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OPTIMIZATION OF THE PER TONE NOISE PROTECTION IN XDSL SYSTEMS EMPLOYING VIRTUAL NOISE

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ABSTRACT

In Digital Subscriber Line (DSL) systems, the concept of Virtual Noise (VN) was introduced to improve the protection against fluctuating crosstalk and to increase link stability. In our previous work [10], we have presented an algorithm that estimates the VN mask and the initialization signal-to-noise-ratio (SNR) margin from noise measurements. In [10] however, perfect bitswapping was assumed. In practical DSL systems bitswapping might be too slow which makes the link sensitive to sudden noise increases. Hence, the outage probability achieved in [10] can only be realized in slowly changing channels. This paper investigates the optimization of the VN mask and the initialization SNR margin presented in [10] in order to improve the robustness against sudden noise increases in terms of outage probability, especially for modems with slow bitswapping procedures.

***Index Terms*—** DSL, Virtual Noise, Margin, Outage

1. INTRODUCTION

Digital Subscriber Lines (DSLs) have gained importance over the last years as services such as video and Voice-over-IP (VoIP), which pose high requirements on data rates, latency and line stability, have become part of our every day. The constantly increasing number of DSL users and trying match the growing bandwidth demand by using high frequencies have made far-end crosstalk (FEXT) between copper wires the dominant impairment in current DSL systems [3, 5].

The noise perceived by a DSL line is dominated by the summation of crosstalk from all other active lines in the bundle. Hence, it fluctuates when DSL users turn on/off their modems. According to [6], the number of active DSL users is strongly daytime dependent and consequently, the distribution of noise on DSL lines is not stationary.

In order to provide a reliable service, it is important to be robust against the fluctuating noise. Traditionally, a fixed 6 dB SNR margin is deducted from the measured SNR during initialization when calculating the bitloading of the discrete multi-tone (DMT) modulated subchannels. The SNR margin is the decrease in the SNR that can be withstood by the system at the same target data rate and target error probability [2]. Moreover, modems have the ability to adapt to slowly changing channels by applying bitswapping. Bitswapping procedures equalize the SNR margin across tones by moving bits from tones with higher energy cost to those with lower energy cost [9].

In [10], it was shown that, due to the non-stationarity of noise on DSL lines, using a fixed SNR margin and applying bitswapping might not be enough to provide a reliable service and that employing the VN can improve not only the protection against fluctuating noise, but also the data rate performance. The VN is a tone-dependent noise power spectral density (PSD) specified in the central office management information base and communicated to the transceivers during initialization [8]. The reference noise PSD, which is the maximum of the VN PSD and the measured noise PSD, is used for calculating the bitloading on each tone during initialization and showtime [7]. An SNR margin much smaller than 6 dB has been shown to be sufficient to compensate increases of the reference noise PSD during showtime [10].

Moreover, in [10], we presented an approach for estimating the VN mask and the SNR margin from noise measurements such that a given outage probability is satisfied and the data rate is maximized. In [10], however, perfect bitswapping was assumed. In practical DSL systems, bitswapping is not perfect and therefore, the outage probability achieved in [10] can only be realized in slowly changing channels.

In practical DSL systems, the received noise power is frequency-dependent and the noise power might increase at some tones such that the per tone SNR margins of the affected tones become negative. Moreover, bitswapping might be too slow to move bits from the affected tones such that non-negative per tone margins are restored [2]. In such cases, if the minimum of the per tone SNR margins remains negative for a certain period of time, the connection is interrupted and modems will have to reinitialize at a lower data rate. To prevent this from happening, the noise protection at tones with large noise variances should be increased such that the occurrence probability of negative per tone margins is reduced while the noise protection at tones with small noise variances should be decreased such that the overall data rate stays unchanged.

In this paper, we optimize the VN mask and the SNR margin presented in [10] such that the noise protection at all tones is equalized. Using our proposed approach will improve the robustness against sudden noise increases in terms of outage probability.

