

# Optimization of the Rate Adaptation Procedures in xDSL Systems

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**Abstract**—In Digital Subscriber Line (DSL) systems, the noise perceived by a DSL line fluctuates when DSL lines sharing the same bundle are switched on/off. The concept of Seamless Rate Adaptation (SRA) was introduced to seamlessly adapt the data rate of a connection to the changing channel while in operation without any service interruption. Interleavers in communication paths impose a constraint on the amount of data rate that can be changed by an SRA procedure. In the case of large changes in the channel, multiple consecutive SRA procedures have to be executed. Moreover, to keep an equal signal-to-noise-ratio margin over all tones, it is very likely that the bit loadings of all tones have to be modified in each SRA procedure. Hence, the state of the art SRA is too slow and provides means of adaptation only to slowly changing channels and will not prevent reinitializations under certain conditions such as large rapid changes in noise. In this paper, we optimize the rate adaptation procedure in terms of adaptation time and average bit error rate. We present an algorithm that solves the stated problem optimally and a sub-optimal low complex algorithm that can be used in practice by modems with limited computing power. Both algorithms lead to significant gains in terms of adaptation time and average bit error rate.

## I. INTRODUCTION

Digital Subscriber Lines (DSLs) have emerged as a key technology over the last years since services which pose high requirements on data rates, latency and line stability such as video and voice-over-IP have become part of our every day life. The constantly increasing number of DSL users and their growing bandwidth demand have made far-end crosstalk (FEXT) between copper wires the dominant impairment in current DSL systems [9], [10]. The noise perceived by a DSL line is comprised of the summation of the crosstalk from all active lines in the bundle. According to [11], the number of active DSL users is strongly daytime dependent and consequently, the distribution of noise on DSL lines is not stationary.

One approach to protect against time-varying non-stationary noise is to set the target bit rate such that it can still be achieved at times with peak noise levels. Clearly, this will lead to a data rate loss at times with low noise levels [3], [12], [13]. Another approach is to adapt the target data rate to the changing channel conditions. ADSL2/ADSL2plus and VDSL2 address this problem by including the ability to seamlessly adapt the data rate on-line [5]. This ability, called Seamless Rate Adaptation (SRA), enables a DSL system to change the data rate of the connection while in operation without any service interruption [5]. An SRA procedure is executed if the signal-to-noise-ratio (SNR) margin reaches a threshold value. The SNR margin is the decrease in the SNR that can be withstood by the system at the same target data rate and target error probability [3]. After an SRA procedure, modems transmit

with a new bit loading where the number of bits transmitted in a Discrete Multi Tone (DMT) symbol is different than before the procedure and the SNR margin and consequently the bit error probability are equal over all tones. SRA uses the sophisticated Online Reconfiguration (OLR) procedures to coordinate the data rate change between the transmitter and receiver. According to [4], one OLR request can modify up to 128 tones. In order to equalize the SNR margin after the channel conditions have changed, it is very likely that more than 128 tones have to be modified in one SRA procedure. In that case, consecutive OLR requests are sent. The transmission with the new bit loading begins after the last OLR request has been acknowledged.

Care must be taken when SRA is implemented in communication paths that include interleavers. Interleavers are used to protect the data against bursts of errors by dispersing errors over the data stream. Interleavers operate on blocks of data. An output cannot be generated until several input blocks have been received. The time required to receive the input blocks causes an interleaver delay, which increases when data rate is reduced and vice versa. Similarly, the degree of impulse noise protection decreases when the data rate is increased since more bits will be corrupted by an impulse noise over a period of time, and vice versa. The way to achieve true seamless behavior when an interleaver is enabled is to change the interleaver depth in proportion to the data rate so that the overall delay and the degree of impulse noise protection remain constant [7]. Changing the interleaver depth introduces an offset in the transmission since it requires emptying the interleaver and filling it up again according to the new depth. Dummy bytes insertion methods like [7] can minimize this offset to the difference between the interleaver delays that correspond to the different depth. In the VDSL2 standard [4], the change of the interleaver depth is limited by setting a maximum value for this offset through the parameter maximum delay variation  $DV_{\max}$ . Consequently, the data rate change in one SRA procedure is also limited by  $DV_{\max}$ . If the data rate change allowed by  $DV_{\max}$  is not sufficient to fully adapt to the current channel, a number of consecutive SRA procedures are executed.

