R. S. Ganesan, H. Al-Shatri, T. Weber and A. Klein, ”Relay-aided interference alignment for bidirectional communications in multi-pair multi-relay networks,” in *Proc. IEEE Workshop on Signal Processing Advances in Wireless Communications*, Jun. 2013.

Relay-aided interference alignment for bidirectional communications in multi-pair multi-relay networks

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***Abstract*—In this paper, bi-directional communication between K node pairs is considered. Each node has N antennas and wants to transmit d data streams to its communication partner. Q relays with R antennas each aid in their communication. Two-way relaying is assumed. The nodes do not have transmit channel state information and the relays do not have enough antennas to spatially separate the data streams. Taking these two constraints into account, the relays and the receive filters are designed based on two different objectives. In the first case, the relays cooperate with each other to design their filters such that interferences are aligned at the receivers. The receive filters are simple zero forcing filters that nullify the interferences. In the second case, the relays and the receive filters jointly minimize the mean square error (MMSE) at the receivers. Through simulation results it is shown that both schemes achieve the same number of degrees of freedom in the system. However, the MMSE based scheme has better sum rate performance than the interference alignment based scheme.**

I. INTRODUCTION

Interference alignment (IA) is a promising technique for achieving high capacity gains in wireless networks. In [1], IA is introduced for a K-user interference channel. In [1], it is shown that $K/2$ degrees of freedom (DoF) are achievable in the K-user interference channel. However, there are several challenges like the bilinear nature of the IA problem, the need for global channel knowledge, infinite symbol extensions over which IA is performed etc. Recently, relays have been used to overcome some of these challenges.

Relay aided IA can simplify the process of IA and reduce the amount of channel state information (CSI) needed at each node. In this paper, bidirectional communication between K node pairs is considered where each node transmits d data streams to its communication partner. We focus on two-way relaying and IA along spatial dimensions. In [2], it is shown that a single relay with $R = Kd$ antennas can decouple the bilinear IA problem into three linear problems, namely signal alignment (SA), channel alignment (CA), and transceive zero forcing. This decoupling achieves exactly the same number of DoF as IA along spatial dimensions in a K-user interference channel without the relay. For the case $R > Kd$, multiple IA solutions exist. Several methods to choose a solution that maximizes a given utility function have been proposed in [3]–[5].

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In all the signal alignment based schemes [2]–[5], the number N of antennas at the nodes should satisfy $2N \geq (K+1)d$. This condition is relaxed in [6]. Here, in order to achieve IA, only the condition $2N + R \geq (2K+1)d$ needs to be satisfied. Note that $R \geq Kd$. Hence, increasing R, the number of antennas at each node can be reduced. Note that, in addition to the K-user interference channel, signal alignment based relay aided IA is also considered in [7], [8] for the MIMO Y channel and in [9] for the MIMO X channel.

In all the schemes discussed above, only a single relay is considered and transmit CSI (TxCSI) is assumed at all the nodes. Relay aided IA is interesting especially for the case where the relay cannot spatially separate the data streams. In the single relay case, if the relay cannot spatially separate the data streams, SA is necessary to achieve IA. Hence, the nodes need Tx CSI. However, for the case of multiple relays, say Q relays, as long as $RQ \geq 2Kd$, SA is not necessary even if the nodes cannot spatially separate the data streams. Hence, IA can be achieved without TxCSI at the nodes.

In this paper, we consider multiple relays and the nodes do not have TxCSI. The relays cannot spatially separate the data streams. However, the relays cooperate with each other in choosing their filters and perform IA at the receivers. The nodes have receive CSI (RxCSI) and perform zero forcing to separate the useful and the interference signals. An iterative algorithm to achieve IA is proposed. Furthermore, the properness condition is derived in terms of K, N, d, R and Q to identify if a given scenario is likely to be feasible. IA aims at maximizing the DoF and, hence, it is optimal at high signal to noise ratios (SNR). To improve the performance at low and medium SNR, in addition we propose an iterative algorithm to minimize the mean squared error (MSE) at the receivers subject to a total power constraint at the relays. This scheme is an extension of the iterative schemes proposed in [10] and [11] for one- and two-way relaying, respectively. In [10], [11] individual power constraints at the relays are considered and hence, the relay filters are optimized one after another. In the current paper, we have a total power constraint and hence, the relay filters are optimized at the same time.

The organization of the paper is as follows. The system model is introduced in Section II. In Section III and Section IV, the proposed iterative IA algorithm and the proposed iterative MMSE algorithm, respectively, are described. Section

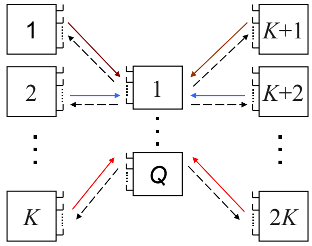

Fig. 1. K-pair two-way relay network

V evaluates the performance of the proposed schemes in terms of the sum rate of the system. Section VI concludes the paper.