2. SYSTEM MODEL

An N-user DSL binder is considered. The N users employ DMT over tones $k = 1, \ldots, K$. Assuming perfect synchronization of the modems and a sufficiently long cyclic prefix, each tone can be modeled as an independent N-user Interference Channel. Let $h_k^{n,n}$ denote the direct channel coefficient of user n on tone k and $h_k^{n,m}$ $(n \neq m)$ denote the FEXT channel coefficient from disturber m to user n. Furthermore, we define $(\sigma_z^n)^2$ as the variance of the white Gaussian noise received by user n and s_k^n as the average transmit power of user n on tone k. Let VN_n^k denote the value of the VN PSD at the receiver of user n on tone k. The VN PSD is communicated to the transceivers during initialization and remains constant during operation.

For the system explained above, the virtual-signal-to-noise-ratio (VSNR) seen at the receiver of user n and used for the bitloading of tone k is given in [10] by

$$VSNR_k^n = \frac{|h_k^{n,n}|^2 s_k^n}{\max\{VN_k^n, \sum_{m \neq n} |h_k^{n,m}|^2 s_k^m + (\sigma_z^n)^2\}}. \quad (1)$$

Moreover, let Γ denote the SNR gap to the capacity of an uncoded QAM system. Γ is a function of the target bit error probability P_e and the coding gain. Let γ_k^n denote the virtual SNR margin on tone k of user n. The virtual SNR margin γ_k^n describes the decrease in the $VSNR_k^n$ that can be withstood at tone k such that the same bit-loading and SNR gap can still be used at that tone. Using the Shannon-gap approximation [1, 4], the bit-loading of user n on tone k results in $b_k^n = \log_2\left(1 + \frac{1}{\Gamma \gamma_k^n} VSNR_k^n\right)$.

3. OUTAGE PROBABILITY & VIRTUAL NOISE

In this section, we present system parameters that are defined in [10] and needed for the remainder of the paper.

To make the derivations in this paper more comprehensible we assume that the noise experienced by user n during initialization is smaller then the VN mask at all tones. Let $\gamma_\text{init}^{\text{dB},n}$ be the SNR margin allocated to user n and $Y_{\text{max},k}^{\text{dB},n}$ be a random variable modeling the maximum noise that can be experienced by user n on tone k over 24 hours in dB scale. With this definitions, and assuming perfect bitswapping (pb) and that the DSL user activity is cyclostationary with the period of one day, the outage probability for a user who is connected for 24 hours was defined in [10] as:

$$\begin{aligned} P_\text{out,pb}^n &= \Pr\{\sum_k \max\{VN_k^{\text{dB},n}, Y_{\text{max},k}^{\text{dB},n}\} \\ &- VN_k^{\text{dB},n} > K\gamma_\text{init}^{\text{dB},n}\}. \end{aligned} \quad (2)$$

Moreover, it was proven in [10] that the K random variables $Y_{\text{max},k}^{\text{dB},n}$ can be assumed to come from a K-variate Gaussian distribution with the K marginal cdfs $F_{Y_{\text{max},k}^{\text{dB},n}}$, $k = 1 \cdots K$.

Now by approximating the minimum of $Y_{\text{max},k}^{\text{dB},n}$ by its 0,1% percentile and denoting the value of the Gaussian cumulative distribution function (cdf) at the 0,1% percentile as $P_{0,1}$, the VN mask derived in [10] is give by

$$VN_k^{\star\text{dB},n} = \min\{Y_{\text{max},k}^{\text{dB},n}\} \approx F_{Y_{\text{max},k}^{\text{dB},n}}^{-1}(P_{0,1}). \quad (3)$$

By defining $J^n = \sum_k Y_{\text{max},k}^{\text{dB},n}$ and F_{J^n} as its cdf, (2) becomes

$$P_\text{out,pb}^n = 1 - F_{J^n}(K\gamma_\text{init}^{\text{dB},n} + \sum_k VN_k^{\star\text{dB},n}) \quad (4)$$

The SNR margin $\gamma_\text{init}^{\star\text{dB},n}$ derived in [10] is then given by

$$\gamma_\text{init}^{\star\text{dB},n} = \frac{F_{J^n}^{-1}(1 - P_\text{out,pb}^n) - \sum_k VN_k^{\star\text{dB},n}}{K}. \quad (5)$$

Note that J^n is a linear combination of the components of a K-variate Gaussian distribution and is therefore also Gaussian distributed. Furthermore, the per tone outage probability over 24 hours at tone k, after initialization where the actual noise power is smaller than the VN mask at all tones and the bitswapping procedures haven't start, was given in [10] by:

$$P_{\text{out},k}^n = 1 - F_{Y_{\text{max},k}^{\text{dB},n}}(VN_k^{\star\text{dB},n} + \gamma_\text{init}^{\star\text{dB},n}). \quad (6)$$

Since $\gamma_\text{init}^{\star\text{dB},n}$ is equal for all tones and since the cdfs $F_{Y_{\text{max},k}^{\text{dB},n}}$, $k = 1 \cdots K$ have different parameters, $P_{\text{out},k}^n$ will vary from one tone to another.

