H. Degenhardt and A. Klein, ”A Network Coding Approach to Non-Regenerative Multi-Antenna Multi-Group Multi-Way Relaying,” in *Proc. of the IEEE Vehicular Technology Conference (VTC2013-Spring), Dresden, Germany, June 2013.*

A Network Coding Approach to Non-Regenerative Multi-Antenna Multi-Group Multi-Way Relaying

Holger Degenhardt[1,2] and Anja Klein[1]
[1] Technische Universität Darmstadt, Institute of Telecommunications, Communications Engineering Lab,
Merckstr. 25, 64283 Darmstadt, Germany, Email: {h.degenhardt, a.klein}@nt.tu-darmstadt.de
[2] Technische Universität Darmstadt, Graduate School CE, Dolivostrasse 15, 64293 Darmstadt, Germany

Abstract—**A multi-group multi-way relaying scenario is considered. Each node has to transmit an individual message and has to receive the messages of all other nodes within its group. These multi-way communications between the multi-antenna nodes are performed via an intermediate non-regenerative multi-antenna relay station. Self- as well as known-interference cancellation are exploited at the nodes and are considered for the derivation of the relay transceive filter. Furthermore, to achieve high spectral efficiency, a transmit strategy which exploits the ideas of network coding is proposed. The proposed transmit strategy combined with the derived relay transceive filter requires less antennas at the relay station and achieves higher sum rates compared to conventional approaches which do not fully exploit the interference cancellation capabilities of the nodes.**

I. Introduction

Relaying techniques can be used to expand the coverage of wireless networks and to increase the achievable throughput.

In [1] and [2], the optimal relay transceive filter is derived for one-way relaying with multiple antennas considering a single-pair scenario. Two-way relaying is proposed in [3] to overcome the duplexing loss of conventional one-way relaying schemes. In [4], non-regenerative multi-antenna two-way relaying in a single-pair scenario is investigated and a minimum mean square error (MMSE) relay transceive filter exploiting self-interference cancellation is derived. Non-regenerative multi-pair two-way relaying with single-antenna nodes and a multi-antenna relay station has been considered in [5]–[7]. The design of network codes for multi-user multi-hop networks has been investigated in [8] and references therein. Considering multi-antenna nodes and exploiting the multiplexing gain increases the achievable sum rates. The authors of [9], [10] investigate a pairwise communication of multi-antenna nodes via an intermediate multi-antenna relay.

Applications such as video conferences or multiplayer gaming as well as emergency applications usually require the data exchange between multiple nodes. If each node of a group wants to share its data with all other nodes within its group, multi-way communications can be performed [11]–[13] to enable spectrally efficient transmissions. To support multiple multi-way communications via an intermediate half-duplex relay station, multi-antenna techniques can be used to spatially separate the communication groups and to enable the simultaneous communication of all nodes. In [11], the full-duplex multi-group multi-way relay channel is investigated. Non-regenerative multi-way relaying via a half-duplex multi-antenna relay station for a single group as well as for a multi-group scenario is considered in [12] and [13], respectively. However, multiple antennas at the nodes are not considered to further increase the sum rate and the proposed transmission schemes do not fully exploit network coding.

In this paper, multiple multi-antenna nodes per group are considered. The multi-way communications are performed via an intermediate half-duplex multi-antenna relay station and the communications between the groups are spatially separated. Network coding is used to exploit the self- and known-interference cancellation capabilities of the nodes. Based on this, a novel transmission strategy is proposed to enable the multi-way communications. The proposed transmission strategy additionally requires a novel relay transceive filter design to fully exploit the self- and known-interference cancellation capabilities at the nodes. Thus, a self- and known-interference aware relay transceive filter is derived and an analytical solution for the relay transceive filter is presented.

