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Non-coherent distributed space–time coding techniques for two-way wireless relay networks ☆

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ABSTRACT

To overcome the overhead involved with channel estimation, several non-coherent distributed space–time coding (DSTC) strategies for two-way wireless relay networks (TWRNs) using the amplify-and-forward and the decode-and-forward protocol have been recently proposed that do not require channel state information (CSI) at any node to decode the information symbols. In this paper, novel differential DSTC strategies for TWRNs using the two- and three-phase protocol are proposed. In our transmission schemes, the relays do not waste power to transmit information known at the respective destination nodes. This is achieved by combining the symbols from both terminals received at the relays into a single symbol of the unaltered constellation. Furthermore, in our strategies, the direct link between the communicating terminals can be naturally incorporated to further improve the diversity gain. Simulations show a substantially improved performance in terms of bit error rate (BER) of the proposed strategies as compared to the existing strategies.

1. Introduction

In scattering environments, cooperative diversity techniques employing multiple single-antenna relays can efficiently be applied to combat the effect of multi-path fading [1–3]. The basic setup of one-way distributed relay networks consists of a source terminal, a destination terminal, and multiple relay nodes. The signals transmitted by the source terminals are processed at the relays and retransmitted emulating a "virtual" antenna array and, therefore, creating multi-antenna transmit diversity.

It is widely established that using multiple relays can dramatically improve the performance of wireless relay networks in terms of data rate and error performance. Therefore, relays have been identified as an integral part of future wireless communication networks [2–9].

Efficient one-way cooperative diversity techniques based on DSTC have been recently developed to exploit the spatial diversity provided by multiple single-antenna relay nodes located spatially distributed in the area in between the communicating terminals [4–11]. DSTC techniques involve processing of the transmitted symbols in the spatial dimension, over multiple antennas, and in time dimension, over multiple time slots. By adding redundant information in space and time, both the reliability and the throughput can be improved at no additional cost of bandwidth or transmitted power and without requiring CSI at the transmitting nodes [2,3]. These benefits have been widely recognized in the international research community and standardization bodies.

Depending on the functionalities of the relays, several transmission protocols have been proposed which specify

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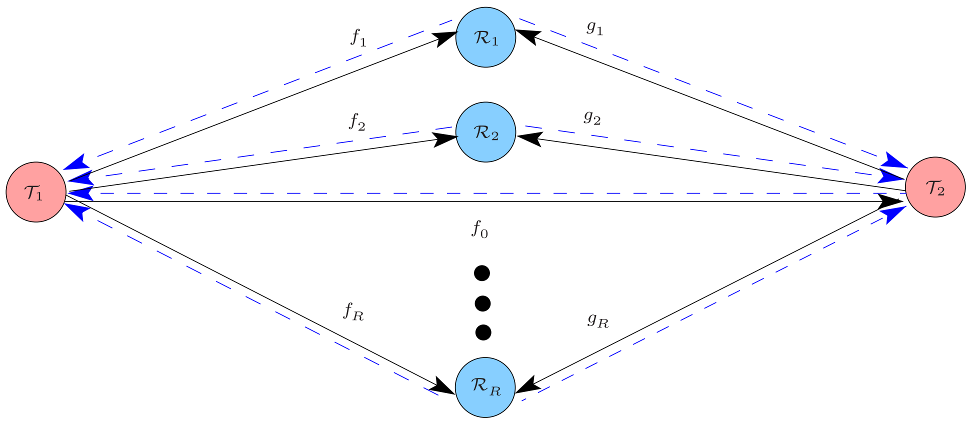

Fig. 1. TWRN with $R+2$ nodes.

the process of processing and retransmission of the received signal at the relays [2,3]. Among them, the most popular protocols are (i) the amplify-and-forward (AF) protocol, where each relay receives a noisy version of the information signal which is amplified at the relays and retransmitted and (ii) the decode-and-forward (DF) protocol, where symbols are decoded and re-encoded at the relay and then forwarded to the destination. Based on the availability of CSI, various cooperative diversity techniques have been considered. Some cooperative diversity techniques are based on the unrealistic assumption of perfect CSI at all nodes [12,13]. Other techniques, such as the DSTC techniques, consider the case of perfect CSI available only at the receiving nodes [6,14–16]. The recently proposed non-coherent differential techniques have been designed based on the assumption of CSI available neither at the terminals nor at the relays. These methods reduce the system complexity by avoiding the overhead involved with the transmission and processing of pilot signals [9–11,16–19].

In TWRNs, the communicating terminals mutually exchange information via a group of relays [16–24]. These relay networks can be categorized according to the number of phases required for the information exchange. There exist three popular classes: the four-phase TWRNs, the three-phase TWRNs, and the two-phase TWRNs. In [16,18–20], coherent and non-coherent DSTC techniques using the AF and the DF protocol have been proposed. It has been shown that DSTC techniques using the two- and three-phase protocol outperform the conventional DSTC techniques using the four-phase protocol due to the effective reduction in the symbol rate associated with the latter protocol [16,19,20]. However, the two-phase protocol in [19,20] cannot exploit the direct link between the communicating terminals since both terminals transmit their information symbols simultaneously to the relay nodes and a half duplex constraint is assumed to apply, i.e., devices can either transmit or receive in a specific time slot.

In this paper, we develop non-coherent two- and three-phase DSTC techniques for two-way relay networks where the direct link between the communicating terminals can be incorporated. The proposed techniques combine received symbols from both terminals at the relays into a single symbol using a simple differential encoding scheme, such that each terminal can decode the transmitted symbol of the other terminal using the information of its own transmitted symbol. Interestingly, in contrast to combination schemes that rely on extended modulations [19], the proposed differential encoding scheme performed at the relays is not associated with any waste in power for transmitting information symbols known at either receiver, resulting in improved BER performance at both terminals.

2. Wireless relay network model

We consider a wireless relay network with $R+2$ half-duplex single-antenna relay nodes as shown in Fig. 1 where the two terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ intend to exchange information and R nodes $(\mathcal{R}_1,\dots,\mathcal{R}_R)$ act as distributed relays for the signals transmitted from the terminals. We denote the channels from $\mathcal{T}_1$ to $\mathcal{T}_2$, from $\mathcal{T}_1$ to the rth relay, and from $\mathcal{T}_2$ to the rth relay as f_0, f_r, and g_r, respectively. We assume channel reciprocity for the transmission from $\mathcal{T}_1$ to $\mathcal{T}_2$ and vice versa. Further, we consider the extended block fading channel model in the two-phase protocol, for which the channels are assumed to remain approximately constant over $2T$ consecutive time slots and to slowly evolve outside this time interval, where T denotes the block length. Similarly, in three-phase schemes, the channels are assumed to remain approximately constant over $3T$ time slots. We further consider that the relays are perfectly synchronized and CSI is not available at any node. The nodes $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{R}_1,\dots,\mathcal{R}_R$ have limited average transmit powers $P_{\mathcal{T}_1}$, $P_{\mathcal{T}_2}$, $P_{\mathcal{R}_1},\dots,P_{\mathcal{R}_R}$, respectively.

Throughout this paper $\measuredangle$, $(\cdot)^*$, $\|\cdot\|$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $\odot$, diag($\mathbf{a}$), $\mathbf{I}_T$, $\mathbf{e}_r$, σ^2, $[\mathbf{a}]_i$, $\mathbf{0}_T$,and $\mathrm{E}\{\cdot\}$ denote the argument of a complex number, the complex conjugate, the Frobenius norm, the matrix transpose, the Hermitian transpose, the Hadamard (or Schur) product, the diagonal matrix whose diagonal elements are the elements of the vector $\mathbf{a}$, the $T\times T$ identity matrix, the rth column of $\mathbf{I}_T$, the noise variance, the ith entry of a vector $\mathbf{a}$, the $T\times T$ matrix with all zero entries, and the statistical expectation, respectively.

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Depending on the used context, $|\cdot|$ denotes the absolute value or the cardinality of a set.

3. Three-phase two-way differential DSTC techniques

Let us assume that $\mathbf{s}_{\mathcal{T}_1}^{(k)} = [[\mathbf{s}_{\mathcal{T}_1}^{(k)}]_1, \ldots, [\mathbf{s}_{\mathcal{T}_1}^{(k)}]_T]^{\mathrm{T}}$ and $\mathbf{s}_{\mathcal{T}_2}^{(k)} = [[\mathbf{s}_{\mathcal{T}_2}^{(k)}]_1, \ldots, [\mathbf{s}_{\mathcal{T}_2}^{(k)}]_T]^{\mathrm{T}}$ denote the $T \times 1$ vectors containing the kth block of information symbols of terminal $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively, where $[\mathbf{s}_{\mathcal{T}_t}^{(k)}]_i$ is taken from a M-PSK constellation denoted by set $\mathcal{S}_{\mathcal{T}_t}$ and T is the number of time slots in each phase which is assumed to be equal to the number of relays, hence $T=R$. In order to facilitate the processing at the relays and the destinations, in the first phase of the kth block, terminal $\mathcal{T}_1$ differentially encodes the information symbol vector $\mathbf{s}_{\mathcal{T}_1}^{(k)}$ as

$$\mathbf{x}_{\mathcal{T}_1}^{(k)} = \mathrm{diag}(\mathbf{x}_{\mathcal{T}_1}^{(k-1)})\mathbf{s}_{\mathcal{T}_1}^{(k)}, \tag{1}$$