In [10], the cdfs $F_{Y_{\text{max},k}^{\text{dB},n}}$, $k = 1 \cdots K$, and F_{J^n} are estimated from showtime measurements. Hence, the VN mask $VN_k^{\star\text{dB},n}$, $k = 1 \cdots K$, the SNR margin $\gamma_\text{init}^{\star\text{dB},n}$, the cdfs $F_{Y_{\text{max},k}^{\text{dB},n}}$, $k = 1 \cdots K$ and F_{J^n} will be assumed to be known for the remainder of the paper.

4. BITSWAPPING RELIANCE FACTOR

In the derivation of $P_\text{out,pb}^n$ in (4) and (2), perfect bitswapping is assumed. Clearly, the DSL system profits from assuming perfect bitswapping. This profit can be understood as the decrease in the outage probability that results from applying perfect bitswapping to a system with fixed data rate, VN mask and SNR margin.

Let out_k describe an outage event at tone k. The outage probability of a system that does not apply bitswapping at any time instant can be expressed by:

$$P_\text{out,nb}^n = \Pr\{out_1 \cup \cdots \cup out_k \cup \cdots \cup out_K\}. \quad (7)$$

Furthermore, for the same system now applying perfect bitswapping, let us define the bitswapping reliance factor Δ as an expression for the reliance of the system on bitswapping for achieving the outage probability $P_\text{out,pb}^n$.

$$\Delta = \frac{P_\text{out,nb}^n - P_\text{out,pb}^n}{1 - P_\text{out,pb}^n}, \quad \Delta \in [0, 1]. \quad (8)$$

Since bitswapping procedures might be too slow in practical DSL systems, the reliance on bitswapping to achieve a certain target outage probability should be held to the necessary minimum. Consequently, the bitswapping reliance factor Δ has to be minimized. To minimize Δ, $P_\text{out,nb}^n$ has to be minimized while the outage probability $P_\text{out,pb}^n$ has to be maintained. Minimizing $P^n_{\text{out,nb}}$ in (7) implies equalizing the probabilities of outage events at all tones. Hence, in order not to strongly rely on bitswapping, the VN mask and the SNR margin have to be optimized such that the per tone outage probabilities $P^n_{\text{out},k}$, $k = 1 \cdots K$ are equalized without having to decrease the target data rate or increase the outage probability $P^n_{\text{out,pb}}$.

5. DATA RATE AND OUTAGE PROBABILITY CONDITIONS

In order to rely less on bitswapping to achieve $P^n_{\text{out,pb}}$, $VN^{\star\text{dB},n}_k$, $k = 1 \cdots K$ and $\gamma^{\star\text{dB},n}_{\text{init}}$ have to be optimized such that the per tone outage probabilities are equalized without having to increase $P^n_{\text{out,pb}}$ or decrease the target data rate. In this section, we will present the conditions that the optimized $VN^{\text{dB},n}_k$, $k = 1 \cdots K$ and $\gamma^{\text{dB},n}_{\text{init}}$ need to satisfy in order to maintain $P^n_{\text{out,pb}}$ and the target data rate.

