

Relay-Aided Interference Alignment for Multiple Partially Connected Subnetworks

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Abstract—In this paper, large partially connected wireless relay interference networks are considered. The entire network is made up of multiple disjoint groups of source-destination node pairs and relays called subnetworks. Each subnetwork is assumed to be fully connected. The different subnetworks are mutually connected by a few inter-subnetwork links. A two-hop relay-aided interference alignment scheme is applied to achieve interference-free communication in the whole network. The feasibility conditions for interference alignment in the considered networks are investigated. To this end, we extend the concept of external constraints, which was proposed in [1] for the two-subnetwork case, and characterize the proper relay antenna configurations in large networks with an arbitrary number of subnetworks.

I. INTRODUCTION

The main result of interference alignment (IA) can be summarized as “everyone gets half the cake” [2]. More specifically, every user in a wireless interference network is able to achieve one half of the degrees of freedom (DoF) that he could achieve in the absence of interferences. However, the study of feasibility conditions shows that the goal of “everyone gets half the cake” is difficult to reach in constant channels, especially when there are lots of users. In constant multiple-input-multiple-output (MIMO) interference networks for instance, the number of antennas required at every user increases linearly with the total number of users [3]–[5]. In contrast to this, IA aided by half-duplex relays guarantees the achievement of 1/2 DoF per user even if each user only has a single antenna, but requires lots of relays or relay antennas depending on the network size [6]–[8]. The challenge we are facing is how to reduce the required number of relays or relay antennas in large networks. One possibility is to exploit partial connectivity. At realistic signal-to-noise-ratios (SNRs), neglecting the relatively weak links is an acceptable approximation [9]. In [9] and [10], partial connectivity has been exploited to reduce the number of required antennas at the nodes for IA in MIMO interference networks as compared to the fully connected case. However, to our knowledge, only few results are published for relay-aided IA in partially connected networks [1], [8].

In our preliminary work [1], the considered network is made up of two subnetworks, which are partially connected to each other. Each subnetwork includes multiple node pairs and multiple single-antenna relays. The question how to deploy the relays among the two subnetworks has been studied using graph theory. We have shown that partial connectivity can indeed help to reduce the total number of required relays in

such a network. However, the method we used in [1] would be invalid if there were more than two subnetworks. In this paper, we will extend the graph-based method to the cases where the entire network is made up of an arbitrary number of subnetworks. The feasibility conditions for IA will be derived.

In Section II, the system model and the IA conditions will be introduced. In Section III, we will redefine the external constraints, which were first introduced in [1]. They are then exploited in Section IV for studying the feasibility conditions for IA in the considered network. Finally, we compare the performances achieved by a few representative scenarios based on simulations and conclude our work.

II. SYSTEM MODEL AND LINEAR IA

We consider a network consisting of K single-antenna source-destination node pairs and Q amplify-and-forward relays with R_q antennas at the q th relay. All nodes and relays operate in half-duplex mode. Each source node transmits a single data symbol intended for the corresponding destination node using two transmission phases. In the first phase, every source node transmits to the relays and to the destination nodes. In the second phase, the source nodes transmit to the destination nodes again while each relay forwards the linearly processed received signals to the destination nodes. For the convenience of discussion, we will use s_j , d_k and r_q to denote the j -th source node, the k -th destination, and the q -th relay, respectively. Let the 2×1 unit vectors $\mathbf{v}^{(j)} = (v_1^{(j)}, v_2^{(j)})^T$ and $\mathbf{u}^{(k)} = (u_1^{(k)}, u_2^{(k)})^T$ denote the transmit filter at s_j and the receive filter at d_k , respectively. Let the $R_q \times R_q$ matrix $\mathbf{G}^{(q)}$ denote the processing matrix at r_q . The wireless communication channel is assumed to be constant during the two transmission phases. Let the scalar $h_{DS}^{(k,j)}$, the $R_q \times 1$ vector $\mathbf{h}_{RS}^{(q,j)}$, and the $1 \times R_q$ vector $\mathbf{h}_{DR}^{(k,q)}$ denote the channels between s_j and d_k , between s_j and r_q , and between r_q and d_k , respectively. All channel coefficients are assumed to be independently drawn from a continuous distribution.

