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Pair-Aware Interference Alignment in Multi-user Two-way Relay Networks

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Abstract—In this paper, K bidirectionally communicating node pairs with each node having N antennas and one amplify and forward relay having R antennas are considered. Each node wants to transmit d data streams to its communication partner. Taking into account that each node can perform self interference cancellation, a new scheme called *Pair-Aware Interference Alignment* is proposed. In this scheme, the transmit precoding matrices and the relay processing matrix are chosen in such a way that at any given receiver all the interfering signals except the self interference are within the interference subspace and the useful signal is in a subspace linearly independent of the interference subspace. If the number of variables is larger than or equal to the number of constraints in the system, the system is classified as proper, else as improper. Through simulations it is shown that for a proper system ($2Kd \leq 2N + R - d$), interferences can be perfectly aligned and the useful signals can be decoded interference-free at the receivers. An iterative algorithm to achieve the interference alignment solution is proposed. Also for the proper system fulfilling a certain additional condition, which will be derived in this paper, a closed form solution is proposed.

Index Terms—Pair-aware interference alignment, two-way relaying, signal alignment, channel alignment.

I. INTRODUCTION

INTERFERENCE between communication links is the major limiting factor in wireless communication networks, especially when the interfering signal is of similar strength as the useful signal. Recently, interference alignment [1]–[3] has been developed as an efficient technique to handle interferences at high Signal to Noise Ratios (SNRs). In [3], interference alignment is introduced for a K -user interference channel, where K transmitters want to communicate *unidirectionally* with their corresponding receivers. The receiver space is divided into two subspaces [3], namely, the useful subspace and the interference subspace. Each of the transmitters chooses its transmit signal such that at each of the receivers, only the useful signal is within the useful subspace and all the

interference signals are within the interference subspace. In contrast to the conventional orthogonal multiplexing schemes where every transmitter can use only $\frac{1}{K}$ of the total resource, using interference alignment, every transmitter can utilize $\frac{1}{2}$ of the total resource [3].

Interference alignment can be performed in the dimensions of time [3], frequency [4], space [5]–[7] or signal level [8]. In this paper, we focus on interference alignment in spatial dimensions. In [5]–[7], iterative methods to achieve interference alignment have been proposed. The iterative algorithms converge only if an alignment solution is feasible. In [9]–[11], the feasibility conditions for interference alignment are derived in terms of the number N of antennas at each transmitter and receiver, the number d of data streams that each transmitter wants to transmit and the number K of transmitters in the system. For a K user interference channel, the interference alignment solution is feasible if

$$N \geq \frac{(K+1)d}{2} \quad (1)$$

holds [9]–[11]. A closed form solution for interference alignment in spatial dimensions is known only in special cases [12]. In [12], each node requires at least $(K-1)d$ antennas for the closed form solution to be feasible.

According to (1), the number of data streams that can be transmitted by each transmitter is limited by the number of antennas at the transmitters and receivers. A large number of time slot extensions is needed for each transmitter to be able to transmit half of the number of data streams that it can transmit in the absence of interference [3]. In [5], for a three user case, it has been shown that with the help of a relay, two time slot extensions are sufficient to achieve this.

Relay aided interference alignment has been considered in [13]–[17]. In [13]–[16], the relaying is performed based on the one-way relaying protocol. In [17], a *bidirectional* communication between K communication partners as shown in Figure 1 is considered and the relaying is performed based on the two-way relaying protocol. Two-way relaying requires only two time slots for bidirectional communication, whereas one-way relaying requires four time slots. At the same time, compared to one-way relaying, two-way relaying introduces new challenges as the number of interferences is almost doubled. However, as we have shown in [17] and in the current paper, the ability of each node to cancel self interference can be utilized to decouple the interference alignment into two steps called signal alignment and channel alignment.

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In [17], each of the $2K$ nodes has N antennas and wants to transmit d data streams to its communication partner, see Figure 1. It is assumed that the number R of relay antennas is $R < 2Kd$ so that the conventional transceive zero forcing or decode-and-forward cannot be performed at the relay. It has been shown in [17] that $R \geq Kd$ antennas are required at the relay to be able to separate the useful signal and the interference signals at the receiving nodes. The case $R = Kd$ is addressed in [17]. In the first time slot called *multiple access* (MAC) phase, all the nodes transmit to the relay. The relay receives the sum of $2Kd$ data streams. These $2Kd$ data streams cannot be spatially separated in a Kd -dimensional space. However, the nodes can perform self interference cancellation and hence, all the nodes transmit their signals to the relay in such a way that at the relay, each node's signal subspace aligns with the signal subspace of its communication partner. This is called *signal alignment* [17]. In the second time slot called *broadcast* (BC) phase, each node designs its receive zero forcing matrix such that the effective channel consisting of the channel between the relay and the node and the receive zero forcing matrix spans the same subspace as that of its communication partner. This is called *channel alignment* [17]. After signal and channel alignment, there are only Kd effective data streams and Kd effective channels. The relay with $R = Kd$ antennas can perform transceive zero forcing [17]. The nodes require at least $\frac{(K+1)d}{2}$ antennas to perform signal and channel alignment [17]. After signal alignment and channel alignment followed by transceive zero forcing, the interferences are aligned perfectly within the ISS at the receivers.

In this paper, a generalization of [17], i.e., $R \geq Kd$ is considered. When $R > Kd$, complete signal alignment is not necessary and hence, the requirement $N \geq \frac{(K+1)d}{2}$ can be relaxed. In this paper, a novel interference alignment scheme called Pair-Aware Interference Alignment (PAIA) is proposed. The term Pair-Aware implies that for the process of interference alignment, only the inter-pair interference needs to be considered. In the PAIA scheme, each node partially aligns its signal at the relay with that of its communication partner. However, perfect signal alignment is not necessary. Partial channel alignment followed by transceive zero forcing is performed to achieve interference alignment in the BC phase. The properness condition [10] is also derived for the considered system. An iterative algorithm is proposed to achieve the interference alignment solution. Additionally, a closed form solution is possible in certain cases. The condition for the applicability of the closed form solution is derived and the closed form solution is given.

The organization of the paper is as follows. The system model is introduced in Section II. In Section III, the proposed pair-aware interference alignment scheme is described. The condition for the properness of the system is derived in Section IV. Closed form and iterative algorithms are described in Section V. Section VI evaluates the performance of the proposed scheme in terms of the sum rate of the system. Section VII concludes the paper.