and transmits this vector after power scaling with $\sqrt{3P_{\mathcal{T}_1}}$. At the beginning of transmission in the first block the recursion in (1) is initialized with the symbol vector $\mathbf{x}_{\mathcal{T}_1}^{(0)}$ which is known at the transmitter and receiver. In the next block of symbols, the last transmitted symbol vector of the previous block can be used as a reference in the decoding procedure. During the second phase from time slot $T+1$ to $2T$ in the kth block, terminal $\mathcal{T}_2$ transmits the differentially encoded $T \times 1$ symbol vector $\mathbf{x}_{\mathcal{T}_2}^{(k)}$ to the relays, given by

$$\mathbf{x}_{\mathcal{T}_2}^{(k)} = \mathrm{diag}(\mathbf{x}_{\mathcal{T}_2}^{(k-1)})\mathbf{s}_{\mathcal{T}_2}^{(k)}, \tag{2}$$

where $\mathbf{x}_{\mathcal{T}_2}^{(0)} = [1,1,\cdots,1]^{\mathrm{T}}$ that is also scaled with $\sqrt{3P_{\mathcal{T}_2}}$. In the first phase of the kth block as shown in Fig. 2, from time slot 1 to T, the $T \times 1$ vector received at the rth relay is given by

$$\mathbf{y}_{\mathcal{R}_1,r}^{(k)} = \sqrt{3P_{\mathcal{T}_1}} f_r^{(k)} \mathbf{x}_{\mathcal{T}_1}^{(k)} + \mathbf{n}_{\mathcal{R}_1,r}^{(k)}, \tag{3}$$

where $f_r^{(k)}$ denotes the channel from terminal $\mathcal{T}_1$ to the rth relay in the kth block, and $\mathbf{n}_{\mathcal{R}_1,r}^{(k)}$ denotes the $T \times 1$ noise vector of the kth block at the rth relay in the first phase. We assume that the noise vector can be modeled as a spatially white independently and identically distributed complex circular Gaussian random variable with zero mean and covariance $\sigma^2 \mathbf{I}_T$. Similarly, in the second phase of the kth block as shown in Fig. 3, from time slot $T+1$ to $2T$, the $T \times 1$ vector received at the rth relay is given by

$$\mathbf{y}_{\mathcal{R}_2,r}^{(k)} = \sqrt{3P_{\mathcal{T}_2}} g_r^{(k)} \mathbf{x}_{\mathcal{T}_2}^{(k)} + \mathbf{n}_{\mathcal{R}_2,r}^{(k)}, \tag{4}$$

where $g_r^{(k)}$ denotes the channel from terminal $\mathcal{T}_1$ to the rth relay in the kth block and $\mathbf{n}_{\mathcal{R}_2,r}^{(k)}$ denotes the $T \times 1$ noise vector at the rth relay in the second phase of the kth block.

In the following, we propose an efficient encoding strategy at the relays that facilitates simple signal separation at the destinations without however involving decoding of the signals received at the relays. During the third phase of the kth block as illustrated in Figs. 4 and 5, the rth relay combines the received signal vectors in (3) and (4) into a single $T \times 1$ signal vector

$$\begin{aligned}\mathbf{y}_{\mathcal{R}_3,r}^{(k)} &= \mathbf{\Phi}_{\mathcal{R},r}^{(k)}\left(\mathbf{y}_{\mathcal{R}_2,r}^{(k)} \odot \mathbf{y}_{\mathcal{R}_1,r}^{(k)}\right), \\ &= 3\sqrt{P_{\mathcal{T}_1}P_{\mathcal{T}_2}} f_r^{(k)} g_r^{(k)} \mathbf{\Phi}_{\mathcal{R},r}^{(k)} \mathbf{x}_{\mathcal{R}}^{(k)} + \mathbf{v}_{\mathcal{R}_3,r}^{(k)},\end{aligned} \tag{5}$$

where the combined signal part is defined as

$$\mathbf{x}_{\mathcal{R}}^{(k)} = \mathbf{x}_{\mathcal{T}_1}^{(k)} \odot \mathbf{x}_{\mathcal{T}_2}^{(k)}, \tag{6}$$

the additive noise terms are combined in

$$\begin{aligned}\mathbf{v}_{\mathcal{R}_3,r}^{(k)} = \mathbf{\Phi}_{\mathcal{R},r}^{(k)}\Big(&\sqrt{3P_{\mathcal{T}_1}} f_r^{(k)} \mathbf{x}_{\mathcal{T}_1}^{(k)} \odot \mathbf{n}_{\mathcal{R}_2,r}^{(k)} + \sqrt{3P_{\mathcal{T}_2}} g_r^{(k)} \mathbf{n}_{\mathcal{R}_1,r}^{(k)} \\ &\odot \mathbf{x}_{\mathcal{T}_2}^{(k)} + \mathbf{n}_{\mathcal{R}_1,r}^{(k)} \odot \mathbf{n}_{\mathcal{R}_2,r}^{(k)}\Big),\end{aligned} \tag{7}$$

and $\mathbf{\Phi}_{\mathcal{R},r}^{(k)}$ denotes a diagonal scaling matrix that adjusts the transmitted power at the rth relay. Let us consider without loss of generality the simple choice of

$$\mathbf{\Phi}_{\mathcal{R},r}^{(k)} = \mathrm{diag}\left(\left[\frac{1}{|[\mathbf{y}_{\mathcal{R}_1,r}^{(k)}]_1[\mathbf{y}_{\mathcal{R}_2,r}^{(k)}]_1|}, \ldots, \frac{1}{|[\mathbf{y}_{\mathcal{R}_1,r}^{(k)}]_T[\mathbf{y}_{\mathcal{R}_2,r}^{(k)}]_T|}\right]^{\mathrm{T}}\right) \tag{8}$$

that turns out to yield excellent decoding performance. However, another possible choice would be to select uniform constant scaling hence $\mathbf{\Phi}_{\mathcal{R},r}^{(k)} = \mathbf{I}_T$.

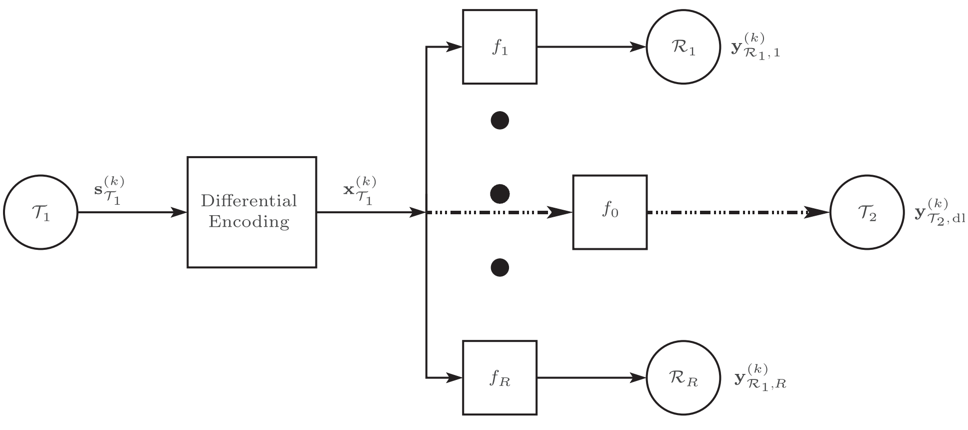

Fig. 2. Block diagram of the first phase.

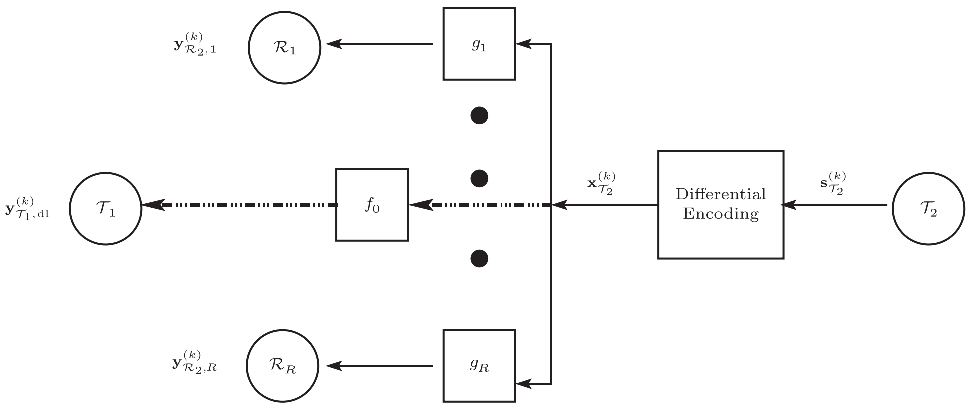

Fig. 3. Block diagram of the second phase.