We use lower case letters for scalars and lower case bold letters and upper case bold letters to denote column vectors and matrices, respectively. $(.)^*$, $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$ denote the complex conjugate, transpose and complex conjugate transpose of the element within the brackets, respectively. $\mathrm{Tr}(.)$ and $\mathrm{vec}(.)$ denote the trace and vectorization operations, respectively.

II. System Model

A K-pair two-way relay network with Q amplify and forward half-duplex relays each having R antennas is considered, see Figure 1. Each of the $2K$ nodes has N antennas and wants to transmit d data streams to its communication partner. Global channel knowledge is assumed at all the relays. No TxCSI is available at the nodes, but RxCSI is available at the nodes. Let node j and node k be the communication partners for $j = 1, \dots, 2K$ and $k = j + K$ if $j \le K$ and $k = j - K$ if $j > K$. Two-way relaying [12] is assumed. In the first time slot called multiple access (MAC) phase, the $2K$ nodes transmit their signals to the relays and in the second time slot called broadcast (BC) phase, the relays broadcast a linearly processed version of the signals received in the previous time slot to the $2K$ nodes. Let $\mathbf{d}_j$ and $\mathbf{V}_j$ denote the data symbols and the transmit filter matrix of node j, respectively. Since TxCSI is not available at the nodes, $\mathbf{V}_j$ is fixed a priori. Each node has a maximum transmit power P_{node}. Let $\mathbf{H}^{\text{sr}}_{jq}$ and $\mathbf{H}^{\text{rd}}_{qj}$ denote the MIMO channel matrix between node j and relay q in the first and the second time slot, respectively. Let $\mathbf{G}_q$ denote the matrix representing the linear signal processing performed at the relay q. The relays have a total transmit power P_{relay} available for transmission. The received signal $\mathbf{y}_k$ at node k is given by

$$\begin{aligned}\mathbf{y}_k = \;& \sum_{q=1}^{Q} \mathbf{H}^{\text{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\text{sr}}_{jq}\mathbf{V}_j\mathbf{d}_j + \sum_{q=1}^{Q} \mathbf{H}^{\text{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\text{sr}}_{kq}\mathbf{V}_k\mathbf{d}_k \\ & + \sum_{\substack{i=1,\\ i\neq j,k}}^{2K} \sum_{q=1}^{Q} \mathbf{H}^{\text{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\text{sr}}_{iq}\mathbf{V}_i\mathbf{d}_i + \widetilde{\mathbf{n}}_k \end{aligned} \tag{1}$$

where $\widetilde{\mathbf{n}}_k = \sum_{q=1}^{Q} \mathbf{H}^{\text{rd}}_{qk}\mathbf{G}_q\mathbf{n}_{1q} + \mathbf{n}_{2k}$ is the effective noise at receiver k with $\mathbf{n}_{1q}$ and $\mathbf{n}_{2k}$ denoting the noise at relay q and node k, respectively. The components of the noise vectors at relay q and node k are i.i.d. complex Gaussian random variables which follow $\mathcal{CN}(0, \sigma^2_{1q})$ and $\mathcal{CN}(0, \sigma^2_{2k})$, respectively. In (1), the first term corresponds to the useful signal. The second and the third terms correspond to the self interference and unknown interferences, respectively. It is assumed that the self interference can be perfectly cancelled. Let $\mathbf{U}^{\mathrm{H}}_k$ denote the receive filter at node k. Then the estimated data symbols for receiving $\mathbf{d}_j$ at receiver k are given by

$$\begin{aligned}\hat{\mathbf{d}}_j = \;& \mathbf{U}^{\mathrm{H}}_k \sum_{q=1}^{Q} \mathbf{H}^{\text{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\text{sr}}_{jq}\mathbf{V}_j\mathbf{d}_j + \\ & \mathbf{U}^{\mathrm{H}}_k \sum_{\substack{i=1,\\ i\neq j,k}}^{2K} \sum_{q=1}^{Q} \mathbf{H}^{\text{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\text{sr}}_{iq}\mathbf{V}_i\mathbf{d}_i + \mathbf{U}^{\mathrm{H}}_k\widetilde{\mathbf{n}}_k. \end{aligned} \tag{2}$$

Two different objectives are considered in this paper. The first one is to perform IA at the receiver. Here, our objective is to align all the interference within an $N - d$ dimensional interference subspace (ISS) and to ensure that the useful signals fully occupy a d dimensional useful subspace (USS) which is linearly independent from ISS. This means the condition

$$\mathbf{U}^{\mathrm{H}}_k \sum_{q=1}^{Q} \mathbf{H}^{\text{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\text{sr}}_{iq}\mathbf{V}_i = \begin{cases} \mathbf{0} & \text{if } i \neq j,k \\ \mathbf{I} & \text{if } i = j \end{cases} \tag{3}$$

needs to be satisfied for $i = 1, \dots, 2K$ and $k = 1, \dots, 2K$.

