IEEE Wireless Communications and Networking Conference, WCNC 2013, Shanghai, China

# Comparison of different multicast strategies in wireless identically distributed channels

Erzim Veshi, Alexander Kuehne, Anja Klein
Communications Engineering Lab
Technische Universität Darmstadt
Merckstrasse 25, 64283 Darmstadt, Germany
Email: [e.veshi, a.kuehne, a.klein] @nt.tu-darmstadt.de

***Abstract*—This paper analyzes multicast (MC) as an efficient approach in transmitting the same information to multiple receivers (RXs). In each transmission time slot (TS), based on the channel realizations and on the specific MC strategy, a subset of RXs to be served is selected. The members of the served subset may change in the next TS. We assume that the channels from the transmitter (TX) to the RXs are independent and identically distributed (i.i.d.). This makes it possible for each RX to get, on average, the same amount of information. Herein, we find a closed form solution regarding the throughput of the Opportunistic Multicast strategy with fixed group size (OppMC-FS) [1] and present a new variant of this strategy called OppMC with optimal group size (OppMC-OS), providing an analytical solution regarding its throughput. Both variants, OppMC-FS and OppMC-OS, require instantaneous channel state information (CSI) at the TX. In addition, we also propose a new MC strategy, named MC based on statistical channel knowledge (StCSI-MC), and find the throughput of this strategy in a closed form. Results show that our strategies outperform broadcast or unicast and also that a good MC strategy can be found without the need of instantaneous CSI.**

***Index*- Multicast, Broadcast, Opportunistic Scheduling, Statistical CSI.**

## I. Introduction

The ever increasing demand for higher data rates in wireless networks has triggered the research of different strategies in order to increase the efficiency and optimize the use of the wireless channel. Many applications, e.g., Multimedia Broadcast and Multicast Service (MBMS) [2], Digital Video Broadcast (DVB) [3], software update, video streaming, etc, require the delivery of the same information to a large group of users. In some mobile systems this is done by copying the information and then sending each one of the copies to the designated receiver (RX) using a unicast (UC) strategy. Another possible solution is to send the same information to all the RXs using a broadcast (BC) strategy. Both of those strategies may exploit the random nature of the wireless channel in different ways. UC creates multi-user diversity gain if the information is transmitted always to the RX under the best channel conditions, while BC produces broadcast gain since the information is transmitted to all the RXs using the same resource.

There has been much research on UC in wireless networks. In [4], the authors propose the Proportional Fair (PF) strategy while in [5], the Max CIR (Carrier to Interference Ratio) strategy has been proposed. A strategy considering both fairness and efficiency of UC has been introduced in [6] and another similar strategy, called FECD (fair and efficient channel dependent), has been introduced in [7]. Typically, the aforementioned UC strategies utilize Time Division Multiplexing (TDM) and, during each transmission time slot (TS), the transmitter (TX) decides which RX to serve based on the channel state information (CSI), which has to be provided to the TX. Once the RX is selected, the TX transmits as much information as only that RX can decode.

In the multicast (MC) strategy, in each TS, the TX transmits to one group of RXs and all the members of that group get as much information as the RX of that group under the worst channel conditions (RXmin) can decode [8]. BC can be seen as a special case of MC where all the RXs in the system are part of the served group.

Various papers [1], [9]–[12] have compared different UC or MC strategies and intensive work has been performed in determining which strategy to use for which scenario. In [9], a comparison between UC and BC is performed under the assumption that the RXs are equidistant from the TX. Therein, it can be seen that BC is good when the RXs are close to the TX or in a high SNR (signal to noise ratio) regime while UC shows good performance for low SNR values. In [10] [1] and [11], opportunistic multicast (OppMC) strategies are introduced. In [10], the authors introduce the Median Scheduling Strategy which divides the number of RXs in half and serves the best group during each TS. In [1], different group sizes have been simulated and it has been shown that the system throughput scales linearly with the number of multicast receivers in the system. The authors in [11] investigate deeper non i.i.d. wireless channels while in [12], quantized rate levels and different wireless channel scenarios, concerning the strategy introduced in [1], have been simulated.

The works in [1], [10], [11] show that MC outperforms BC and UC for a large range of SNR values. Herein, we extend their research by providing a closed form solution regarding the system throughput of their strategies and also proposing a way to find the best MC group size given the SNR distributions. Furthermore we investigate the case when the TX does not have instantaneous CSI.

