made in [1]–[15] is realistic for narrowband time division duplex (TDD) systems [31]. We further consider in our derivations a block fading channel model in which the channels remain constant during two consecutive transmission blocks. This assumption is however relaxed in the simulations, where we consider both time-variant and time-invariant channels.

The bi-directional communication between $\mathcal{T}_1$ and $\mathcal{T}_2$ is organized in consecutive blocks, where in each block a four time slot protocol is used. During the first two time-slots of this scheme, communication from $\mathcal{T}_1$ to $\mathcal{T}_2$ is established. The last two time slots are for communication from $\mathcal{T}_2$ to $\mathcal{T}_1$.

In the first time slot of the mth transmission block, terminal $\mathcal{T}_1$ transmits the signal $\sqrt{P_1}x_1^m$ to the RS, where P_1 is the transmission power of terminal $\mathcal{T}_1$ and x_1^m is the transmitted data symbol of the mth transmission block. In the mth and $(m-1)$th transmission block we denote $\mathbf{f}$ and $\mathbf{g}$ as the $R \times 1$ vectors containing the channel coefficients characterizing the transmission between terminal $\mathcal{T}_1$ and the RS and between the RS and terminal $\mathcal{T}_2$, respectively. Then, the signals received at the RS in the first time slot of the mth transmission

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block can be expressed as the vector $\mathbf{v}_1^m = [v_{1,1}^m, \ldots, v_{R,1}^m]^T$, given by

$$\mathbf{v}_1^m = \sqrt{P_1} x_1^m \mathbf{f} + \boldsymbol{\eta}_1^m, \quad (1)$$

where $\boldsymbol{\eta}_t^m = [\eta_{1,t}^m, \ldots, \eta_{R,t}^m]^T$ and where $\eta_{r,t}^m$ denotes the noise at the rth RS antenna in the mth transmission block at time slot $t \in \{1, 2, 3, 4\}$. The receive signal vector $\mathbf{v}_1^m$ is then weighted by the $R \times R$ beamforming matrix $\mathbf{W}_2$ and the resulting $R \times 1$ signal vector

$$\mathbf{t}_2^m = \mathbf{W}_2 \mathbf{v}_1^m \quad (2)$$

is transmitted to terminal $\mathcal{T}_2$ in the second time slot of the mth transmission block. The corresponding received signal at terminal $\mathcal{T}_2$ is then given by

$$y_2^m = \mathbf{g}^T \mathbf{t}_2^m + \nu_2^m = \mathbf{g}^T \mathbf{W}_2 \mathbf{v}_1^m + \nu_2^m, \quad (3)$$

where ν_2^m is the noise at terminal $\mathcal{T}_2$.

Let us assume that the noise in the network can be modeled as a spatially and temporally uncorrelated random processes with zero mean and variance $\mathrm{E}\{|\eta_{r,t}^m|^2\} = \sigma_R^2$, $\mathrm{E}\{|\nu_2^m|^2\} = \sigma_2^2$, $\mathrm{E}\{|\nu_4^m|^2\} = \sigma_1^2$, $\forall r, m, t$. Moreover, we assume without any loss of generality that $\mathrm{E}\{|x_1^m|^2\} = \mathrm{E}\{|x_2^m|^2\} = 1$.

We first regard the ideal case that full CSI is available at the RS. The optimization problem of designing $\mathbf{W}_2$ such that the SNR at terminal $\mathcal{T}_2$ is maximized has been treated in [21]. It has been shown that the ideal AF beamforming matrix is given by

$$\mathbf{W}_2^\star = c_2 \mathbf{g}^* \mathbf{f}^H, \quad (4)$$

where $c_2 = \sqrt{P_{R,2} / \left((\sigma_R^2 + P_1 \|\mathbf{f}\|^2) \, \|\mathbf{f}\|^2 \|\mathbf{g}\|^2 \right)}$ is a power scaling factor to ensure an average transmit power of $P_{R,2}$ at the RS. This leads to a maximum SNR of

$$\mathrm{SNR}_2^\star = \frac{P_1 P_{R,2} \|\mathbf{f}\|^2 \|\mathbf{g}\|^2}{\sigma_R^2 \sigma_2^2 + \sigma_2^2 P_1 \|\mathbf{f}\|^2 + \sigma_R^2 P_{R,2} \|\mathbf{g}\|^2}. \quad (5)$$

In the third and the fourth time slot, the communication from terminal $\mathcal{T}_2$ to terminal $\mathcal{T}_1$ is accomplished. In the third time slot of the mth transmission block, terminal $\mathcal{T}_2$ transmits the signal $\sqrt{P_2} x_2^m$ to $\mathcal{T}_1$, where P_2 denotes the transmission power of terminal $\mathcal{T}_2$ and x_2^m is the transmitted data symbol. Similar as in (1), the received $R \times 1$ signal vector at the RS in the third time slot of the mth transmission block is given by

$$\mathbf{v}_3^m = \sqrt{P_2} x_2^m \mathbf{g} + \boldsymbol{\eta}_3^m. \quad (6)$$

