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Interference Alignment Aided by Locally Connected Relays

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Abstract—In this paper, we consider a class of relay networks made up of two partially connected subnetworks. Every subnetwork consists of several relays along with the nearby source-destination node pairs being connected to the relays and, is assumed to be fully connected. The two subnetworks are coupled through the so called inter-subnetwork direct links. A linear interference alignment approach exploiting the one-way relaying protocol is applied. The feasibility conditions for interference alignment, i.e., the required numbers of relays in the whole network and in every subnetwork, are investigated. To this end, we develop a graph-based method to model the network topology and introduce the external constraints, from which an upper bound and a lower bound of the minimum required number of relays in a single subnetwork are obtained. Furthermore, we characterize the feasible region for achieving interference alignment in the considered networks using these bounds.

I. INTRODUCTION

Recent research on interference management techniques in relay interference channel has focused on the required number of relays/relay antennas for achieving a certain number of degrees of freedom (DoF). Interference alignment (IA) aided by relays is a promising technique being able to achieve high per-user DoF with only few time extensions as well as few antennas at the source and at the destination nodes [1]–[4]. However, state of the art relay-aided IA algorithms assume fully connected relay networks, i.e., the networks with all communications links between the nodes and the relays having non-negligible channel gains. To achieve IA in fully connected networks, many relays/relay antennas are required [2]–[4], especially in large networks with lots of node pairs, which is one of the major difficulties for the implementation of relay-aided IA. In realistic scenarios, some of the communications links may be relatively weak as compared to the other links and, can be even neglected at reasonable signal-to-noise-ratios (SNRs), e.g., a relay may be only accessible by the nearby nodes. Such networks can be assumed to have partial connectivity. Partial connectivity has been exploited for IA without relays to increase the achievable DoF [5]–[7], but it has not yet been considered for relay-aided IA to reduce the required number of relays/relay antennas.

In this paper, we consider the relay networks consisting of several single-antenna source-destination node pairs and several single-antenna amplify-and-forward relays. Information shall be transmitted from the source nodes towards the destination nodes exploiting both the direct links and the relay links. Partial connectivity is introduced, i.e., some links with negligibly small channel gains are assumed to be absent. More specifically, the whole network are made up of two partially connected subnetworks. Each subnetwork includes a subset of the relays along with the nearby node pairs being connected to these relays and, is assumed to be fully connected. The two subnetworks have neither common relays nor common node pairs. However, the so called inter-subnetwork direct links between the source nodes in one subnetwork and the destination nodes in the other one may exist. For instance, Fig. 1 illustrates a network made up of two partially connected subnetworks. Due to the presence of the inter-subnetwork links, the IA problems in the two subnetworks are coupled, e.g., the required number of relays in a single subnetwork is influenced by the number of available relays in the other subnetwork. The influence will be interpreted as external constraints. We develop a graph-based method to investigate the external constraints. Furthermore, we also derive an upper bound and a lower bound of the minimum required number of relays in a single subnetwork and characterize the feasible region for IA using these bounds.

In Section II, the system model and the IA conditions are introduced. In Section III, we study the external constraints. These external constraints will then be exploited in Section IV for investigating the required numbers of relays in the considered networks. Finally, we compare the performances achieved by a few representative scenarios based on simulations and conclude our work.

II. SYSTEM MODEL AND LINEAR IA

Recall the relay network made up of two partially connected subnetworks as introduced in Section I. The whole network consists of a set of \( K \) single-antenna node pairs \( \{(s_1, d_1), \ldots, (s_K, d_K)\} \) and a set of \( Q \) single-antenna relays \( \{r_1, \ldots, r_Q\} \). Let \( K_n \) and \( Q_n \) denote the number of node pairs and the number of relays in the \( n \)-th subnetwork, respectively. Every subnetwork is assumed to have at least three node pairs. Each source node \( s_k \) transmits a single data symbol intended for the corresponding destination node \( d_k \) through a constant interference channel using two time slots. In the first time slot, each source node transmits to the connected destination nodes and all relays in the corresponding subnetwork. In the second time slot, the source nodes retransmit to the connected destination nodes while every relay forwards a scaled version
of its received signal to all the destination nodes in the corresponding subnetwork. Let $h_{DS}^{(k,j)}, h_{RS}^{(q,j)}$, and $h_{DR}^{(k,q)}$ denote the channel coefficients of the links between $s_j$ and $d_k$, between $s_j$ and $d_q$, and between $r_q$ and $d_k$, respectively. The channel coefficients of the present links are assumed to be continuously and independently distributed over the complex field. The other channel coefficients are set to zero. Global channel coefficients of the present links are assumed to be zero. The other channel coefficients are set to zero. Global channel coefficients of the present links are assumed to be zero. The other channel coefficients are set to zero.