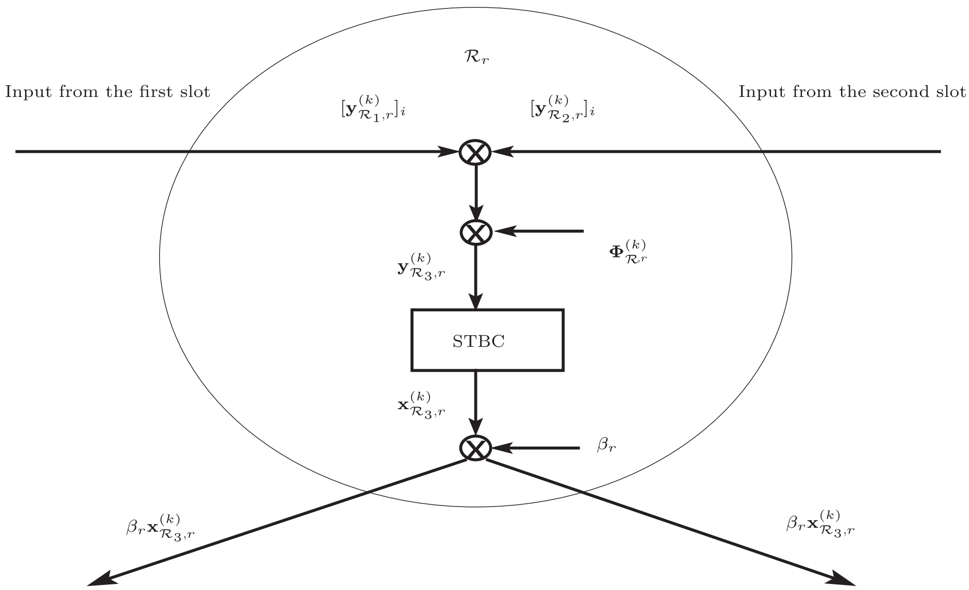

Fig. 4. Combination procedure at the rth relay.

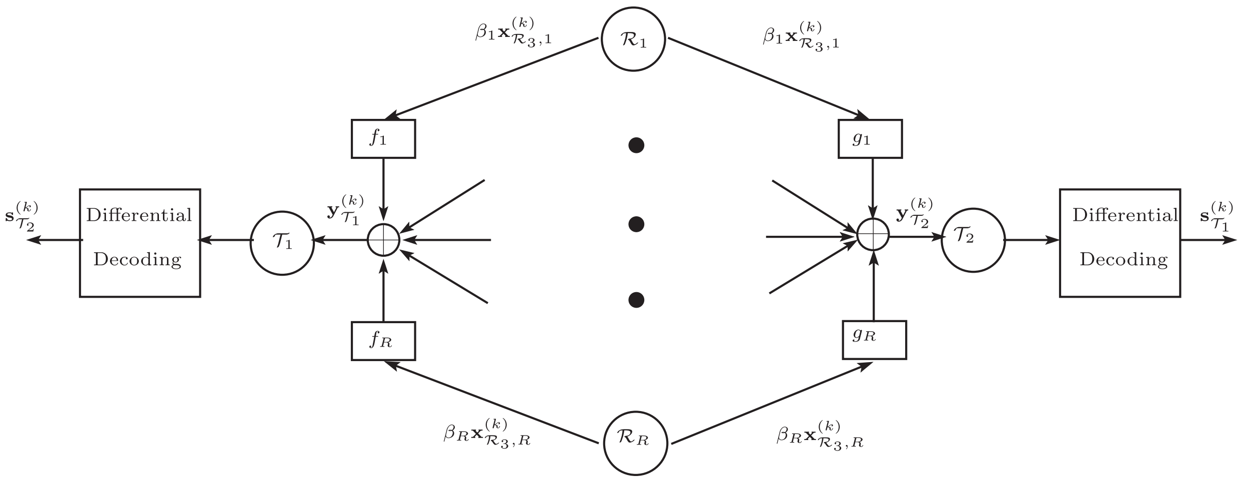

Fig. 5. Block diagram of the third phase.

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We define

$$[h_r^{(k)}]_i = \frac{3\sqrt{P_{\mathcal{T}_1}P_{\mathcal{T}_2}}f_r^{(k)}g_r^{(k)}}{|[\mathbf{y}_{\mathcal{R}_1,r}^{(k)}]_i|\,|[\mathbf{y}_{\mathcal{R}_2,r}^{(k)}]_i|}, \tag{9}$$

we observe from (3) and (4) that for sufficiently large SNR and with the scaling in (8)

$$|[h_r^{(k)}]_i| = \frac{3\sqrt{P_{\mathcal{T}_1}P_{\mathcal{T}_2}}|f_r^{(k)}|\,|g_r^{(k)}|}{\left|[\mathbf{y}_{\mathcal{R}_1,r}^{(k)}]_i\right|\left|[\mathbf{y}_{\mathcal{R}_2,r}^{(k)}]_i\right|} \approx 1. \tag{10}$$

Making use of the approximation in (10), Eq. (5) can be rewritten as

$$\mathbf{y}_{\mathcal{R}_3,r}^{(k)} \approx e^{j\measuredangle(f_r^{(k)}g_r^{(k)})}\mathbf{x}_{\mathcal{R}}^{(k)} + \mathbf{v}_{\mathcal{R}_3,r}^{(k)}, \tag{11}$$

which can be viewed as a scaled version of (6) that is corrupted by the additive noise of the rth relay defined in (7). From (5) and (6), we observe that the rth relay combines the received symbol vectors $\mathbf{x}_{\mathcal{T}_1}^{(k)}$ and $\mathbf{x}_{\mathcal{T}_2}^{(k)}$ from both terminals into a single symbol vector $\mathbf{x}_{\mathcal{R}}^{(k)}$ using a specific type of differential encoding scheme that does not require decoding of the symbols at the relays as illustrated in Fig. 4. This combining scheme enables each terminal to decode the transmitted symbols of the opposite terminal using the information of its own transmitted symbols. It can also be observed that the symbols $\mathbf{x}_{\mathcal{R}}^{(k)}$ in (6) belong to the same constellation as the symbols originally transmitted from both terminals. As a consequence, using the proposed differential encoding strategy the relays do not waste power to transmit information that is already known at the individual receivers. This results in an improved overall system performance in terms of BER as compared to conventional combination in [19] that rely on extended constellations. From Eq. (11), we observe that the combined symbol vectors $\mathbf{y}_{\mathcal{R}_3,r}^{(k)}$ at the relays are approximately equal up to an unknown phase rotation introduced by the complex channels $h_r^{(k)}$ and the additive noise. This means that the relay network can be considered as a "virtual" centralized multi-antenna network with R transmitters in which CSI is not available at the transmitter side. In this case, space–time block coding (STBC) techniques conventionally applied in centralized MISO systems can straightforwardly be applied. Hence, the rth relay encodes the combined symbol vector in (5) using STBC precoding scheme, such that

$$\mathbf{X}_{\mathcal{R}_3,r}^{(k)} = \mathcal{X}(\mathbf{y}_{\mathcal{R}_3,r}^{(k)}) = [\mathbf{A}_1\mathbf{y}_{\mathcal{R}_3,r}^{(k)} + \mathbf{B}_1(\mathbf{y}_{\mathcal{R}_3,r}^{(k)})^*, \ldots, \mathbf{A}_T\mathbf{y}_{\mathcal{R}_3,r}^{(k)} + \mathbf{B}_T(\mathbf{y}_{\mathcal{R}_3,r}^{(k)})^*], \tag{12}$$

where the code matrix

$$\mathcal{X}(\mathbf{x}) = [\mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{x}^*, \ldots, \mathbf{A}_T\mathbf{x} + \mathbf{B}_T\mathbf{x}^*], \tag{13}$$

with precoding matrices $\mathbf{A}_1, \ldots, \mathbf{A}_T$ and $\mathbf{B}_1, \ldots, \mathbf{B}_T$ defines the applied STBC scheme. Throughout this paper, we assume that the STBC precoding matrices exhibit the mutual exclusivity property that either $\mathbf{A}_r = \mathbf{0}_T$ or $\mathbf{B} = \mathbf{0}_T$. This property is valid for a large number of commonly used STBCs. For instance, let us consider a wireless relay network with two relay nodes $R = T = 2$ for which the popular Alamouti scheme can be applied [9,16,18,27] where $\mathbf{A}_r$ and $\mathbf{B}_r$ are chosen as

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{B}_1 = \mathbf{0}_T,$$
$$\mathbf{A}_2 = \mathbf{0}_T, \quad \mathbf{B}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{14}$$

In the case of a wireless relay network with four relay nodes, i.e., $T = R = 4$, quasi-orthogonal space–time codes [9,25–27] can be applied where $\mathbf{A}_r$ and $\mathbf{B}_r$ are chosen, e.g., as

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$
$$\mathbf{A}_3 = \mathbf{0}_T, \quad \mathbf{A}_4 = \mathbf{0}_T, \quad \mathbf{B}_1 = \mathbf{0}_T, \quad \mathbf{B}_2 = \mathbf{0}_T,$$
$$\mathbf{B}_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{B}_4 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{15}$$