Following the relaying procedure for the second time slot given in (2)-(5), the RS weights $\mathbf{v}_3^m$ by the $R \times R$ beamforming matrix $\mathbf{W}_4$ and transmits the $R \times 1$ signal vector $\mathbf{t}_4^m$, given by $\mathbf{t}_4^m = \mathbf{W}_4 \mathbf{v}_3^m$ to terminal $\mathcal{T}_1$ in the fourth time slot. The ideal relay weighting matrix $\mathbf{W}_4^\star$ is given by $\mathbf{W}_4^\star = c_4 \mathbf{f}^* \mathbf{g}^H$, where $c_4 = \sqrt{P_{R,4} / \left((\sigma_R^2 + P_2 \|\mathbf{g}\|^2) \, \|\mathbf{g}\|^2 \|\mathbf{f}\|^2 \right)}$ is a constant and $P_{R,4}$ is the transmission power of the RS.

Utilizing the ideal relay beamforming matrices requires the perfect knowledge of the instantaneous channel vectors $\mathbf{f}$ and $\mathbf{g}$. Here, we address the problem of choosing the beamforming matrices in the case that CSI is not available.

III. The differential beamforming scheme

In this section, we present the DBF scheme which does not require knowledge of the CSI neither a the RS nor at the terminals.

At the terminals, we apply differential phase shift keying (PSK) where the transmitted data symbols x_1^m and x_2^m are generated from the information bearing symbols s_1^m and s_2^m by differential coding as [32]

$$x_1^m = x_1^{m-1} s_1^m, \qquad x_2^m = x_2^{m-1} s_2^m. \quad (7)$$

x_1^{m-1} and x_2^{m-1} are the transmitted data symbols of the previous transmission block $m-1$, and s_1^m and s_2^m are drawn from PSK constellations $\mathcal{M}_1$ and $\mathcal{M}_2$, respectively. We assume, without loss of generality, that $|s_1^m|^2 = |s_2^m|^2 = 1$ holds true for all m and that the DBF scheme is initialized with the symbols $x_1^0 = 1$ and $x_2^0 = 1$. Then, it follows by induction that $|x_1^m|^2 = |x_2^m|^2 = 1$ holds true for all $m \geq 1$.

In the 2nd time slot of the mth transmission block, the goal is to approximate the ideal beamforming matrix $\mathbf{W}_2^\star$ of (4). In the DBF scheme, the vectors

$$\tilde{\mathbf{f}} = \mathbf{v}_1^{m-1}, \quad (8)$$

$$\tilde{\mathbf{g}} = \mathbf{v}_3^{m-1} \quad (9)$$

are used instead of the true channel vectors channel vectors $\mathbf{f}$ and $\mathbf{g}$. Then, the approximated beamforming matrix is given by

$$\tilde{\mathbf{W}}_2^\star = \tilde{c}_2 \tilde{\mathbf{g}}^* \tilde{\mathbf{f}}^H = \tilde{c}_2 \left(\sqrt{P_1 P_2} (x_1^{m-1} x_2^{m-1})^* \mathbf{g}^* \mathbf{f}^H + \mathbf{N}_2^m \right), \quad (10)$$

where

$$\tilde{c}_2 = \sqrt{P_{R,2}} / \|\tilde{\mathbf{g}}^* \tilde{\mathbf{f}}^H \mathbf{v}_1^m\|, \quad (11)$$

$$\mathbf{N}_2^m = (\boldsymbol{\eta}_3^{m-1})^* (\boldsymbol{\eta}_1^{m-1})^H + (x_2^{m-1})^* \sqrt{P_2} \mathbf{g}^* (\boldsymbol{\eta}_1^{m-1})^H + (x_1^{m-1})^* \sqrt{P_1} (\boldsymbol{\eta}_3^{m-1})^* \mathbf{f}^H. \quad (12)$$

Here, $\tilde{c}_2$ is a scaling factor such that the power at the RS results in $\|\mathbf{t}_2^m\|^2 = P_{R,2}$ and $\mathbf{N}_2^m$ is a noise matrix.

The transmit signal vector $\mathbf{t}_2^m$ at the RS is given by

$$\mathbf{t}_2^m = \hat{s}_{1,R}^m \tilde{c}_2 (\mathbf{v}_3^{m-1})^* = \tilde{c}_2 (\mathbf{v}_3^{m-1})^* (\mathbf{v}_1^{m-1})^H \mathbf{v}_1^m = \tilde{\mathbf{W}}_2^\star \mathbf{v}_1^m. \quad (13)$$

Let us define $\sqrt{P_1} \mathbf{f} = \mathbf{h}$, where $\mathbf{h}$ is unknown. Then, the Maximum Likelihood detection problem of finding the transmitted data symbols from the received signal vectors $\mathbf{v}_1^{m-1}$ and $\mathbf{v}_1^m$ at the RS corresponds to the following Least Squares problem [33]

$$\min_{\mathbf{h}, x_1^m \in \mathcal{M}_1, x_1^{m-1} \in \mathcal{M}_1} \|\mathbf{h} x_1^m - \mathbf{v}_1^m\|^2 + \|\mathbf{h} x_1^{m-1} - \mathbf{v}_1^{m-1}\|^2. \quad (14)$$