In this paper, we first investigate OppMC presenting a closed form solution regarding the throughput of the strategies considered in [10] and [1] which are OppMC strategies with fixed group size (OppMC-FS). Based on that solution, we find the best group size in the OppMC-FS strategy, given the SNR distribution at the RXs and the number of RXs in the system. After that, we propose a new variant of OppMC strategy, called OppMC with optimal group size (OppMC-OS), in which the group size may change in each TS in order to maximize the throughput. Furthermore, we propose a new strategy, called MC based on statistical channel knowledge (StCSI-MC), which does not require instantaneous channel knowledge at the TX. For each strategy, we always investigate the achievable throughput under the assumption of i.i.d. (independent and identically distributed) channel conditions. This assumption leads to the fact that, given one of our MC strategies, all the RXs in the system will get the same average throughput, named average user throughput. If we change the strategy, the average user throughput changes, but remains the same for all the RXs in the system.

The rest of the paper is organized as follows. In Section II, the system model is presented. Section III refers to the OppMC strategy, introducing also the OppMC-OS variant. Section IV describes the StCSI-MC strategy. In Section V, a performance comparison between our strategies, and also the UC and BC strategies, is performed. In Section VI, conclusions are drawn.

## II. System model

In this section, the system model is introduced. We are considering one TX and a number $N$ of RXs. Each RX has a distance $d$ from the TX, see Fig. 1, and wants to receive the same information from the TX. The TX and all the RXs are equipped with omnidirectional antennas. The transmission takes place in TSs. In each TS, depending on the MC strategy that we are using, only certain RXs are selected to be served. Each RX will have the same probability to be selected on average, since all the RXs have the same channel statistics. The scenario can, e.g., be seen as a downlink of a cellular network.

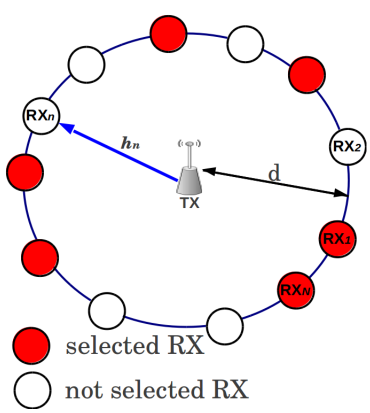


Fig. 1. Scenario

Regarding the wireless channel, with $h_n$ we denote the channel coefficients between the TX and RX $n$, $n = 1..N$. We suppose that $\forall n = 1..N$, $h_n$ are i.i.d. complex circularly symmetric Gaussian distributed random variables with zero mean and variance 1. Furthermore, block flat fading will be assumed, i.e., the channel coefficients are constant for the entire duration of the TS and for the whole bandwidth of the transmitted signal. In addition, we assume a path loss channel model. We also assume additive white Gaussian noise (AWGN) at each RX. This noise can be thermal noise or interference coming from other TXs close to this RX. Because of these assumptions, we will have a memory-less channel.

If the path loss coefficient is denoted by $\alpha$, the transmit power is denoted by $P_{\mathrm{T}}$ and the noise power at each RX is denoted by $P_{\mathrm{Noise}}$, then the instantaneous SNR value at RX $n$ will be

$$\gamma_n = \left(\frac{d}{d_0}\right)^{-\alpha} \cdot \frac{P_{\mathrm{T}}|h_n|^2}{P_{\mathrm{Noise}}} = \frac{P_{\mathrm{T}}|h_n|^2}{d_n^{\alpha} P_{\mathrm{Noise}}} = \overline{\gamma} \cdot |h_n|^2 \quad (1)$$

assuming that $d_0 = 1$ m. $\overline{\gamma}$ is the average SNR value that RX $n$ is experiencing. $\overline{\gamma}$ is not dependent on the RX's index $n$ since it is the same for all the RXs.

Due to the assumptions above, the channel power coefficient $|h_n|^2$ is exponentially distributed, making also $\gamma_n$ exponentially distributed with an average value of $\overline{\gamma}$. The probability density function (PDF) of the SNR for RX $n$ is

$$f_\gamma(\gamma) = f_{\gamma_n}(\gamma_n) = \begin{cases} \frac{1}{\overline{\gamma}} \exp\left(\frac{-\gamma}{\overline{\gamma}}\right) & \text{if } \gamma \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

and it is independent of the index $n$ of the RX.