Considering (6), the mutual exclusivity property that either $\mathbf{A}_r = \mathbf{0}_T$ or $\mathbf{B} = \mathbf{0}_T$, and the approximation in (11), Eq. (12) can be rewritten as

$$\begin{aligned}\mathbf{X}_{\mathcal{R}_3,r}^{(k)} &\approx [\mathbf{A}_1\mathbf{x}_{\mathcal{R}}^{(k)}h_r + \mathbf{B}_1(\mathbf{x}_{\mathcal{R}}^{(k)})^*h_r^*, \ldots, \mathbf{A}_T\mathbf{x}_{\mathcal{R}}^{(k)}h_r + \mathbf{B}_T(\mathbf{x}_{\mathcal{R}}^{(k)})^*h_r^*],\\ &\quad + [\mathbf{A}_1\mathbf{v}_{\mathcal{R}_3,r}^{(k)} + \mathbf{B}_1(\mathbf{v}_{\mathcal{R}_3,r}^{(k)})^*, \ldots, \mathbf{A}_T\mathbf{v}_{\mathcal{R}_3,r}^{(k)} + \mathbf{B}_T(\mathbf{v}_{\mathcal{R}_3,r}^{(k)})^*],\\ &= \mathbf{X}_{\mathcal{R}}^{(k)}\boldsymbol{\Delta}_{f,r}^{(k)}\boldsymbol{\Delta}_{g,r}^{(k)} + \mathbf{V}_{\mathcal{R}_3,r}^{(k)},\end{aligned} \tag{16}$$

where $\boldsymbol{\Delta}_{f,r}^{(k)} = \mathrm{diag}(\mathbf{f}_r^{(k)})$ and $\boldsymbol{\Delta}_{g,r}^{(k)} = \mathrm{diag}(\mathbf{g}_r^{(k)})$ for $\mathbf{f}_r^{(k)}$ and $\mathbf{g}_r^{(k)}$ denoting a $R \times 1$ vectors with the ith element defined as

$$[\mathbf{f}_r^{(k)}]_i = \begin{cases} e^{j\measuredangle(f_r^{(k)})} & \text{if } \mathbf{B}_i = \mathbf{0}_T, \\ e^{-j\measuredangle(f_r^{(k)})} & \text{if } \mathbf{A}_i = \mathbf{0}_T \end{cases} \tag{17}$$

and

$$[\mathbf{g}_r^{(k)}]_i = \begin{cases} e^{j\measuredangle(g_r^{(k)})} & \text{if } \mathbf{B}_i = \mathbf{0}_T, \\ e^{-j\measuredangle(g_r^{(k)})} & \text{if } \mathbf{A}_i = \mathbf{0}_T, \end{cases} \tag{18}$$

respectively. Further

$$\mathbf{X}_{\mathcal{R}}^{(k)} = \mathcal{X}(\mathbf{x}_{\mathcal{R}}^{(k)}) = [\mathbf{A}_1\mathbf{x}_{\mathcal{R}}^{(k)} + \mathbf{B}_1(\mathbf{x}_{\mathcal{R}}^{(k)})^*, \ldots, \mathbf{A}_T\mathbf{x}_{\mathcal{R}}^{(k)} + \mathbf{B}_T(\mathbf{x}_{\mathcal{R}}^{(k)})^*], \tag{19}$$

and $\mathbf{V}_{\mathcal{R}_3,r}^{(k)} = \mathcal{X}(\mathbf{v}_{\mathcal{R}_3,r}^{(k)})$ contain the combined signal and noise component in the STBC matrix (16), respectively. The STBC matrix in (19) describes the STBC matrix as it is used in a conventional, i.e., centralized multi-antenna system with $R = T$ antennas transmitting the combined symbol vector $\mathbf{x}_{\mathcal{R}}^{(k)}$ in (6). Note that with (6) and (19) the code matrix can also be expressed as

$$\mathbf{X}_{\mathcal{R}}^{(k)} = \mathbf{X}_{\mathcal{T}_1}^{(k)} \odot \mathbf{X}_{\mathcal{T}_2}^{(k)}, \tag{20}$$

where $\mathbf{X}_{\mathcal{T}_t}^{(k)} = \mathcal{X}(\mathbf{x}_{\mathcal{T}_t}^{(k)})$ denotes the STBC matrix in (19) corresponding to the symbol vector $\mathbf{x}_{\mathcal{T}_t}^{(k)}$ transmitted by terminal $\mathcal{T}_t$ in the tth phase. We observe from (16) that the rth relay recovers the STBC matrix of the centralized multi-antenna system in (20), up to scaling of the columns that depends on the composite channels h_r (or its conjugate h_r^*) in (9) and addition of a noise component obtained from the combined relay noise $\mathbf{v}_{\mathcal{R}_3,r}^{(k)}$ in (7). In the kth block of the third phase from time slot $2T+1$ to $3T$, the R relays jointly transmit a STBC matrix where the rth relay transmits the rth column of its respective code matrix $\mathbf{X}_{\mathcal{R}_3,r}^{(k)}$ in (12). Hence, the vector

$$\mathbf{x}_{\mathcal{R}_3,r}^{(k)} = \mathbf{X}_{\mathcal{R}_3,r}^{(k)}\mathbf{e}_r \tag{21}$$

is transmitted by the rth relay after scaling with power coefficient $\beta_r = \sqrt{P_{\mathcal{R}_r}}$ for $\mathbf{\Phi}_{\mathcal{R},r}^{(k)}$ defined according to (8) to satisfy the power constraint. In the following, we consider only the received signals at terminal $\mathcal{T}_2$. The signal received at terminal $\mathcal{T}_1$ can be computed correspondingly. The received signal vector at terminal $\mathcal{T}_2$ in the kth block is given by

$$\begin{aligned}\mathbf{y}_{\mathcal{T}_2}^{(k)} &= \sum_{r=1}^{R}\beta_r g_r^{(k)}\mathbf{x}_{\mathcal{R}_3,r}^{(k)} + \mathbf{n}_{\mathcal{T}_2}^{(k)},\\ &= \sum_{r=1}^{R}\beta_r g_r^{(k)}\mathbf{X}_{\mathcal{R}_3,r}^{(k)}\mathbf{e}_r + \mathbf{n}_{\mathcal{T}_2}^{(k)},\\ &= \sum_{r=1}^{R}\beta_r g_r^{(k)}(\mathbf{X}_{\mathcal{R}}^{(k)}\mathbf{\Delta}_{f,r}^{(k)}\mathbf{\Delta}_{g,r}^{(k)}\mathbf{e}_r + \mathbf{V}_{\mathcal{R}_3,r}^{(k)}\mathbf{e}_r) + \mathbf{n}_{\mathcal{T}_2}^{(k)},\\ &= (\mathbf{X}_{\mathcal{T}_1}^{(k)} \odot \mathbf{X}_{\mathcal{T}_2}^{(k)})\mathbf{\Delta}_g^{(k)}\mathbf{\Delta}_f^{(k)}\mathbf{c}_g^{(k)} + \mathbf{v}_{\mathcal{T}_2}^{(k)},\\ &= (\mathbf{S}_{\mathcal{T}_1}^{(k)} \odot \mathbf{S}_{\mathcal{T}_2}^{(k)} \odot \mathbf{X}_{\mathcal{T}_1}^{(k-1)} \odot \mathbf{X}_{\mathcal{T}_2}^{(k-1)})\mathbf{\Delta}_g^{(k)}\mathbf{\Delta}_f^{(k)}\mathbf{c}_g^{(k)} + \mathbf{v}_{\mathcal{T}_2}^{(k)},\end{aligned} \tag{22}$$

where $\mathbf{n}_{\mathcal{T}_2}^{(k)}$ denotes the $T\times 1$ vector containing the receiver noise of the kth block at terminal $\mathcal{T}_2$, $\mathbf{S}_{\mathcal{T}_t}^{(k)} = \mathcal{X}(\mathbf{s}_{\mathcal{T}_t}^{(k)})$ and

$$\mathbf{v}_{\mathcal{T}_2}^{(k)} = \sum_{r=1}^{R}\beta_r g_r^{(k)}\mathbf{V}_{\mathcal{R}_3,r}^{(k)}\mathbf{e}_r + \mathbf{n}_{\mathcal{T}_2}^{(k)}, \tag{23}$$

$$\mathbf{\Delta}_f^{(k)} = \sum_{r=1}^{R}\mathbf{\Delta}_{f,r}^{(k)}\mathbf{e}_r, \tag{24}$$

$$\mathbf{\Delta}_g^{(k)} = \sum_{r=1}^{R}\mathbf{\Delta}_{g,r}^{(k)}\mathbf{e}_r, \tag{25}$$

$$\mathbf{c}_g^{(k)} = [\beta_1 g_1^{(k)},\ldots,\beta_R g_R^{(k)}]^{\mathrm{T}}. \tag{26}$$