Throughout this paper, we use lower case letters for scalars and lower case bold letters and upper case bold letters for column vectors and matrices, respectively. $(\cdot)^H$ denotes the

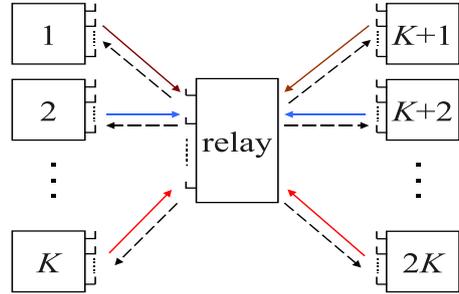


Fig. 1. K user pair relay interference channel

complex conjugate transpose operation. We define two subspaces to be linearly independent if no non-zero vector in one subspace can be expressed as a linear combination of the basis vectors of the other subspace. Let A_1 and A_2 denote two N -dimensional subspaces in an R -dimensional vector space W . The union of the two subspaces is defined as

$$A_1 \cup A_2 := \{c_1 \mathbf{p}_1 + c_2 \mathbf{p}_2 \in W : \forall \mathbf{p}_1 \in A_1, \forall \mathbf{p}_2 \in A_2 \text{ and } \forall c_1, c_2 \in \mathbb{C}\}.$$

II. SYSTEM MODEL

Figure 1 shows the multi-user two-way relay network with K communication pairs and one relay. Without loss of generality, we assume that the nodes j and $j + K$ for $j = 1, \dots, K$ are communication partners. Each node has N antennas and the relay has R antennas. Each of the $2K$ nodes wants to transmit d data streams to its communication partner, i.e., a bidirectional communication is assumed. There is no direct link between the nodes. The relay and the nodes are assumed to operate in half duplex mode and to have a transmit power constraint. The two-way relaying protocol [18] is considered. In the first phase called MAC phase, all the nodes transmit their signals to the relay and in the second phase called BC phase, the relay broadcasts the signals to the nodes.

Let \mathbf{d}_i and \mathbf{V}_i for $i = 1, \dots, 2K$ denote the data vector and the transmit precoding matrix of node i , respectively. Note that j and $j + K$ for $j = 1, \dots, K$ are used as indices to identify the communication partners while $i, i = 1, \dots, 2K$, is used to define quantities for all $2K$ nodes. Let the matrix \mathbf{H}_{ri} denote the channel between transmitter i and the relay. The noise at the relay is denoted by the vector \mathbf{n}_r . The components of the noise vector \mathbf{n}_r are assumed to be i.i.d. complex Gaussian random variables which follow the complex normal distribution $\mathcal{CN}(0, \sigma_r^2)$ with zero mean and variance σ_r^2 . The signal received at the relay in the MAC phase is given by

$$\mathbf{r} = \sum_{i=1}^{2K} \mathbf{H}_{ri} \mathbf{V}_i \mathbf{d}_i + \mathbf{n}_r. \quad (2)$$

Amplify and Forward relaying is assumed. The BC phase is considered now. Let \mathbf{G} denote the linear signal processing matrix at the relay. Let \mathbf{n}_i denote the noise at node i , $i = 1, \dots, 2K$. The components of the noise vector \mathbf{n}_i are i.i.d. complex Gaussian random variables which follow $\mathcal{CN}(0, \sigma_i^2)$. Let the matrix \mathbf{H}_{ir} denote the channel between the relay and

node i . Let $\tilde{\mathbf{n}}_i = \mathbf{H}_{ir}\mathbf{G}\mathbf{n}_r + \mathbf{n}_i$ denote the effective noise at receiver i . The received signal at receiver j is

$$\begin{aligned} \mathbf{y}_j = & \mathbf{H}_{jr}\mathbf{G}\mathbf{H}_{r(j+K)}\mathbf{V}_{j+K}\mathbf{d}_{j+K} + \mathbf{H}_{jr}\mathbf{G}\mathbf{H}_{rj}\mathbf{V}_j\mathbf{d}_j \\ & + \mathbf{H}_{jr}\mathbf{G}\sum_{\substack{i=1, \\ i \neq j, j+K}}^{2K} \mathbf{H}_{ri}\mathbf{V}_i\mathbf{d}_i + \tilde{\mathbf{n}}_j \end{aligned} \quad (3)$$

for $j = 1, \dots, K$. The received signal at receiver $j + K$ for $j = 1, \dots, K$ can be obtained by interchanging j and $j + K$ in the above equation. In (3), the first and the second term of the sum correspond to the useful part and the self-interference, respectively. The third term corresponds to the unknown interference. It is assumed that self-interference can be perfectly canceled at the receiver. Let \mathbf{U}_i^H denote the receive filter matrix at receiver i , $i = 1, \dots, 2K$. Then, the estimated data stream at node j , $j = 1, \dots, K$, is given by

$$\begin{aligned} \hat{\mathbf{d}}_j = & \mathbf{U}_j^H\mathbf{H}_{jr}\mathbf{G}\mathbf{H}_{r(j+K)}\mathbf{V}_{j+K}\mathbf{d}_{j+K} \\ & + \mathbf{U}_j^H\mathbf{H}_{jr}\mathbf{G}\sum_{\substack{i=1, \\ i \neq j, j+K}}^{2K} \mathbf{H}_{ri}\mathbf{V}_i\mathbf{d}_i + \mathbf{U}_j^H\tilde{\mathbf{n}}_j. \end{aligned} \quad (4)$$

In order to decode the desired signal successfully, the unknown interference should be within the interference subspace and the useful signal subspace should be linearly independent of the interference subspace. The self interference can be in both the subspaces. The dimension of the useful subspace should be larger than or equal to the size of the data vector \mathbf{d}_{j+K} . Then the receive filter \mathbf{U}_j^H can be designed as the zero forcing filter that suppresses all the unknown interferences. This means that in (4), the following conditions need to be satisfied:

$$\text{rank}(\mathbf{U}_j^H\mathbf{H}_{jr}\mathbf{G}\mathbf{H}_{r(j+K)}\mathbf{V}_{j+K}) = d, \quad (5)$$

$$\mathbf{U}_j^H\mathbf{H}_{jr}\mathbf{G}\mathbf{H}_{ri}\mathbf{V}_i = 0 \quad \forall i \neq j, j + K. \quad (6)$$

Assuming that the input symbols denoted by the elements of the vector \mathbf{d}_{j+K} are independent and zero mean complex Gaussian distributed with variance one, the achievable rate with which node $j + K$ can transmit is given by

$$\begin{aligned} R_{j+K} = & \frac{1}{2} \log_2 \left| \mathbf{I} + (\mathbf{U}_{j+K}^H \mathbf{R}_{\tilde{\mathbf{n}}_{j+K}} \mathbf{U}_{j+K})^{-1} \right. \\ & \left. \mathbf{U}_{j+K}^H \mathbf{H}_{(j+K)r} \mathbf{G} \mathbf{H}_{rj} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{rj}^H \mathbf{G}^H \mathbf{H}_{(j+K)r} \mathbf{U}_{j+K} \right|, \end{aligned} \quad (7)$$

where $|\cdot|$ denotes the determinant operation and $\mathbf{R}_{\tilde{\mathbf{n}}_{j+K}}$ is the covariance matrix of $\tilde{\mathbf{n}}_{j+K}$. Note that for the receivers $j + K$ for $j = 1, \dots, K$, the interference alignment conditions and achievable rates are obtained by interchanging j and $j + K$ in the above equations (4)-(7).