The cumulative distribution function of $\gamma$ is

$$F_\gamma(\gamma) = \begin{cases} 1 - \exp\left(\frac{-\gamma}{\overline{\gamma}}\right) & \text{if } \gamma \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

We will study the system from the information theoretical point of view and allow infinite delay. This assumption is acceptable if we consider a large buffer at each RX and make use of the fountain codes where every transmitted bit counts as useful bit since each RX can reconstruct the original message from the received encoded bits [13]. The average throughput which can be transmitted to each RX will be derived from the Shannon-Hartley theorem [14], in form of the ergodic capacity (in bits/s) and, for simplicity, it will be assumed that $B = 1$ Hertz.

## III. Opportunistic Multicasting (OppMC) Strategy

In this section, we will analyze OppMC more in details. In the OppMC strategy, in each TS, after having the CSI of all the channels between the TX and all the RXs, the TX decides which group of RXs should be served. In Section III-A, we will examine the OppMC with fixed group size $K$ (OppMC-FS) strategy, in which the number of RXs in the served group is fixed in each TS, while the members of this group may change [1]. In Section III-B, we propose an OppMC variant, called OppMC with optimized group size (OppMC-OS), in which the cardinality of the served group may change in each TS. BC and UC can be considered as special cases of the OppMC-FS strategy, for group sizes of $K = N$ and $K = 1$, respectively. The gain of this strategy with respect to UC or

BC is that, by modifying the cardinality of the MC group that we serve, we can profit from both multi-user diversity gain and broadcast gain. The larger the group size, the higher the broadcast gain, but the lower the multi-user diversity gain.

### A. OppMC-FS Strategy

In this section, the OppMC-FS strategy is analysed in details and a closed form solution is provided regarding the average throughput of this strategy. In OppMC-FS, we serve $K$ out of $N$ receivers in a multicast way and, in every TS, the cardinality of the served subset is always fixed to $K$. The members of this subset are the $K$ RXs which have the highest instantaneous SNR values in a given TS and we send as much information as RXmin of this subset can decode. Since we assumed i.i.d. channels, each RX has the same probability to be part of the served subset.

In order to determine the average user throughput, which is the throughput that each RX gets on average, we first order the instantaneous SNR values $\gamma_n$ based on their realizations, from the highest to the smallest. We denote with $y_K$ the random variable expressing the $K^{\text{th}}$ maximum ordered statistics, i.e., for every TS, $(K-1)$ $\gamma_n$ values are higher than that value and $(N-K)$ $\gamma_n$ values are lower than it. The PDF of $y_K$ is found as

$$f_K(\gamma) = \frac{N!}{(N-K)!(K-1)!} F_\gamma^{N-K}(\gamma)[1-F_\gamma(\gamma)]^{K-1} f_\gamma(\gamma) \quad (4)$$

[15], where $F_\gamma(\gamma)$ and $f_\gamma(\gamma)$ are given in (2) and (3), respectively.

In this case, since we are sending to all the RXs of the served subset as much as RXmin of this subset can decode, to obtain the average user throughput, we have to average over the SNR distribution of RXmin in the served subset. Thus, the average user throughput is

$$D_K = \mathbb{E}(D_{\text{OppMC-FSK}}) = \frac{K}{N}\int_0^\infty \log_2(1+\gamma) f_K(\gamma) \mathrm{d}\gamma \quad (5)$$

In (5), the factor $K$ takes into account the fact that $K$ RXs are served in each TS, while the factor $1/N$ expresses the average user throughput. For (5), we can find a closed form solution if we apply the binomial theorem for the expansion of the power of a series [16, Eq 1.111] and by solving the integral [16, Eq 4.337.2]. Finally, the average user throughput of this strategy, is

$$D_K = C\sum_{l=0}^{N-K}\binom{N-K}{l}\frac{-1^l}{l+K}\exp\left(\frac{l+K}{\overline{\gamma}}\right)\mathrm{E}_1\left(\frac{(l+K)}{\overline{\gamma}}\right) \quad (6)$$

where $C = \frac{K(N-1)!}{(K-1)!(N-K)!ln(2)}$ and $\mathrm{E}_1(x) = -\mathrm{Ei}(-x)$ is the exponential integral function [17]. For any combination of $\overline{\gamma}$ and $N$, an optimal $K$ can be found which maximizes $D_K$.