Considering the above, in the case of large changes in the channel conditions, e.g. if a strong VDSL2 interferer is switched on, adapting the data rate to the new channel will have to be done in multiple SRA procedures in order to satisfy the maximum delay variation constraint. Furthermore, in order to have an equal SNR margin over all tones after each procedure, multiple OLR requests each modifying 128 tones have to be sent within each SRA procedure. Obviously, the adaption will require a long time. If the SRA procedures fail to restore a

positive SNR margin over all tones within a certain period of time, the connection is interrupted and modems will have to reinitialize at a lower data rate. Hence, the state of the art SRA provides means of adapting only to slowly changing channels and will not prevent reinitializations under certain conditions such as large rapid changes in noise [5], [6].

In this paper we optimize the rate adaptation procedures in terms of adaptation time and average bit error rate during the adaptation. We show that in the case of multiple consecutive SRA procedures, modifying each tone only once throughout the whole adaptation procedure and minimizing the average bit error rate over all tones instead of equalizing the SNR margin after each SRA procedure leads to a tremendous decrease in the adaption time and therefore less occurred errors. We present an algorithm that solves the stated problem optimally and a sub-optimal low complex algorithm that can be used in practice by modems with limited computing power. Both algorithms lead to significant gains in terms of adaptation time and average bit error rate.

## II. SYSTEM MODEL

An  $N$ -user DSL binder is considered. The  $N$  users employ DMT over tones  $k = 1, \dots, K$ . Assuming perfect synchronization of the modems and a sufficiently long cyclic prefix, each tone can be modeled as an independent  $N$ -user Interference Channel. Let  $h_k^{n,n}$  denote the direct channel coefficient of user  $n$  on tone  $k$  and  $h_k^{n,m}$  ( $n \neq m$ ) denote the FEXT channel coefficient from disturber  $m$  to user  $n$ . Furthermore, we define  $(\sigma^n)^2$  as the variance of the white Gaussian noise received by user  $n$  and  $s_k^n$  as his average transmit power on tone  $k$ . For the system explained above, the signal-to-noise-ratio (SNR) seen at the receiver of the victim user  $n$  and used for the bit loading of tone  $k$  is given by

$$SNR_k = \frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m \neq n} |h_k^{n,m}|^2 s_k^m + (\sigma^n)^2}. \quad (1)$$

Moreover, let  $\Gamma$  denote the SNR gap to the capacity of an uncoded QAM system. Let  $\gamma_k$  denote the SNR margin on tone  $k$ . The SNR margin  $\gamma_k$  describes the decrease in the  $SNR_k$  that can be withstood at tone  $k$  such that the same bit-loading and SNR gap can still be used at that tone. Using the Shannon-gap approximation [2], [1], the bit-loading on tone  $k$  results in  $b_k = \log_2 \left( 1 + \frac{1}{\Gamma \gamma_k} SNR_k \right)$ . With  $f_s$  being the symbol rate, the achievable total data rate  $R$  of user  $n$  is then found by

$$R = f_s \sum_k b_k. \quad (2)$$

Moreover, assuming a M-Quadrature Amplitude Modulation (M-QAM), and defining  $M_k$  as the constellation size at tone  $k$  and  $Q(\cdot)$  as the Gaussian  $Q$  function, the symbol error rate of an M-QAM symbol can be approximated by [14]