For given x_1^m and x_1^{m-1}, $\hat{\mathbf{f}} = \frac{1}{2} \left(\mathbf{v}_1^m (x_1^m)^* + \mathbf{v}_1^{m-1} (x_1^{m-1})^* \right)$ is a solution to (14) with respect to $\mathbf{h}$. Inserting $\hat{\mathbf{f}}$ in problem (14) and applying (7) results in

$$\max_{s_1^m \in \mathcal{M}_1} \Re\{(s_1^m)^* (\mathbf{v}_1^{m-1})^H \mathbf{v}_1^m\},$$

which leads to the *soft decoded* symbol at the RS

$$\hat{s}_{1,R}^m = (\mathbf{v}_1^{m-1})^H \mathbf{v}_1^m = \sum_{r=1}^{R} (v_{r,1}^{m-1})^* v_{r,1}^m \quad (15)$$

Remark 1. $(v_{r,1}^{m-1})^* v_{r,1}^m$ in (15) can be regarded as soft differential PSK decoding of the symbol s_1^m at the rth RS antenna. In the distributed differential (DD) scheme of [19], where relays cannot exchange their received symbols, each relay computes $(v_{r,1}^{m-1})^* v_{r,1}^m$, $r \in \{1, \ldots, R\}$. The summation of the products as in (15) is therefore not performed. It is simple to prove that our centralized solution in (15) to the estimation problem (14) is in general more accurate than a distributed solution in which the received signals of the antennas are processed independently at each relay.

Remark 2. The vector $\mathbf{v}_1^{m-1}$ in (15) is used as a receive beamforming vector and can be regarded as an approximation of $\mathbf{f}$ contained in the ideal AF relay weighting matrix of (4). The concept of using a received signal vector to perform beamforming has also been proposed in [33]–[35], however in a different context where receive beamforming has been applied to detect differentially encoded symbols.

Remark 3. It is easy to adapt the proposed DBF scheme which uses the AF relaying protocol to a decode-and-forward relaying scheme.

The relay can decode the information symbol s_1^m from (15) and than retransmit it. Another alternative is to transmit a block of symbols from terminal $\mathcal{T}_1$ to the relay and then apply error correction to the received signals according to the utilized channel coding scheme. Afterwards, the decoded symbols can be re-encoded and transmitted to terminal $\mathcal{T}_2$.

To transmit $\hat{s}_{1,R}^m$ from the RS to terminal $\mathcal{T}_2$, we approximate the transmit beamforming vector $\mathbf{g}$ contained in the beamforming matrix of (4) by $\mathbf{v}_3^{m-1}$. Note that, the concept of using the received signal vectors for the transmit beamforming in TDD systems can also be found in [36]–[38].

Making use of (1)-(3) and (13)–(10), the signal y_2^m received at terminal $\mathcal{T}_2$ in the second time slot of the mth transmission block is given by

$$y_2^m = \mathbf{g}^T \tilde{\mathbf{W}}_2^\star \mathbf{v}_1^m + \nu_2^m = \underbrace{d_2^m (x_2^{m-1})^* s_1^m}_{\text{desired signal}} + \underbrace{n_2^m}_{\text{noise signal}}, \tag{16}$$

where we have used the definitions

$$\begin{aligned} d_2^m =& \tilde{c}_2 P_1 \sqrt{P_2} \|\mathbf{f}\|^2 \|\mathbf{g}\|^2, & (17)\\ n_2^m =& \tilde{c}_2 \mathbf{g}^T \left(\sqrt{P_1 P_2} (x_1^{m-1} x_2^{m-1})^* \mathbf{g}^* \mathbf{f}^H + \mathbf{N}_2^m \right) \boldsymbol{\eta}_1^m \\ &+ \tilde{c}_2 \sqrt{P_1} x_1^m \mathbf{g}^T \mathbf{N}_2^m \, \mathbf{f} + \nu_2^m. & (18) \end{aligned}$$

Note that according to (7), the product $x_1^m \cdot (x_1^{m-1})^*$ in (16) results in s_1^m.

The desired signal in (16) contains the transmitted symbol x_2^{m-1} of terminal $\mathcal{T}_2$. This is a consequence of using $\mathbf{v}_3^{m-1}$ of (6) as the transmit beamforming vector at the RS. However, similar as in the analog network coding techniques of [22] and [23], terminal $\mathcal{T}_2$ can cancel the unwanted phase shift of the desired signal caused by $(x_2^{m-1})^*$ as this symbol is known. Towards this goal, terminal $\mathcal{T}_2$ multiplies its received signal by x_2^{m-1}, since $(x_2^{m-1})^* x_2^{m-1} = |x_2^{m-1}|^2 = 1$ and obtains

$$\hat{s}_{1,\mathcal{T}_2}^m = y_2^m x_2^{m-1} = d_2^m s_1^m + n_2^m x_2^{m-1}. \tag{19}$$

$\hat{s}_{1,\mathcal{T}_2}^m$ can be viewed as the *soft decoded* symbol at terminal $\mathcal{T}_2$. From (19), a symbol $\hat{s}_1^m$ is detected by the rule

$$\hat{s}_1^m = \arg \min_{s \in \mathcal{M}_1} \left| \hat{s}_{1,\mathcal{T}_2}^m / |\hat{s}_{1,\mathcal{T}_2}^m| - s \right|,$$

as the amplitude in PSK constellations does not convey information.