III. PAIR-AWARE INTERFERENCE ALIGNMENT (PAIA) SCHEME

In this section, the proposed *Pair-Aware Interference Alignment* (PAIA) scheme is introduced. The objective of this scheme is to choose the coefficients of the precoding matrices \mathbf{V}_j and \mathbf{V}_{j+K} for $j = 1, 2, \dots, K$ and the relay processing matrix \mathbf{G} such that (5) and (6) are satisfied. This is achieved in three steps: partial signal alignment, partial channel alignment and transceive zero forcing.

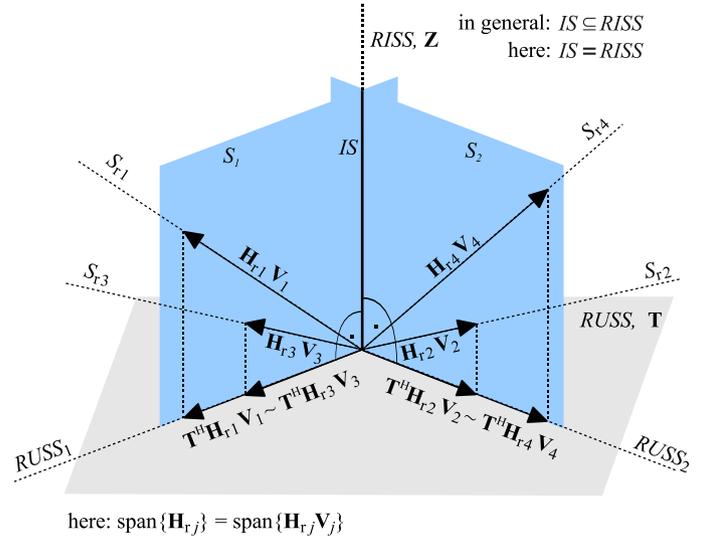


Fig. 2. Illustration of partial signal alignment for $K = 2$, $R = 3$, $N = 1$, and $d = 1$

A. Partial Signal Alignment

In the MAC phase, the relay receive space is divided into two *orthogonal* subspaces, namely, the relay receive useful subspace *RUSS* of dimension Kd and the relay receive interference subspace *RISS* of dimension $R - Kd$. *RUSS* has to be chosen such that it is possible that each node aligns its signal with that of its communication partner in the *RUSS*. In the *RISS*, the signal alignment does not need to be feasible. This is called partial signal alignment. Let \mathbf{T} denote the projection matrix that projects the received signal at the relay to *RUSS*. By this projection operation, the signal components in *RISS* are nullified. The signal alignment of each pair $(j, j + K)$, $j = 1, 2, \dots, K$ within the *RUSS* is represented by

$$\text{span}(\mathbf{T}^H \mathbf{H}_{rj} \mathbf{V}_j) = \text{span}(\mathbf{T}^H \mathbf{H}_{r(j+K)} \mathbf{V}_{j+K}). \quad (8)$$

For illustration, let us consider the example $N = 1$, $d = 1$, $R = 3$ and $K = 2$. In this case, the 3-dimensional relay receive space and the subspaces and operations mentioned above are visualized in Figure 2. As the nodes have only a single antenna each, signal alignment in the 3-dimensional relay receive space is not feasible. However, as shown in Figure 2, the signals received at the relay can be projected to a properly chosen 2-dimensional *RUSS*, such that signal alignment is achieved. There are also additional variables given in Figure 2 which will be described as soon as they are introduced in the later sections in the paper.

Let $RUSS_j$ denote the d -dimensional alignment subspace of the pair $(j, j + K)$ within the *RUSS*, see also Figure 2. Then, the *RUSS* is given by $RUSS = \bigcup_{j=1}^K RUSS_j$. In order to be able to separate the useful signal from the interference at the receiver, $RUSS_j$ should be linearly independent of $\bigcup_{\substack{j'=1 \\ j' \neq j}}^K RUSS_{j'}$.

B. Partial Channel Alignment

After partial signal alignment followed by nullifying the interferences in the *RISS*, there are only Kd effective data

streams corresponding to the pair-wise aligned signals of the K pairs. In the BC phase, the Kd effective data streams are transmitted such that at each of the receivers, all the interference signals are within the interference subspace and the useful signal is within the useful subspace. This is achieved through partial channel alignment and transceive zero forcing. Partial channel alignment is dual to partial signal alignment and is explained in the following.

Let $\mathbf{F}_{jr} = (\mathbf{U}_j^H \mathbf{H}_{jr})^H$ denote the effective channel between the relay and receiver j . Consider the pair $(j, j + K)$. The effective channels of these two nodes are said to be pair-wise aligned if they span the same subspace at the relay. Similar to partial signal alignment in the MAC phase, now, the transmit signal space of the relay is divided into two orthogonal subspaces: the relay transmit useful subspace $TUSS$ and a common relay transmit interference subspace $TISS$. The Kd -dimensional subspace $TUSS$ is chosen such that pair-wise channel alignment is possible in the $TUSS$. However, the components within the $TISS$ do not need to align pair-wise.

Let \mathbf{Q} denote the projection matrix that projects the transmit signal at the relay to $TUSS$. Then in $TUSS$, the effective channels of partner nodes align pair-wise. This is given by

$$\text{span} \left((\mathbf{U}_j^H \mathbf{H}_{jr} \mathbf{Q})^H \right) = \text{span} \left((\mathbf{U}_{j+K}^H \mathbf{H}_{(j+K)r} \mathbf{Q})^H \right) \quad (9)$$

for $j = 1, 2, \dots, K$. Each of the partner nodes $(j, j + K)$ chooses its receive zero forcing matrix such that the effective channel aligns with the effective channel of its communication partner within the $TUSS$. The alignment subspace of each pair $(j, j + K)$ should be linearly independent of the union of the useful subspaces of all the other pairs so that a zero forcing filter can be designed for each of the node pairs.