$$SER_k \approx 2Q \left( \sqrt{\frac{3}{M_k - 1} SNR_k} \right). \quad (3)$$

According to [8], the bit error rate (BER) at tone  $k$  is given by

$$BER_k = \frac{2^{b_k - 1}}{2^{b_k} - 1} SER_k. \quad (4)$$

The average bit error rate over  $K$  tones is then given by

$$BER_{\text{avg}} = \frac{\sum_k BER_k b_k}{\sum_k b_k}. \quad (5)$$

## III. SEAMLESS RATE ADAPTATION

As already mentioned in section I, in order to keep the interleaver delay and the degree of impulse noise protection constant, the interleaver depth  $D$  has to be changed in proportion to the data rate  $R$ . The parameter  $DV_{\text{max}}$  limits the change of the depth  $D$  and therefore limits the change in  $R$ . By defining the interleaver delay  $d_{\text{int}}$  and defining the higher data rate in the data rate change  $R_{\text{high}}$ , the change in the data rate allowed by an SRA procedure is given by [4]

$$\Delta R \leq \frac{DV_{\text{max}} R_{\text{high}}}{d_{\text{int}}} = \Delta R_{\text{max}}. \quad (6)$$

Furthermore, SRA uses the OLR procedure to coordinate the change in the transmission parameters that leads to the new data rate. Before the first OLR request is sent, the receiver modem measures the channel over a time period  $T_{\text{meas}}$  and calculates the new bit loading over a time period  $T_{\text{cal}}$ . The OLR request carrying the new bit loading parameters for up to 128 tones is then sent over the time  $T_{\text{req}}$  to the transmitter. After the reception of the request at the transmitter side, the requested change in the transmission parameters is processed consuming the time  $T_{\text{pr}}$ . The transmitter can either reject or acknowledge the request. In the second case, an acknowledgment specifying the time when the transmission with the new parameters should take place is sent back to the receiver modem. The transmission time of the acknowledgment is denoted by  $T_{\text{ack}}$  and the time between receiving the acknowledgment and the beginning of transmission with the new parameters is denoted by  $T_{\text{syn}}$ .

In the case where more than 128 tones have to be modified, consecutive OLR procedures are executed, where a request can only be sent when the acknowledgment of the previous request has been received. Moreover, the transmission with the new parameters begins  $T_{\text{syn}}$  after the reception of the last acknowledgment.

Assuming all the overhead traffic is transmitted over a separate robust overhead channel (ROC) as described in [4], one can assume that all the transmitted messages will be received correctly at the other end. By defining  $N_{T,i}$  as the number of tones modified by SRA procedure  $i$  and  $\lceil \cdot \rceil$  as the ceiling function, the time needed by SRA procedure  $i$  is given by

$$T_{\text{sra}, N_{T,i}} = T_{\text{meas}} + T_{\text{cal}} + \sum_{l=1}^{\lceil \frac{N_{T,i}}{128} \rceil} T_{\text{req},l} + \lceil \frac{N_{T,i}}{128} \rceil (T_{\text{pr}} + T_{\text{ack}}) + T_{\text{syn}}. \quad (7)$$

Note that  $T_{\text{req},l}$  increases linearly with the number of tones modified in OLR procedure  $l$ . Furthermore, by defining the number of SRA procedures  $N_{\text{sra}}$  needed to fully adapt to the channel while each procedure fulfills the data rate constraint in (6), the total adaptation time is given by

$$T_{\text{adapt}} = \sum_{i=1}^{N_{\text{sra}}} T_{\text{sra}, N_{T,i}}. \quad (8)$$

In the case of large changes in the channel conditions, e.g. a strong VDSL2 interferer is switched on, the state of the art SRA procedure will have to modify all  $K$  tones which requires  $\lceil \frac{K}{128} \rceil$  OLR procedures in order to equalize the SNR margin after every procedure  $i$ . Obviously, the adaptation time will be very large.