The signal vector transmitted at the RS in the fourth time slot of the mth transmission block can be expressed as $\mathbf{t}_4^m = \tilde{\mathbf{W}}_4^\star \mathbf{v}_3^m$, using the beamforming matrix $\tilde{\mathbf{W}}_4^\star = \tilde{c}_4 (\mathbf{v}_1^m)^* (\mathbf{v}_3^{m-1})^H$ and the constant $\tilde{c}_4 = \sqrt{P_{R,4}} / \|(\mathbf{v}_1^m)^* (\mathbf{v}_3^{m-1})^H \mathbf{v}_3^m\|$. Similarly as in (19), the signal received at terminal $\mathcal{T}_1$ is multiplied by x_1^{m-1} before the symbol detection.

Remark 4. The DBF scheme can easily be modified to a *one-directional* two-time-slot scheme, where data is transferred from terminal $\mathcal{T}_1$ to terminal $\mathcal{T}_2$. Assuming that the channels vary slowly, we can replace $\mathbf{v}_3^{m-1}$ in (13) by a vector of received signals which is updated after a certain number of blocks when $\mathcal{T}_2$ transmits a reference symbol to the relay.

IV. Analysis for the high power regime and power allocation

In this section, an approximate expression for the SNR of the DBF scheme is derived. We assume that the block fading assumption of the previous section is valid and that the transmission powers of the terminals are large compared to the noise power. The simulations will demonstrate that this approximate expression is also accurate for moderate transmission powers.

If the CSI is available and for $P_1, P_2, P_{R,2} \to \infty$, the maximum achievable SNR at terminal $\mathcal{T}_2$ is well approximated by the expression

$$\overline{\mathrm{SNR}}_2^\star = \frac{P_1 P_{R,2} \|\mathbf{f}\|^2 \|\mathbf{g}\|^2}{\sigma_2^2 P_1 \|\mathbf{f}\|^2 + \sigma_R^2 P_{R,2} \|\mathbf{g}\|^2}, \tag{20}$$

where we have dropped the term $\sigma_R^2 \sigma_2^2$ in the denominator of $\mathrm{SNR}_2^\star$ given in (5) as $\sigma_R^2 \sigma_2^2 \ll \sigma_2^2 P_1 \|\mathbf{f}\|^2 + \sigma_R^2 P_{R,2} \|\mathbf{g}\|^2$.

To derive an asymptotic approximation of the performance of the proposed DBF scheme, let us focus on the scaling factor $\tilde{c}_2$ in the relay weighting matrix $\tilde{\mathbf{W}}_2^\star$ of (11). Neglecting the noise terms in the vectors $\mathbf{v}_3^{m-1} = \sqrt{P_2} x_2^{m-1} \mathbf{g} + \boldsymbol{\eta}_3^m$, $\mathbf{v}_1^{m-1} = \sqrt{P_1} x_1^{m-1} \mathbf{f} + \boldsymbol{\eta}_1^{m-1}$, and $\mathbf{v}_1^m = \sqrt{P_1} x_1^m \mathbf{f} + \boldsymbol{\eta}_1^m$, we obtain

$$\tilde{c}_2 \approx \sqrt{P_{R,2}} / \left(P_1 \sqrt{P_2} \|\mathbf{g}\| \|\mathbf{f}\|^2 \right), \tag{21}$$

where we have made use of the fact that $|x_1^{m-1}| = |x_2^{m-1}| = |x_1^m| = 1$. Then, using (12)–(19) and (21) leads to

$$\begin{aligned} d_2^m \approx& \sqrt{P_{R,2}} \|\mathbf{g}\|, & (22)\\ n_2^m x_2^{m-1} \approx& \sqrt{P_{R,2}} \left(\frac{x_1^{m-1} \|\mathbf{g}\| \mathbf{f}^H \boldsymbol{\eta}_1^m}{\sqrt{P_1} \|\mathbf{f}\|^2} + \frac{x_1^m \|\mathbf{g}\| (\boldsymbol{\eta}_1^{m-1})^H \mathbf{f}}{\sqrt{P_1} \|\mathbf{f}\|^2} \right. \\ &+ \left. \frac{s_1^m \mathbf{g}^T (\boldsymbol{\eta}_3^{m-1})^* x_2^{m-1}}{\sqrt{P_2} \|\mathbf{g}\|} \right) + \nu_2^m x_2^{m-1}. & (23) \end{aligned}$$