The paper is organized as follows. In Section II, the system model is given. A multi-way transmit strategy which exploits network coding is presented in Section III. A self- and known-interference aware transceive filter is derived in Section IV. Simulation results in Section V confirm the analytical investigations and Section VI concludes the paper. [3]

II. System Model

As shown in Figure 1, a multi-group multi-way relaying scenario with multi-antenna nodes and a multi-antenna relay station RS is considered. The scenario in Figure 1 consists of $G = 2$ groups marked by different colors and $N = 3$ nodes per group. Nodes S_1, S_2 and S_3 form the first communication group G_1 and nodes S_4, S_5 and S_6 form the second communication group G_2. The communications are performed via a

[3] Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate and complex conjugate transpose, respectively. The operators $\mathrm{tr}(\cdot)$, $\mathrm{diag}[\cdot]$, $\otimes$ denote the sum of the main diagonal elements of a matrix, the construction of a diagonal matrix with the elements contained in the vector and the Kronecker product of matrices, respectively. The operators $\Re[\cdot]$, $||\cdot||_2$ denote the real part of a scalar or a matrix and the Frobenius norm of a matrix, respectively. The vectorization operator $\mathrm{vec}(Z)$ stacks the columns of matrix Z into a vector. The operator $\mathrm{vec}^{-1}_{M,N}(\cdot)$ is the revision of the operator $\mathrm{vec}(\cdot)$, i.e., a vector of length MN is sequentially divided into N smaller vectors of length M which are combined to a matrix with M rows and N columns. The operator $\mathrm{mod}_y x$ returns the modulus of x after division by y and $\mathbf{I}_M$ denotes an identity matrix of size M.

single subcarrier and, in general, $G \geq 1$ groups and $N \geq 2$ nodes per group are considered. The total number of nodes is given by $K = G \cdot N$ and the term S_k, $k = 1, 2, ..., K$, is used to label the nodes. In each group, the nodes are equipped with $M_g \geq 1$, $g = 1, ..., G$, antennas and simultaneously transmit one data stream per antenna. With L the number of antennas at RS, the following constraint has to be fulfilled for the transmission on one subcarrier to suppress the interferences between the groups and to enable a proper relay transceive filter design within each group:

$$L \geq \sum_{g=1}^{G} N \cdot M_g - \min_{\forall g}(M_g). \tag{1}$$

The required number of antennas L at RS can be smaller than the total number of simultaneously transmitted data streams because network coding is applied to cancel self- as well as known-interferences.

The transmit power at each node and at the relay station RS is limited by $P_{\mathrm{MS,max}}$ and $P_{\mathrm{RS,max}}$, respectively. The channel $\mathbf{H}_k \in \mathbb{C}^{L \times M_g}$ from node S_k of group G_g to RS is assumed to be constant during one transmission cycle of the multi-way scheme which is described in the following and channel reciprocity is assumed. RS is assumed to have perfect channel state information (CSI) and the nodes have receive CSI and can subtract self- and known-interferences. All signals are assumed to be statistically independent and the noise at RS and at the nodes is assumed to be additive white Gaussian with variances $\sigma^2_{\mathrm{n,RS}}$ and σ^2_{n}, respectively. The system equations in baseband for multi-way relaying are presented in the following where all nodes are simultaneously transmitting to RS in the first time slot $t = 1$. The transmitted symbols of S_k are contained in the vector $\mathbf{x}_{\mathrm{S}_k}$ and the transmit filter at S_k is assumed to be $\mathbf{Q}_k = \sqrt{\frac{P_{\mathrm{MS,max}}}{M_g}} \cdot \mathbf{I}_{M_g}$. Thus, the received signal at RS is given by

$$\mathbf{y}_{\mathrm{RS}} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{Q}_k \mathbf{x}_{\mathrm{S}_k} + \mathbf{n}_{\mathrm{RS}}, \tag{2}$$

where $\mathbf{n}_{\mathrm{RS}}$ represents the complex white Gaussian noise vector at RS. Afterwards, the transmissions from RS to the nodes are performed in $N-1$ time slots. Thus, N time slots are required to perform the multi-way communications of all nodes. In time slots $t = 2$, $t = 3$,..., $t = N$, RS linearly processes the received signal using the transceive filter matrices $\mathbf{G}_2$, $\mathbf{G}_3$,..., $\mathbf{G}_N$, respectively. The relay transceive filter for the tth time slot is given by