### B. OppMC-OS Strategy

In this paragraph, the OppMC-OS variant is explained. Since we have the full instantaneous CSI knowledge at the TX, in each TS, we can also form a subset of RXs to be served without a fixed cardinality. This subset may change its cardinality and also its members in each TS in order to maximize the average user throughput. In a specific TS, if we want to transmit to the RX with the $j^{\text{th}}$ highest SNR value $\gamma_j$, we should send $D_j = log_2(1+\gamma_j)$ bits/s and the total system throughput, from all the $N$ RXs in the system, will be $j * D_j$, since the number of RXs that can decode $D_j$ is $j$. In order to find the maximum achievable system throughput $D_{\text{Opt-Sys}}$, in each TS, we have to find:

$$D_{\text{Opt-Sys}} = \max_{j=1..N} j * \log_2(1+\gamma_j) \quad (7)$$

and send $D_{\text{Opt-Sys}}$ to all the RXs, but only to the RXs that are in condition to decode $D_{\text{Opt-Sys}}$ will get information and contribute to the system throughput. Even here, in each TS, every RX will have the same probability to be served since they have i.i.d. channels from the TX.

The average user throughput of this strategy is

$$\mathbb{E}(D_{\text{OppMC-OS}}) = \frac{\mathbb{E}(D_{\text{Opt-Sys}})}{N}$$
$$= \frac{1}{N}\int_0^\infty \Bigg[1 - \prod_{K=1}^{N}\sum_{l=N-K+1}^{N}\sum_{i=0}^{l}\binom{N}{l}\binom{l}{i}(-1)^i$$
$$\exp\left(\frac{-(2^{\frac{x}{K}}-1)(i(N-l))}{\overline{\gamma}}\right)\Bigg]\mathrm{d}x \quad (8)$$

For the derivation of (8) see the Appendix.

## IV. StCSI-MC Strategy

Having instantaneous CSI at the TX has always a cost in terms of resources used to find it, cost which increases with the number of RXs in the system. In some cases, it is much simpler to have only statistical CSI at the TX, e.g., the distance of the RXs, the SNR distribution at each of them and the correlation between the channel coefficients. Because of this, in this section, a MC strategy, called StCSI-MC, is proposed for the case when the TX has only statistical CSI.

In StCSI-MC, the TX will determine its throughput based on statistical CSI. Some RXs, under good instantaneous channel conditions will be able to decode and will be part of the served subset (SSB), while other RXs, under bad channel conditions, will not be able to decode. In each TS, the TX does not know the cardinality and the members of the SSB. Based on the transmitted throughput, the TX can estimate an average cardinality of the SSB.

The idea of this strategy is as follows. In each TS, the TX transmits $D_\Gamma = \log_2(1+\Gamma)$ bits/s, where $\Gamma$ is a fixed quantity which does not depend on the channel conditions. Thus, $D_\Gamma$ does not depend on the instantaneous SNR values on the RXs. In each TS, the RXs with an instantaneous SNR value $\gamma_n$ higher than $\Gamma$ will be able to decode the transmitted information and the amount of throughput that each one of them will get is $\log_2(1+\Gamma)$, independent on their actual SNR values. If $D_\Gamma$ increases, the average cardinality of SSB decreases while the throughput of each one of the SSB members increases. Hence, the best trade-off has to be found in order to maximize the throughput in terms of average user throughput.

If we apply StCSI-MC, the average user throughput is

$$\begin{aligned}\mathbb{E}(D_{\text{StCSI-MC}}) &= \frac{1}{N}\sum_{i=1}^{N}\int_{\Gamma}^{\infty}\log_2(1+\Gamma)f_{\gamma}(x)\mathrm{d}x\\ &= \exp\left(\frac{-\Gamma}{\overline{\gamma}}\right)\log_2(1+\Gamma) \qquad (9)\end{aligned}$$

In Fig. 2, we plot the average user throughput of (9) as a function of $\Gamma$, for a given value of $\overline{\gamma}$.

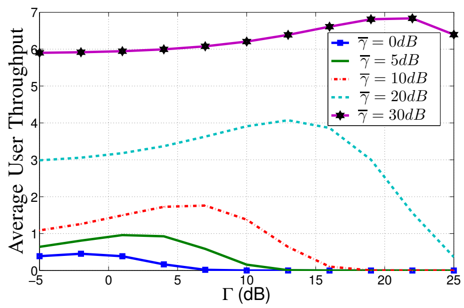


Fig. 2. Average user throughput of the StCSI-MC strategy versus $\Gamma$ for different values of $\overline{\gamma}$