Our second objective is to minimize the MSE subject to the power constraints at the nodes and at the relays. The MSE of the estimated data symbols at receiver k is given by

$$MSE_k = \mathbb{E}\left\{\|\hat{\mathbf{d}}_j - \mathbf{d}_j\|^2\right\}. \tag{4}$$

The IA and the minimization of MSE subject to power constraints are non-convex [13] problems. In the following, we propose iterative schemes to obtain sub-optimum solutions.

III. Proposed Interference Alignment Algorithm

In this section, an iterative algorithm to achieve IA is described. Our objective is to design $\mathbf{G}_q$ for $q = 1, \dots, Q$ such that all the interferences are aligned at the receive nodes within the ISS and the useful signals are within the USS and to design $\mathbf{U}_k$ for $k = 1, \dots, 2K$ to zero force the interferences in order to satisfy (3). The basic idea of the proposed iterative scheme is as follows. The condition of (3) is a set of bilinear equations in $\mathbf{U}_k$ and $\mathbf{G}_q$. Fixing one of the two matrices results in a set of linear equations. Hence, we alternatingly optimize $\mathbf{U}_k$ and $\mathbf{G}_q$ to satisfy (3).

First, we arbitrarily fix $\mathbf{U}_k$ for $k = 1, \dots, 2K$. Vectorizing (3) and using the identity $\mathrm{vec}(\mathbf{YXZ}) = \left(\mathbf{Z}^{\mathrm{T}} \otimes \mathbf{Y}\right)\mathrm{vec}(\mathbf{X})$ we get

$$\sum_{q=1}^{Q} \left(\mathbf{H}^{\text{sr}}_{iq}\mathbf{V}_i\right)^{\mathrm{T}} \otimes \left(\mathbf{U}^{\mathrm{H}}_k\mathbf{H}^{\text{rd}}_{qk}\right)\mathrm{vec}(\mathbf{G}_q) = \begin{cases} \mathrm{vec}(\mathbf{0}) & \text{if } i \neq j,k \\ \mathrm{vec}(\mathbf{I}) & \text{if } i = j \end{cases} \tag{5}$$

for $i, k = 1, \dots, 2K$. Let $\mathbf{D}_{i,q,k} = \left(\mathbf{H}^{\text{sr}}_{iq}\mathbf{V}_i\right)^{\mathrm{T}} \otimes \left(\mathbf{U}^{\mathrm{H}}_k\mathbf{H}^{\text{rd}}_{qk}\right)$ and $\mathbf{g}_q = \mathrm{vec}(\mathbf{G}_q)$. Then (5) can be written as

$$\underbrace{\begin{pmatrix} \mathbf{D}_{j,1,k} & \cdots & \mathbf{D}_{j,Q,k} \\ \mathbf{D}_{1,1,k} & \cdots & \mathbf{D}_{1,Q,k} \\ \vdots & & \vdots \\ \mathbf{D}_{i,1,k} & \cdots & \mathbf{D}_{i,Q,k} \\ \vdots & & \vdots \\ \mathbf{D}_{2K,1,k} & \cdots & \mathbf{D}_{2K,Q,k} \end{pmatrix}_{i\neq j,k}}_{\mathbf{D}_k} \begin{pmatrix} \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_Q \end{pmatrix} = \begin{pmatrix} \mathrm{vec}(\mathbf{I}) \\ \mathrm{vec}(\mathbf{0}) \\ \vdots \\ \mathrm{vec}(\mathbf{0}) \end{pmatrix} \tag{6}$$

for $k = 1, \ldots, 2K$ with $i \neq j, k$ means that the rows corresponding to these two indices are not part of $\mathbf{D}_k$. Further with $\mathbf{D} = \left[\mathbf{D}_1^{\mathrm{T}} \cdots \mathbf{D}_{2K}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\mathbf{g} = \left[\mathbf{g}_1^{\mathrm{T}} \ldots \mathbf{g}_Q^{\mathrm{T}}\right]^{\mathrm{T}}$, $\mathbf{b}_k = \left[\mathrm{vec}\left(\mathbf{I}\right)^{\mathrm{T}} \; \mathrm{vec}\left(\mathbf{0}\right)^{\mathrm{T}} \; \ldots \; \mathrm{vec}\left(\mathbf{0}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, and $\mathbf{b} = \left[\mathbf{b}_1^{\mathrm{T}} \; \mathbf{b}_2^{\mathrm{T}} \; \ldots \; \mathbf{b}_{2K}^{\mathrm{T}}\right]^{\mathrm{T}}$. (6) can be written as