C. Transceive Zero Forcing

After partial channel alignment followed by a projection onto the subspace orthogonal to the $TISS$, the effective channel of each node spans the same subspace as the effective channel of its communication partner. Hence, zero forcing the effective channel of one node by the relay forces also the one of its communication partner to zero. In the BC phase, there are Kd effective data streams and there are Kd effective channels. As the $TUSS$ is of dimension Kd , transceive zero forcing can be performed at the relay. Let \mathbf{G}_s denote the transceive zero forcing matrix at the relay. Then the relay processing matrix \mathbf{G} is given by

$$\mathbf{G} = \mathbf{Q} \mathbf{G}_s \mathbf{T}^H. \quad (10)$$

After partial signal and channel alignment and transceive zero forcing, there will be no unknown interference at the receivers and the d data streams of the desired signal will be linearly independent of each other, i.e., (5) and (6) are satisfied.

Partial signal alignment and partial channel alignment are dual problems. As can be seen from (8) and (9), partial signal alignment and partial channel alignment are bilinear problems with the same number of variables and equations. Hence, for the consideration of the properness condition and for the algorithms described in this paper, only the MAC phase will be considered. These algorithms for the MAC phase can be

directly applied for the BC phase by replacing the matrix \mathbf{T} by the matrix \mathbf{Q} , the precoding matrices by the receive zero forcing matrices and the MAC channel matrices by the corresponding BC channel matrices.

IV. PROPERNESS CONDITION

In this section, the properness condition for the proposed PAIA scheme is derived. In [10], the properness condition for a K -user interference channel is derived by counting the number M_v of variables and the number M_c of constraints in the system. If $M_v \geq M_c$, then the system is considered as proper, otherwise as improper. This method of counting M_v and M_c is applied to the proposed PAIA method in the following subsections.

Properness is not a sufficient condition for the feasibility of the system and there exist proper systems that are not feasible [19]. However, the intuition is that proper systems are likely to be feasible [10]. In [10], Bernstein's Theorem is used to verify if the proper systems are feasible by calculating the mixed volume of the polynomials. However, the constraint of (8) for the partial signal alignment problem is a system of polynomial equations with correlated coefficients and hence, applying Bernstein's Theorem, the mixed volume of the polynomials gives only an upper bound on the number of solutions [10]. In Section VI, through simulation results on the leakage interference at the receivers it will be shown that in our considered multi-user two-way relay networks, typically, proper systems are also feasible.

A. Number M_v of Variables

In this subsection, we count the number M_v of variables in the system. There are two kinds of variables in the system. M_{vn} denotes the number of variables corresponding to the antennas at the nodes and M_{vr} denotes the number of variables corresponding to the antennas at the relay. Each node has N antennas and transmits d data streams. The precoding matrix at each node is of size $N \times d$. Hence, Nd variables are available in each precoding matrix. In order to be able to decode the d data streams, it is necessary that d columns of the precoding matrix are linearly independent of each other. d^2 variables are required to make the columns of the precoding matrix linearly independent. Hence, $(N - d)d$ free variables are available. In other words, choosing a d -dimensional subspace in an N -dimensional space results in $(N - d)d$ free variables. There are $2K$ nodes in the system, this leads to

$$M_{vn} = 2K(N - d)d. \quad (11)$$

In the R -dimensional relay receive space, a Kd -dimensional subspace has to be reserved for the useful signals. Hence, $R - Kd$ dimensions are left for the $RISS$. Choosing a subspace of dimension $R - Kd$ in an R -dimensional space results in $(R - Kd)(R - (R - Kd)) = (R - Kd)Kd$ free variables. Given the $RISS$, the $RUSS$ is uniquely defined. Each of the communication pairs can choose its d -dimensional useful signal space within this Kd -dimensional $RUSS$. Choosing a d -dimensional subspace in a Kd -dimensional subspace results in $(Kd - d)d$ free variables. There are K pairs, hence, $K(Kd - d)d$ free variables in choosing the subspaces $RUSS_j$ within

RUSS. The $RUSS_j$ of the node pair $(j, j + K)$ should be linearly independent of the subspace spanned by the union of the useful subspaces of all the other node pairs. It will be described in Section V that the choice of $RUSS_j$ depends on the channel matrices corresponding to the pair $(j, j + K)$. As the channel matrices of the pair $(j, j + K)$ are assumed to be independent of those of all the other pairs, the probability of two pairs choosing the same $RUSS_j$ is zero. This leads to

$$M_{vr} = (R - Kd)Kd + K(K - 1)d^2. \quad (12)$$

With this, the total number of variables is given by

$$M_v = M_{vn} + M_{vr} = 2K(N - d)d + (R - Kd)Kd + K(K - 1)d^2. \quad (13)$$

B. Number M_c of Constraints

Now we count the number of constraints in the system. The constraints consider that the data streams transmitted by each node pair shall be within its useful subspace at the relay or within the common interference subspace, but not in the useful subspace of the other pairs as described in Section III. Now, consider one of the d data streams transmitted by node j . $RISS$ is of dimension $R - Kd$ and $RUSS_j$ is of dimension d . Hence, the considered data stream from node j should be within the $(R - Kd) + d$ -dimensional subspace formed by $RISS$ and $RUSS_j$. This introduces $R - ((R - Kd) + d) = (K - 1)d$ constraints in the system. There are d data streams per node and $2K$ nodes in the system. Hence, the number of constraints in the system is given by

$$M_c = 2K(K - 1)d^2. \quad (14)$$

C. Properness Condition

For a proper system, the number of variables should be greater than or equal to the number of constraints in the system i.e., $M_v \geq M_c$. This leads to

$$2(N - d) + R + d \geq 2Kd. \quad (15)$$

Eq. (15) implies that when two antennas are added to the relay, one antenna can be removed from each of the $2K$ nodes and the system will still be proper. The feasibility conditions derived in [20] and [17] are special cases of (15). In case of $N = d$, (15) becomes $R \geq (2K - 1)d$, which is the condition for pair-aware transceive zero forcing [20]. In case of $R = Kd$, (15) becomes $N \geq \frac{(K+1)d}{2}$, which is the condition for perfect signal alignment [17].