$$\mathbf{G}_t = \gamma \widetilde{\mathbf{G}}_t, \tag{3}$$

where $\widetilde{\mathbf{G}}_t$ is the transceive filter at RS which does not implicitly fulfill the power constraint and γ is a scalar value to satisfy the relay power constraint. It is given by

$$\gamma = \sqrt{\frac{P_{\mathrm{RS,max}}}{\sum_{k=1}^{K} ||\widetilde{\mathbf{G}}_t \mathbf{H}_k \mathbf{Q}_k||_2^2 + ||\widetilde{\mathbf{G}}_t||_2^2 \sigma^2_{\mathrm{n,RS}}}}. \tag{4}$$

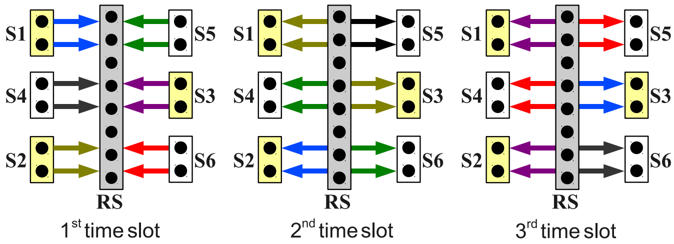

Fig. 1. Desired signals of the proposed NCMW transmit strategy for a multi-group multi-way relaying scenario with $G = 2$ groups, $N = 3$ nodes per group and $M_1 = M_2 = 2$ antennas per node.

The relay transmits the linearly processed version of $\mathbf{y}_{\mathrm{RS}}$ to all nodes. The received signal $\mathbf{y}_{\mathrm{S}_{k,t}}$ using the receive filter $\mathbf{D}_{k,t}$ at node S_k in time slot t is given by

$$\mathbf{y}_{\mathrm{S}_{k,t}} = \mathbf{D}_{k,t}(\mathbf{H}_k^{\mathrm{T}} \mathbf{G}_t \mathbf{y}_{\mathrm{RS}} + \mathbf{n}_{k,t}), \tag{5}$$

where $\mathbf{n}_{k,t}$ represents the complex white Gaussian noise vector at S_k. In this paper, $\mathbf{D}_{k,t} = \mathbf{I}_{M_g}$ is assumed for the receive filtering at the nodes.

III. Hybrid uni-/multicasting strategy exploiting the utilization of network coding

In [13], a hybrid uni-/multicasting strategy as well as a multicasting strategy are proposed to perform multi-way relaying in N time slots. However, both strategies do not fully exploit the utilization of network coding. For the hybrid uni-/multicasting strategy, self- and known-interference cancellation capabilities at the nodes are not considered for the relay transceive filter design. For the multicasting strategy, self- and known-interference cancellation are considered, but network coding is not fully exploited. Furthermore, the transmit power at the relay station is not efficiently used for the multicasting strategy and the strategy suffers from an increase of the forwarded noise power at RS.

Thus, we propose an enhanced hybrid uni-/multicasting strategy which exploits the utilization of network coding. This strategy is termed network coding multi-way (NCMW) strategy in the following. The overall communication is performed in N time slots. In the first time slot, all nodes simultaneously transmit to RS and in the remaining $N-1$ time slots, RS retransmits linearly processed versions of the received signals back to the nodes. Based on the hybrid uni-/multicasting strategy of [13], in each time slot $t = 2, 3, ..., N$, one signal is unicasted to one node and another signal is multicasted to the remaining nodes of the group. However, in contrast to the hybrid uni-/multicasting strategy of [13], we do not consider the unicasted signal as interference at the nodes which receive the multicasted signal and vice versa. Thus, the transmission of the unicasted signal to the nodes which intentionally receive the multicasted signal and vice versa has not to be suppressed by the relay transceive filter. To simplify the descriptions, we assume that the signal of the first node in each group is always unicasted and the signals of the remaining nodes are multicasted. Optimizing the selection of the unicasted signal can further improve the performance.