We exploit the linear algorithm proposed in [4] to achieve IA. On the one hand, IA requires that the interferences at all destination nodes shall be nulled:

$$\sum_{q=1}^{Q} h_{DR}^{(k,q)} g^{(q)} h_{RS}^{(q,j)} + h_{DS}^{(k,j)} \left( \frac{v_2^{(j)}}{v_1^{(j)}} + \frac{u_1^{(k)}}{u_2^{(k)}} \right) = 0, \forall k,j, k \neq j, \quad (1)$$

where the relay scaling factors $g^{(q)}$ and the ratios of the filter coefficients $v_2^{(j)}/v_1^{(j)}$ and $u_1^{(k)}/u_2^{(k)}$ are chosen as variables. Let a solution to the interference-nulling conditions of (1) be written in the vector form $x = (x_1^T, x_2^T)^T$, where $x_n$ is a vector including the $R_n + 2K_n$ variables of the $n$-th subnetwork. We refer to the solution space $W$ of (1) as the interference-nulling solution space. Note that if relay links are not available between two directly connected nodes $s_j$ and $d_k$, the interference propagating through the direct link can only be suppressed by choosing the corresponding filters orthogonal, i.e., $v_2^{(j)}/v_1^{(j)} = -u_1^{(k)}/u_2^{(k)}$, almost surely. On the other hand, if an interference-nulling solution $x \in W$ also fulfills the equality

$$\sum_{q=1}^{Q} h_{DR}^{(k,q)} g^{(q)} h_{RS}^{(q,j)} + h_{DS}^{(k,j)} \left( \frac{v_2^{(j)}}{v_1^{(j)}} + \frac{u_1^{(k)}}{u_2^{(k)}} \right) = 0, \quad (2)$$

the solution $x$ is called an invalid solution with respect to the $k$-th node pair since the useful signal at $d_k$ will be nulled by the receive filter. Otherwise, we refer to $x$ as a valid solution with respect to the $k$-th node pair. The invalid solution subspace with respect to the $k$-th node pair, which is implicitly defined by equality (2) in $W$, can be either a strict subspace of $W$ having codimension one if (2) is linearly independent of the interference-nulling conditions, or be identical to $W$. IA is feasible if and only if a valid solution with respect to all node pairs exists. This requires sufficient relays to ensure that the invalid solution subspace with respect to every node pair is a strict subspace of $W$, e.g., in a fully connected network,

$$Q \geq K^2 - 3K + 2 \quad (3)$$

relays are required [3], [4]. For random channel coefficients, the required number of relays is usually derived in the almost sure sense.

### III. External Constraints

In this section, we first define the external constraints. This involves the following vector spaces. Define orthogonal projections $P_1$ and $P_2$ in $\mathbb{C}^{Q+2K}$:

$$P_1 = \begin{pmatrix} I_{Q+2K_1} & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad P_2 = \begin{pmatrix} 0 & 0 \\ 0 & I_{Q+2K_2} \end{pmatrix}, \quad (4)$$

where $I$ is identity matrix. Performing $P_n$ on the interference-nulling solution space $W$ results in a space $P_n W$. In other words, for any vector $(x_1^T, x_2^T)^T \in W$, there are $(x_1^T, 0)^T \in P_1 W$ and $(0, x_2^T)^T \in P_2 W$. Furthermore, let a subsystem of the interference-nulling conditions be formed by the equations corresponding to the intra-subnetwork interferences in the $n$-th subnetwork. Denote the solution space of this subsystem by $W_n$. Thus $W_n$ consists of all the vectors that only null the intra-subnetwork interferences in the $n$-th subnetwork, leaving the remaining interferences unconsidered. Performing $P_n$ on $W_n$ yields a space $P_n W_n$. Obviously, $P_n W$ is a subspace of $P_n W_n$, because $W$ is a common subspace of $W_1$ and $W_2$. With the help of these vector spaces, a set of external constraints of the $n$-th subnetwork can be defined as a system of linear equations such that any vector $x \in P_n W_n$ also belongs to $P_n W$ if and only if $x$ satisfies the system of linear equations.