We remark that Eq. (22) also applies for scaling matrices different from the one chosen in (8). However, this requires corresponding modification in the definitions of the vectors in (17) and (18), as well as the power coefficient β_r used for the transmission of the code vector (21) at the rth relay. For example, if uniform constant scaling is used, hence if $\mathbf{\Phi}_{\mathcal{R},r}^{(k)} = \mathbf{I}_T$, then

$$[\mathbf{f}_r^{(k)}]_i = \begin{cases} f_r^{(k)} & \text{if } \mathbf{B}_i = \mathbf{0}_T,\\ (f_r^{(k)})^* & \text{if } \mathbf{A}_i = \mathbf{0}_T,\end{cases} \tag{27}$$

$$[\mathbf{g}_r^{(k)}]_i = \begin{cases} g_r^{(k)} & \text{if } \mathbf{B}_i = \mathbf{0}_T,\\ (g_r^{(k)})^* & \text{if } \mathbf{A}_i = \mathbf{0}_T\end{cases} \tag{28}$$

are considered instead of (17) and (18), respectively, and $\beta_r = \sqrt{P_{\mathcal{R}_r}/(9P_{\mathcal{T}_1}P_{\mathcal{T}_2} + 3P_{\mathcal{T}_1} + 3P_{\mathcal{T}_2})}$. In the kth block, the previous differentially encoded symbol matrix $\mathbf{X}_{\mathcal{T}_2}^{(k-1)}$ at terminal $\mathcal{T}_2$, the current symbol matrix $\mathbf{S}_{\mathcal{T}_2}^{(k)}$ of terminal $\mathcal{T}_2$, and the estimate of the previous differentially encoded matrix $\mathbf{X}_{\mathcal{R}}^{(k-1)}$, denoted by $\hat{\mathbf{X}}_{\mathcal{R}}^{(k-1)}$, can be considered as known at terminal $\mathcal{T}_2$. Making use of the extended block fading assumption where $\mathbf{\Delta}_g^{(k)} = \mathbf{\Delta}_g^{(k-1)}$, $\mathbf{\Delta}_f^{(k)} = \mathbf{\Delta}_f^{(k-1)}$, and $\mathbf{c}_g^{(k)} = \mathbf{c}_g^{(k-1)}$, the received signal vector at terminal $\mathcal{T}_2$ defined in (22) during the third phase of the $(k-1)$th and kth block can be expressed as

$$\begin{aligned}\mathbf{y}_{\mathcal{T}_2}^{(k)} &= \mathbf{X}_{\mathcal{R}}^{(k)}\mathbf{\Delta}_g\mathbf{\Delta}_f\mathbf{c}_g + \mathbf{v}_{\mathcal{T}_2}^{(k)},\\ &= (\mathbf{S}_{\mathcal{T}_1}^{(k)} \odot \mathbf{S}_{\mathcal{T}_2}^{(k)} \odot \mathbf{X}_{\mathcal{R}}^{(k-1)})\mathbf{\Delta}_g\mathbf{\Delta}_f\mathbf{c}_g + \mathbf{v}_{\mathcal{T}_2}^{(k)},\end{aligned} \tag{29}$$

$$\mathbf{y}_{\mathcal{T}_2}^{(k-1)} = \mathbf{X}_{\mathcal{R}}^{(k-1)}\mathbf{\Delta}_g\mathbf{\Delta}_f\mathbf{c}_g + \mathbf{v}_{\mathcal{T}_2}^{(k-1)}, \tag{30}$$

$$\begin{aligned}\tilde{\mathbf{y}}_{\mathcal{T}_2}^{(k-1)} &= (\hat{\mathbf{X}}_{\mathcal{R}}^{(k-1)})^{-1}\mathbf{y}_{\mathcal{T}_2}^{(k-1)},\\ &= \mathbf{\Delta}_g\mathbf{\Delta}_f\mathbf{c}_g + (\hat{\mathbf{X}}_{\mathcal{R}}^{(k-1)})^{-1}\mathbf{v}_{\mathcal{T}_2}^{(k-1)}.\end{aligned} \tag{31}$$

From (29) to (31), the decoder at terminal $\mathcal{T}_2$ in the absence of a direct link between the communicating terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ can be expressed as

$$\arg\min_{\mathbf{s}_{\mathcal{T}_1}^{(k)}}\|\mathbf{y}_{\mathcal{T}_2}^{(k)} - (\mathbf{S}_{\mathcal{T}_1}^{(k)} \odot \mathbf{S}_{\mathcal{T}_2}^{(k)} \odot \hat{\mathbf{X}}_{\mathcal{R}}^{(k-1)})\tilde{\mathbf{y}}_{\mathcal{T}_2}^{(k-1)}\|^2, \tag{32}$$

where $\hat{\mathbf{X}}_{\mathcal{R}}^{(k-1)} = \hat{\mathbf{X}}_{\mathcal{T}_1}^{(k-1)} \odot \mathbf{X}_{\mathcal{T}_2}^{(k-1)}$. We remark that for our distributed differential STBC coding scheme similar properties as in conventional centralized systems apply. In particular, if orthogonal or quasi-orthogonal STBCs are used in (13) then the decoding procedure in (32) can be carried out symbol-wise or pair-wise, respectively, resulting in substantially reduced decoding complexity at the terminals [6,9,27].

In the case that the direct link between the two terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ is available, the received signal vector at terminal $\mathcal{T}_2$ during the first transmission phase is given by

$$\begin{aligned}\mathbf{y}_{\mathcal{T}_2,\mathrm{dl}}^{(k)} &= \sqrt{3P_{\mathcal{T}_1}}f_0^{(k)}\mathbf{x}_{\mathcal{T}_1}^{(k)} + \mathbf{n}_{\mathcal{T}_2}^{(k)},\\ &= \sqrt{3P_{\mathcal{T}_1}}f_0^{(k)}\mathbf{S}_{\mathcal{T}_1}^{(k)}\mathbf{x}_{\mathcal{T}_1}^{(k-1)} + \mathbf{n}_{\mathcal{T}_2}^{(k)},\end{aligned} \tag{33}$$

where $\mathbf{n}_{\mathcal{T}_2}^{(k)}$ denotes the $T\times 1$ vector containing the receiver noise of the kth block at terminal $\mathcal{T}_2$ and $\mathbf{x}_{\mathcal{T}_1}^{(k)}$ is defined in (1). Making use of the symbol vector $\mathbf{y}_{\mathcal{T}_2,\mathrm{dl}}^{(k)}$ defined in (33), the decoder at terminal $\mathcal{T}_2$ can be expressed as

$$\begin{aligned}&\arg\min_{\mathbf{s}_{\mathcal{T}_1}^{(k)}}\|\mathbf{y}_{\mathcal{T}_2}^{(k)} - (\mathbf{S}_{\mathcal{T}_1}^{(k)} \odot \mathbf{S}_{\mathcal{T}_2}^{(k)} \odot \hat{\mathbf{X}}_{\mathcal{R}}^{(k-1)})\tilde{\mathbf{y}}_{\mathcal{T}_2}^{(k-1)}\|^2\\ &\quad + \|\mathbf{y}_{\mathcal{T}_2,\mathrm{dl}}^{(k)} - \mathbf{S}_{\mathcal{T}_1}^{(k)}\mathbf{y}_{\mathcal{T}_2,\mathrm{dl}}^{(k-1)}\|^2.\end{aligned} \tag{34}$$

We observe that in our scheme each relay combines the received signal vectors from both terminals into a single signal vector without decoding them and broadcasts the resulting vector to the destination. Each terminal can decode the transmitted symbols of the other terminal from its received signals of the relays using the information of its own transmitted symbols.

On the other hand, the DF protocol can also be applied to the proposed technique where each relay can differentially decode the information symbol vectors $\mathbf{s}_{\mathcal{T}_1}^{(k)}$ and $\mathbf{s}_{\mathcal{T}_2}^{(k)}$ of the first and second terminal, respectively, defined in (1) and (2) without requiring CSI. In this case, similarly as in (6), the decoded and re-encoded symbol vectors could be combined at the rth relay into a single symbol vector. In the next phase, the rth relay encodes the combined symbol vector and then broadcasts it to both terminals. We remark that our proposed technique in combination with the DF protocol enjoys a substantially lower relay decoding complexity as compared to the previously proposed two-phase DSTC techniques which has the order of $|\mathcal{S}_{\mathcal{T}_1}||\mathcal{S}_{\mathcal{T}_2}|$ [20]. To apply DSTC at the relay nodes using the DF protocol, the relays have to ideally decode the received signal correctly, i.e., $\hat{\mathbf{x}}^{(k)}_{\mathcal{R}_1} = \hat{\mathbf{x}}^{(k)}_{\mathcal{R}_2} = \hat{\mathbf{x}}^{(k)}_{\mathcal{R}_3} = \cdots = \mathbf{x}^{(k)}_{\mathcal{R}}$. Thus, in order to achieve the full diversity, the relays should decode the information symbols correctly, otherwise the decoder at the destination terminal suffers from a poor error performance. Therefore, to achieve the full diversity the authors in [20] proposed the use of cyclic redundancy check (CRC) at the relay nodes at the cost of reducing the spectral efficiency.

4. Two-phase two-way differential DSTC technique

In the two-phase protocol, e.g., proposed in [19,20], the terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ simultaneously transmit during the first phase from time slot 1 to T, their information symbol vectors to the relays. In the second phase, from time slot $T+1$ to $2T$, the relays forward the received signals to the destinations using either the AF or DF transmission. Although the protocol proposed in [19,20] allows cooperation in two phases, a major drawback is that it can not exploit the direct link between the communicating terminals. Moreover, the protocol proposed in [20] suffers from a high relay decoding complexity and is often associated with poor error performance. It was shown in [20] that due to the increase in the symbol rate the two-phase protocols perform better than the three-phase protocols, specifically for transmitted symbols with high modulation orders.

In this section, we reduce the three-phase protocol to a two-phase protocol that exhibits a symbol rate equivalent to that of the two-phase protocols of [16,19,20]. The proposed two-phase protocol enjoys improved error performance and low decoding complexity without requiring CSI at any node and allows the use of the direct link between the communicating terminals.