V. PROPOSED PAIA ALGORITHM

In this section, an iterative algorithm and a closed form solution to obtain partial signal alignment are proposed. In order to achieve partial signal alignment, the matrices \mathbf{T} , \mathbf{V}_j , and \mathbf{V}_{j+K} have to be chosen such that (8) is satisfied. First, \mathbf{T} is chosen such that signal alignment given by (8) is feasible. Finding \mathbf{T} is a bilinear problem. An iterative method is proposed to find the solution. If the relays and the nodes have certain numbers of antennas higher than the minimum required numbers of antennas given by (15), then the problem of finding \mathbf{T} can be reformulated into a linear problem and,

hence, a closed form solution is possible. Once \mathbf{T} is known, the precoding matrices \mathbf{V}_j and \mathbf{V}_{j+K} can be calculated in closed form.

In the following, some notations and definitions which are needed for the algorithm description are introduced in Section V-A. In Section V-B, the problem of partial signal alignment is reformulated so that the calculation of \mathbf{T} becomes a problem of finding the subspace $RISS$ intersecting with several subspaces. An iterative algorithm and a closed form solution to find the $RISS$ are proposed in Section V-C, where the condition for the applicability of the closed form solution is also derived. After finding the $RISS$ and hence \mathbf{T} , the precoding matrices of all nodes are obtained in Section V-D.

A. Notations and definitions

In this subsection, we introduce the definition of the intersection of two subspaces and the method to find the intersection subspace. The dimension of the intersection subspace is derived. Also an extension to the intersection subspace of many subspaces is given.

Let A_1 and A_2 denote two F -dimensional subspaces in an R -dimensional space W . The intersection of the two subspaces A_1 and A_2 is defined as

$$A_1 \cap A_2 := \{\mathbf{q} \in W : \mathbf{q} \in A_1 \text{ and } \mathbf{q} \in A_2\}. \quad (16)$$

In the following, if we say that two subspaces do not intersect, we mean that their intersection subspace is of dimension 0. Let the columns of the matrices \mathbf{A}_1 and \mathbf{A}_2 represent the basis vectors of the subspaces A_1 and A_2 , respectively. Let \mathbf{X} be a matrix of size $F \times I$ with $0 < I \leq F$. For an arbitrary choice of \mathbf{X} with rank I , the product $\mathbf{A}_1\mathbf{X}$ gives a basis for an I -dimensional subspace within A_1 . Let $\mathbf{A}_1\mathbf{X}_1$ and $\mathbf{A}_2\mathbf{X}_2$ denote two bases for the intersection subspace $A_1 \cap A_2$. Then

$$A_1 \cap A_2 = \text{span}\{\mathbf{A}_1\mathbf{X}_1\} = \text{span}\{\mathbf{A}_2\mathbf{X}_2\}, \quad (17)$$

where \mathbf{X}_1 and \mathbf{X}_2 are $F \times I$ matrices of rank I , with I being the dimension of the intersection subspace. If the span of two subspaces is equal, then any basis for one subspace is also a basis for the other subspace. Hence, we can choose bases such that $\mathbf{A}_1\mathbf{X}_1 = \mathbf{A}_2\mathbf{X}_2$. Therefore,

$$\begin{pmatrix} \mathbf{A}_1 & -\mathbf{A}_2 \end{pmatrix} \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} = \mathbf{0}. \quad (18)$$

The dimension of the null space of the matrix $(\mathbf{A}_1 - \mathbf{A}_2)$ gives the dimension of the intersection subspace and is given by $I \geq 2F - R$. In this paper, the subspaces A_1 and A_2 are defined by the matrices denoting the channel between the transmitter and receiver nodes. As the channel coefficients between different nodes are uncorrelated, with a probability of one, the dimension of the intersection subspace is given by $I = 2F - R$. In general, the intersection of K such F -dimensional subspaces A_1, A_2, \dots, A_K results in an intersection subspace $A_1 \cap A_2 \dots \cap A_K$ of dimension

$$I_K = KF - (K - 1)R. \quad (19)$$

B. Reformulation of Partial Signal Alignment

In this section, we reformulate the partial signal alignment problem (8). Note that $RISS$ uniquely determines $RUSS$ and, hence, uniquely determines \mathbf{T} . Therefore, our objective is to choose $RISS$ in such a way that signal alignment given by (8) is feasible. For the communication partners $(j, j + K)$, let S_{rj} and $S_{r(j+K)}$ represent two subspaces spanned by the columns of the channel matrices \mathbf{H}_{rj} and $\mathbf{H}_{r(j+K)}$, respectively. Let $S_j = S_{rj} \cup S_{r(j+K)}$. In our example in Figure 2, we have $K = 2$ node pairs with $N = 1$ and their corresponding subspaces S_1 and S_2 are shown by the two vertical planes. In general, S_j for $j = 1, 2, \dots, K$ are $2N$ -dimensional subspaces and the signals of the node pairs $(j, j + K)$ span a $2d$ -dimensional subspace in S_j . The $RUSS_j$ corresponding to this pair is of dimension d . Hence, to make sure that the signals from this pair do not interfere with the signals from the other pairs, d dimensions of the subspace corresponding to the signal received from this pair should be within the $RUSS_j$ and the other d dimensions should be within the $RISS$. This means that the $RISS$ has to be chosen such that the intersection subspace between S_j and $RISS$ is at least d -dimensional. This needs to hold for each pair $(j, j + K)$ with $j = 1, 2, \dots, K$. In Figure 2, the dimension of $RISS$ is $R - Kd = d = 1$ and, hence, $RISS$ is directly obtained as the intersection subspace between S_1 and S_2 . However, in general $RISS$ is of dimension $R - Kd \geq d$ and the subspaces S_j for $j = 1, 2, \dots, K$ do not need to intersect with each other. Only $RISS$ needs to intersect with each of the K subspaces S_j . Thus, the problem of determining \mathbf{T} that makes (8) feasible is reformulated as the problem of finding $RISS$ that has at least a d -dimensional intersection subspace with each of the subspaces S_j for $j = 1, 2, \dots, K$.