From Fig. 2, it can be seen that given $\overline{\gamma}$, there is always a value of $\Gamma$, named $\Gamma_{\text{OPT}}$, for which $\mathbb{E}(D_{\text{StCSI-MC}})$ reaches its maximum. We can find this value if we set the derivative of $\mathbb{E}(D_{\text{StCSI-MC}})$ to zero as:

$$\left(\frac{1}{\ln 2}\frac{1}{1+\Gamma} - \frac{1}{\overline{\gamma}}\ln(1+\Gamma)\right)\exp\left(\frac{-\Gamma}{\overline{\gamma}}\right) = 0 \qquad (10)$$

The solution of (10) is given by:

$$\Gamma_{\text{OPT}} = -1 + \frac{\overline{\gamma}}{W(\overline{\gamma})} \qquad (11)$$

where $W(x)$ is the Lambert function [18]. In the following, the StCSI-MC using $\Gamma_{\text{OPT}}$ will be called OStCSI-MC, where O stands for optimal.

## V. Numerical results and comparisons

This section is divided in two parts. In the first part, numerical results will be shown regarding the different OppMC variants while in the second part, a comparison of the best OppMC variants with the OStCSI-MC strategy is shown. Since the UC and BC strategies can be seen as special cases of the OppMC-FS strategy, they are also included in the comparisons. The system parameters are given in Table I.

### *A. Numerical results for OppMC*

The performance of our proposed OppMC strategies can be shown in terms of throughput gain that our variants produce with respect to the UC case (OppMC-FS for $K = 1$). This gain is defined as the ratio between the average user throughput of the specific OppMC strategy divided by the average user throughput of the UC strategy. $G_K$ is the gain of the OppMC-FS variant with respect to UC and $G_b$ is the gain of the OppMC-OS variant with respect to UC. These gains are:

TABLE I
System parameters

| Nomenclature | Value | Meaning |
|---|---|---|
| $P_T$ | 38 dBm | Transmission Power |
| $P_{\text{Noise}}$ | -80 dBm | Noise Power |
| $\alpha$ | 3.5 | Path Loss Coefficient |
| $d$ | 50m — 300 m | Distance between RXs and TX |
| $\overline{\gamma}$ | 22.8 dB — -4.2 dB | Resulting average SNR values |

$$G_K = \frac{\mathbb{E}(D_K)}{\mathbb{E}(D_1)} \text{ and } G_b = \frac{\mathbb{E}(D_{OppMC-OS})}{\mathbb{E}(D_1)} \qquad (12)$$

In Fig. 3, $G_K$ is plotted. Here, $K$ takes values 4, 8, 12, 16, 20 and also $N$ (BC), which, in this case, is chosen to be 25.

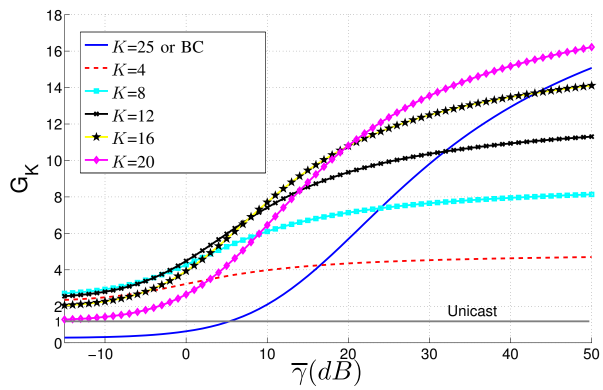


Fig. 3. Gain of the OppMC-FS strategy with respect to UC versus the average SNR value on each RX ($\overline{\gamma}$). In this case, $N = 25$.

From Fig. 3, we can see that by using the OppMC-FS strategy we can have higher average user throughput values than by using UC or BC strategies, especially in the middle and in the low SNR regime. Furthermore, the higher the $\overline{\gamma}$, the better the BC strategy. Also for different $\overline{\gamma}$, different values of $K$ lead to the maximum $G_K$.

In Fig. 4, we can see $G_K$ and $G_b$ as a function of $K$ for given average SNR values $\overline{\gamma}$. It can be seen that the higher $\overline{\gamma}$, the higher the gain, for both OppMC-FS and OppMC-OS. A gain of 10 means that each RX will get on average 10 times more information than if we were using the UC strategy. The gain, $G_b$ or $G_K$, is always between 0 and $N$, while the gap between $G_b$ and $G_K$ with the optimal $K$ is rather small.

In Fig. 5, $\mathbb{E}(D_K)$ is plotted as a function of $N$ for different values of $\overline{\gamma}$, under the condition that $\frac{K}{N}$ stays constant (in our case $\frac{1}{3}$). In the same figure, also the average user throughput provided from the UC or BC strategies is plotted.