From (22) and (23), the proposed differential scheme approximately achieves an SNR given by

$$\tilde{\mathrm{SNR}}_2 = \frac{P_1 P_2 P_{R,2} \|\mathbf{f}\|^2 \|\mathbf{g}\|^2}{\sigma_R^2 \left(2 P_{R,2} P_2 \|\mathbf{g}\|^2 + P_{R,2} P_1 \|\mathbf{f}\|^2\right) + \sigma_2^2 P_1 P_2 \|\mathbf{f}\|^2}. \tag{24}$$

Similar to the derivation of $\tilde{\mathrm{SNR}}_2$, one can show that the SNR at terminal $\mathcal{T}_1$ is approximately given by

$$\tilde{\mathrm{SNR}}_4 = \frac{P_1 P_2 P_{R,4} \|\mathbf{f}\|^2 \|\mathbf{g}\|^2}{\sigma_R^2 \left(2 P_{R,4} P_1 \|\mathbf{f}\|^2 + P_{R,4} P_2 \|\mathbf{g}\|^2\right) + \sigma_1^2 P_1 P_2 \|\mathbf{g}\|^2}. \tag{25}$$

Based on the SNR expressions in (24) and (25), we propose to distribute an amount of total power P_T among the RS and the terminals to maximize the minimum SNR at the terminals. The latter *SNR balancing* approach is mathematically formulated as the optimization problem [11]

$$\begin{aligned} \max_{P_1, P_2, P_{R,2}, P_{R,4}} \quad & \min(\tilde{\mathrm{SNR}}_2, \tilde{\mathrm{SNR}}_4) \\ \text{s.t.} \quad & P_1 + P_2 + P_{R,2} + P_{R,4} \le P_T, \end{aligned} \tag{26}$$

where $P_1, P_2, P_{R,2}$ and $P_{R,4}$ are the optimization variables and where the constraint guarantees that the sum power does not exceed P_T. It is easy to show that at an optimum point of problem (26) i) $\tilde{\mathrm{SNR}}_2 = \tilde{\mathrm{SNR}}_4$ holds true; ii) the inequality constraint will be satisfied with equality and $P_{R,2} + P_{R,4} = P_T - P_2 - P_1$ holds true. Let us define $P_R = P_{R,2} + P_{R,4}$ and introduce the power allocation factor $0 \le \alpha \le 1$ such that $P_{R,2} = \alpha P_R$ and $P_{R,4} = (1 - \alpha) P_R$. For the sake of simplicity, let us first assume that P_1 and P_2 are fix. Then, the power allocation can be found by solving the equation $\tilde{\mathrm{SNR}}_2 = \tilde{\mathrm{SNR}}_4$ which results in solving the quadratic form

$$\begin{aligned} &\alpha^2 P_R \sigma_R^2 \left(P_2 \|\mathbf{g}\|^2 - P_1 \|\mathbf{f}\|^2\right) + \alpha \Big(P_R \sigma_R^2 \left(P_1 \|\mathbf{f}\|^2 - P_2 \|\mathbf{g}\|^2\right) \\ &\quad + \sigma_2^2 P_1 P_2 \|\mathbf{f}\|^2 + \sigma_1^2 P_1 P_2 \|\mathbf{g}\|^2 \Big) - \sigma_2^2 P_1 P_2 \|\mathbf{f}\|^2 = 0. \end{aligned} \tag{27}$$

For the symmetric case

$$P_1 = P_R \sigma_R^2 / (2\sigma_1^2), \qquad P_2 = P_R \sigma_R^2 / (2\sigma_2^2), \tag{28}$$

$$P_R = P_T / \left(1 + \sigma_R^2/(2\sigma_1^2) + \sigma_R^2/(2\sigma_2^2)\right), \tag{29}$$

the solution to (27) is given by $\alpha = 1/2$ leading to

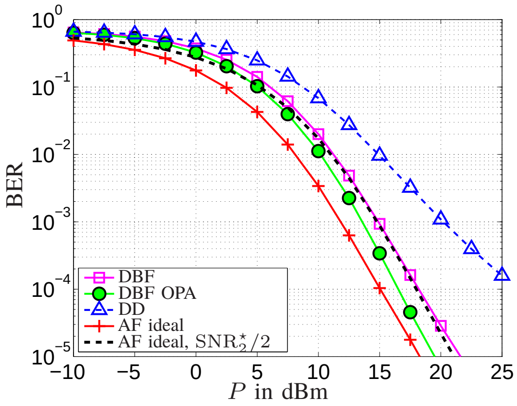

Fig. 1. BER versus transmitted power P for differential and non-differential schemes using QPSK in an urban micro scenario, v =0 km/h.