C. $RISS$ - Relay Receive Interference Subspace

Iterative Method: For the iterative method, we introduce the following terminology. Let $RISS_j$ denote the d -dimensional intersection subspace between $RISS$ and S_j for $j = 1, 2, \dots, K$. Thus, $RISS_j$ is a d -dimensional subspace of both $RISS$ and S_j , i.e., $RISS_j \subseteq RISS$ and $RISS_j \subseteq S_j$. Furthermore, the square of the Frobenius norm of the projection of the orthonormal basis vectors of a subspace A on a subspace B is termed similarity measure of (A, B) . This similarity measure of (A, B) is inversely related to the minimum principal angle between the subspaces A and B . Assume that the dimension of A is smaller than or equal to that of B . If $A \subseteq B$, then the similarity measure takes its maximum value, which is equal to the dimension of A . In this case, the minimum principal angle between A and B is zero.

The basic idea of the iterative method for finding $RISS$ is the following. Initially, we arbitrarily choose an $(R - Kd)$ -dimensional subspace $RISS^{(0)}$. Then, in iteration step m , first we find a number K of d -dimensional subspaces $RISS_j^{(m)} \subseteq S_j$, for $j = 1, 2, \dots, K$, such that the K similarity measures of $(RISS_j^{(m)}, RISS^{(m-1)})$ for $j = 1, 2, \dots, K$ are maximized. This is equivalent to maximizing the sum of the K similarity measures. Note that $RISS_j^{(m)}$ for $j = 1, 2, \dots, K$ is chosen as a d -dimensional subspace of S_j , but in general is not yet a d -dimensional subspace of $RISS^{(m-1)}$, so all

or some of the K similarity measures will be smaller than d before convergence of the iterative algorithm. Secondly, in iteration step m , we find a new $(R - Kd)$ -dimensional subspace $RISS^{(m)}$ such that the sum of the K similarity measures of $(RISS_j^{(m)}, RISS^{(m)})$ for $j = 1, 2, \dots, K$ is maximized. These two operations are repeated iteratively. As in each iteration, the sum of the K similarity measures is maximized, the subspaces $RISS_j^{(m)}$ $j = 1, 2, \dots, K$ and $RISS^{(m)}$ will move in such directions that for increasing m , all $RISS_j^{(m)}$, $j = 1, 2, \dots, K$ will finally be d -dimensional subspaces of $RISS^{(m)}$ and the sum of the similarity measures will converge to the value Kd .

Now we will give the mathematical description of the iterative algorithm. Let the columns of the unitary matrix \mathbf{S}_j of size $R \times 2N$ denote a basis of S_j . Since $RISS_j^{(m)} \subseteq S_j$, there exists a unitary matrix $\mathbf{X}_j^{(m)}$ of size $2N \times d$ and rank d , such that the columns of the product $\mathbf{S}_j \mathbf{X}_j^{(m)}$ give a basis of $RISS_j^{(m)}$. Let the columns of the unitary matrix $\mathbf{Z}^{(m)}$ of size $R \times (R - Kd)$ denote a basis of $RISS^{(m)}$. Initially, $\mathbf{Z}^{(0)}$ is chosen to be an arbitrary $(R - Kd)$ -dimensional subspace. In the m^{th} iteration step, the sum of the squares of the Frobenius norms of the projection of $\mathbf{S}_j \mathbf{X}_j^{(m)}$ on $\mathbf{Z}^{(m-1)}$ is denoted by

$$p^{(m,1)} := \sum_{j=1}^K \text{trace} \left(\mathbf{X}_j^{(m)\text{H}} \mathbf{S}_j^{\text{H}} \mathbf{Z}^{(m-1)} \mathbf{Z}^{(m-1)\text{H}} \mathbf{S}_j \mathbf{X}_j^{(m)} \right). \quad (20)$$

For $\mathbf{Z}^{(m-1)}$ determined in the $(m - 1)^{\text{th}}$ iteration step, $\mathbf{X}_j^{(m)}$ which maximizes (20) is given by

$$\mathbf{X}_j^{(m)} = \lambda_{\max, d} \left(\mathbf{S}_j^{\text{H}} \mathbf{Z}^{(m-1)} \mathbf{Z}^{(m-1)\text{H}} \mathbf{S}_j \right), \quad (21)$$

where $\lambda_{\max, d}(\cdot)$ represents the matrix containing as its columns the eigenvectors corresponding to the first d largest eigenvalues of the matrix within the brackets [21]. Using the identity $\text{trace}(\mathbf{A}\mathbf{B}) = \text{trace}(\mathbf{B}\mathbf{A})$, the sum of the K similarity measures of $(RISS_j^{(m)}, RISS^{(m)})$ for $j = 1, 2, \dots, K$ can be written as

$$p^{(m,2)} = \text{trace} \left(\mathbf{Z}^{(m)\text{H}} \sum_{j=1}^K \left(\mathbf{S}_j \mathbf{X}_j^{(m)} \mathbf{X}_j^{(m)\text{H}} \mathbf{S}_j^{\text{H}} \right) \mathbf{Z}^{(m)} \right). \quad (22)$$

Next, for given $\mathbf{X}_j^{(m)}$, the $\mathbf{Z}^{(m)}$ that maximizes (22) is given by

$$\mathbf{Z}^{(m)} = \lambda_{\max, (R-Kd)} \left(\sum_{j=1}^K \mathbf{S}_j \mathbf{X}_j^{(m)} \mathbf{X}_j^{(m)\text{H}} \mathbf{S}_j^{\text{H}} \right) \quad (23)$$

[21]. (21) and (23) are repeated iteratively until convergence. Finally, the span of the matrix \mathbf{Z} gives $RISS$. $RISS$ uniquely determines the matrix \mathbf{T} and \mathbf{T} can be calculated as $\mathbf{T} = \text{null}(\mathbf{Z}^{\text{H}})$.

As $\mathbf{S}_j \mathbf{X}_j$ and \mathbf{Z} are unitary matrices, the sum of the K similarity measures is upper bounded by Kd . In each iteration step, the sum of the K similarity measures is maximized and, hence, the algorithm converges. However, due to the non-concave nature of the problem, convergence to the global maximum cannot be guaranteed. From the simulations, it is observed that iteratively optimizing $\mathbf{X}_j^{(m)}$ and $\mathbf{Z}^{(m)}$ using

(21) and (23), the value of the objective function typically converges to Kd and, hence, the algorithm converges to $RISS$ and $RISS_j$.

Closed Form Solution: In this section, the closed form solution to find the $RISS$ and the condition for the applicability of the closed form solution are derived.

For illustration, consider the example in Figure 2 with $K = 2$, $N = 1$, $R = 3$, and $d = 1$. In this example, $RISS$ is of dimension $R - Kd = d = 1$ and $RISS$ should have a $d = 1$ dimensional intersection with each of the $2N = 2$ dimensional subspaces S_1 and S_2 . Hence, $RISS$ can be found directly by determining the intersection of S_1 and S_2 .