From Fig. 5 it can be seen that for high values of $N$, $\mathbb{E}(D_K)$ stays constant if we increase the number of RXs in our system, under the condition that the fraction of served RXs over the total number of RXs stays constant. For the UC or BC strategies, if the total number of RXs in the system increases,

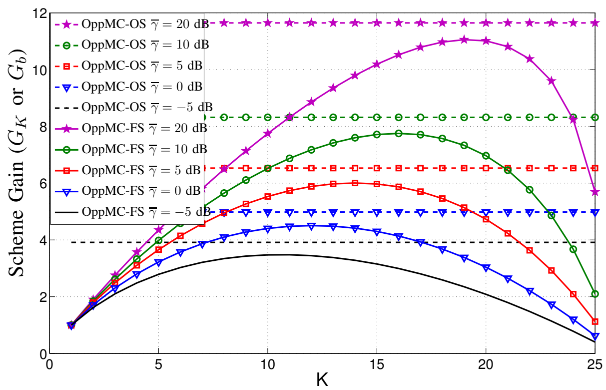


Fig. 4. $G_K$ versus the number of RXs in the served subset $K$ in the OppMC-FS strategy. In the same graph, also $G_b$ is shown. Different values of $\overline{\gamma}$ are taken into consideration. For this case, the number of RXs in the system is $N = 25$.

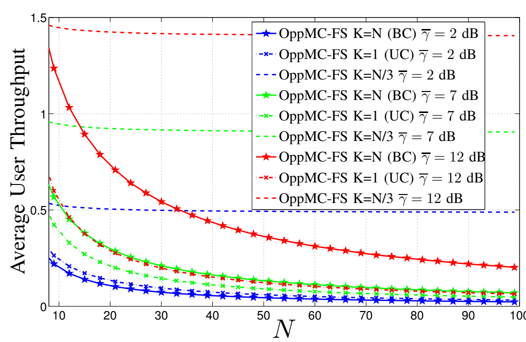


Fig. 5. Average user throughput versus number of users in the system $N$ for the OppMC-FS strategy

the average user throughput decreases. In the UC case, this is due to the fact that we serve just one (the best) RX per TS and what each RX will get is proportionally inverse with $N$ and proportional with the PDF of the best RX, see (4). If $N$ increases, also $f_K(y)$ increases, but still the factor $\frac{1}{N}$ is dominant for this case and drops the $\mathbb{E}(D_1)$ down. For the BC case, if $N$ increases, $\mathbb{E}(D_N)$ goes down because we have to serve the RX under the worst instantaneous SNR conditions and the higher $N$, the worse are the conditions of this RX.

Also, from Fig. 4, we can see that there is always one value of $K$ for which $\mathbb{E}(D_K)$ achieves a maximum and this $K$ value depends only on $\overline{\gamma}$ and $N$. Because of this, if we know $\overline{\gamma}$, we can always find a value of $\frac{K}{N}$ in order for our variant to outperform all the others OppMC-FS variants. This variant is called Optimal OppMC-FS (OOppMC-FS). The best values of $\frac{K}{N}$, for different values of $N$, as a function of $\overline{\gamma}$ of all the RXs are given in Figure 6.

In Fig. 6, it can be noticed that the best $\frac{K}{N}$ ratio, regarding OppMC-FS, depends in general on $\overline{\gamma}$ tending to 1 for large $\overline{\gamma}$. Furthermore, it can be seen that for a given $\overline{\gamma}$, the best $\frac{K}{N}$ ratio is almost independent from the number $N$ of users in the system.

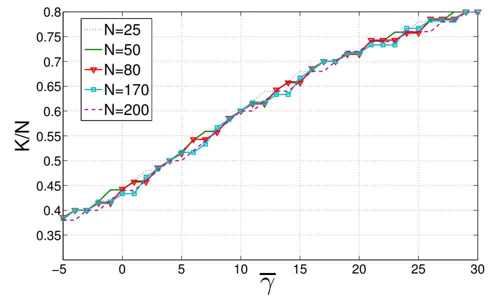


Fig. 6. Best $\frac{K}{N}$ value versus $\overline{\gamma}$ for OppMC-FS for different values of $N$.

## B. Comparison OppMC and OStCSI-MC strategies

This paragraph is dedicated to a comparison of all the strategies discussed in this paper. In the case of OppMC-FS and StCSI-MC, we have taken into consideration only the best performing variant of those strategies which are OOppMC-FS and OStCSI-MC respectively. In addition, also a comparison with the UC and BC schemes is made.