#### IV. PROBLEM FORMULATION

As shown in the previous section, in order to equalize the SNR margin after every SRA procedure  $i$ , each tone has to be modified  $N_{\text{sra}}$  times. Moreover, equalizing the SNR margin prevents from reinitializations only if a positive margin across all tones can be restored by the equalization. As discussed above, this is not necessarily the case when multiple SRA procedures are executed.

For the purpose of accelerating the adaptation procedure, in this paper we propose to modify each tone only once throughout the whole adaptation procedure. After each SRA procedure, the target bit loading of the tones modified by that procedure is achieved and only after the last SRA procedure, an equal margin is restored. The crux here is to choose the tones to be modified by every SRA procedure such that the average BER is decreased as rapidly as possible while fulfilling the data rate constraint in (6).

Let us define  $\mathbb{V}$  as the set of  $V$  tones that have not yet been modified during the adaptation procedure and  $b_v^*$  and  $\text{BER}_v^*$  the target bit loading and the target BER at tone  $v$ , respectively. Furthermore, we define  $\mathbf{x}$  as a vector with  $V$  binary integer variables, where  $x_v = 1$  and  $x_v = 0$  indicate that tone  $v$  will be modified and will not be modified by the next SRA procedure, respectively. By denoting the average BER before an SRA procedure as  $\text{BER}_{\text{avg},0}$ , the problem stated above can then be formulated as the following binary integer minimization problem with  $\mathbf{x}$  as an optimization parameter:

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} \quad \frac{\sum_v b_v \text{BER}_v (1-x_v) + \sum_v b_v^* \text{BER}_v^* x_v}{\sum_v b_v (1-x_v) + \sum_v b_v^* x_v} - \text{BER}_{\text{avg},0} \\ & \text{subject to} \quad f_s \cdot \sum_v (b_v - b_v^*) x_v \leq \Delta R_{\text{max}} \end{aligned} \quad (9)$$

#### V. PROPOSED ALGORITHMS

In this section, we will present two algorithms. The first one solves the minimization problem in (9) optimally. The second one is a low complex sub-optimal algorithm.

##### A. Optimal SRA

Considering Equations (7) and (8), the minimum number of OLR procedures throughout the whole adaptation procedure is  $\lceil \frac{K}{128} \rceil$ . This number is achieved when  $z \cdot 128$  tones, where  $z = 1, \dots, Z$ , are modified in each SRA procedure. If  $K$  is not a multiple of 128, the number of tones modified by the last SRA procedure will be less than  $z \cdot 128$ . With this observation, the SRA time in (9) becomes constant for every  $z$  and the optimization problem (9) splits into  $Z$  optimization problems that are given, after rearranging the objective function, by

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} \quad \frac{\sum_v (b_v^* \text{BER}_v^* - b_v \text{BER}_v) x_v + \sum_v b_v \text{BER}_v}{\sum_v (b_v^* - b_v) x_v + \sum_v b_v} \\ & \text{subject to} \quad f_s \cdot \sum_v (b_v - b_v^*) x_v \leq \Delta R_{\text{max}}, \\ & \quad \sum_v x_v = z \cdot 128, \quad z = 1, \dots, Z. \end{aligned} \quad (10)$$

The  $Z$  minimization problems above can be solved using the bisection method on the objective function described by Algorithm 1.