Normally, the external constraints of a single subnetwork, e.g., the first one, depend on the channel realization. However, we will show that in special cases, e.g., if $Q_2 = 0$ and $Q_2 \geq K_2 (K_2 - 1)$, the external constraints only depend on the network topology. In the following, we propose a graph-based method to identify a set of external constraints of the first subnetwork in each of the above cases. The same approach applies for the second subnetwork. Define graph $G$ to be an undirected bipartite graph with the vertices being the source and the destination nodes of a given network and the edges being the direct interference links, i.e., the direct links except for the ones between the node pairs $(s_k, d_k)$. Particularly, we define an external path to be a path on the graph $G$ with both ends belonging to the same subnetwork and all intermediate vertices belonging to the other subnetwork. We also introduce
the following notations. If \( v_{(j)} / u_{(i)}^{*} = -u_{1}^{(k)} / u_{2}^{(k)} \) holds, the filter \( v_{(j)} \) is orthogonal to \( u_{(k)} \). Let this be denoted by \( v_{(j)} \perp u_{(k)} \). If \( v_{(j)} / u_{1}^{(k)} = e_{2}^{(k)} / e_{1}^{(k)} \) holds, the filter \( v_{(j)} \) is aligned with \( e_{(j)} \). Let this be denoted by \( v_{(j)} \parallel e_{(j)} \).

In the case of \( Q_{2} = 0 \), nulling the intra-subnetwork interference between two nodes in the second subnetwork requires the corresponding transmit and receive filters being orthogonal, almost surely. Consequently, if two nodes in the first subnetwork, e.g., \( s_{j} \) and \( d_{k} \), are connected by an external path, the filters at any two neighboring nodes in the external path are almost surely orthogonal. Furthermore, the total number of edges in any external path connecting \( s_{j} \) and \( d_{k} \) must be odd because \( G \) is a partite graph. Therefore, if \( s_{j} \) and \( d_{k} \) are connected by at least one external path, \( v_{(j)} \perp u_{(k)} \) almost surely follows. Similarly, two source nodes \( s_{j} \) and \( s_{k} \) or two destination nodes \( d_{j} \) and \( d_{k} \) in the first subnetwork being connected by at least one external path respectively results in the constraint \( v_{(j)} \parallel u_{(k)} \) or \( v_{(j)} \parallel u_{(k)} \), almost surely. We claim that in the case of \( Q_{2} = 0 \), all the constraints resulting from the external paths of the first subnetwork as discussed above form a set of external constraints specifying the subspace \( P_{1}W \) in \( P_{1}W_{1} \). To prove this, we need to show: (a) if \( (x_{1}^{T}, x_{2}^{T})^{T} \) is an interference-nulling solution, i.e., \( (x_{1}^{T}, x_{2}^{T})^{T} \) belongs to \( W_{1} \), then \( (x_{1}^{T}, 0)^{T} \) satisfies these constraints; (b) for any vector \( (x_{1}^{T}, 0)^{T} \in W_{1} \) satisfying these constraints, there exists a vector \( x_{2} \) such that \( (x_{1}^{T}, x_{2}^{T})^{T} \) belongs to \( W \). The necessity, i.e., (a), is trivial since these constraints are deduced from the interference-nulling conditions and only involve the filters in the first subnetwork. To show the sufficiency, i.e., (b), we can first choose the filter at the end of an inter-subnetwork link in the second subnetwork to be orthogonal to the filter at the other end of the link. Afterwards, the remaining filters in the second subnetwork can be pairwise orthogonalized according to the previous one. This yields the required vector \( x_{2} \).

In the case of \( Q_{2} \geq K_{2}(K_{2} - 1) \), a set of external constraints specifying the subspace \( P_{1}W \) in \( P_{1}W_{1} \) can be formed by the constraints following from the external paths of the first subnetwork consisting only of the inter-subnetwork links. Similar to the case of \( Q_{2} = 0 \), the necessity is trivial. To show the sufficiency, we first find all the external paths of the first subnetwork consisting only of the inter-subnetwork links. Each of these paths only has a single intermediate node in the second subnetwork, otherwise it includes at least one intra-subnetwork link. The filter at the intermediate node of any of these external paths can then be chosen orthogonal to the filters at the ends of the path. Finally, since the intra-subnetwork interference-nulling conditions in the second subnetwork form \( K_{2}(K_{2} - 1) \) linear equations, \( K_{2}(K_{2} - 1) \) relays are sufficient for solving these equations with arbitrarily fixed filters. Hence, the remaining filters and the relay scaling factors in the second subnetwork can be obtained. This yields the required vector \( x_{2} \).