In general, $RISS$ can be of dimension $R - Kd \geq d$. For simplicity of the following notation, assume $R - Kd$ is an integer multiple of d , say $R - Kd = nd$, $n \in \mathbb{N}$, and K is an integer multiple of n , say $K = K_0n$, $K_0 \in \mathbb{N}$. For an $RISS$ of dimension $R - Kd > d$, it is not necessary that all the K subspaces S_j for $j = 1, 2, \dots, K$ have a common d -dimensional intersection subspace. Our task is to find a $R - Kd = nd$ dimensional $RISS$ that has at least a d -dimensional intersection subspace with each of the K subspaces S_j for $j = 1, 2, \dots, K$. In this section, we propose one possible approach to achieve this. First we split up the K subspaces S_j for $j = 1, 2, \dots, K$ into n disjoint groups with $K_0 = \frac{K}{n}$ subspaces in each group. The n disjoint groups can be formed arbitrarily. With (19), the dimension of the intersection subspace of all the K_0 subspaces of a certain group is given by $2NK_0 - (K_0 - 1)R$. In the following, we want to guarantee that the intersection subspace of all the K_0 subspaces of a group is at least d -dimensional, and to this purpose the condition

$$2NK_0 - (K_0 - 1)R \geq d \quad (24)$$

has to hold. For $\tau = 1, 2, \dots, n$, we define $IS_\tau^{(d)}$ as an arbitrary d -dimensional subspace of the intersection of all the K_0 subspaces in the corresponding group τ . Then the $RISS$ can be found as the union of all $IS_\tau^{(d)}$ for $\tau = 1, 2, \dots, n$, i.e.,

$$RISS = \bigcup_{\tau=1}^n IS_\tau^{(d)}. \quad (25)$$

By its construction, this $RISS$ is guaranteed to have an at least d -dimensional intersection with each S_j for $j = 1, 2, \dots, K$. For the $RISS$ to have a d -dimensional intersection subspace with each of the K subspaces, (24) needs to be true for all the n groups. Hence, multiplying (24) on both sides with n leads to the condition

$$2NK - (K - n)R \geq nd. \quad (26)$$

Note that (26) is more strict than (15) and hence, the closed form solution is possible only if the nodes and/or the relays have more antennas than required by the properness condition (15).

Remark 1: Note that for the case $R - Kd = d$, there is only a single group, i.e., $n = 1$. In this case, both (26) and the properness condition of (15) yield the same result, $2N + d \geq R$. Therefore, for the case $R - Kd = d$, the closed form solution is possible whenever the system is proper. In this case, the properness condition is also a sufficient condition.

Remark 2: If $R - Kd$ is not an integer multiple of d , then one approach is to form only an integer number $\lfloor \frac{R-Kd}{d} \rfloor$

of groups. This means, only a $(\lfloor \frac{R-Kd}{d} \rfloor d)$ -dimensional subspace of the $RISS$ will be considered to guarantee an at least d -dimensional intersection subspace with each of the K subspaces S_j for $j = 1, 2, \dots, K$. The remaining $((R - Kd) - (\lfloor \frac{R-Kd}{d} \rfloor Kd))$ -dimensional subspace of the $RISS$ will be arbitrarily chosen.

Remark 3: If K is not an integer multiple of n , then the arbitrarily chosen disjoint groups will have a different number of subspaces and not the same number K_0 .

D. Precoding Matrices

In Section V-C, the $RISS$ has been determined either in closed form or through the iterative method. In this section, the precoding matrices \mathbf{V}_j and \mathbf{V}_{j+K} are obtained in closed form. In Figure 2, $N = d = 1$ and hence, the projection of the received signals on the $RUSS$ results in partial signal alignment. However, in general, the precoding matrices need to be chosen such that partial signal alignment is achieved. Consider the node pair $(j, j + K)$. The $RISS$ calculated in the subsection V-C has an at least d -dimensional intersection subspace with the subspace S_j . Let $RISS_j^{(d)}$ denote an arbitrary d -dimensional subspace in the intersection subspace. Recollect from Section V-B that $S_j = S_{rj} \cup S_{r(j+K)}$.

Now we need to choose a d -dimensional subspace from S_{rj} and a d -dimensional subspace from $S_{r(j+K)}$ such that the corresponding $2d$ -dimensional subspace in S_j includes the d -dimensional intersection subspace $RISS_j^{(d)}$. Let the columns of the matrix \mathbf{Z}_j of size $R \times d$ span the intersection subspace $RISS_j^{(d)} \subseteq (RISS \cap S_j)$. The columns of the matrices $\mathbf{H}_{rj}\mathbf{V}_j$ and $\mathbf{H}_{r(j+K)}\mathbf{V}_{j+K}$ span the d -dimensional subspaces chosen from S_{rj} and $S_{r(j+K)}$, respectively. Then the subspace spanned by the columns of $[\mathbf{H}_{rj}\mathbf{V}_j \quad \mathbf{H}_{r(j+K)}\mathbf{V}_{j+K}]$ has to contain \mathbf{Z}_j . This can be written as

$$\mathbf{Z}_j = \mathbf{H}_{rj}\mathbf{V}_j + \mathbf{H}_{r(j+K)}\mathbf{V}_{j+K}. \quad (27)$$

Since by design, $RISS_j^{(d)} \subseteq (S_{rj} \cup S_{r(j+K)})$, the above equation has a unique solution for a given $RISS_j^{(d)}$ and choosing a different $RISS_j^{(d)} \subseteq RISS_j$ will result in another solution. In addition, the nodes j and $j + K$ can transmit in any direction spanned by the columns of the matrix \mathbf{V}_j and \mathbf{V}_{j+k} , respectively. The direction can be chosen such that some utility function is maximized and this optimization is left for future work.

VI. PERFORMANCE ANALYSIS

In this section, the sum rate performance of the PAIA scheme is compared with a conventional multi-user two-way relaying scheme based on [20]. Also, the leakage interference at the receiver is plotted to show that the proposed methods converge to interference alignment solutions.

Let P denote the transmit power at each of the nodes. The relay has a transmit power KP . The noise power at each node is assumed to be the same and is denoted by $\sigma_k^2 = \sigma_r^2 = \sigma^2$. The channel matrices corresponding to the channel between the nodes and the relay are generated randomly using the i.i.d. frequency flat Rayleigh MIMO Channel Model [22] and channel reciprocity is assumed. The channel matrices are

TABLE I
SIMULATION SCENARIOS

Scenario Name	d	R	N	K
A	1	5	2	4
B	1	9	3	7
C	2	10	4	4
D	2	12	5	5
E	3	17	5	4

normalized such that the average received power is the same as the average transmit power. Five different simulation scenarios are considered as shown in Table I.