Let $b^n_k = \log_2\left(1 + \frac{1}{\Gamma\gamma^n_{\text{init}}}\frac{|h^{n,n}_k|^2 s^n_k}{VN^n_k}\right)$ be the number of bits transmitted on tone k calculated during initialization. Furthermore, let $b^{\star n}_k$ be the number of bits transmitted on tone k calculated during initialization when using $VN^{\star\text{dB},n}_k$, $k = 1 \cdots K$ and $\gamma^{\star\text{dB},n}_{\text{init}}$. To maintain the data rate performance, the condition $\sum_k b^n_k = \sum_k b^{\star n}_k$ must hold. This implies in linear scale

$$\prod_k \left(1 + \frac{1}{\Gamma\gamma^n_{\text{init}}}\frac{|h^{n,n}_k|^2 s^n_k}{VN^n_k}\right) = \prod_k \left(1 + \frac{1}{\Gamma\gamma^{\star n}_{\text{init}}}\frac{|h^{n,n}_k|^2 s^n_k}{VN^{\star n}_k}\right). \tag{9}$$

Since high SNR is typical for DSL systems, the addition of 1 on both sides can be neglected [2]. We obtain the following data rate condition in dB scale:

$$\sum_k VN^{\text{dB},n}_k + \gamma^{\text{dB},n}_{\text{init}} = \sum_k VN^{\star\text{dB},n}_k + \gamma^{\star\text{dB},n}_{\text{init}}. \tag{10}$$

Moreover, let $V^n = \sum_k \max\{VN^{\text{dB},n}_k, Y^{\text{dB},n}_{\text{max},k}\}$ and F_{V^n} be its cdf. Similar to (4), the outage probability is given by $P^n_{\text{out,pb}} = 1 - F_{V^n}(K\gamma^{\text{dB},n}_{\text{init}} + \sum_k VN^{\text{dB},n}_k)$. Hence, to maintain the outage probability, the following condition must hold:

$$F^{-1}_{J^n}(1 - P^n_{\text{out,pb}}) = F^{-1}_{V^n}(1 - P^n_{\text{out,pb}}). \tag{11}$$

6. EQUALIZATION OF THE PER TONE OUTAGE PROBABILITIES

In this section, we find the optimum VN mask as a function of the SNR margin such that equal per tone outage probabilities are achieved while fulfilling condition (10). Then, in the following Section, the minimum SNR margin that satisfies condition (11) is determined.

Forming (6) with $VN^{\text{dB},n}_k$ and $\gamma^{\text{dB},n}_{\text{init}}$ and expressing $P^n_{\text{out},k}$ by the tone-independent variable P, yielding $P^n_{\text{out},k} = 1 - P$, leads to $F^{-1}_{Y^{\text{dB},n}_{\text{max},k}}(P) = VN^{\text{dB},n}_k + \gamma^{\text{dB},n}_{\text{init}}$. Now the problem of equalizing the per tone outage probabilities while maintaining the data rate can be be solved by finding P that satisfies the following Equation:

$$\sum_k F^{-1}_{Y^{\text{dB},n}_{\text{max},k}}(P) = \sum_k VN^{\star\text{dB},n}_k + \gamma^{\star\text{dB},n}_{\text{init}}. \tag{12}$$

Since the inverse cdfs $F^{-1}_{Y^{\text{dB},n}_{\text{max},k}}(P)$, $k = 1 \cdots K$ are monotonically increasing, the optimal solution of (12) can be obtained using bisection on the scalar P, yielding P_{eq}. Now, the VN mask is given by:

$$VN^{\text{dB},n}_k = F^{-1}_{Y^{\text{dB},n}_{\text{max},k}}(P_{\text{eq}}) - \gamma^{\text{dB},n}_{\text{init}},\ k = 1 \cdots K \tag{13}$$

7. MINIMIZATION OF THE SNR MARGIN

In this section we find the minimum $\gamma^{\text{dB},n}_{\text{init}}$ that satisfies condition (11). Let us explain first, how the choice of the SNR margin has an effect on the bitswapping reliance factor during showtime. For that purpose, let us assume that the per tone outage probabilities $P^n_{\text{out},k}$, $k = 1 \cdots K$ have been equalized at initialization. During showtime at time T_0 the actual noise might exceed the VN mask at only some tones. Bitswapping will distribute the available SNR margin equally on all tones. Consequently, tones that were distorted by the noise increase will have larger per tone SNR margins and therefore, a lower per tone outage probability than before the bitswap procedure and tones that were not distorted by the noise increase will have smaller per tone SNR margins and therefore, a higher per tone outage probability than before the bitswap procedure. Obviously, the bitswapping reliance factor calculated with per tone outage probabilities at T_0 is larger than the one calculated with the initial equalized per tone outage probabilities. Therefore, the tone-wise worst case outage probability at tone k, where the SNR margin is used up by noise increases at other tones, is given by

$$P^{\text{worst},n}_{\text{out},k} = 1 - F_{Y^{\text{dB},n}_{\text{max},k}}(VN^{\text{dB},n}_k) \tag{14}$$