$$\mathbf{D}\mathbf{g} = \mathbf{b}. \tag{7}$$

The matrix $\mathbf{D}$ is of dimension $2K(2K-1)d^2 \times QR^2$. In this paper, it is assumed that the relays cannot spatially separate the data streams and hence, $QR^2 < 2K(2K-1)d$. In this case, the least squares solution for (7) is given by

$$\mathbf{g} = \mathbf{D}^{\dagger}\mathbf{b} \tag{8}$$

where $\mathbf{D}^{\dagger}$ is the pseudo inverse of $\mathbf{D}$. Now $\mathbf{G}_q$ for $q = 1, \ldots, Q$ is obtained for fixed receive filters.

In the next step, we fix $\mathbf{G}_q$ for $q = 1, \ldots, Q$ and optimize the receive filters as follows: Vectorization of (3) results in

$$\mathbf{H}_{ik}^{\mathrm{eff}} \mathrm{vec}\left(\mathbf{U}_k^{\mathrm{H}}\right) = \begin{cases} \mathbf{0} & \text{if } i \neq j, k \\ \mathbf{I} & \text{if } i = j \end{cases} \tag{9}$$

where $\mathbf{H}_{ik}^{\mathrm{eff}} = \left(\left(\sum_{q=1}^{Q} \mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i\right)^{\mathrm{T}} \otimes \mathbf{I}\right)$. With

$$\begin{aligned} \mathbf{c} &= \left[\mathrm{vec}\left(\mathbf{I}\right)^{\mathrm{T}} \; \mathrm{vec}\left(\mathbf{0}\right)^{\mathrm{T}} \; \ldots \; \mathrm{vec}\left(\mathbf{0}\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\ \mathbf{u}_k &= \mathrm{vec}\left(\mathbf{U}_k^{\mathrm{H}}\right), \\ \mathbf{H}_k &= \left[\mathbf{H}_{jk}^{\mathrm{effT}} \; \mathbf{H}_{1k}^{\mathrm{effT}} \; \cdots \; \mathbf{H}_{ik}^{\mathrm{effT}} \; \cdots \; \mathbf{H}_{2k}^{\mathrm{effT}}\right]_{i \neq j,k}^{\mathrm{T}}. \end{aligned} \tag{10}$$

(9) can be expressed as $\mathbf{H}_k \mathbf{u}_k = \mathbf{c}$. Then the least squares solution for the receive filter is obtained as

$$\mathbf{u}_k = \mathbf{H}_k^{\dagger}\mathbf{c} \tag{11}$$

for $k = 1, \ldots, 2K$. Iteratively optimizing the relay filters $\mathbf{G}_q$ for $q = 1, \ldots, Q$ and the receive filters $\mathbf{U}_k^{\mathrm{H}}$ for $k = 1, \ldots, 2K$ using (8) and (11), respectively, a least squares solution for (3) can be found. As in each steps of the iterations, the remaining error is reduced and the error is lower bounded by zero, the algorithm converges to a local optimum. Convergence to a global optimum cannot be guaranteed.

Properness condition: In order to identify if IA is possible for a given scenario, we classify the system into proper and improper systems [14]. This classification is performed by counting the number N_{v} of variables and the number N_{e} of equations in the system. If $N_{\mathrm{v}} \geq N_{\mathrm{e}}$, then the system is proper [14]. Otherwise, it is improper. The intuition is that proper systems are likely to be feasible [14]. The variables in the system are the relay and receive filter coefficients. Each relay processing matrix is of dimension $R \times R$ and the receive filters are of dimension $d \times N$. Hence, there are $2KNd + QR^2$ variables. From (3), there are $2K(2K-1)d^2$ equations. From the total power constraint at the relays, we have one equation. Therefore, for the system to be proper, the condition

$$2KNd + QR^2 \geq 2K(2K-1)d^2 + 1 \tag{12}$$

should be satisfied. It has to be noted that in contrast to [14], where the number of variables corresponding to each receive filter is counted as $Nd - d^2$ to make sure that the receive filter spans a d dimensional subspace, in this paper, we count the number of variables corresponding to each receive filter as Nd. This is due to the fact that in (3), we explicitly make the dimension of the subspace spanned by the columns of the receiver filter matrix to be d.