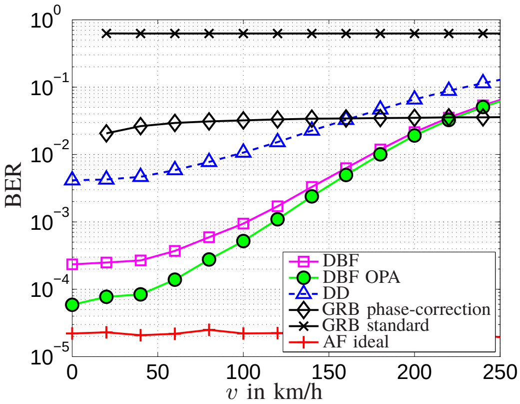

Fig. 2. BER versus velocity v for differential and non-differential schemes using QPSK in an urban micro scenario, P = 17dBm.

$$P_{R,2} = P_{R,4} = P_R/2. \qquad (30)$$

Interestingly, plugging (28)–(30) in (20) and (24), we obtain

$$\tilde{\mathrm{SNR}}_2 = \overline{\mathrm{SNR}}_2^\star/2. \qquad (31)$$

We observe from (31) that the proposed DBF scheme achieves approximately half the SNR of the ideal AF beamforming scheme if the communication channels remain constant during two consecutive transmission blocks and if we use (28)–(30).

In general, the power allocation according to (28)–(30) is not the optimum solution to the problem (26). However, the optimization problem (26) is highly non-linear and might be difficult to solve exactly in reasonable time. Furthermore, the optimum power allocation (OPA) depends on the channel vectors $\mathbf{f}$ and $\mathbf{g}$ and has to be recomputed if $\mathbf{f}$ and $\mathbf{g}$ change. The advantage of the *constant* power allocation in (28)–(30) is that it does not depend on the time-variant channel vectors but only on the constant noise powers and the total available power P_T. Therefore, it is sufficient to compute (28)–(30) once at the RS using $\sigma_1^2, \sigma_2^2, \sigma_R^2$ and P_T.

V. Simulation Results

Throughout our simulations, we regard a RS equipped with $R\!=\!5$ antennas arranged in a uniform linear array in which neighboring antennas have a distance of two wavelengths. The information bearing symbols are taken from a QPSK constellation. We divide the transmission power such that $P \triangleq P_1 = P_2 = P_{R,2} = P_{R,4} = P_T/4$ and set the noise powers $\sigma_R^2 = \sigma_1^2 = \sigma_2^2 = -132$ dBm. This choice satisfies (28)–(30). To test our scheme under realistic conditions, the channel coefficients are created by using the urban micro scenario of [30]. We do not regard shadowing effects in our simulations to avoid strong fluctuations in the channel strengths. The following system parameters are chosen according to the Long Term Evolution (LTE) standard for mobile communication [39]. The system works at a carrier frequency and the duration of one time slot. Then, the bandwidth is given by $1/T_S$ which corresponds to the bandwidth of a subcarrier in a multi-carrier LTE system. Our simulation results are averaged over 20000 runs and each simulation run comprises $M = 300$ transmission blocks.

We compare the DBF scheme with the ideal AF relaying scheme (AF ideal), the ideal AF relaying scheme where P_T is scaled such that the SNR at terminal $\mathcal{T}_2$ is halved (AF ideal, $\mathrm{SNR}_2^\star/2$), the DD scheme of [19], where every relay transmits with the power P/R, and the general rank beamforming (GRB) scheme proposed in [20]. The GRB scheme is suitable for time-variant channels as it does not use the instantaneous CSI. In the GRB scheme, beamformers are designed based on the channel covariance matrices corresponding to $\mathbf{f}$ and $\mathbf{g}$. In practice, the channel covariance matrices have to be estimated by using training symbols. In the simulations, we use $1/(4M)\sum_{m=1}^{M}\sum_{t=1}^{4}\mathbf{f}_{m,t}\mathbf{f}_{m,t}^H$ and $1/(4M)\sum_{m=1}^{M}\sum_{t=1}^{4}\mathbf{g}_{m,t}\mathbf{g}_{m,t}^H$ as estimates of the channel covariance matrices where $\mathbf{f}_{m,t}$ and $\mathbf{g}_{m,t}$ denote channel vectors in the tth time slot of the mth transmission block for the channels between terminal $\mathcal{T}_1$ and the RS and between the RS and terminal $\mathcal{T}_2$, respectively. Note that the GRB scheme does not explicitly regard a phase correction in the desired signal at the terminals. We examine the GRB scheme of [20] without correcting a phase shift, i.e. GRB standard, and the GBR scheme where terminals know and correct the phase shift perfectly, i.e. GRB phase-correction. Moreover, we compare our results to the DBF scheme with OPA (DBF OPA) where P_T is divided by solving (26) via grid search. To model time variation of the channel we consider motion of the terminals with different velocities v including the case of $v = 0$ km/h in which the channels remain constant. As the GRB scheme for constant channels leads to the ideal AF relaying scheme, we regard the GRB scheme if $v\!>\!0$ km/h.