We introduce partial connectivity into this network by assuming only a few strong links to be non-negligible and setting the relatively weak ones to zero. More specifically, it is assumed that each relay is only connected to the node pairs close by. Unlike ??, a single relay and the node pairs being connected to this relay form a so called subnetwork in this paper. We also assume that the different subnetworks do not have common nodes. Hence, the entire network can be partitioned into Q disjoint subnetworks. Let K_q denote the size

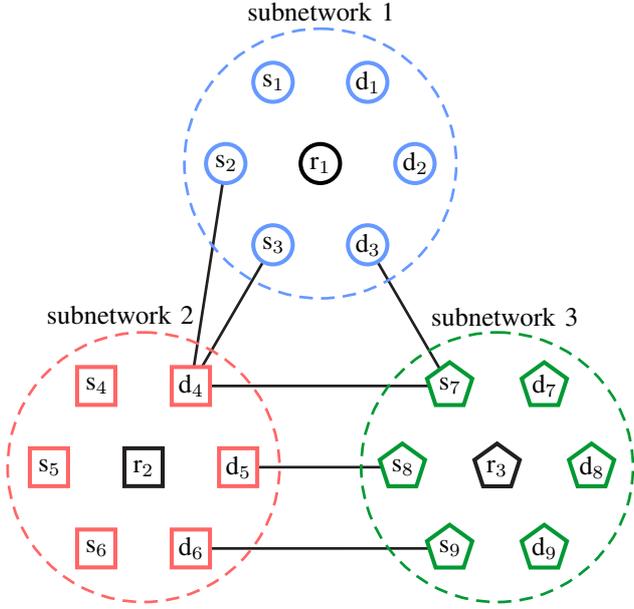


Fig. 1. A partially connected network made up of three subnetworks

of the q -th subnetwork, i.e., the number of node pairs being connected to the q -th relay. Throughout this paper, $K_q \geq 3$ is always assumed to exclude some special cases where the use of relays may be unnecessary for IA. Furthermore, we assume that each source node is connected to all the destination nodes in its own subnetwork by the so called intra-subnetwork direct links. But source nodes may also be connected to some destination nodes in other subnetworks by the so called inter-subnetwork direct links. In other words, an individual subnetwork is fully connected and different subnetworks may be partially connected to each other. For instance, a partially connected network made up of three subnetworks is illustrated in Fig. 1. The intra-subnetwork links are not shown for simplicity, but they are still assumed to be non-negligible. This paper focuses on finding the minimum numbers of relay antennas required for IA in such networks and, therefore, global channel state information is assumed to be available at every node and relay.

To perform IA, the processing matrices at the relays and the filters at the source/destination nodes shall be adapted to the channel to satisfy the interference-nulling conditions

$$\mathbf{u}^{(k)*T} \begin{pmatrix} h_{\text{DS}}^{(k,j)} & 0 \\ h_{\text{RL}}^{(k,j)} & h_{\text{DS}}^{(k,j)} \end{pmatrix} \mathbf{v}^{(j)} = 0, \quad \forall k, j, \quad k \neq j, \quad (1)$$

with the term $h_{\text{RL}}^{(k,j)}$ given by

$$h_{\text{RL}}^{(k,j)} = \sum_{q=1}^Q \mathbf{h}_{\text{DR}}^{(k,q)} \mathbf{G}^{(q)} \mathbf{h}_{\text{RS}}^{(q,j)}. \quad (2)$$

In order to utilize the relays, we assume that all source nodes do transmit in the first phase and that all destination nodes do receive in the second phase. Therefore, the interference-nulling conditions (1) can be linearized as

$$h_{\text{RL}}^{(k,j)} + h_{\text{DS}}^{(k,j)} \left(\frac{v_2^{(j)}}{v_1^{(j)}} + \frac{u_1^{(k)*}}{u_2^{(k)*}} \right) = 0, \quad \forall k, j, \quad k \neq j, \quad (3)$$