However, the sets of external constraints derived above may be linearly dependent. To find a set of linearly independent external constraints in the case of \( Q_{2} = 0 \), we define another graph \( G_{1} \). The vertices of \( G_{1} \) are the source and the destination nodes in the first subnetwork. Every external constraint in the case of \( Q_{2} = 0 \) corresponds to an edge of \( G_{1} \). For instance, a diagram of the graph \( G_{1} \) for the network shown in Fig. 1 illustrates Fig. 2. By graph theory, a set of linearly independent external constraints corresponds to the edges of a maximal forest of \( G_{1} \), i.e., a maximal acyclic subgraph of \( G_{1} \) including all vertices [8]. In Fig. 2, a maximal forest of \( G_{1} \) has three edges corresponding to, e.g., \( v_{(1)} \parallel v_{(2)} \), \( v_{(2)} \parallel v_{(3)} \), and \( v_{(3)} \parallel u_{(3)} \), which are linearly independent. Similarly, define \( H_{1} \) to be a graph with the same vertices as \( G_{1} \) and the edges being the external constraints in the case of \( Q_{2} \geq K_{2}(K_{2} - 1) \). For instance, the graph \( H_{1} \) for the network shown in Fig. 1 has only one edge \( v_{(1)} \parallel v_{(2)} \). The number of edges in a maximal forest of \( G_{1} \) or \( H_{1} \) can be denoted by \( \text{rank}(G_{1}) \) or \( \text{rank}(H_{1}) \), respectively.

IV. REQUIRED NUMBERS OF RELAYS

We then consider the required number of relays in a single subnetwork. Let a valid solution with respect to the \( n \)-th subnetwork be defined as an interference-nulling solution which is valid with respect to all node pairs belonging to the \( n \)-th subnetwork. Thus a valid solution with respect to a single subnetwork exists if every invalid solution subspace with respect to a node pair belonging to the subnetwork is a strict subspace of \( W \), and vice versa. Hence, \( 1A \) is feasible, i.e., all invalid solution subspaces are strict subspaces in \( W \), if and only if there is a valid solution with respect to every single subnetwork. Thus, the number of relays in every subnetwork shall be sufficient for guaranteeing the existence of a valid solution with respect to the subnetwork. On the one hand, since each individual subnetwork is fully connected, the required number of relays in the \( n \)-th subnetwork without external constraints is \( K_{n}^{2} - 3K_{n} + 2 \) (by (3)). On the other hand, the external constraints do not involve the relays. Thus from an engineering point of view, they are almost surely linearly independent of the intra-subnetwork interference-nulling conditions if there are sufficient relays. Therefore, satisfying each external constraint besides nulling the intra-subnetwork interferences requires one more relay. Hence, the required number of relays in the first subnetwork is at least \( Q_{1} = K_{1}^{2} - 3K_{1} + 2 + \text{rank}(G_{1}) \) if \( Q_{2} = 0 \) holds, and \( Q_{1} = K_{1}^{2} - 3K_{1} + 2 + \text{rank}(H_{1}) \) if \( Q_{2} \geq K_{2}(K_{2} - 1) \) holds. Furthermore, in the case of \( Q_{2} \geq K_{2}(K_{2} - 1) \), the external constraints of the first subnetwork only depends on the inter-subnetwork connectivity. In other words, these constraints
always needs to be satisfied, almost surely, regardless of the number of available relays in the second subnetwork. Consequently, $Q_1$ is a lower bound of the minimum required number of relays in the first subnetwork. Accordingly, if the first subnetwork has at least $Q_1$ relays, a valid solution with respect to the first subnetwork exists for any number of relays in the second subnetwork. Thus $Q_1$ is an upper bound of the minimum required number of relays in the first subnetwork. Similarly, we can define $Q_2$ and $Q_3$, which are the upper and the lower bound of the minimum required number of relays in the second subnetwork, respectively.