Fig. 1 depicts the BER at terminal $\mathcal{T}_2$ versus P for $v = 0$ km/h and, therefore, for time-invariant channels. The proposed DBF scheme achieves the performance of the ideal AF relaying scheme at $\mathrm{SNR}_2^\star/2$ which confirms the theoretical result of (31). Especially for $P > 5$dBm, the approximation of (31) is highly accurate. For time-variant channels, the DBF scheme outperforms the GRB scheme with perfect phase correction at velocities below 220 km/h, see Fig. 2 (here, perfect phase correction is a generous assumption if the terminals have a very high velocity). The GRB standard without phase correction is not practical. From Fig. 1 and Fig. 2, one can see that the centralized DBF scheme offers significant performance gains compared to the DD system. Moreover, the DBF scheme with OPA performs slightly better than the DBF scheme with constant power allocation.

VI. Conclusion

In this work, we have introduced a novel DBF scheme for the bi-directional communication between two terminals via a multi-antenna RS. The scheme does not require CSI at any node in the network and it is therefore particularly suitable for environments with time-variant channels. For time-invariant channels, we have shown that the performance of the DBF scheme degrades by approximately 3dB compared to the ideal AF relaying scheme which requires perfect knowledge of the CSI. In the simulations, this analytical result is highly accurate for transmission powers above 5dBm. For time-variant channels, the simulation results have demonstrated that the proposed scheme dramatically outperformed the covariance based beamforming scheme of the literature for velocities below 220 km/h.

References

[1] J. Joung and A. H. Sayed, “Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems,” *IEEE Transactions on Signal Processing,* vol. 58, pp. 1833–1846, Mar. 2010.

[2] R. Wang and M. Tao, “Joint source and relay precoding design for MIMO two-way relaying based on MSE criteria,” *IEEE Transactions on Signal Processing,* vol. 6, no. 3, pp. 1352–1365, Mar. 2012.

[3] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, “Optimal beamforming for two-way multi-antenna relay channel with analogue network coding,” *IEEE Journal on Selected Areas in Communications,* vol. 27, no. 5, pp. 699–712, June 2009.

[4] X. Wang and X.-D. Zhang, “Optimal beamforming in MIMO two-way relay channels,” *Proceedings IEEE Global Telecommunications Conference,* pp. 1–5, Miami, USA, 2010.

[5] F. Roemer and M. Haardt, “Algebraic Norm-Maximizing (ANOMAX) transmit strategy for Two-Way relaying with MIMO amplify and forward relays,” *IEEE Signal Processing Letters,* vol. 16, pp. 909–912, Oct. 2009.

[6] F. Roemer and M. Haardt “A low-complexity relay transmit strategy for two-way relaying with MIMO amplify and forward relays,” *Proceedings IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP’10),* pp. 3254–3257, Dallas, TX, Mar. 2010.

[7] J. Zhang, F. Roemer, and M. Haardt, “Beamforming design for multi-user two-way relaying with mimo amplify and forward relays,” *Proceedings IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP’11),* pp. 2824–2827, Prague, Czech Republic, May 2011.

[8] A. Khabbazibasmenj, F. Roemer, S. A. Vorobyov, and M. Haardt, “Sum-rate maximization in two-way AF MIMO relaying: polynomial time solutions to a class of DC programming problems,” *IEEE Transactions on Signal Processing,* vol. 60, no. 10, pp. 5478–5493, Oct. 2012.

[9] G. Li, Y. Wang, and P. Zhang, “Optimal linear MMSE beamforming for two way multi-antenna relay systems,” *IEEE Communications Letters,* vol. 15, no. 5, pp. 533–535, May 2011.

[10] V. Havary-Nassab, S. Shahbazpanahi and A. Grami, “Optimal distributed beamforming for two-way relay networks,” *IEEE Transactions on Signal Processing,* vol. 58, no. 3, pp. 1238–1250, Mar. 2010.

[11] S. Shahbazpanahi and M. Dong, “A semi-closed form solution to the SNR balancing problem of two-way relay network beamforming,” *Proceedings IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP’10),* pp. 2514–2517, Dallas, TX, Mar. 2010.

[12] S. Shahbazpanahi and M. Dong, “Achievable rate region and sum-rate maximization for network beamforming for bi-directional relay networks,” *Proceedings IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP’10),* pp. 2510-2513, Dallas, TX, Mar. 2010.

[13] A. Schad, A.B. Gershman, and S. Shahbazpanahi, “Capacity maximization for distributed beamforming in one- and bi-directional relay networks,” *Proceedings IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP’11),* pp. 2804–2807, Prague, Czech Republic, May 2011.

[14] A. Schad and M. Pesavento, “Multiuser bi-directional communications in cooperative relay networks,” *Proceedings IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP’11)*, pp. 217–220, San Juan, Puerto Rico, Dec. 2011.

[15] C. Wang, H. Chen, Q. Yin, A. Feng, and A. Molisch, “Multi-user two-way relay networks with distributed beamforming,” *IEEE Transactions on Wireless Communications,* vol. 10, no. 10, pp. 3460–3471, Oct 2011.

[16] M. Zeng, R. Zhang, and S. Cui, “On design of collaborative beamforming for two-way relay networks,” *IEEE Transactions on Signal Processing,* vol. 59, no. 5, pp. 2284–2295, May 2011.