IV. Proposed Iterative MMSE Algorithm

In this section, an iterative algorithm to minimize the MSE subject to a total power constraint at the relays is described. First, we formulate the optimization problem. Then we propose an iterative algorithm to obtain a local minimum. The algorithm consist of two steps. First we arbitrarily fix the relay filters and derive the optimum receive filters that minimize the MSE at the receivers. In the second step, we fix the receive filters and using the Lagrange multiplier method, we derive the optimum relay filters that minimize the MSE subject to a total power constraint at the relays. The receive filters and relay filters are iteratively optimized until the algorithm converges to a local optimum.

A. Formulation of MMSE problem

In this subsection, the problem of minimizing the MSE subject to a total power constraint at the relays is formulated. As we have a total power constraint, it is favourable to represent the linear processing at the relays by one single matrix $\mathbf{G}$ with block diagonal structure given by $\mathbf{G} = \mathrm{blkdiag}\left(\mathbf{G}_1, \; \mathbf{G}_2, \; \ldots, \; \mathbf{G}_Q\right)$. The channels in the MAC and BC phases are rewritten as $\mathbf{H}_{i\mathrm{r}} = \left[\mathbf{H}_{i1}^{\mathrm{T}} \; \mathbf{H}_{i2}^{\mathrm{T}} \; \cdots \; \mathbf{H}_{iQ}^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\mathbf{H}_{\mathrm{r}k} = \left[\mathbf{H}_{1k} \; \mathbf{H}_{2k} \; \cdots \; \mathbf{H}_{Qk}\right]$, respectively. Similarly, the noise at the relays can be denoted by a single vector as $\mathbf{n}_1 = \left[\mathbf{n}_{11}^{\mathrm{T}} \; \mathbf{n}_{12}^{\mathrm{T}} \; \cdots \; \mathbf{n}_{1Q}^{\mathrm{T}}\right]^{\mathrm{T}}$. Then $\widetilde{\mathbf{n}}_k = \mathbf{H}_{\mathrm{r}k}\mathbf{G}\mathbf{n}_1 + \mathbf{n}_{2k}$. It is assumed that the data symbols are independent. Hence, $\mathbb{E}\left\{\mathbf{d}_j\mathbf{d}_j^{\mathrm{H}}\right\} = \mathbf{R}_{\mathbf{d}_j}$ and $\mathbb{E}\left\{\mathbf{d}_j\mathbf{d}_i^{\mathrm{H}}\right\} = \mathbf{0}$ for $i \neq j$. Let $\mathbf{S}_k = \mathbf{H}_{\mathrm{r}k}\mathbf{G}\mathbf{H}_{j\mathrm{r}}\mathbf{V}_j$ and $\mathbf{e}_k = \sum_{\substack{i=1 \\ i \neq j,k}}^{2K} \mathbf{H}_{\mathrm{r}k}\mathbf{G}\mathbf{H}_{i\mathrm{r}}\mathbf{V}_i\mathbf{d}_i$. Then (4) can be expressed as

$$\begin{aligned} MSE_k = &\mathrm{Tr}\left(\left(\mathbf{U}_k^{\mathrm{H}}\mathbf{S}_k - \mathbf{I}\right)\mathbf{R}_{\mathbf{d}_j}\left(\mathbf{S}_k^{\mathrm{H}}\mathbf{U}_k - \mathbf{I}\right)\right) + \\ &\mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}}\mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\}\mathbf{U}_k\right) + \mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}}\mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}\mathbf{U}_k\right). \end{aligned} \tag{13}$$

With $\mathbf{R}_Q = \sum_{i=1}^{2K}\left(\mathbf{H}_{i\mathrm{r}}\mathbf{V}_i\right)\mathbf{R}_{\mathbf{d}_i}\left(\mathbf{H}_{i\mathrm{r}}\mathbf{V}_i\right)^{\mathrm{H}} + \mathbb{E}\left\{\mathbf{n}_1\mathbf{n}_1^{\mathrm{H}}\right\}$ the optimization problem is given by

$$\begin{aligned} \underset{\mathbf{U}_k, \mathbf{G}}{\text{minimize}} \quad & \overline{MSE} = \sum_{k=1}^{2K} MSE_k \\ \text{subject to} \quad & \mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) \leq P_{\mathrm{relay}}. \end{aligned} \tag{14}$$

The optimization problem is non-convex [13]. In the following, an iterative algorithm to obtain a local minimum is proposed.