**Proposition 1:** In a given network made up of two subnetworks with at least three node pairs each, $Q_n$ and $Q_n'$ satisfy

$$Q_1 - Q_1 = Q_2 - Q_2. \tag{5}$$

**Proof:** First consider two disconnected subnetworks. Thus $Q_n = Q_n' = K_n^2 - 3K_n + 2$ holds for both subnetworks in this case. Then we modify the network by adding inter-subnetwork links to it. Adding the first inter-subnetwork link does not produce any external path in the modified network. Thus equality (5) still holds. Without loss of generality, we assume that an inter-subnetwork link $e_0$ with the ends $s_i$ and $d_k$ in the first subnetwork and $d_k$ in the second subnetwork is added to a network with at least one inter-subnetwork link, which corresponds to the graphs $G, G_n$ and $H_n$. The resulting network corresponds to the graphs $G', G'_n$ and $H'_n$. Then the following three cases shall be distinguished.

**Case I.** Neither $s_i$ nor $d_k$ is an end of the previously added inter-subnetwork links. If the second subnetwork has at least three node pairs, any two nodes in the second subnetwork are connected by a path in $G$. Therefore, adding $e_0$ results in at least one external path in $G'$ between $s_i$ and every end of the previously added links in the first subnetwork. Accordingly, new edges following from these paths shall be included in the modified graph $G'_n$. However, only one more edge ending at $s_i$ is included in a maximal forest of $G'_n$. Thus, adding $e_0$ results in increasing $Q_1$ by one. On the other hand, since $d_k$ is not an end of any previously added link, all the new external paths resulting from adding $e_0$ involve intra-subnetwork links of the second subnetwork. Hence, the graph $H'_n$ is identical to $H_n$ and $Q_1$ remains unchanged. Accordingly, $Q_2$ is increased by one and $Q_3$ remains unchanged.

**Case II.** Either $s_i$ or $d_k$ is an end of a previously added inter-subnetwork link. Firstly, assume that $d_k$ is an end of a previously added link $e_1$ which has the other end $s_i$ in the first subnetwork. For the same reason as in Case I, adding $e_0$ results in increasing $Q_1$ by one. However, $e_0$ and $e_1$ form a new external path in $G'$ consisting only of inter-subnetwork links. Therefore, the graph $H'_n$ includes a new edge between $s_i$ and $s_j$. Although several previously added links may have $d_k$ as a common end, only one more edge ending at $d_k$ shall be included in a maximal forest of graph $H'_n$. Therefore, $Q_1$ is increased by one. However, adding $e_0$ does not affect $Q_2$ and $Q_3$, because $e_0$ has the common end $d_k$ with $e_1$ and, therefore, does not produce new external constraints for the second subnetwork. Secondly, if $s_j$ instead of $d_k$ is a common end with $e_1$, $Q_1$ and $Q_3$ remain unchanged whereas $Q_2$ are both increased by one.

**Case III.** Both $s_j$ and $d_k$ are ends of previously added inter-subnetwork links. Assume that $d_k$ is an end of $e_1$ which has the other end $s_i$ in the first subnetwork, and $s_j$ is an end of $e_2$ which has the other end $d_k$ in the second subnetwork. Then $Q_1$ and $Q_2$ remain unchanged, because $G_n'$ is identical to $G_n$ for both subnetworks. We only consider the influence of adding $e_0$ on $Q_1$ and $Q_2$. (a) If $s_i$ and $d_k$ are already connected by a previously added link $e_3$, then $e_1$, $e_2$ and $e_3$ form a path between $s_j$ and $d_k$ in $G$, which results in $\nu^{(j)} \perp u^{(k)}$. Thus, adding $e_0$ does not introduce a linearly independent interference-nulling condition to the network. Consequently, $Q_1$ and $Q_2$ remain unchanged. (b) If for any choice of $e_1$ and $e_2$, the ends $s_j$ and $d_k$ are not connected by a previously added link, then adding $e_0$ result in a new edge ending at $s_j$ in a maximal forest of $H'_n$. Therefore, $Q_2$ is increased by one. Accordingly, $Q_3$ is increased by one as well.

To conclude, the equality of (5) will not be affected by adding an arbitrary number of inter-subnetwork links between two disconnected subnetwork, in arbitrary order.

We will then characterize the feasible region for IA, i.e., the pairs of the required relay numbers $(Q_1, Q_2)$ such that IA is feasible, in the considered networks.

**Proposition 2:** In a network consisting of two subnetworks with at least three node pairs each, the feasible region for IA is given by

$$\begin{align*}
Q_1 + Q_2 &\geq Q_1 + Q_2 - 2 \quad \text{(6a)} \\
Q_n &\geq Q_n, \quad n = 1, 2 \quad \text{(6b)}
\end{align*}$$

in the almost sure sense.