[17] C. Li, L. Yang, and W.-P. Zhu, “Two-way MIMO relay precoder design with channel state information,” *IEEE Transactions on Communications,* vol. 58, no. 12, pp. 3358–3363, Dec. 2010.

[18] K.-J. Lee, K. W. Lee, H. Sung, and I. Lee, “Sum-rate maximization for two-way MIMO amplify-and-forward relaying systems,” *Proceedings IEEE Vehicular Technology Conference (VTC’09),* pp. 1–5, Spring, 2009.

[19] S. Alabed and M. Pesavento, “A simple distributed differential transmit beamforming technique for two-way wireless relay networks,” *Proceedings IEEE/ITG Workshop Smart Antennas (WSA’12),* pp. 243–247, Dresden, Mar. 2012.

[20] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, “Joint receive-transmit beamforming for multi-antenna relaying schemes,” *IEEE Transactions on Signal Processing,* vol. 58, pp. 4966–4972, Sep. 2010.

[21] B. Khoshnevis, W. Yu, and R. Adve, “Grassmannian beamforming for MIMO amplify-and-forward relaying,” *Journal on Selected Areas in Communications,* vol. 26, no. 8, pp. 1397–1407, Oct. 2008.

[22] S. Alabed, M. Pesavento, and A. Klein, “Non-coherent distributed space-time coding techniques for two-way wireless relay networks,” *EURASIP special issue on Sensor Array Processing,* Jan. 2013.

[23] S. Alabed, M. Pesavento, and A. Klein, “Distributed differential space-time coding for two-way relay networks using analog network coding,” *Proceedings IEEE European Signal Processing Conference (EUSIPCO’13),* Marrakech, Morocco, March 2013.

[24] T. Himsoon, W. Siriwongpairat, W. Su, and K. Liu, “Differential modulations for multinode cooperative communications,” IEEE Transactions on Signal Processing, vol. 56, no. 7, July 2008.

[25] T. Himsoon, W. Su, and K. Liu, “Differential modulation for multinode amplify-and-forward wireless relay networks,” Wireless Communications and Networking Conference, vol. 2, pp. 1195–1200, 2006.

[26] T. Himsoon, W. Su, and K. Liu, “Differential transmission for amplify-and-forward cooperative communications,” *IEEE Signal Processing Letters,* vol. 12, no. 9, Sep. 2005.

[27] Q. Zhao and H. Li, “Differential modulation for cooperative wireless systems,” *IEEE Transactions on Signal Processing,* vol. 55, pp. 2273–2283, May 2007.

[28] Q. Liu, W. Zhang, X. Ma, and G. T. Zhou, “A practical amplify-and-forward relaying strategywith an intentional peak power limit,” *ICASSP’10,* pp. 2518–2521, Dallas, TX, Mar. 2010.

[29] Q. Liu, W. Zhang, and X. Ma, “Practical and general amplify-and-forward designs for cooperative networks,” *INFOCOM’2010*, pp. 1999–2007, San Diego, CA, Mar. 2010.

[30] D. Baum, J. Hansen, and J. Salo, “An interim channel model for beyond-3G systems: Extending the 3GPP spatial channel model,” *Proceedings IEEE Veh. Technol. Conf.,* vol. 5, pp. 3132–3136, Jun. 2005.

[31] J. C. Haartsen, “Impact of non-reciprocal channel conditions in broadband TDD systems,”*Proc. IEEE PIMRC’08,* pp. 1–5, Sep. 2008.

[32] J. Boccuzzi, "Signal processing for wireless communications,“ McGraw Hill, New York, 2008.

[33] M. K. Varanasi, “A Systematic Approach to the Design and Analysis of Optimum DPSK Receivers for Generalized Diversity Communications over Rayleigh Fading Channels,” *IEEE Transactions on Communications,* vol. 47, No. 9, pp. 1365–1375 Sept. 1999.

[34] F. Adachi, “Postdetection optimal diversity combiner for DPSK differential detection,” *IEEE Transactions on Vehicular Technology,* vol. 42, pp. 326–337, Aug. 1993.

[35] C. Chung, “Differentially amplitude and phase-encoded QAM for the correlated Rayleigh-fading channel with diversity reception,” *IEEE Transactions on Communications,* vol. 45, pp. 309–321, Mar. 1997.

[36] H. Liu and G. Xu, “Multiuser blind channel estimation and spatial channel pre-equalization,” *Proc. ICASSP’95,* pp. 1756–1759, Detroit, MI, May 1995.

[37] G. G. Raleigh, S. D. Diggavi, V. K. Jones, and A. Paulraj, “A blind adaptive transmit antenna algorithm for wireless communications,” *in Proc. ICC’95,* vol. 3, p. 1949, 1995.

[38] P. Mongensen, F. Frederiskson, J. Wigard, and S. Peterson, “A research study of antenna diversity and data receivers for DECT,” *Proc. Nordic Radio Symp.,* Slatsjobaden, Sweden, Apr. 1995.

[39] E. Dahlman, S. Parkvall, and J. Sköld, "LTE/LTE-Advanced for mobile broadband,“ Academic Press, 2011.