B. Receive filter design

In this subsection, for fixed relay filters the optimum receive filters are derived. The derivation of the optimum receive filters is similar to the derivation presented in [11] and it is briefly repeated here for completeness. First we initalize the relay filters arbitrarily. For fixed relay filters, the optimization problem in (14) is an unconstrained quadratic optimization problem [11]. For the optimum $\mathbf{U}_k$ the condition

$$\frac{\partial \overline{MSE}}{\partial \mathbf{U}_k^*} \stackrel{!}{=} \mathbf{0} \tag{15}$$

holds. Substituting (14) in (15), the optimum $\mathbf{U}_k$ [11] that minimizes $\overline{MSE}$ is given by

$$\mathbf{U}_k = \left[\mathbf{S}_k \mathbf{R}_{\mathbf{d}_j} \mathbf{S}_k^{\mathrm{H}} + \mathbb{E}\left\{\mathbf{e}_k \mathbf{e}_k^{\mathrm{H}}\right\} + \mathbb{E}\left\{\widetilde{\mathbf{n}}_k \widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}\right]^{-1} \mathbf{S}_k \mathbf{R}_{\mathbf{d}_j}. \tag{16}$$

C. Relay filter design

In this subsection, for fixed receive filters, using the Lagrange multiplier method the optimum relays filter subject to a total power constraint at the relays are derived. For fixed receive filters, the optimization problem of (14) is a quadratically constrained quadratic minimization problem. This is a convex problem whose optimum can be obtained using the Lagrange multiplier method. The Lagrangian function is given by

$$L(\mathbf{G}, \lambda) = \overline{MSE} + \lambda\left(\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) - P_{\text{relay}}\right). \tag{17}$$

Substituting $\overline{MSE}$ in (17) using (14) and (13) results in

$$\begin{aligned} L(\mathbf{G}, \lambda) =& \mathrm{Tr}\left[\mathbf{G}^{\mathrm{H}} \sum_{k=1}^{2K} \left(\mathbf{F}_{2k}\mathbf{G}\mathbf{F}_{1k} - \mathbf{F}_{jk} + \mathbf{F}_{2k}\mathbf{G}\mathbf{R}_{\mathrm{n}1}\right)\right] \\ &+ \mathrm{Tr}\left[\mathbf{G}^{\mathrm{H}}\lambda\mathbf{G}\mathbf{R}_Q\right] + \mathrm{Tr}\left[\mathbf{C}\right], \end{aligned} \tag{18}$$

where

$$\mathbf{F}_{1k} = \sum_{i=1,\ i\neq k}^{2K} \mathbf{H}_{i\mathrm{r}} \mathbf{V}_i \mathbf{R}_{\mathbf{d}\mathbf{d}i} \mathbf{V}_i^{\mathrm{H}} \mathbf{H}_{i\mathrm{r}}^{\mathrm{H}}, \quad \mathbf{F}_{2k} = \mathbf{H}_{\mathrm{r}k}^{\mathrm{H}} \mathbf{U}_k \mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{\mathrm{r}k},$$

$$\mathbf{F}_{jk} = \mathbf{H}_{\mathrm{r}k}^{\mathrm{H}} \mathbf{U}_k \mathbf{R}_{\mathbf{d}\mathbf{d}j} \mathbf{V}_j^{\mathrm{H}} \mathbf{H}_{j\mathrm{r}}^{\mathrm{H}}, \quad \mathbf{R}_{\mathrm{n}1} = \mathbb{E}\left\{\mathbf{n}_1\mathbf{n}_1^{\mathrm{H}}\right\}, \tag{19}$$

and $\mathbf{C}$ consists of the terms independent of $\mathbf{G}^*$. The optimum $\mathbf{G}$ and λ satisfy the KKT conditions given by

$$\frac{\partial L(\mathbf{G}, \lambda)}{\partial \mathbf{G}^*} = 0 \tag{20}$$

$$\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) \leq P_{\text{relay}} \tag{21}$$

$$\lambda\left(\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) - P_{\text{relay}}\right) = 0 \tag{22}$$

$$\lambda \geq 0. \tag{23}$$

In (20), the partial derivative is take only with respect to the block diagonal elements of $\mathbf{G}$. Let $\mathbf{B}$ denote a block diagonal matrix of same dimension and same block diagonal structure as $\mathbf{G}$, but with all the block diagonal elements being equal to one. Then (20) implies

$$\mathbf{B} \circ \left(\sum_{k=1}^{2K} \left(\mathbf{F}_{2k}\mathbf{G}\mathbf{F}_{1k} - \mathbf{F}_{jk} + \mathbf{F}_{2k}\mathbf{G}\mathbf{R}_{\mathrm{n}1}\right) + \lambda\mathbf{G}\mathbf{R}_Q\right) = \mathbf{0}, \tag{24}$$

where $\mathbf{A} \circ \mathbf{B}$ denotes the Hadamard product of $\mathbf{A}$ and $\mathbf{B}$. With $\mathbf{F}_{4k} = \mathbf{F}_{1k} + \mathbf{R}_{\mathrm{n}1}$ and $\mathbf{Z} = \sum_{k=1}^{2K} \mathbf{F}_{jk}$, (24) can be written as

$$\mathbf{B} \circ \left(\sum_{k=1}^{2K} \mathbf{F}_{2k}\mathbf{G}\mathbf{F}_{4k} + \lambda\mathbf{G}\mathbf{R}_Q\right) = \mathbf{B} \circ \mathbf{Z} \tag{25}$$