**Proof:** If the two subnetworks are disconnected, then $Q_n = Q_n'$ holds for both subnetworks. Consequently, the inequality of (6b) implies (6a). Besides, if (6b) holds, $Q_n \geq Q_n'$ holds as well in this case. Therefore, IA is almost surely feasible. Otherwise, IA is almost surely infeasible.

If there is at least one inter-subnetwork link, we will obtain (6a) by counting the number of variables $N_V$ and the number of constraints $N_C$ while taking the invalid solution subspaces into account. In other words, $N_V > N_C + 1$ shall hold so that the equality of (2) with respect to each node pair is linearly independent of the interference-nulling conditions. The total number of free variables $N_V$ is simply $Q + 2K$. The total number of constraints consists of two parts. Firstly, the intra-subnetwork interference-nulling conditions correspond to $K_1(K_1 - 1) + K_2(K_2 - 1)$ constraints. Secondly, adding every inter-subnetwork link as we did in the proof of Proposition 1 produces an additional constraint that the filters at its ends need to be orthogonal, except for Case III(a), because it does not introduce a linearly independent constraint. Let $N_1, N_2$ and $N_3$ denote the numbers of times that Case I, II and III(b) occur when adding the inter-subnetwork links, respectively. Since the first link added between two disconnected subnetworks shall be counted additionally, the inter-subnetwork links introduce
$N_1 + N_2 + N_3 + 1$ constraints. Furthermore, the equations

$$N_1 - N_3 = Q_1 - Q_2$$

$$N_2 + 2N_3 = Q_3 - (K_1^2 - 3K_1 + 2) + Q_4 - (K_2^2 - 3K_2 + 2)$$

can be summarized from the proof of Proposition 1. Hence, the total number of constraints is

$$N_C = Q_1 + Q_2 + 2K - 3.$$ (9)

Comparing $N_V$ and $N_C$ obtained above yields condition (6a). Furthermore, inequality (6b) is a necessary condition. Hence, Proposition 2 follows.

Proposition 2 also implies that the bounds $Q_1$ and $Q_2$ are tight for two subnetworks. Recall the example shown in Fig. 1. It is already derived in Section III that rank($G_1$) = 3 and rank($H_1$) = 1 hold. This results in $Q_1 = 5$ and $Q_2 = 3$. We can also derive $Q_3 = 4$ and $Q_4 = 2$ using the same approach. By Proposition 2, the feasible region for IA as partly shown in Table I is obtained.

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✓: Valid solutions w.r.t. both subnetworks exist.
×: All solutions are invalid w.r.t. both subnetworks.

1st/2nd: Valid solutions w.r.t. only one subnetwork exist.

### V. Simulation Results

In this section, we evaluate the performance achieved in the network illustrated in Fig. 1 based on simulations. The present links are assumed to be i.i.d. Rayleigh fading with unit average gain. Zero mean additive white Gaussian noise is assumed at the relays and at the destination nodes. Equal power allocation is assumed among the source nodes. The total transmit power at the relays is assumed to be equal to the total power at the source nodes. The filters and the relay scaling factors are obtained from a randomly picked interference-nulling solution of (1). The performance is measured by the average sum-rate per time-slot $C$ as a function of the pseudo SNR $\gamma_{PSNR}$, which is defined to be the ratio of the total transmitted energy by both the source nodes and the relays to the noise variance [4]. Fig. 3 shows the achieved performances with different relay numbers. With (3, 3) or (4, 2) relays, IA is almost surely feasible. The total number of achieved DoF is 3. There is no qualitative sum-rate difference between these two cases. With (5, 0) or (0, 4) relays, valid solutions with respect to only one subnetwork can be obtained and IA is almost surely infeasible. The total number of DoF is 1.5. As a reference scenario, we also assume that each relay is accessible by all node pairs. Then 20 global relays are required to achieve 3 DoF in the network.

### VI. Conclusion

In this paper, we apply a linear relay-aided IA algorithm to a class of networks made up of two partially connected subnetworks. We introduce the external constraints to investigate the coupling of the two subnetworks. A graph-based method is proposed to identify the sets of external constraints as well as the required numbers of relays. We show that using locally connected relays can help to achieve IA with less relays as compared to the one required in the fully connected networks.

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