where the R_q^2 elements of every relay processing matrix $\mathbf{G}^{(q)}$ and the $2K$ ratios of the filter coefficients $v_2^{(j)}/v_1^{(j)}$ and $u_1^{(k)*}/u_2^{(k)*}$ are chosen as variables [6]. We refer to the solution subspace W of (3) as the interference-nulling solution space. In particular, the term $h_{\text{RL}}^{(k,j)}$ is zero if s_j and d_k belong to different subnetworks. Therefore, the inter-subnetwork interference signal propagating through a non-negligible inter-subnetwork direct link can be suppressed only if $v_2^{(j)}/v_1^{(j)} = -u_1^{(k)*}/u_2^{(k)*}$ holds, namely if $\mathbf{v}^{(j)}$ is orthogonal to $\mathbf{u}^{(k)}$. Let this be denoted by $\mathbf{v}^{(j)} \perp \mathbf{u}^{(k)}$.

Interestingly, one can always choose the filters at all the source and destination nodes to be pairwise orthogonal to satisfy (3) without any relay. However, this solution also nullifies all the useful signals. More precisely, the interference-nulling solutions which also satisfy the equality

$$h_{\text{RL}}^{(k,k)} + h_{\text{DS}}^{(k,k)} \left(\frac{v_2^{(k)}}{v_1^{(k)}} + \frac{u_1^{(k)*}}{u_2^{(k)*}} \right) = 0 \quad (4)$$

result in the useful signal of the k -th node pair being aligned with the interferences. We refer to such solutions as the invalid solutions with respect to the k -th node pair. Obviously, they form a subspace of W . Hence, IA is feasible if and only if the equality of (4) for each node pair is linearly independent from the interference-nulling conditions of (3). As the channel coefficients are random, the feasibility conditions for IA are usually obtained in the almost sure sense.

III. EXTERNAL CONSTRAINTS

In this section, we will lay the validity issue aside and study the mutual coupling among the subnetworks using the concept of external constraints (ECs). The following two examples may help to understand this concept.

Example 1: In the network shown in Fig. 1, the node d_4 receives non-negligible inter-subnetwork interferences from both s_2 and s_3 . Hence, condition $v_2^{(2)}/v_1^{(2)} = v_1^{(3)}/v_2^{(3)}$ shall hold to align these interferences at d_4 , namely $\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}$ shall be parallel. Let this be denoted by $\mathbf{v}^{(2)} \parallel \mathbf{v}^{(3)}$. In other words, the variables of subnetwork 1 shall be chosen to satisfy $\mathbf{v}^{(2)} \parallel \mathbf{v}^{(3)}$ in addition to its intra-subnetwork interference-nulling conditions.

Example 2: In the same network, d_3 receives non-negligible inter-subnetwork interference from s_7 . Therefore, the variables of subnetwork 1 and subnetwork 3 shall be chosen to satisfy $\mathbf{v}^{(7)} \perp \mathbf{u}^{(3)}$ in addition to the intra-subnetwork interference-nulling conditions in each individual subnetwork so that the inter-subnetwork interference could be nullified at d_3 .

Different from [1], we will redefine the ECs in this paper for studying networks being made up of an arbitrary number of subnetworks. Firstly, let $S \subseteq \{1, \dots, Q\}$ denote the set of subnetworks indexed by the elements of S . Secondly, let W_S denote the solution space of a subsystem of (3) which is formed by the intra-subnetwork interference-nulling conditions of each individual subnetwork in S . In other words, W_S consists of all the $2K + \sum_{q=1}^Q R_q^2$ dimensional vectors that only align the intra-subnetwork interferences of the subnetworks in S , whereas leaving the intra-subnetwork interferences of the

other subnetworks and all the inter-subnetwork interferences unconsidered. For convenience, let the elements of a vector \mathbf{x} in W_S or W be arranged such that $\mathbf{x} = (\mathbf{x}_1^T, \dots, \mathbf{x}_Q^T)^T$, where \mathbf{x}_q includes the $2K_q + R_q^2$ variables in the q -th subnetwork. Thirdly, define the projection p_S with the projection matrix

$$\begin{pmatrix} \mathbf{D}_S^{(1)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{D}_S^{(Q)} \end{pmatrix}, \quad S \subseteq \{1, \dots, Q\}, \quad (5)$$

where $\mathbf{D}_S^{(q)}$ is a $(2K_q + R_q^2) \times (2K_q + R_q^2)$ identity matrix if $q \in S$ and a zero matrix otherwise. Applying p_S on the solution spaces W_S and W results in the image spaces $p_S(W_S)$ and $p_S(W)$, respectively. Note that $p_S(W) \subseteq p_S(W_S)$ always holds because W is a subspace of every W_S . Based on these vector spaces, the ECs are defined as follows.