In (25), $\mathbf{F}_{2k},\ \mathbf{F}_{4k},\ \mathbf{R}_Q$, and $\mathbf{Z}$ are matrices of dimension $QR \times QR$. Each of these matrices is composed of Q^2 block matrices of dimension $R \times R$ each. Let $\mathbf{F}_{2k}^{l,q},\ \mathbf{F}_{4k}^{l,q},\ \mathbf{R}_Q^{l,q}$ and $\mathbf{Z}^{l,q}$ denote the $(l,q)^{\text{th}}$ block of the matrices $\mathbf{F}_{2k},\ \mathbf{F}_{4k},\ \mathbf{R}_Q$, and $\mathbf{Z}$, respectively. Then (25) becomes

$$\sum_{q=1}^{Q} \sum_{k=1}^{2K} \mathbf{F}_{2k}^{l,q} \mathbf{G}_q \mathbf{F}_{4k}^{q,l} + \lambda \mathbf{G} \mathbf{R}_Q^{l,l} = \mathbf{Z}^{l,l} \tag{26}$$

for $l = 1, \ldots, Q$. With $\mathbf{X}_q^l = \sum_{k=1}^{2K} \mathbf{F}_{4k}^{q,l\mathrm{T}} \otimes \mathbf{F}_{2k}^{l,q}$ and $\mathbf{Y}^l = \mathbf{R}_Q^{l,l\mathrm{T}} \otimes \mathbf{I}$, vectorizing (26) we get

$$\sum_{q=1}^{Q} \mathbf{X}_q^l \mathrm{vec}\left(\mathbf{G}_q\right) + \lambda \mathbf{Y}^l \mathrm{vec}\left(\mathbf{G}_l\right) = \mathrm{vec}\left(\mathbf{Z}^{l,l}\right). \tag{27}$$

Let $\mathbf{X}$ denote a block matrix whose $(l,q)^{\text{th}}$ block is $\mathbf{X}_q^l$ and $\mathbf{Y}$ denote a block diagonal matrix whose $(l,l)^{\text{th}}$ block is $\mathbf{Y}^l$. Also, let $\mathbf{z} = \left[\mathrm{vec}\left(\mathbf{Z}^{1,1}\right)^{\mathrm{T}}\ \ldots\ \mathrm{vec}\left(\mathbf{Z}^{Q,Q}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$. Then (27) becomes

$$\left(\mathbf{X} + \lambda\mathbf{Y}\right)\mathbf{g} = \mathbf{z}. \tag{28}$$

From the above equation, we get

$$\mathbf{g} = \left(\mathbf{X} + \lambda\mathbf{Y}\right)^{-1}\mathbf{z}. \tag{29}$$

Now we have the optimum $\mathbf{G}$ in closed form. However, λ needs to be determined such that (21), (22) and (23) are satisfied. From (23), $\lambda \geq 0$. First set $\lambda = 0$. If (21) is satisfied, then the optimum λ is equal to zero. If (21) is not satisfied, then $\lambda > 0$. Hence, in order to satisfy (21) and (22), the condition

$$\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) - P_{\text{relay}} = 0. \tag{30}$$

needs to hold. From (28), we know that $\mathrm{Tr}\left(\mathbf{G}\mathbf{R}_Q\mathbf{G}^{\mathrm{H}}\right) - P_{\text{relay}}$ is a decreasing function of λ. Using the fact that $\mathbf{X}$ is a positive semidefinite matrix and setting $\mathbf{X} = \mathbf{0}$, it can be proven that λ is bounded by

$$0 \leq \lambda \leq \sqrt{\frac{\mathbf{z}^{\mathrm{H}}\mathbf{Y}^{-1}\mathbf{z}}{P_{\text{relay}}}}. \tag{31}$$

Hence, (30) can be solved using the bisection method. The receive and relay filters are optimized iteratively either till the MSE does not change significantly or till a specified number of iterations is reached. Since at each iteration step, the MSE is minimized, the algorithm is guaranteed to converge to a minimum, though not necessarily to a global minimum.