In each time slot, the receive signals at S_k should consist of a strong desired signal. Furthermore, the power of intra-group and inter-group interferences as well as the noise power should be as low as possible in relation to the receive power of the desired signal. The power of self- and known-interferences can be ignored because these interference are assumed to be perfectly canceled at each node. Nevertheless, no power should be wasted by the relay transceive filter to suppress or forward these interferences.

To describe the NCMW transmission strategy in detail, a scenario consisting of $G \geq 1$ groups with $N = 4$ nodes per group is exemplary considered. For simplicity, we focus on the first group G_1 which consists of the nodes S_1, S_2, S_3 and S_4 to describe the transmissions within each group. Similar to the hybrid uni-/multicasting strategy of [13], in time slot t, the signal of S_1 is unicasted to S_t and the signal of S_t is multicasted to the remaining nodes of the group. However, as already mentioned before, the unicasted signal is not considered as interference at the nodes which receive the multicasted signal and vice versa. Thus, the network coding capabilities at the nodes are exploited. The considered desired signals, self- and known-interferences are summarized in Table I. In each time slot $t = 2, 3, ..., N$, it is assumed that the nodes can subtract the back propagated self-interferences. Furthermore, it is assumed that the nodes can subtract known interferences, i.e., interferences which are known at the nodes due to successful decoding of the corresponding signals in a previous time slot, e.g., each node knows the multicasted signals of the previous time slots. Additionally, the unicasted signal is always assumed to be known at the nodes which receive the multicasted signal. This assumption is possible because the unicasted signal is received and estimated at S_k in time slot $t = k$ and afterwards known-interference cancellation can be applied to the signals received in previous or subsequent time slots.

The indices of the nodes whose signals are assumed to be self- or known-interferences at node S_k in time slot t are collected in the subset $\mathcal{N}_{k,t}$. Self- and known-interferences can be subtracted at each node S_k assuming that the overall channels $\mathbf{H}_k^{\mathrm{T}}\mathbf{G}_t\mathbf{H}_l, \forall l \in \mathcal{N}_{k,t}$ are perfectly known at S_k. The signal, interference and noise covariance matrices after self- and known-interference cancellation for the reception of the desired signal of S_k at S_l in time slot t are given by

$$\begin{aligned} \mathbf{A}_t^{\mathrm{S}_{k,l}} &= \mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{H}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\mathbf{Q}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_l^{*}, \\ \mathbf{B}_t^{\mathrm{S}_{k,l}} &= \mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\left(\sum_{j=1,j\notin\mathcal{N}_{l,t}}^{K}\mathbf{H}_j\mathbf{Q}_j\mathbf{R}_{\mathbf{x}_{\mathrm{S}_j}}\mathbf{Q}_j^{\mathrm{H}}\mathbf{H}_j^{\mathrm{H}}\right)\mathbf{G}_t^{\mathrm{H}}\mathbf{H}_l^{*}, \\ \mathbf{C}_t^{\mathrm{S}_{k,l}} &= \mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}_t^{\mathrm{H}}\mathbf{H}_l^{*} + \mathbf{R}_{\mathbf{n}_{\mathrm{S}k}}, \end{aligned} \tag{6}$$

respectively, with $\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}$ the signal covariance matrix of $\mathbf{x}_{\mathrm{S}_k}$ and $\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}$, $\mathbf{R}_{\mathbf{n}_{\mathrm{S}_l}}$ the noise covariance matrices at RS and S_l, respectively.

Assuming that optimal Gaussian codebooks are used for each data stream, the achievable data rate from S_k to S_l in time slot t is given by

$$C_{\mathrm{S}_{k,l,t}} = \frac{1}{N}\log_2 |(\mathbf{I}_M + \mathbf{A}_t^{\mathrm{S}_{k,l}}(\mathbf{B}_t^{\mathrm{S}_{k,l}} + \mathbf{C}_t^{\mathrm{S}_{k,l}})^{-1})|, \tag{7}$$

where N is the number of required time slots to perform all multi-way transmissions. The maximum achievable multi-way rate for the transmission of S_k is determined by the minimum over all achievable rates from S_k to any other node within the same group. Thus, it is given by