Definition 1: The set of ECs for a set of subnetworks S is a system of linear equations such that any vector $\mathbf{y} \in p_S(W_S)$ also belongs to $p_S(W)$ if and only if \mathbf{y} satisfies these equations.

In plain words, the variables in a set of subnetworks shall satisfy not only its intra-subnetwork interference-nulling conditions but also the ECs for it so that all the interferences in the entire network could be nullified. Generally speaking, the ECs depend on the channel realization. However, we will show that a set of necessary ECs can be found using the knowledge of the topology of the inter-subnetwork links only.

Let G_{inter} be an undirected bipartite graph. The two sets of vertices of G_{inter} are the source and the destination nodes of the whole network, respectively. The edges of G_{inter} are the non-negligible inter-subnetwork direct links. A path in G_{inter} is called an external path of the set of subnetworks S if both terminal vertices of the path are nodes in S . To nullify the inter-subnetwork interferences, the filters at any two neighboring vertices in such an external path have to be orthogonal as discussed in Section II. Therefore, an external path results in one of the following two kinds of constraints on the filters at its terminal nodes. If two source nodes s_i and s_j or two destination nodes d_k and d_l are connected by an external path, the number of edges of the path must be even because G_{inter} is bipartite. Hence, $\mathbf{v}^{(i)} \parallel \mathbf{v}^{(j)}$ or $\mathbf{u}^{(k)} \parallel \mathbf{u}^{(l)}$ shall be fulfilled, respectively, as in Example 1. If a source node s_j and a destination node d_k are connected by an external path, the number of edges of the path must be odd. Therefore, $\mathbf{v}^{(j)} \perp \mathbf{u}^{(k)}$ shall be fulfilled, as in Example 2. Note that the same constraint follows even if two nodes are connected by more than one external path. Obviously, all the constraints resulting from the external paths of S are necessary ECs for S .

However, some of the above ECs may be redundant and we are only interested in a set of linearly independent ones. To this purpose, define G_S to be a graph with its edges being the set of the necessary ECs for S and its vertices being the nodes involved in these ECs. Using graph theory [11], it is easy to see that the number of linearly independent ECs for S equals the rank of the incidence matrix of G_S , which will be simply denoted by $\text{rank}(G_S)$. A set of linearly independent necessary ECs for S can therefore be represented by a maximal forest in G_S . In Fig. 2, the graphs $G_{\{1\}}$, $G_{\{2,3\}}$ and $G_{\{1,2,3\}}$ in the

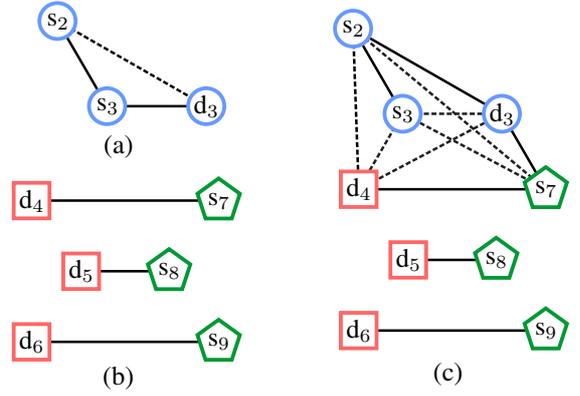


Fig. 2. Examples of the graphs of the necessary ECs in the network shown in Fig. 1: (a) $G_{\{1\}}$, (b) $G_{\{2,3\}}$, and (c) $G_{\{1,2,3\}}$. The solid lines indicate the edges of a maximal forest in each graph.

network shown in Fig. 1 are illustrated as examples. The solid lines indicate the edges of a maximal forest in each graph.