Let us assume that $T \geq 2$ is an even number and $\mathbf{s}^{(k)}_{\mathcal{T}_t} = [[\mathbf{s}^{(k)}_{\mathcal{T}_t}]_1, \ldots, [\mathbf{s}^{(k)}_{\mathcal{T}_t}]_T]^{\mathrm{T}}$ denotes the kth information symbol vector of dimension $T \times 1$ corresponding to $\mathcal{T}_t$ where the symbols $\mathbf{s}^{(k)}_{\mathcal{T}_t} \in \mathcal{S}_{\mathcal{T}_t}$. Every two information symbols $[\mathbf{s}^{(k)}_{\mathcal{T}_t}]_{2i-1}$ and $[\mathbf{s}^{(k)}_{\mathcal{T}_t}]_{2i}$ are combined at the terminal $\mathcal{T}_t$ into a single "super-symbol" $[\mathbf{d}^{(k)}_{\mathcal{T}_t}]_i$ of constellation $\mathcal{X}_{\mathcal{T}_t}$, with $M_{\mathcal{T}_t} = |\mathcal{X}_{\mathcal{T}_t}| = |\mathcal{S}_{\mathcal{T}_t}|^2$, using the mapping $\mathbf{d}^{(k)}_{\mathcal{T}_t} = \mathcal{G}_{\mathcal{T}_t}(\mathbf{s}^{(k)}_{\mathcal{T}_t})$ with the combination function where

$$[\mathbf{d}^{(k)}_{\mathcal{T}_t}]_i = \mathcal{G}_{\mathcal{T}_t}([\mathbf{s}^{(k)}_{\mathcal{T}_t}]_{2i-1}, [\mathbf{s}^{(k)}_{\mathcal{T}_t}]_{2i}) = \exp\left(j\frac{(\measuredangle[\mathbf{s}^{(k)}_{\mathcal{T}_t}]_{2i-1} + M_{\mathcal{T}_t}\measuredangle[\mathbf{s}^{(k)}_{\mathcal{T}_t}]_{2i})}{M_{\mathcal{T}_t}}\right), \tag{35}$$

$t=1,\ 2$, and $\mathbf{d}^{(k)}_{\mathcal{T}_t} = [[\mathbf{d}^{(k)}_{\mathcal{T}_t}]_1, \ldots, [\mathbf{d}^{(k)}_{\mathcal{T}_t}]_{T/2}]^{\mathrm{T}}$ denotes the kth "super-symbol" vector of dimension $T/2 \times 1$. The terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ encode their "super-symbols" $\mathbf{d}^{(k)}_{\mathcal{T}_1}$ and $\mathbf{d}^{(k)}_{\mathcal{T}_2}$ differentially into the symbols $\mathbf{x}^{(k)}_{\mathcal{T}_t} \in \mathcal{X}_{\mathcal{T}_t}$ to facilitate simple decoding at the relays without requiring CSI. In the first period of the first phase from time slot 1 to $T/2$, the terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ simultaneously transmit to the relays in the kth block the differentially encoded "super-symbol" vectors $\sqrt{2P_{\mathcal{T}_1}}\mathbf{x}^{(k)}_{\mathcal{T}_1}$ and $\sqrt{2P_{\mathcal{T}_2}}\mathbf{x}^{(k)}_{\mathcal{T}_2}$, respectively, where

$$\mathbf{x}^{(k)}_{\mathcal{T}_t} = \mathrm{diag}\left(\mathbf{d}^{(k)}_{\mathcal{T}_t}\right)\mathbf{x}^{(k-1)}_{\mathcal{T}_t}, \tag{36}$$

$\mathbf{x}^{(k)}_{\mathcal{T}_t} = \left[[\mathbf{x}^{(k)}_{\mathcal{T}_t}]_1, \ldots, [\mathbf{x}^{(k)}_{\mathcal{T}_t}]_{T/2}\right]^{\mathrm{T}}$ denotes the kth transmitted differentially encoded "super-symbol" vector of dimension $T/2 \times 1$ transmitted by terminal $\mathcal{T}_t$, $\mathbf{x}^{(0)}_{\mathcal{T}_t} = [1,1,\ldots,1]^{\mathrm{T}}$ defines the initial transmitted symbol vector in the first transmission that can be used as a reference at the receiver to start the differential decoding procedure, and $[\mathbf{x}^{(k)}_{\mathcal{T}_t}]_i \in \mathcal{X}_{\mathcal{T}_t}$. During the first period of the first phase from time slot 1 to $T/2$, the received signal vector of the kth block at the rth relay is given by

$$\mathbf{y}^{(k)}_{\mathcal{R}1,r} = \sqrt{2P_{\mathcal{T}_1}}f^{(k)}_r\mathbf{x}^{(k)}_{\mathcal{T}_1} + \sqrt{2P_{\mathcal{T}_2}}g^{(k)}_r\mathbf{x}^{(k)}_{\mathcal{T}_2} + \mathbf{n}^{(k)}_{\mathcal{R}1,r}, \tag{37}$$

where $\mathbf{n}^{(k)}_{\mathcal{R}1,r}$ denotes the $T/2 \times 1$ noise vector of the kth block at the receiver of the rth relay in the first period of the first phase. Similarly, in the second period of the first phase, from time slot $T/2+1$ to T, the terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ transmit simultaneously $\sqrt{2P_{\mathcal{T}_1}}\mathbf{x}^{(k)}_{\mathcal{T}_1}$ and $-\sqrt{2P_{\mathcal{T}_2}}\mathbf{x}^{(k)}_{\mathcal{T}_2}$, respectively. The only difference with respect to the first period is that in the second period the terminal $\mathcal{T}_2$ transmits the same symbols multiplied by -1 to enable simple separation of the received symbols at each relay, as illustrated below. The received signal vector at the rth relay during the second period of the first phase is given by

$$\mathbf{y}^{(k)}_{\mathcal{R}2,r} = \sqrt{2P_{\mathcal{T}_1}}f^{(k)}_r\mathbf{x}^{(k)}_{\mathcal{T}_1} - \sqrt{2P_{\mathcal{T}_2}}g^{(k)}_r\mathbf{x}^{(k)}_{\mathcal{T}_2} + \mathbf{n}^{(k)}_{\mathcal{R}2,r}, \tag{38}$$

where $\mathbf{n}^{(k)}_{\mathcal{R}2,r}$ denotes the $T/2 \times 1$ noise vector of the kth block at the receiver of the rth relay in the second period of the first phase. Making use of the block fading assumption, i.e., the channel remains constant over $2T$ time slots, we can replace in (37) and (38) the channel coefficients $f^{(k)}_r$ and $g^{(k)}_r$ by f_r and g_r, respectively. From (37) and (38), the rth relay obtains

$$\tilde{\mathbf{y}}^{(k)}_{\mathcal{R}_1,r} = \frac{\mathbf{y}^{(k)}_{\mathcal{R}_1,r} + \mathbf{y}^{(k)}_{\mathcal{R}_2,r}}{2}, \tag{39}$$

$$\tilde{\mathbf{y}}^{(k)}_{\mathcal{R}_2,r} = \frac{\mathbf{y}^{(k)}_{\mathcal{R}_1,r} - \mathbf{y}^{(k)}_{\mathcal{R}_2,r}}{2}, \tag{40}$$

from which "soft" estimates of the information symbol vectors $\hat{\mathbf{s}}^{(k)}_{\mathcal{T}_1,r}$ and $\hat{\mathbf{s}}^{(k)}_{\mathcal{T}_2,r}$ of the terminals $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively, can be computed by splitting the received signals that contain the transmitted symbols as follows:

$$[\hat{\mathbf{s}}^{(k)}_{\mathcal{T}_t,r}]_{2i-1} = \exp(j2\pi\ \mathrm{mod}\{(\measuredangle[\tilde{\mathbf{y}}^{(k)}_{\mathcal{R}t,r}]_i - \measuredangle[\tilde{\mathbf{y}}^{(k-1)}_{\mathcal{R}_t,r}]_i)M_{\mathcal{T}_t}, 2\pi\}), \tag{41}$$

$$[\hat{\mathbf{s}}^{(k)}_{\mathcal{T}t,r}]_{2i} = \exp\left(j\left(\measuredangle[\tilde{\mathbf{y}}^{(k)}_{\mathcal{R}_t,r}]_i - \measuredangle[\tilde{\mathbf{y}}^{(k-1)}_{\mathcal{R}_t,r}]_i - \frac{\measuredangle[\hat{\mathbf{s}}^{(k)}_{\mathcal{T}_t,r}]_{2i-1}}{M_{\mathcal{T}_t}}\right)\right), \quad (42)$$

where $i = 1,2,\ldots,T$. Note that the separation procedure carried out at the relays according to (41) and (42) is associated with negligible complexity as compared to the "hard-decision" relay decoding complexity of the two-phase coherent DSTC using the DF protocol [20]. Similar to Eq. (5), the rth relay combines the separated symbols $\hat{\mathbf{s}}^{(k)}_{\mathcal{T}_1,r}$ and $\hat{\mathbf{s}}^{(k)}_{\mathcal{T}_2,r}$ from the two terminals into a single symbol as

$$\hat{\mathbf{s}}^{(k)}_{\mathcal{R},r} = \hat{\mathbf{s}}^{(k)}_{\mathcal{T}_1,r} \odot \hat{\mathbf{s}}^{(k)}_{\mathcal{T}_2,r}. \quad (43)$$

Similarly as in Section 3, we observe from (43) that the proposed encoding strategy allows the relays to combine the separated symbols $\hat{\mathbf{s}}^{(k)}_{\mathcal{T}_1,r}$ and $\hat{\mathbf{s}}^{(k)}_{\mathcal{T}_2,r}$ from the two terminals into a single symbol $\hat{\mathbf{s}}^{(k)}_{\mathcal{R},r}$ belonging to the same constellation as the separated symbols. In this way, the relays do not waste power to transmit known information to either side. Each terminal can then decode the transmitted symbol of the opposite terminal using the information of its own transmitted symbol resulting in overall improved the BER performance.