By defining the vector  $\mathbf{a}$  containing  $b_v^* \text{BER}_v^* - b_v \text{BER}_v$  and  $\mathbf{q}$  containing  $b_v^* - b_v$  for  $v = 1, \dots, V$ , Algorithm 1 is given by

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#### Algorithm 1 Bisection method to find the optimum $\mathbf{x}$

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- 1: Initialize  $\text{UP}_b = \text{BER}_{\text{avg},0}$ ,  $\text{LOW}_b = \text{target BER}_{\text{avg}}$
  - 2: Choose a tolerance  $\epsilon \geq 0$
  - 3: **repeat**
  - 4: Find  $\mathbf{x}$  that satisfies the following linear constraints
  - 5:  $\mathbf{a}^T \mathbf{x} + \sum_v b_v \text{BER}_v \leq \text{UP}_b (\mathbf{q}^T \mathbf{x} + \sum_v b_v)$
  - 6:  $\mathbf{a}^T \mathbf{x} + \sum_v b_v \text{BER}_v \geq \text{LOW}_b (\mathbf{q}^T \mathbf{x} + \sum_v b_v)$
  - 7:  $f_s \cdot \sum_v (b_v - b_v^*) x_v \leq \Delta R_{\text{max}}$
  - 8:  $\sum_v x_v = z \cdot 128$
  - 9: **if**  $\mathbf{x}$  exists **then**
  - 10:  $\text{UP}_b = \frac{\text{UP}_b + \text{LOW}_b}{2}$
  - 11: **else**
  - 12:  $\delta = \text{LOW}_b$ ,  $\text{LOW}_b = \text{UP}_b$
  - 13:  $\text{UP}_b = \text{UP}_b + \frac{\text{UP}_b - \delta}{2}$
  - 14: **end if**
  - 15: **until**  $\text{UP}_b - \text{LOW}_b \leq \epsilon$
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The  $Z$  minimization problems yield the solution vectors  $\mathbf{x}_z$ ,  $z = 1, \dots, Z$ . The solution vector that corresponds to the smallest value of the objective function in (9) is the best choice of tones to be modified by an SRA procedure such that the average BER is decreased as rapidly as possible while fulfilling the data rate constraint in (6).

##### B. Low Complex SRA

Solving the previous binary integer problems is a very complex task that requires high computing power. In practice, the computing power of modems is limited and, therefore, low complex algorithms are needed. The following Algorithm 2 chooses to first modify the tones that will lead to the largest decrease in the average BER.

By defining  $\text{BER}_{\text{avg},x_v}$  as the average BER when only tone  $v$  is modified to the target bit loading, Algorithm 2 is given by

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#### Algorithm 2 Low complex algorithm to find a sub-optimal $\mathbf{x}$

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- 1: Compute the vector  $\Delta \text{BER}_{\text{avg},v} = \text{BER}_{\text{avg},0} - \text{BER}_{\text{avg},x_v}$ ,  $v = 1, \dots, V$
  - 2: Sort  $\Delta \text{BER}_{\text{avg},v}$  in descending order yielding  $\Delta \text{BER}_{\text{avg},\text{sorted}}$
  - 3: Compute the permutation vector  $\mathbf{W}$  containing the indices of  $\Delta \text{BER}_{\text{avg},v}$  in their order in  $\Delta \text{BER}_{\text{avg},\text{sorted}}$ , with entries  $w_1, \dots, w_W$
  - 4: Initialize  $y = 1$
  - 5: **while**  $\sum_{p=1}^y b_{w_p} - b_{w_p}^* \leq \Delta R_{\text{max}}$  **do**
  - 6:  $y = y + 1$
  - 7:  $x_{w_p} = 1$
  - 8: **end while**
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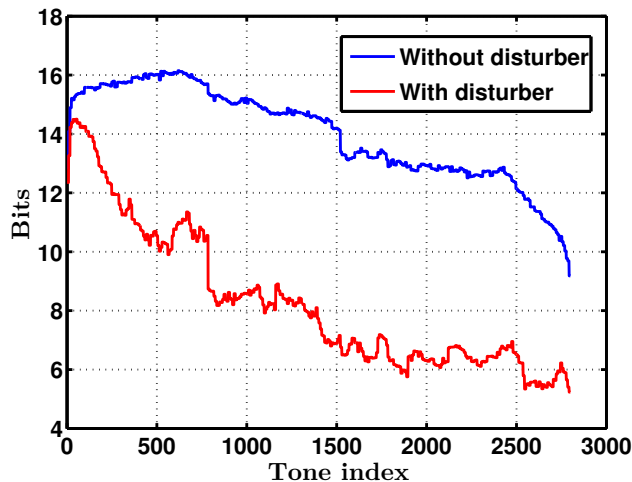


Fig. 1. Bit loading of the victim line when the disturber is switched off and when the disturber is switched on.