IV. NUMBERS OF REQUIRED RELAY ANTENNAS

In this section, we will show that finding all the necessary ECs for every set of subnetworks is then sufficient for characterizing the numbers of required relay antennas for IA in the considered network. We refer to the tuple of the numbers of relay antennas in a set of subnetworks S as a relay antenna configuration in S . A relay antenna configuration is called proper in S if it satisfies

$$\sum_{q \in S'} R_q^2 \geq \sum_{q \in S'} K_q(K_q - 3) + \text{rank}(G_{S'}) + 2, \quad \forall S' \subseteq S. \quad (6)$$

We first argue that IA is almost surely infeasible if the relay antenna configuration is improper in any $S \subseteq \{1, \dots, Q\}$. The number of intra-subnetwork interference-nulling conditions in the subnetworks belonging to S is $\sum_{q \in S} K_q(K_q - 1)$ while the number of linearly independent ECs for S is $\text{rank}(G_S)$. Note that the ECs do not involve the channels between the nodes and the relays, whose coefficients are randomly drawn from a continuous distribution. We therefore conclude that the total number of constraints for S is

$$N_{c,S} = \sum_{q \in S} K_q(K_q - 1) + \text{rank}(G_S). \quad (7)$$

Moreover, the number of variables in S is

$$N_{v,S} = \sum_{q \in S} (R_q^2 + 2K_q). \quad (8)$$

If the constraints for S can be fulfilled, a solution space of dimension $N_{v,S} - N_{c,S}$ is obtained. However, as the ECs do not take the invalid solutions into account, there are two more requirements to be fulfilled. Firstly, each equality of (4) corresponding to a node pair in S shall be linearly independent from the $N_{c,S}$ constraints. Secondly, there always exists a one-dimensional invalid solution subspace, as explained in Section II. In other words, the $N_{v,S} - N_{c,S}$ dimensional solution space shall have a strict subspace which is at least of dimension one, thus

$$N_{v,S} - N_{c,S} \geq 2 \quad (9)$$

shall hold. Substituting (7) and (8) into (9) yields that the proper configuration of relay antennas in every S is a necessary condition for IA being feasible.

We then argue that IA is almost surely feasible if the relay antenna configuration in the whole network is proper. In fact, we will show the following stronger statement.

Proposition 1: If the relay antenna configuration in the set of subnetworks S is proper, there is at least one vector \mathbf{y} in the space $p_S(W)$ such that \mathbf{y} is valid with respect to all node pairs in S , almost surely.

Proof: Every vector $\mathbf{y} \in p_S(W_S)$ satisfying all the ECs for S also belongs to $p_S(W)$ by Definition 1. If S consists only of a single subnetwork, namely $|S| = 1$, the proposition simply follows from comparing the numbers of variables and constraints in S . Suppose the proposition is true for S' , where $|S'| < Q$. Then one can select a vector $\mathbf{y}_1 \in p_{S'}(W)$ which is valid with respect to all node pairs in S' . Consider the set of subnetworks S consisting of S' and the q -th subnetwork, where $q \notin S'$. Then $\text{rank}(G_S) \geq \text{rank}(G_{S'}) + \text{rank}(G_{\{q\}})$ holds because every EC for either S' or $\{q\}$ is also an EC for S . If there is at most a single non-negligible inter-subnetwork direct link between S' and the q -th subnetwork, i.e., $\text{rank}(G_S) \leq \text{rank}(G_{S'}) + \text{rank}(G_{\{q\}}) + 1$ holds, the relay antenna configuration being proper in both S' and $\{q\}$ implies that it is proper in S . In this case, one can pick any vector $\mathbf{y}_2 \in p_{\{q\}}(W)$ which is valid with respect to every node pair in $\{q\}$ while scaling \mathbf{y}_1 to satisfy the EC between S' and $\{q\}$. Therefore, $\alpha\mathbf{y}_1 + \mathbf{y}_2$ is the desired vector in $p_S(W)$. We then consider the case where there are more than one non-negligible inter-subnetwork direct links between S' and the q -th subnetwork, i.e., $\text{rank}(G_S) \geq \text{rank}(G_{S'}) + \text{rank}(G_{\{q\}}) + 2$ holds. Since \mathbf{y}_1 can satisfy $\sum_{r \in S'} (R_r^2 + 2K_r) - 2$ constraints and one of the EC between S' and $\{q\}$ can be satisfied by the scaling factor α , the number of remaining constraints in S is

$$N_c = \sum_{r \in S} K_r(K_r - 1) + \text{rank}(G_S) - \sum_{r \in S'} (R_r^2 + 2K_r) + 1. \quad (10)$$