In the second phase of the kth block from time slot $T+1$ to $2T$, the rth relay precodes the symbol vector and its conjugate with the $T\times T$ unitary matrices $\mathbf{A}_r$ and $\mathbf{B}_r$ and scales the resulting vector before broadcasting it to the terminals. Similar to (12), the rth relay encodes the combined symbol vector in (43) using a linear STBC precoding scheme, hence

$$\mathbf{S}^{(k)}_{\mathcal{R},r} = \mathcal{X}(\hat{\mathbf{s}}^{(k)}_{\mathcal{R},r}) = [\mathbf{A}_1\hat{\mathbf{s}}^{(k)}_{\mathcal{R},r} + \mathbf{B}_1(\hat{\mathbf{s}}^{(k)}_{\mathcal{R},r})^*, \ldots, \mathbf{A}_T\hat{\mathbf{s}}^{(k)}_{\mathcal{R},r} + \mathbf{B}_T(\hat{\mathbf{s}}^{(k)}_{\mathcal{R},r})^*]. \quad (44)$$

In the second phase of the kth block, the R relays jointly transmit a differentially encoded STBC matrix similarly as in Section 3 where the rth relay transmits the rth column of its differentially encoded STBC matrix, given by

$$\mathbf{x}^{(k)}_{\mathcal{R},r} = \mathbf{S}^{(k)}_{\mathcal{R},r}\mathbf{X}^{(k-1)}_{\mathcal{R},r}\mathbf{e}_1, \\ = \mathbf{S}^{(k)}_{\mathcal{R},r}\mathbf{x}^{(k-1)}_{\mathcal{R},r}, \quad (45)$$

where $\mathbf{X}^{(0)}_{\mathcal{R},r} = \mathbf{I}_T$, $\mathbf{x}^{(0)}_{\mathcal{R},r} = \mathbf{e}_1$, and the structure of $\mathbf{S}^{(k)}_{\mathcal{R},r}$ depends on the used STBC precoding matrices defined in (14) and (15). Similar to (22), the received signal vector at terminal $\mathcal{T}_2$ is given by

$$\mathbf{y}^{(k)}_{\mathcal{T}_2} = \sum_{r=1}^{R} \beta_r g_r \mathbf{x}^{(k)}_{\mathcal{R},r} + \mathbf{n}^{(k)}_{\mathcal{T}_2}, \quad (46)$$

where $\beta_r = \sqrt{P_{\mathcal{R}_r}}$ and $\mathbf{n}^{(k)}_{\mathcal{T}_2}$ denotes the noise vector of the kth block at terminal $\mathcal{T}_2$. In the second phase of the kth block, the received signal vector at terminal $\mathcal{T}_2$ can be expressed as

$$\mathbf{y}^{(k)}_{\mathcal{T}_2} = \mathbf{X}^{(k)}\mathbf{c}_g + \mathbf{n}^{(k)}_{\mathcal{T}_2}, \quad (47)$$

where

$$\mathbf{c}_g = [\beta_1 g_1, \beta_2 g_2, \ldots, \beta_R g_R]^{\mathrm{T}}, \quad (48)$$

$$\mathbf{X}^{(k)} = \hat{\mathbf{S}}^{(k)}_{\mathcal{R}}\mathbf{X}^{(k-1)}. \quad (49)$$

Then from (49), we can rewrite (47) as

$$\mathbf{y}^{(k)}_{\mathcal{T}_2} = \hat{\mathbf{S}}^{(k)}_{\mathcal{R}}\mathbf{X}^{(k-1)}\mathbf{c}_g + \mathbf{n}^{(k)}_{\mathcal{T}_2}, \\ = \hat{\mathbf{S}}^{(k)}_{\mathcal{R}}(\mathbf{y}^{(k-1)}_{\mathcal{T}_2} - \mathbf{n}^{(k-1)}_{\mathcal{T}_2}) + \mathbf{n}^{(k)}_{\mathcal{T}_2}. \quad (50)$$

From (50), the decoder at terminal $\mathcal{T}_2$ in the absence of a direct link between the communicating terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ can be expressed as

$$\arg\min_{\mathbf{s}^{(k)}_{\mathcal{R}}} \left\| \mathbf{y}^{(k)}_{\mathcal{T}_2} - \hat{\mathbf{S}}^{(k)}_{\mathcal{R}}\mathbf{y}^{(k-1)}_{\mathcal{T}_2} \right\|^2, \quad (51)$$

where the elements of the decoding matrix $\hat{\mathbf{S}}^{(k)}_{\mathcal{R}}$ are a function of the transmitted symbols, i.e., $[\mathbf{s}^{(k)}_{\mathcal{R}}]_i = [\mathbf{s}^{(k)}_{\mathcal{T}_1}]_i[\mathbf{s}^{(k)}_{\mathcal{T}_2}]_i$. A similar decoding procedure can be applied at terminal $\mathcal{T}_1$. In order to allow the use of the direct link between the communicating terminals which provides a higher diversity order, an alternative transmission scheme can be applied where in the first phase from time slot 1 to $T/2$ terminal $\mathcal{T}_1$ transmits $\sqrt{4P_{\mathcal{T}_1}}\mathbf{x}^{(k)}_{\mathcal{T}_1}$ while terminal $\mathcal{T}_2$ remains muted and then terminal $\mathcal{T}_2$ transmits $\sqrt{4P_{\mathcal{T}_2}}\mathbf{x}^{(k)}_{\mathcal{T}_2}$ while terminal $\mathcal{T}_1$ remains muted from $T/2+1$ to T. In this case, the terminal that remains muted can receive the transmission of the terminal that transmits such that the direct link can be easily incorporated. We remark that in the latter scheme, the respective terminals transmit with twice the power as compared to the former scheme such that the total transmitted power in both schemes is the same. Similar to (33), the received signal vector at terminal $\mathcal{T}_2$ during the first transmission phase in the presence of the direct link between the communicating terminals is given by

$$\mathbf{y}^{(k)}_{\mathcal{T}_2,\mathrm{dl}} = \sqrt{4P_{\mathcal{T}_1}}f_0\mathbf{x}^{(k)}_{\mathcal{T}_1} + \mathbf{n}^{(k)}_{\mathcal{T}2}, \\ = \sqrt{4P_{\mathcal{T}_1}}f_0 \,\mathrm{diag}(\mathcal{G}_{\mathcal{T}_1}(\mathbf{s}^{(k)}_{\mathcal{T}_1}))\mathbf{x}^{(k-1)}_{\mathcal{T}_1} + \mathbf{n}^{(k)}_{\mathcal{T}2}, \quad (52)$$

where $\mathbf{n}^{(k)}_{\mathcal{T}_2}$ denotes the $T/2\times 1$ vector containing the receiver noise of the kth block at terminal $\mathcal{T}_2$ and $\mathbf{x}^{(k)}_{\mathcal{T}_1}$ is defined in (36). Similar to the decoder of (34) applied in the case where the direct link between the communicating terminals is used with the alternative transmission, the decoder at terminal $\mathcal{T}_2$ can be expressed as

$$\arg\min_{\mathbf{s}^{(k)}_{\mathcal{T}_1}} \|\mathbf{y}^{(k)}_{\mathcal{T}_2} - \hat{\mathbf{S}}^{(k)}_{\mathcal{R}}\mathbf{y}^{(k-1)}_{\mathcal{T}_2}\|^2 + \|\mathbf{y}^{(k)}_{\mathcal{T}_2,\mathrm{dl}} - \mathrm{diag}(\mathcal{G}_{\mathcal{T}_1}(\mathbf{s}^{(k)}_{\mathcal{T}_1}))\mathbf{y}^{(k-1)}_{\mathcal{T}_2,\mathrm{dl}}\|^2. \quad (53)$$

Note that each element of the matrix $\hat{\mathbf{S}}^{(k)}_{\mathcal{R}} = \mathcal{X}(\hat{\mathbf{s}}^{(k)}_{\mathcal{R}})$ is a function of the elements of the information symbol vectors $\mathbf{s}^{(k)}_{\mathcal{T}_1}$ and $\mathbf{s}^{(k)}_{\mathcal{T}_2}$, i.e., $[\mathbf{s}^{(k)}_{\mathcal{R}}]_i = [\mathbf{s}^{(k)}_{\mathcal{T}_1}]_i[\mathbf{s}^{(k)}_{\mathcal{T}_2}]_i$ and $\mathbf{s}^{(k)}_{\mathcal{T}_2}$ is known at terminal $p\mathcal{T}_2$. The decoder of (51) can be implemented using the sphere decoder [28], or in the case of orthogonal DSTCs, a symbol-wise decoder can be applied to decode the received symbols at terminal $\mathcal{T}_2$ [6,9]. The use of the direct link increases the decoding complexity at the destination terminal exponentially with the increase of the constellation size or number of transmitted symbols, as generally a full search over $\mathbf{S}^{(k)}_{\mathcal{T}_1}$ is required. However, this increase is accompanied by a higher diversity order

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provided by the direct link. A similar decoding procedure can be applied at $\mathcal{T}_1$.