$$C_{\mathrm{S}_{k,\max}} = (N-1)\cdot \min_{\forall l\in\mathcal{N}_g} C_{\mathrm{S}_{k,l,t_l}}, \tag{8}$$

where $\mathcal{N}_g$ contains the indices of all nodes within the group of S_k and t_l is the time slot where the signal transmitted by S_k is the desired signal at S_l. The achievable sum rate is given by

$$C_{\mathrm{sum}} = \sum_{k=1}^{K} C_{\mathrm{S}_{k,\max}}. \tag{9}$$

TABLE I

NCMW TRANSMIT STRATEGY FOR A GROUP OF $N = 4$ NODES

	receiving node	S_1	S_2	S_3	S_4
$t=2$	desired signal	S_2	S_1	S_2	S_2
	self-interference	S_1	S_2	S_3	S_4
	known-interference	-	-	S_1	S_1
$t=3$	desired signal	S_3	S_3	S_1	S_3
	self-interference	S_1	S_2	S_3	S_4
	known-interference	S_2	S_1	-	S_1, S_2
$t=4$	desired signal	S_4	S_4	S_4	S_1
	self-interference	S_1	S_2	S_3	S_4
	known-interference	S_2, S_3	S_1, S_3	S_1, S_2	-

IV. TRANSCEIVE FILTER DESIGN AT RS

To consider the proposed NCMW transmit strategy, conventional zero-forcing (ZF) or minimum mean square error (MMSE) relay transceive filters as presented in [13] cannot be applied because these filters do not facilitate the novel consideration of self- and known-interferences. Thus, a self- and known-interference aware relay transceive filter is derived in the following to maximize the achievable sum rate C_{sum} of (9) for the considered multi-user multi-antenna scenario under the given transmit power constraints. The sum rate maximization is a non-convex problem and an analytical solution cannot be obtained. To tackle this problem, the minimization of the mean square error (MSE) is considered for the relay transceive filter design. For given transmit and receive filters at the nodes, the problem is convex and an analytical relay transceive filter solution can be derived. For the derivation of the self- and known-interference aware relay transceive filter, termed MMSE-SKI, the self- and known-interferences are only considered in the power constraint at RS and are not intentionally suppressed by the transceive filter design. The optimization problem for the relay transceive filter design with respect to the transmit power constraint at RS in time slot t is given by

$$\mathbf{G}_t = \arg\min_{\mathbf{G}_t} \mathrm{E}\left\{\sum_{l=1}^{K}\left\|\mathbf{x}_{\mathrm{S}_k} - \hat{\mathbf{x}}_{\mathrm{S}_{k,l}}\right\|_2^2\right\}, \tag{10}$$

where k is the index of the desired signal at S_l in time slot t and

$$\hat{\mathbf{x}}_{\mathrm{S}_{k,l}} = \mathbf{D}_{l,t}\mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t \sum_{\substack{j=1\\ j\notin\mathcal{N}_{l,t}}}^{K} \mathbf{H}_j\mathbf{Q}_j\mathbf{x}_{\mathrm{S}_j} + \mathbf{D}_{l,t}(\mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{n}_{\mathrm{RS}} + \mathbf{n}_{l,t}). \tag{11}$$