In Fig. 7, the average user throughput as a function of $\overline{\gamma}$ for all the different strategies is shown. Here, it can be seen that each one of our introduced strategies, OOppMC-FS, OppMC-OS, OStCSI-MC, outperforms the UC or BC strategy for the given $\overline{\gamma}$ values. In addition to that, OStCSI-MC needs less CSI knowledge than the other strategies, including UC and BC. Furthermore the performance of our discussed strategies (OOppMC-FS, OppMC-OS, OStCSI-MC) are very close to each other. Of course, the best performance is reached by OppMC-OS, which requires instantaneous CSI knowledge and has the most complex solution.

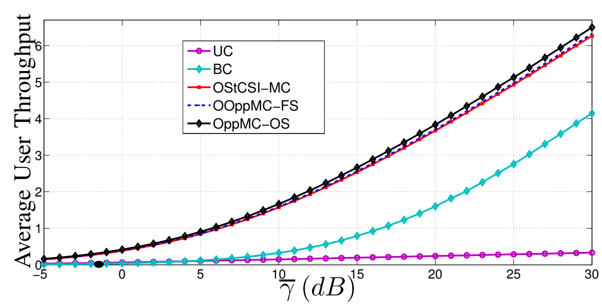


Fig. 7. Strategies comparison in terms of average user throughput versus the average SNR value at each receiver ($\overline{\gamma}$). In this example, $N = 25$.

In Fig. 8, the average user throughput, for each one of our strategies as a function of the total number $N$ of users is plotted assuming $\overline{\gamma} = 20$ dB. Even here it can be seen that both of our closed formed solution strategies outperform UC or BC and they are very close to OppMC-OS. In addition to that, the average user throughput for our proposed strategies does not depend on the number of RXs in the system. In this way, the TX will manage its resources based on what it wants to send to them and not on how many RXs are in the system. This means that if a new RX enters into our system, it will not influence what the others are receiving. The only thing that should be done at the TX is to find again a new value of $\frac{K}{N}$, if it wants to be very rigorous, or just a new value of $K$, if it is performing OppMC-FS scheduling or nothing if it is performing StCSI-MC.

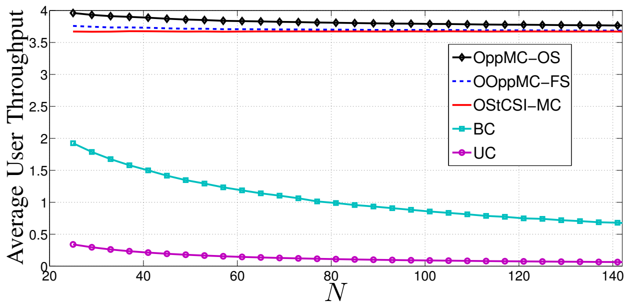


Fig. 8. Strategies comparison in terms of average user throughput versus the total number of RXs in the system ($N$) with $\overline{\gamma} = 20$ dB.

## VI. Conclusions

In this paper, we investigated different multicast strategies in i.i.d. Rayleigh fading channels. We have derived a closed form solution for the opportunistic multicast with fixed group size (OppMC-FS) strategy for the average user throughput. We have also proposed two new strategies. We have given an analytical solution for the average user throughput of the OppMC-OS strategy and a closed form solution for the average user throughput of the StCSI-MC strategy. The StCSI-MC strategy requires less channel knowledge than the other strategies. Results show that the average user throughput of the OppMC-OS strategy is slightly higher than the average user throughput of the OppMC-FS strategy and that the OppMC-FS average user throughput is slightly higher than the StCSI-MC average user throughput. Moreover, the average user throughput of our analyzed strategies is much higher than the average user throughput of unicast or broadcast. Furthermore, the average user throughput in our strategies does not depend on the number of receivers in the system, in contrast to unicast or broadcast strategies. The investigation of the scenarios where the receivers are under different average SNR conditions is left for future work.

## Acknowledgment

This work was performed within the Collaborative Research Centre MAKI (SFB 1053) of the Deutsche Forschungsgemeinschaft (DFG).

## Appendix

In this appendix, the formula regarding average user throughput of the OppMC-OS strategy, given in (8), is derived.