On the other hand and similarly as in Section 3, the DF protocol can also be applied to the proposed two-phase technique. In this case, each relay first decodes the information symbols $\mathbf{s}_{\mathcal{T}_1}^{(k)}$ and $\mathbf{s}_{\mathcal{T}_2}^{(k)}$ of the first and second terminals, respectively, defined in (41) and (42) without requiring CSI. Then, similarly as in (43), the decoded information symbol vectors could be combined at the rth relay into a single symbol vector $\hat{\mathbf{s}}_{\mathcal{R}_r}^{(k)}$. In the next phase, the rth relay differentially encodes the combined symbol vector $\hat{\mathbf{s}}_{\mathcal{R}_r}^{(k)}$ using the $T \times T$ precoding matrices $\mathbf{A}_1, \ldots, \mathbf{A}_R$ and $\mathbf{B}_1, \ldots, \mathbf{B}_R$ before broadcasting the resulting vector to both terminals similar to (44) and (45). The proposed two-phase technique using the DF protocol enjoys a substantially lower relay decoding complexity as compared to the previously proposed two-phase DSTC techniques [20].

Note that both transmission strategies described above in this section, i.e., the simultaneous and the alternating achieve exactly the same BER in the case that the direct link between the communicating terminals is not used. We remark however the simultaneous transmission is beneficial in practical hardware implementations as the requirements on the power amplifiers can be relaxed as compared to the alternating transmission, due to a reduced peak-to-average power ration (PAPR). However, the direct link between the communicating terminals cannot be exploited. On the other hand, the alternative transmission can be used to allow the direct link between the communicating terminals.

5. Simulation results

In our simulations, we have assumed a wireless relay network with two single-antenna relay nodes and independent flat Rayleigh fading channels where the power is distributed among the two terminals and relays as $P_{\mathcal{T}_1} = P_{\mathcal{T}_2} = \sum_{r=1}^{R} P_{\mathcal{R}_r}$ similarly as in [16,20]. For fair comparison of the BER performance of all techniques, the same total transmitted power ($P_T = P_{\mathcal{T}_1} + P_{\mathcal{T}_2} + \sum_{r=1}^{R} P_{\mathcal{R}_r}$, $P_{\mathcal{R}_1} = P_{\mathcal{R}_2} = \cdots = P_{\mathcal{R}_R}$) and bit rate are used.

In Fig. 6, the BER at terminal $\mathcal{T}_2$ is displayed versus the SNR and the proposed two-phase non-coherent DSTC technique using BPSK modulation is compared with the four-phase non-coherent distributed Alamouti space–time coding technique using 4-PSK modulation [9] and the two-phase non-coherent DSTC technique proposed in [19] using BPSK modulation for a total rate of 0.5 bit per channel use (bpcu). The four-phase protocol is the conventional one way relaying scheme applied twice where in the first phase the relays receive the signal from the first terminal, process, and forward it in the second phase to the second terminal. Similarly in the third phase, the relays receive the signal from the second terminal, process, and forward it in the fourth phase to the first terminal. In the two-phase protocol, both terminals transmit their information symbols simultaneously to the relays in the first phase and then the relays forward the received signal in the second phase to the destination terminals using either the AF or DF transmission. From Fig. 6, it can be observed that the proposed two-phase technique outperforms the known two- and four-phase technique and enables the terminals to use the direct link to improve the diversity gain where the abbreviation "DL" stands for the use of the direct link in the technique.

In Fig. 7, the label "First proposed 3-phase scheme" stands for the proposed three-phase scheme using $\mathbf{\Phi}_{\mathcal{R},r}^{(k)} = \mathbf{I}_T$ and $\beta_r = \sqrt{P_{\mathcal{R}_r}/(9P_{\mathcal{T}_1}P_{\mathcal{T}_2} + 3P_{\mathcal{T}_1} + 3P_{\mathcal{T}_2})}$, while the label "Second proposed 3-phase scheme" stands for the proposed three-phase scheme using $\mathbf{\Phi}_{\mathcal{R},r}^{(k)} = \mathrm{diag}([1/|[\mathbf{y}_{\mathcal{R}_1,r}^{(k)}]_1[\mathbf{y}_{\mathcal{R}_2,r}^{(k)}]_1|, \ldots, 1/|[\mathbf{y}_{\mathcal{R}_1,r}^{(k)}]_T[\mathbf{y}_{\mathcal{R}_2,r}^{(k)}]_T|]^{\mathrm{T}})$ and $\beta_r = \sqrt{P_{\mathcal{R}_r}}$. In Fig. 7, the BER at terminal $\mathcal{T}_2$ is displayed versus the SNR and the proposed three-phase non-coherent DSTC technique using 8-PSK modulation is compared with the proposed two-phase non-coherent DSTC technique using

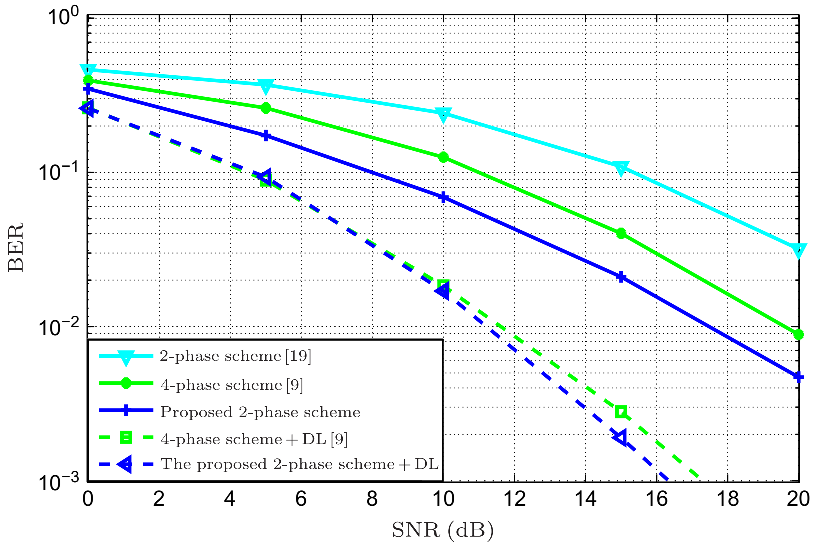

Fig. 6. BER versus SNR for several differential schemes with $R = 2$ and a rate of 0.5 bpcu.

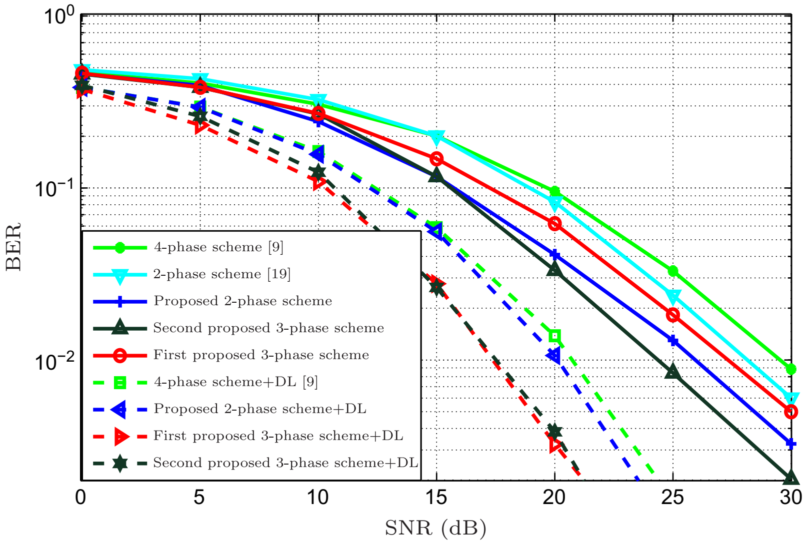

Fig. 7. BER versus SNR for several differential schemes with $R=2$ and a rate of 1 bpcu.

4-PSK modulation, the four-phase non-coherent distributed Alamouti space–time coding technique using 16-PSK modulation [9], and the two-phase non-coherent DSTC technique proposed in [19] using 4-PSK modulation for a total rate of 1 bpcu. From Fig. 7, it can be observed that the proposed two- and three-phase techniques outperform the existing two- and four-phase techniques and also allow the communicating terminals to use the direct link between them to increase the diversity order. In the absence of the direct link between the communicating terminals, the BER performance achieved by the first and the second proposed three-phase scheme is different. This is due to the fact that in the first scheme the relays amplify the noise combined with the received symbol in the transmission while in the second scheme the received signal is properly scaled to avoid noise amplification. From Fig. 7, the techniques that use the direct link between the communicating terminals outperform those without direct link since the direct link increases the diversity gain. Moreover, in the presence of the direct link between the communicating terminals, the proposed three-phase strategy outperforms the proposed two-phase strategy since during the first two phases, the proposed three-phase strategy transmits symbols taken from lower constellation size than those transmitted by the proposed two-phase strategy in the direct link.

6. Conclusion

In this paper, we propose novel differential DSTC techniques for two-way relay networks that do not require CSI at the terminals or relays. Our proposed techniques enjoy a substantially lower relay decoding complexity and provide more coding gain than the state-of-the art techniques by combining the received symbols from both terminals at the relays into a single symbol of the same constellation. Furthermore, our transmission schemes allow the communicating terminals to use the direct link between them to increase the diversity order which is not valid for other simultaneous bidirectional transmission schemes.

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