The MSE for the transmission from S_k to S_l in time slot t is given by

$$\begin{aligned}
&\mathrm{E}\left\{\left\|\mathbf{x}_{\mathrm{S}_k} - \hat{\mathbf{x}}_{\mathrm{S}_{k,l}}\right\|_2^2\right\} \\
&= \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\right) - 2\Re\left[\mathrm{tr}\left(\mathbf{D}_{l,t}\mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{H}_{\mathrm{k}}\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\right)\right] \\
&+ \sum_{\substack{j=1\\ j\notin\mathcal{N}_{l,t}}}^{K} \mathrm{tr}\left(\mathbf{D}_{l,t}\mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{H}_j\mathbf{Q}_j\mathbf{R}_{\mathbf{x}_{\mathrm{S}_j}}\mathbf{Q}_j^{\mathrm{H}}\mathbf{H}_j^{\mathrm{H}}\mathbf{G}_t^{\mathrm{H}}\mathbf{H}_l^{*}\mathbf{D}_{l,t}^{\mathrm{H}}\right) \\
&+ \mathrm{tr}\left(\mathbf{D}_{l,t}\mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}_t^{\mathrm{H}}\mathbf{H}_l^{*}\mathbf{D}_{l,t}^{\mathrm{H}} + \mathbf{D}_{l,t}\mathbf{R}_{\mathbf{n}_{\mathrm{S}k}}\mathbf{D}_{l,t}^{\mathrm{H}}\right), \qquad (12)
\end{aligned}$$

where k is again the index of the desired signal at S_l. The MSE of (10) in combination with the power constraint of RS results in a convex problem with respect to $\mathbf{G}_t$. This problem can be solved by using Lagrangian optimization. Let matrices $\mathbf{\Upsilon}^{(k)}$ and $\mathbf{\Upsilon}$ be given by

$$\mathbf{\Upsilon}^{(k)} = \mathbf{H}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\mathbf{Q}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}, \tag{13a}$$

$$\mathbf{\Upsilon} = \sum_{k=1}^{K}\mathbf{H}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\mathbf{Q}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}} + \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}. \tag{13b}$$

Using matrices $\mathbf{\Upsilon}^{(k)}$ and $\mathbf{\Upsilon}$ of (13) in (10) and considering the power constraint at RS, the Lagrangian function with the Lagrangian multiplier η results in

$$L\left(\mathbf{G}_t,\eta\right) = \sum_{l=1}^{K} F(\mathbf{G}_t,k,l) - \eta\left(\mathrm{tr}\left(\mathbf{G}_t\mathbf{\Upsilon}\mathbf{G}_t^{\mathrm{H}}\right) - P_{\mathrm{RS,max}}\right), \tag{14}$$

with

$$\begin{aligned}
F(\mathbf{G}_t,k,l) &= \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\right) - 2\Re\left[\mathrm{tr}\left(\mathbf{D}_{l,t}\mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{H}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\right)\right] \\
&+ \mathrm{tr}\left(\sum_{\substack{j=1\\ j\notin\mathcal{N}_{l,t}}}^{K} \mathbf{D}_{l,t}\mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{\Upsilon}^{(j)}\mathbf{G}_t^{\mathrm{H}}\mathbf{H}_l^{*}\mathbf{D}_{l,t}^{\mathrm{H}}\right) \\
&+ \mathrm{tr}\left(\mathbf{D}_{l,t}\left(\mathbf{H}_l^{\mathrm{T}}\mathbf{G}_t\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}_t^{\mathrm{H}}\mathbf{H}_l^{*} + \mathbf{R}_{\mathbf{n}_{\mathrm{S}_l}}\right)\mathbf{D}_{l,t}^{\mathrm{H}}\right).
\end{aligned} \tag{15}$$

where k is again the index of the desired signal at S_l. From the Lagrangian function, the Karush-Kuhn-Tucker (KKT) conditions can be derived:

$$\frac{\partial L}{\partial \mathbf{G}_t} = \sum_{l=1}^{K} f(\mathbf{G}_t,k,l) - \eta\ \mathbf{G}_t^{*}\mathbf{\Upsilon}^{\mathrm{T}} = \mathbf{0}, \tag{16a}$$

$$\eta\left(\mathrm{tr}\left(\mathbf{G}_t\mathbf{\Upsilon}\mathbf{G}_t^{\mathrm{H}}\right) - P_{\mathrm{RS,max}}\right) = 0, \tag{16b}$$