Based on (4), we can find that the CDF of the $K^{\text{th}}$ ordered statistic $\gamma_K$ is given by:

$$F_K(\gamma_K) = \sum_{l=N-K+1}^{N} \binom{N}{l} F_\gamma^l(\gamma_K)[1 - F_\gamma(\gamma_K)]^{N-l} \quad (13)$$

We define the random variable $t_K = Klog_2(1 + y_K)$. Its CDF can be found as:

$$\begin{aligned} F_{t_K}(\gamma_K) &= P[t_K < \gamma_K] = P[Klog_2(1 + y_K) < \gamma_K] \\ &= P[y_K < 2^{\frac{\gamma_K}{K}} - 1] = F_K(2^{\frac{\gamma_K}{K}} - 1) \end{aligned} \quad (14)$$

Now, we define $T_{max} = \max_{K=1..N}(t_K)$ and we have:

$$\begin{aligned} F_{T_{max}}(\gamma) &= P[T_{max} < \gamma] = \prod_{K=1}^{N} P[t_K < \gamma] \\ &= \prod_{K=1}^{N} F_K(2^{\frac{\gamma}{K}} - 1) \end{aligned} \quad (15)$$

Since $F_{T_{max}}(\gamma) = 0$ if $y < 0$ we can find:

$$\mathbb{E}(D_{\text{OppMC-OS}}) = \frac{1}{N} \int_0^\infty 1 - F_{T_{max}}(x) \mathrm{d}x \quad (16)$$

which can be written as shown in (8).

## References

[1] U. Kozat, "On the Throughput Capacity of Opportunistic Multicasting with Erasure Codes," in *Proc. of INFOCOM*, 2008.
[2] T. Lohmar, H. Wiemann, F. Hundscheidt, M. Meyer, and R. Keller, "Support of Multicast Services in 3GPP," *Praxis der Informationsverarbeitung und Kommunikation*, pp. 167–172, 2004.
[3] F. Miller, A. Vandome, and M. John, *Digital Video Broadcasting*. VDM Verlag Dr. Mueller e.K., 2010.
[4] A. Jalali, R. Padovani, and R. Pankaj, "Data throughput of CDMA-HDR a high efficiency-high data rate personal communication wireless system," in *Proc. of Vehicular Technology Conference*, 2000.
[5] S. Borst, "User-level performance of channel-aware scheduling algorithms in wireless data networks," in *Proc. of INFOCOM*, 2003.
[6] Y. Liu and E. Knightly, "Opportunistic fair scheduling over multiple wireless channels," in *Proc. of INFOCOM*, 2003.
[7] B. Al-Manthari, N. Nasser, and H. Hassanein, "Fair Channel Quality-Based Scheduling Scheme for HSDPA System," in *Computer Systems and Applications*, 2006.
[8] M. Nagy and S. Singh, "Multicast scheduling algorithms in mobile networks," *Cluster Computing*, vol. 1, pp. 177–185, 1998.
[9] N. El Heni and X. Lagrange, "Multicast vs Multiple Unicast Scheduling in High-Speed Cellular Networks," in *Vehicular Technology Conference*, 2008.
[10] P. K. Gopala and H. Gamal, "Opportunistic multicasting," in *Signals, Systems and Computers*, 2004.
[11] T. ping Low, M. on Pun, Y.-W. Hong, and C.-C. Kuo, "Optimized opportunistic multicast scheduling (OMS) over wireless cellular networks," *IEEE Transactions on Wireless Communications*, 2010.
[12] J. Miroll, M. Beyer, and T. Herfet, "An OFDM WLAN Multicast Hybrid Erasure Correction Prototype with Feedback Aggregation and Rate Adaptation," *Proc. of 17th International OFDM Workshop*, 2012.
[13] J. Byers, M. Luby, and M. Mitzenmacher, "A digital fountain approach to asynchronous reliable multicast," *IEEE Journal on Selected Areas in Communications*, vol. 20, pp. 1528 – 1540, 2002.
[14] C. E. Shannon, "A mathematical theory of communication," *SIGMOBILE Mob. Comput. Commun. Rev.*, vol. 5, pp. 3–55, (1949, reprented 2001).
[15] A. Papoulis, *Probability, random variables, and stochastic processes (pp. 170-177)*. McGraw-Hill, 1984.
[16] I. Gradshten, I. Ryzhik, A. Jeffrey, and D. Zwillinger, *Table of Integrals, Series, And Products*. Elsevier/Academic Press, Amsterdam, 2007.
[17] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions with formulas, graphs, and mathematical tables*. U.S. Government Printing Office, 1964.
[18] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, "On the Lambert W Function," in *Advances in Computational Mathematics*, 1996.