Obviously, to keep the variation of the per tone outage probabilities to a minimum, the VN mask has to be set as high as possible, which implies minimizing the SNR margin in (13). This problem can be formulated as

$$\begin{aligned} &\underset{\gamma^{\text{dB},n}_{\text{init}}}{\text{minimize}} && \gamma^{\text{dB},n}_{\text{init}} \\ &\text{subject to} && F^{-1}_{V^n}(1 - P^n_{\text{out,pb}}) = F^{-1}_{J^n}(1 - P^n_{\text{out,pb}}) \end{aligned} \tag{15}$$

To solve the problem above, F_{V^n} has to be expressed as a function of $\gamma^{\text{dB},n}_{\text{init}}$. In contrast to F_{J^n}, the shape of F_{V^n} is not known. Hence, F_{V^n} cannot be estimated from showtime measurements. In the following, we will express F_{V^n} as a function of $\gamma^{\text{dB},n}_{\text{init}}$, $F_{Y^{\text{dB},n}_{\text{max},k}}$, $k = 1 \cdots K$, F_{J^n} and a measurement of the noise day maxima of the K tones.

Let U be a uniform (0,1) distributed random variable and let $J^n_{\text{up}} = \sum_k F^{-1}_{Y^{\text{dB},n}_{\text{max},k}}(U)$ and $F_{J^n_{\text{up}}}$ be its cdf. According to [11], the upper bound of the sum $\sum_k Y^{\text{dB},n}_{\text{max},k}$ is given by:

$$F^{-1}_{J^n_{\text{up}}}(p) = \sum_k F^{-1}_{Y^{\text{dB},n}_{\text{max},k}}(p),\ \ p \in [0, 1]. \tag{16}$$

The summands in (16) are non-decreasing functions of p and therefore, fully dependent. To improve the approximation in (16), the independence between the summands has to be considered. For that purpose we define the following coefficients. Let $c_{\text{corr}}(p)$ be a correction coefficient to the true cdf F_{J^n} given by:

$$c_{\text{corr}}(p) = \frac{F^{-1}_{J^n}(p)}{\sum_k F^{-1}_{Y^{\text{dB},n}_{\max,k}}(p)}, \quad p \in [0,1]. \tag{17}$$

Moreover, let $y^{\text{dB},n}_{\max,k}$, $k = 1 \cdots K$, be a measurement of the noise day maxima over all K tones and let $\alpha = F_{J^n}(\sum_k y^{\text{dB},n}_{\max,k})$. The coefficient $c_{\text{dev},k}(\alpha)$ models the deviation of $y^{\text{dB},n}_{\max,k}$ from the corresponding corrected realizations $c_{\text{corr}}(\alpha) F^{-1}_{Y^{\text{dB},n}_{\max,k}}(\alpha)$, and is therefore given by

$$c_{\text{dev},k}(\alpha) = \frac{y^{\text{dB},n}_{\max,k}}{F^{-1}_{Y^{\text{dB},n}_{\max,k}}(\alpha)} - c_{\text{corr}}(\alpha), \quad k = 1 \cdots K \tag{18}$$

Furthermore, we define $c_k(p) = c_{\text{corr}}(p) + c_{\text{dev},k}(p)$ and assume that $c_{\text{dev},k}(p) \approx c_{\text{dev},k}(\alpha)$, $p \in [0,1]$, and hence, $c_k(p) \approx c_{\text{corr}}(p) + c_{\text{dev},k}(\alpha)$. An approximation of the sum $\sum_k Y^{\text{dB},n}_{\max,k}$ is now given by:

$$F^{-1}_{J^{n\,\prime}_{\text{up}}}(p) = \sum_k c_k(p) F^{-1}_{Y^{\text{dB},n}_{\max,k}}(p), \quad p \in [0,1]. \tag{19}$$