In scenario A, $R = 5$ and $N = 2$. According to (15), at most $K = 4$ pairs can be served interference-free. From (26), the closed form solution for PAIA is possible. In the reference scheme, using the idea of pair-wise transceive zero forcing [20], at most three pairs can be served at the same time. The multiple antennas at the nodes are utilized to transmit the data streams in the direction corresponding to the largest singular value. Time Division Multiple Access (TDMA) is assumed between different sets of pairs in order to serve all the four pairs. Figure 3 shows the sum rate performance as a function of P/σ^2 . It can be seen that the proposed PAIA (PAIA_A) scheme performs better than the reference scheme (pairAwareZF_A) at high SNR. This is due to the fact that the PAIA scheme utilizes the antennas at the relay and at the nodes to maximize the number of transmitted data streams and does not care about the useful signal power in the *RUSS*. In the reference scheme, the multiple antennas at the transmitters are used to maximize the received signal power and, hence, it performs better at low SNR.

In scenario B, $N = 3$ and $R = 9$. According to (15), at most $K = 7$ pairs can be served interference-free. The closed form solution cannot be determined, as this scenario does not satisfy (26). The iterative method proposed in Section V-C can be used to find the interference alignment solution. In the reference scheme, only five user pairs can be served interference-free and TDMA is assumed between different sets of node pairs in order to serve all the seven pairs. Figure 3 shows that the PAIA scheme performs better than the reference scheme at high SNR.

In Figure 4, three different scenarios C, D and E for the case $d > 1$ are considered. 8, 10 and 12 DoF (total number of interference free data streams transmitted per time slot) are achieved in the scenarios C, D and E, respectively. One can clearly see in Figure 4 that at high SNR values, the slope of the curves corresponds to the DoF achieved. Till 40 dB, the achieved sum rate of PAIA_D is better than that of the PAIA_E. This is due to the fact that the nodes have a total transmit power constraint. Hence, an overall transmit power of $5P$ is available for transmitting 10 data streams in case of PAIA_D, whereas an overall transmit power of $4P$ is available for transmitting 12 data streams in case of PAIA_E.

Figure 5 shows the normalized leakage interference power at the receiver versus the total number of data streams for all the five scenarios. This way of displaying the result is the same as in [10]. On the abscissa, DoF+0 corresponds to the maximum DoF achievable as defined by (15). DoF+1

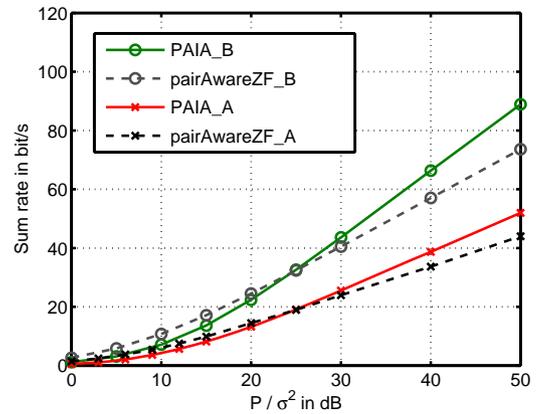


Fig. 3. Sum rate performance for $d = 1$

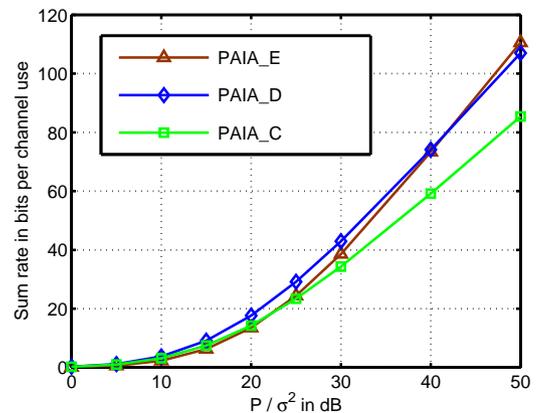


Fig. 4. Sum rate performance for $d > 1$

and DoF+2 represent the cases when 1 and 2 additional data streams, respectively, are transmitted per channel use. The iterative scheme proposed in this paper is used for the simulation. Note that the number of non-zero eigen-values in each step of the iterative scheme is limited by the dimension of *RISS*. Hence, if the number of data streams transmitted by any of the node pairs is increased to a value which is larger than the dimension of *RISS*, then the least squares solutions for (8) and (9) are used for the additional data streams. Figure 5 shows that for the case DoF+0, the leakage power is zero. By increasing the number of data streams per channel use by 1 or 2, the leakage interference at the receiver increases. This shows that the properness of the system typically is the sufficient condition for partial signal alignment.

VII. CONCLUSION

In this paper, a new scheme called Pair-Aware Interference Alignment (PAIA) has been proposed to achieve interference alignment in a multi-user two-way relay network. Taking into account that each node can cancel its self-interference perfectly, the interference alignment is done in three steps, namely, partial signal alignment and partial channel alignment followed by transceive zero forcing. The condition for the properness of the system $2Kd \leq 2(N-d) + R + d$ has been derived. An iterative method to achieve the interference alignment solution has been proposed. For $2KN - (K-n)R \geq nd$,

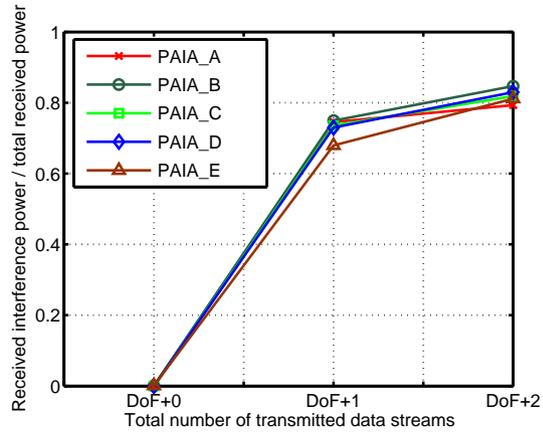


Fig. 5. Ratio of received interference power to total received power

also a closed form solution is given. Through simulations, it has been shown that the properness condition typically gives the sufficient condition for PAIA. It is also shown that the proposed PAIA scheme outperforms conventional schemes at high SNR.

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