If the relay antenna configuration is proper in S , the number of variables provided by $\{q\}$ satisfies

$$\begin{aligned} N_v &\geq 2K_q + \sum_{r \in S} K_r(K_r - 3) + \text{rank}(G_S) + 2 - \sum_{r \in S'} R_r^2 \\ &= N_c + 1. \end{aligned} \quad (11)$$

Hence, a vector $\mathbf{y}_2 \in p_{\{q\}}(W)$ can be found for the given \mathbf{y}_1 . The inequality (11) ensures that equality (4) for each node pair in the q -th subnetwork is almost surely linearly dependent from these constraints. Therefore, the proposition follows. By choosing S as the largest set of subnetworks $\{1, \dots, Q\}$, it follows that the relay antennas configuration being proper in the whole networks is a sufficient condition for IA being feasible. ■

Remark 1: Note that G_q has $2K_q$ vertices and $\text{rank}(G_q)$ is therefore at most $2K_q - 1$ [11]. It is easy to conclude that every relay having $\sqrt{K_q^2 - K_q} + 1$ antennas suffices to perform IA in the entire network even if all the inter-subnetwork direct links are non-negligible. That is to say, even in the worst case, the number of required antennas at a single relay is only related

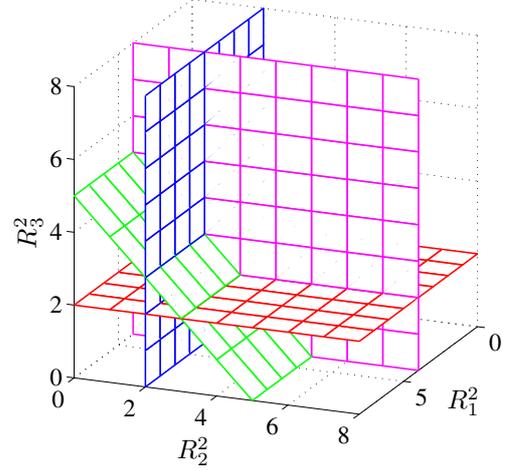


Fig. 3. Boundary of the proper relay antenna configurations in the network shown in Fig. 1.

to the size of the subnetwork it belongs to, regardless of the size of the entire network.

V. SIMULATION RESULTS

Recall the example shown in Fig. 1. Using the graph-based method introduced in Section III, one can find the necessary ECs for every set of subnetwork. Then by (6), the proper relay antenna configurations in this network can be characterized by

$$\begin{aligned} &(R_1^2 \geq 4) \wedge (R_2^2 \geq 2) \wedge (R_3^2 \geq 2) \\ &\wedge (R_1^2 + R_2^2 \geq 5) \wedge (R_1^2 + R_3^2 \geq 5) \wedge (R_2^2 + R_3^2 \geq 5) \\ &\wedge (R_1^2 + R_2^2 + R_3^2 \geq 8). \end{aligned} \quad (12)$$

In a network including Q relays, i.e., including Q subnetworks, there are $2^Q - 1$ such inequalities. Depending on the network topology, some of these inequalities may be implied by the other ones. In this particular example, the region of the proper relay antenna configurations is only bounded by four surfaces, as illustrated by Fig. 3. Since the numbers of relay antennas shall be integers, every relay shall have at least two antennas for performing IA.