TABLE I. SYSTEM PARAMETERS

symbol rate $f_s$	4kHz
number of downstream tones $K$	2784
SNR gap	9.8dB
SNR margin	1dB
Data rate of overhead channel	256 bits/ms
$DV_{max}$	1 ms
$d_{int}$	20.3 ms
$T_{meas}$	64 ms
$T_{cal}$	100 ms
$T_{req}$	$(12 + 4 \cdot N_T)/256$ ms
$T_{pr}$	140 ms
$T_{ack}$	0.1 ms
$T_{syn}$	16.25 ms

## VI. SIMULATION RESULTS

This section presents simulation results to show the benefits of using the proposed SRA Algorithms over the state of the art SRA.

For the simulations, a downstream scenario with a VDSL2 victim line and a VDSL2 disturber line is assumed. The SNRs used for calculating the bit loadings were measured at ADTRAN's DSL lab [13]. The transmit PSD masks used were according to VDSL2 band plan 998ADE17M2xB [4]. Moreover, the parameters in Table I were assumed. To evaluate the SRA Algorithms, the bit loading of the victim line is calculated when the disturber is switched off and then again when the disturber is switched on, as shown in Figure 1. The SRA Algorithms are then used to seamlessly decrease the first bit loading until the second one is reached.

Figure 2 shows the average BER over the adaptation time when using the state of the art SRA and when using the proposed Algorithms. Clearly, both proposed Algorithms outperform the state of the art SRA in terms of average BER and adaptation time. Moreover, as expected the optimal SRA outperforms the low Complex SRA. However, the difference in performance between the optimal and the low complex SRA is small which makes the Low complex SRA a very good candidate to be used by modems with limited computing power.

## VII. CONCLUSION

In this paper, we have shown that in the case of multiple consecutive SRA procedures, modifying each tone only once

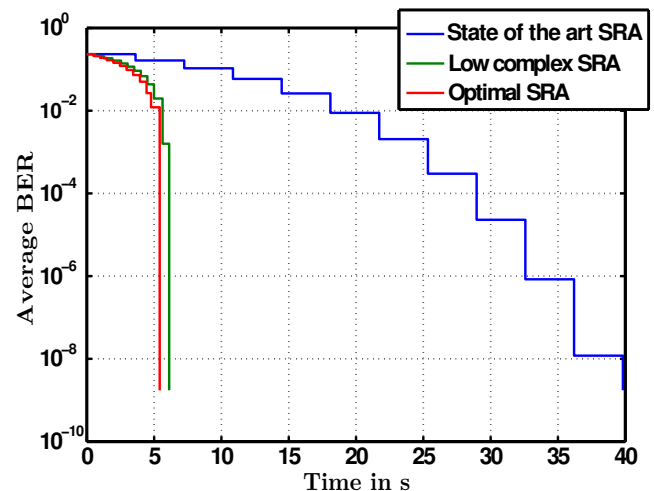


Fig. 2. Average BER over the adaptation time when using the state of the art SRA and when using the proposed Algorithms.

throughout the whole adaptation procedure leads to a tremendous decrease in the adaption time and therefore less occurred errors. Moreover, we have optimally solved the problem of choosing the tones to be modified by each SRA procedure such that the average BER is decreased as rapidly as possible while fulfilling the data rate constraint in (6) and presented a low complex algorithm that achieves similar performance as the optimal one. The low complex algorithm can be used in practice by modems with limited computing power to seamlessly adapt to changing channels.

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