By adding the maximum operator to the summands in (19), (15) is now given by

$$\begin{aligned} \underset{\gamma^{\text{dB},n}_{\text{init}}}{\text{minimize}} \quad & \gamma^{\text{dB},n}_{\text{init}} \\ \text{subject to} \quad & \sum_k \max\Big\{ F^{-1}_{Y^{\text{dB},n}_{\max,k}}(P_{\text{eq}}) - \gamma^{\text{dB},n}_{\text{init}}, \\ & c_k (1 - P^n_{\text{out,pb}}) F^{-1}_{Y^{\text{dB},n}_{\max,k}}(1 - P^n_{\text{out,pb}}) \Big\} \\ & = F^{-1}_{J^n}(1 - P^n_{\text{out,pb}}) \end{aligned} \tag{20}$$

The problem above can be solved by setting $\gamma^{\text{dB},n}_{\text{init}} = 0$ and increasing $\gamma^{\text{dB},n}_{\text{init}}$ gradually until (20) is fulfilled.

8. SIMULATION RESULTS & CONCLUSIONS

This section presents simulation results to show the benefits of optimizing the VN mask and the SNR margin in DSL systems. To model the user activity, we use the user activity model we presented in [10]. In [10], the status of a user is modeled by a Markov chain. The transition probabilities between the states of the Markov chain were determined based on statistics from [6]. Here, a downstream scenario with a VDSL2 victim line and 39 VDSL2 disturber lines is assumed. The transmit PSD masks used are according to VDSL2 band plan 998ADE17M2xB [8]. The non-stationary noise is generated according to the user activity model combined with FEXT measurements of single disturbers made at ADTRAN's DSL lab.

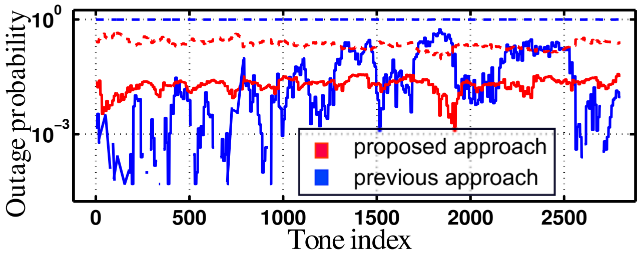

Fig. 1: Previous approach [10] compared with the proposed approach. Dashed line indicates the tone-wise worst case.

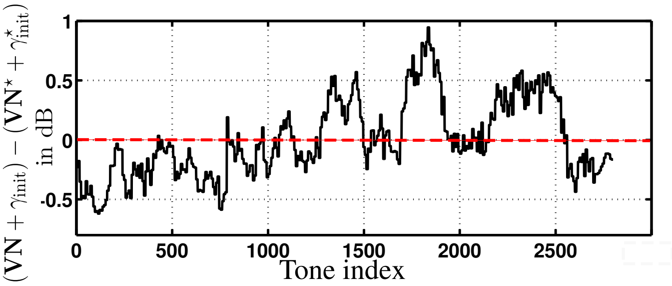

Fig. 2: The change in shape of the Virtual Noise mask.

Figure 1 shows the per tone outage probabilities and the tone-wise worst case outage probabilities when using the VN mask and SNR margin in [10] (previous approach) and when using the optimized VN mask and SNR margin (proposed approach). The reason why the per tone outage probabilities are not exactly equalized when using the proposed approach is that the true distribution of the noise day maxima at some tones deviate from the Gaussian distributions assumed in [10]. The proposed approach reduces the bitswapping reliance factor from 0.7 of the previous approach to 0.2.

The average SNR margin achieved by the proposed approach is equal to 0.4 dB which is much smaller the 1.75 dB of the previous approach. Consequently, the tone-wise worst case outage probabilities in Figure 1 are much lower with the proposed approach than with the previous approach which indicates less variation of the per tone outage probabilities during operation. Moreover, the average SNR margin achieved by the proposed approach complies with the SNR margin determined by simulation from 20000 noise day maxima.

Since the SNR margin is equal over all tones in both approaches, the equalization of the per tone outage probabilities is achieved by changing the shape of the VN mask. This change is illustrated in Figure 2. Obviously, the shape is increased at tones which initially had a high outage probability and vice versa.

To conclude, by equalizing the per tone outage probabilities and minimizing the SNR margin, the proposed approach ensures better robustness against sudden noise increases, especially for modems with slow bitswapping procedures.

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