In the following, we will evaluate the achievable sum-rate in the network shown in Fig. 1 by simulations. The channel coefficients are independently Rayleigh distributed. The average gain of the strong, non-negligible links is normalized to one. Define ρ to be the ratio of the average gain of the links being neglected in Fig. 1 to that of the non-negligible ones. The cases where ρ takes various values from -6 dB to 0 are considered. Zero mean additive white Gaussian noise with common variance σ^2 is assumed at both relays and destination nodes. We assume that all source nodes have equal transmit powers. A randomly selected solution of (3) will be scaled to fulfill a total power constraint. Then the power allocation among the source nodes and the relays will be determined by the chosen interference-nulling solution too. The total average power consumed by the system per channel use is represented by the pseudo SNR

$$\gamma_{\text{pSNR}} = \frac{1}{2} \frac{K P_S + P_R}{\sigma^2}, \quad (13)$$

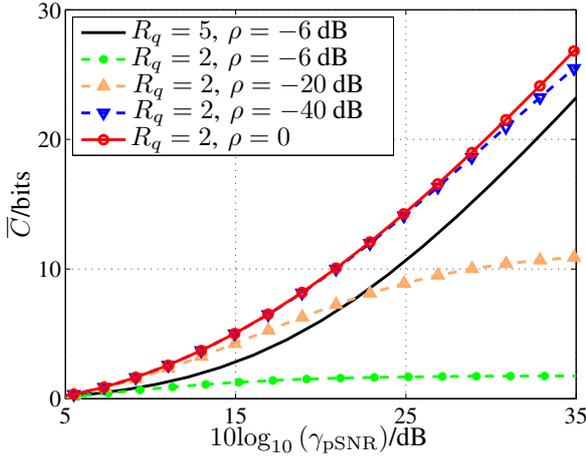


Fig. 4. Achieved average sum rate per channel use as a function of the pseudo SNR in the network shown in Fig. 1

where P_S is the transmit power of every source node and P_R is the total transmit power of the relays. The sum rate per channel use is calculated by

$$C = \frac{1}{2} \sum_{k=1}^K \text{ld} \left(1 + \frac{|h_{\text{eff}}^{(k,k)}|^2 P_S}{\sum_{j \neq k} |h_{\text{eff}}^{(k,j)}|^2 P_S + \sigma_k^2} \right), \quad (14)$$

where

$$h_{\text{eff}}^{(k,j)} = \mathbf{u}^{(k)*T} \begin{pmatrix} h_{\text{DS}}^{(k,j)} & 0 \\ h_{\text{RL}}^{(k,j)} & h_{\text{DS}}^{(k,j)} \end{pmatrix} \mathbf{v}^{(j)} \quad (15)$$

and σ_k^2 is the effective noise variance at the k -th destination node and reads

$$\sigma_k^2 = \sigma^2 |u_2^{(k)}|^2 \sum_{q=1}^Q \mathbf{h}_{\text{DR}}^{(k,q)} \mathbf{G}^{(q)} \mathbf{G}^{(q)*T} \mathbf{h}_{\text{DR}}^{(k,q)*T} + \sigma^2. \quad (16)$$

As a reference, we first assume that the links being neglected in Fig. 1 are relatively strong, namely ρ is -6 dB. We will treat this reference scenario as a fully connected network. To perform IA in the whole network, all the interference-nulling conditions of (3) will be considered. Consequently, each relay needs at least five antennas. The average performance over a large number of channel realizations is shown by the plain solid curve in Fig. 4. It shows that $1/2$ DoF per channel use can be achieved by every user. But if the number of antennas at each relay is reduced to two, IA is infeasible because no user is able to obtain its useful signal. Therefore, we choose to align the interferences propagating through the strong, non-negligible links and ignore the rest. As a result, the sum rate saturates due to the remaining interferences. The achieved performances when ρ is chosen as -6 dB, -20 dB and -40 dB are shown by the dashed curves, respectively. Note that when the neglected links are significantly weaker than the non-negligible ones, remarkable performance gain can be achieved at moderate SNRs. In the ideal case where ρ is zero, all the interferences can be perfectly aligned at every destination node as shown by the solid curve with circles.

VI. CONCLUSION

In this paper, we apply the linear relay-aided IA approach to a class of partially connected networks made up of an arbitrary number of subnetworks. The concept of ECs is extended to investigate the coupling among the subnetworks due to inter-subnetwork direct links. We show that by exploiting the partial connectivity, the number of required antennas at each relay for IA can be reduced as compared to the one in the fully connected network. A graph-based method is employed to characterize the proper relay antenna configurations. Even in very large networks, the number of required antennas at a single relay is only limited by a value related to the size of the subnetwork it belongs to, regardless of the size of the entire network.

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