V. Performance Analysis

In this section, the sum rate performance of the proposed iterative interference alignment (ItrIA) and iterative MMSE (Itr-MMSE) schemes are compared with the pair-aware interference alignment scheme (PAIA) from [6]. The simulation setting is as follows: $N = 2, R = 5, Q = 2$, $d = 1$ and $K = 5$. From the properness condition, if only $K_{\mathrm{s}} \leq K$ node pairs can be served simultaneously, then time sharing is assumed between different sets of pairs in order to serve all the K pairs. The PAIA scheme requires global CSI at all the nodes and is designed for the case of a single relay only [6]. Hence, for the simulation of the PAIA scheme, global CSI is assumed and only one of

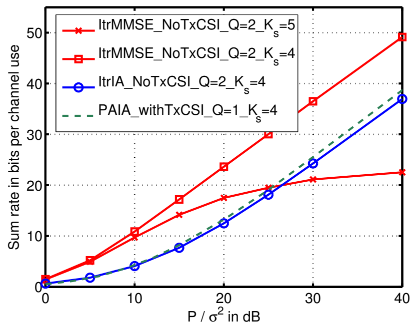

Fig. 2. Sum rate performance for a scenario with $N = 2,\ R = 5$ and $d = 1$

the two relays is used. However, for the proposed schemes no TxCSI is assumed at the nodes and both relays are used for the transmission. Figure 2 shows the sum rate performance of each method as a function of P/σ^2. P is the transmit power available at each node. The relays have a total transmit power of $K_{\mathrm{s}}P$. σ^2 is the receive noise power at each of the relays and each of the receive nodes in the MAC and BC phases, respectively. The transmit filter matrices are chosen as identity matrices of appropriate size and are normalized to satisfy the transmit power constraint. The channel matrices are normalized such that, on average, the transmitted signal power is the same as the received signal power. The sum rate is calculated as an average value of 1000 channel realizations generated using the i.i.d. frequency-flat Rayleigh fading channel model.

In PAIA and in the proposed ItrIA scheme, only $K_{\mathrm{s}} = 4$ node pairs can be served simulataneously without interference. ItrMMSE minimizes the MSE at the receivers and the number of node pairs that can be served without interference is not known. So first we consider the case $K_{\mathrm{s}} = 4$ for all the three schemes. From Figure 2 it can be seen that the proposed ItrIA scheme achieves almost similar performance as the PAIA. The ItrIA schemes do not have TxCSI, but it utilizes both the relays to perform IA. Both the curves have the same slope which implies that the second relay compensates for the absence of TxCSI at the nodes and the ItrIA schemes achieves the same number of degrees of freedom as the PAIA scheme. The ItrMMSE scheme, in addition to achieving the same number of degrees of freedom, also achieves a higher sum rate than the other two schemes. This is due to the fact that for $K_{\mathrm{s}} = 4$, the properness condition is satisfied with inequality sign and there are 9 additional variables which gives multiple solutions to IA problem. The ItrIA scheme chooses one solution arbitrarily, but ItrMMSE chooses the one that minimizes the MSE. Hence, it achieves a higher sum rate performance at all the SNR values.

As the number of node pairs that can be served simultaneously without interference by the ItrMMSE scheme is not known, we consider the case $K_{\mathrm{s}} = 5$. From Figure 2, it can be seen that the performance of the ItrMMSE scheme degrades at the medium and high SNR regime due to residual interferences at the receivers. Hence, only $K_{\mathrm{s}} = 4$ node pairs can be served without interference. This implies that ItrIA and ItrMMSE achieves the same number of DoF. However, ItrMMSE has a better sum rate performance. Hence, ItrIA can be used to identify the DoF using the properness condition and ItrMMSE can be used to maximize the sum rate in this proper system.

VI. Conclusion

In this paper, a multi-pair two-way relaying network with multiple relays is considered. The nodes do not have transmit CSI and the relays do not have a sufficient number of antennas to spatially separate the data streams. For this scenario, two algorithms one based on IA and one based on MMSE are proposed. In the first case, the relays cooperate with each other in chosing their filters and perform IA at the receivers. The properness condition has been derived as $2KNd + QR^2 \geq 2K(2K-1)d^2+1$. Secondly, an iterative algorithm to minimize the MSE at the receivers has been proposed. Simulation results show that both schemes achieve the same number of degrees of freedom. However, the iterative MMSE scheme has better sum rate performance than iterative IA scheme. Hence, IA scheme can be used to identify the DoF and iterative MMSE scheme can be used to maximize the sum rate achieved in the system.

Acknowledgment

This work is funded by the Deutsche Forschungsgesellschaft (DFG) under Grant No. Kl907/5-1 and WE2825/11-1.

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