$$\begin{aligned}
\text{with}\quad f(\mathbf{G}_t,k,l) = &-\mathbf{H}_l\mathbf{D}_{l,t}^{\mathrm{T}}\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}^{\mathrm{T}}\mathbf{Q}_k^{\mathrm{T}}\mathbf{H}_k^{\mathrm{T}} \\
&+ \sum_{\substack{j=1\\ j\notin\mathcal{N}_{l,t}}}^{K} \mathbf{H}_l\mathbf{D}_{l,t}^{\mathrm{T}}\mathbf{D}_{l,t}^{*}\mathbf{H}_l^{\mathrm{H}}\mathbf{G}_t^{*}\mathbf{\Upsilon}^{(j)^{\mathrm{T}}} \\
&+ \mathbf{H}_l\mathbf{D}_{l,t}^{\mathrm{T}}\mathbf{D}_{l,t}^{*}\mathbf{H}_l^{\mathrm{H}}\mathbf{G}_t^{*}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}^{\mathrm{T}}
\end{aligned} \tag{17}$$

The KKT conditions can be used to determine the optimal transceive filter according to (10), because the optimization problem is convex for fixed transmit and receive filters at the nodes. In the following, matrix $\mathbf{K}_t$ is defined as

$$\begin{aligned}
\mathbf{K}_t = &\sum_{l=1}^{K}\sum_{\substack{j=1\\ j\notin\mathcal{N}_{l,t}}}^{K}\left[\mathbf{\Upsilon}^{(j)^{\mathrm{T}}} \otimes \left(\mathbf{H}_l^{*}\mathbf{D}_{l,t}^{\mathrm{H}}\mathbf{D}_{l,t}\mathbf{H}_l^{\mathrm{T}}\right)\right] \\
&+ \sum_{l=1}^{K}\left[\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}^{\mathrm{T}} \otimes \left(\mathbf{H}_l^{*}\mathbf{D}_{l,t}^{\mathrm{H}}\mathbf{D}_{l,t}\mathbf{H}_l^{\mathrm{T}}\right)\right] \\
&+ \left[\mathbf{\Upsilon}^{\mathrm{T}} \otimes \frac{KM\sigma_n^2}{P_{\mathrm{RS,max}}}\mathbf{I}_L\right].
\end{aligned} \tag{18}$$

Using Eqs. (3), (4) and (18), the MMSE-SI filter at RS which solves problem (10) is given by

$$\mathbf{G}_t = \gamma\cdot\mathrm{vec}_{L,L}^{-1}\left(\mathbf{K}_t^{-1}\ \mathrm{vec}\left(\sum_{l=1}^{K}\mathbf{H}_l^{*}\mathbf{D}_{l,t}^{\mathrm{H}}\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\mathbf{Q}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\right)\right), \tag{19}$$

where k is again the index of the desired signal at S_l in time slot t.

V. Simulation results

In this section, numerical results on the achievable sum rates for the proposed NCMW transmission strategy are presented. It is assumed that $P_{\mathrm{MS,max}} = P_{\mathrm{RS,max}}$ and $\sigma_{\mathrm{RS}}^2 = \sigma_{\mathrm{n}}^2$. The path-loss on the i.i.d. Rayleigh fading channels is represented by an average receive signal to noise ratio (SNR) at RS. An average receive SNR at RS of 15dB is assumed.

For comparison, the MMSE and ZF relay transceive filters of [13] are considered using the hybrid uni-/multicasting strategy presented in [13]. For the MMSE relay transceive filter of [13], two cases are distinguished. First, it is assumed that only self-interference cancellation can be performed at the nodes, termed "MMSE, SI@nodes". Secondly, it is assumed that self- and known-interference cancellation can be performed at the nodes, termed "MMSE, SKI@nodes". For the proposed MMSE-SKI relay transceive filter, two different cases are investigated, too. First, only a self-interference aware relay transceive filter design is considered combined with only self-interference cancellation at the nodes, termed "MMSE-SI, SI@nodes". Secondly, a self- and known-interference aware relay transceive filter design as described in Section IV is considered combined with considering self- and known-interference cancellation at the nodes, termed "MMSE-SKI, SKI@nodes".

The average achievable sum rates over different numbers L of antennas at RS for the scenario with $G = 2$ groups

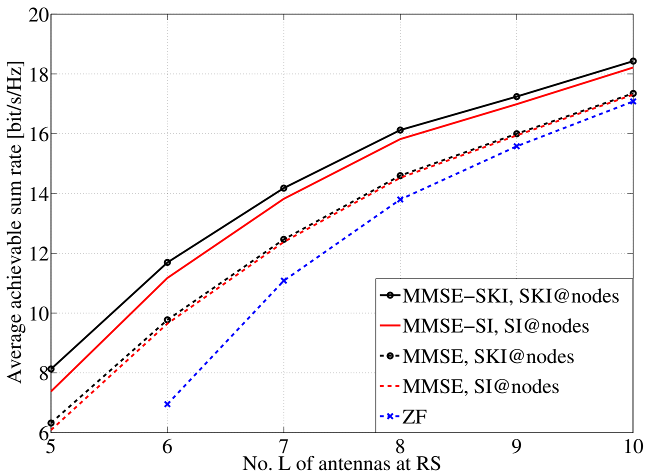

Fig. 2. Average achievable sum rates over number L of antennas at RS for single-antenna nodes, $G = 2$, $N = 3$.

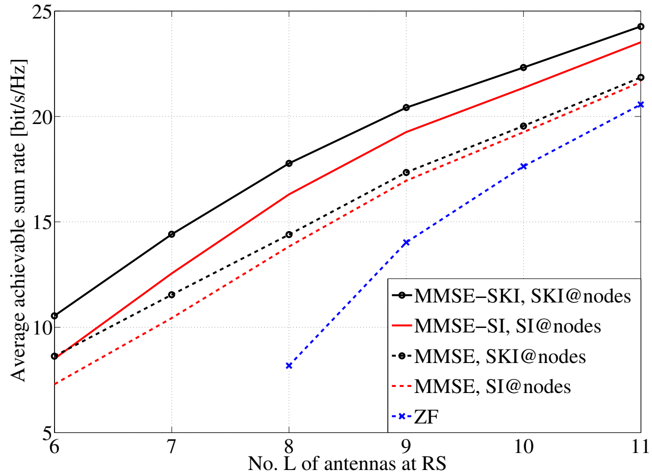

Fig. 3. Average achievable sum rates over number L of antennas at RS for multi-antenna nodes, $G = 1$, $N = 4$, $M_1 = 2$.

and $N = 3$ single-antenna nodes per group are shown in Figure 2. The ZF relay transceive filter requires $L \geq 6$ antennas. The proposed NCMW strategy combined with the MMSE-SKI relay transceive filter "MMSE-SKI, SKI@nodes" clearly outperforms the other approaches because it better exploits the network coding capabilities of multi-way relaying by considering self- and known-interference cancellation at the nodes and for the relay transceive filter design. The performance of only considering self-interference cancellation "SI@nodes" is worse than considering self- and know-interference cancellation "SKI@nodes".

The average achievable sum rates over different numbers L of antennas at RS for the scenario with $G = 1$ group, $N = 4$ nodes per group and $M_1 = 2$ antennas per node are shown in Figure 3. In this scenario, the ZF relay transceive filter requires $L \geq 8$ antennas. In comparison to the aforementioned results, the performance gap between only considering self-interference cancellation and considering self- and know-interference cancellation is increased due to considering an increased number of nodes per group and no inter-group interference. Similar to the aforementioned results, the proposed NCMW strategy combined with the MMSE-SKI relay transceive filter outperforms the other approaches.

VI. Conclusions

A non-regenerative multi-group multi-way relaying scenario has been investigated. A transmission strategy is proposed which exploits network coding to increase the achievable sum rates. Furthermore, an analytical solution for a self- and known-interference aware relay transceive filter considering multi-antenna nodes is derived. The derived relay transceive filter is applied to the proposed transmission strategy which significantly increases the achievable sum rates compared to conventional approaches.

Acknowledgment

The work of Holger Degenhardt is funded by the Deutsche Forschungsgemeinschaft (DFG) under Grant No. Kl907/2-2 and is supported by the LOEWE Priority Program Cocoon and by the Graduate School of Computational Engineering at Technische Universität Darmstadt.

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