H. Degenhardt, F. Hohmann and A. Klein, ”Pilot Transmission Scheme and Robust Filter Design for Non-Regenerative Multi-Pair Two-Way Relaying,” in *Proc. of the IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'12), Sydney, Australia, September 2012.*

Pilot Transmission Scheme and Robust Filter Design for Non-Regenerative Multi-Pair Two-Way Relaying

Holger Degenhardt, Fabian Hohmann and Anja Klein
Institute of Telecommunications, Communications Engineering Lab,
Technische Universität Darmstadt, Merckstr. 25, 64283 Darmstadt, Germany
Email: {h.degenhardt, a.klein}@nt.tu-darmstadt.de

***Abstract*—In this paper, a pilot transmission scheme and a robust self-interference aware relay transceive filter for multi-pair two-way relaying are introduced. The bidirectional pairwise communications of multiple single-antenna nodes are simultaneously performed via an intermediate non-regenerative multi-antenna relay station and a pilot transmission scheme is proposed to obtain channel state information at the relay station and at the nodes. It is assumed that the nodes can use the available channel state information to subtract the back-propagated self-interference and the cases of perfect and imperfect self-interference cancellation are investigated. The relay station performs linear signal processing based on the estimated channels and a robust relay transceive filter approach is introduced which utilizes the fact that the nodes can perform self-interference cancellation. The proposed pilot transmission scheme requires less resources for channel estimation than conventional schemes and the proposed robust filter design increases the achievable sum rate in case of imperfect channel state information.**

I. Introduction

In wireless networks, relaying techniques can be used to expand the coverage and to increase the achievable throughput. To perform multiple bidirectional communications via an intermediate half-duplex relay station RS, multi-antenna techniques can be used to spatially separate the communicating node pairs and to enable the simultaneous communication of all pairs. Within each pair, the two-way relaying protocol of [1] can be applied to overcome the duplexing loss of conventional one-way relaying schemes. Using the two-way relaying protocol and performing a spatial separation of the node pairs at RS, only two time slots are required to perform the overall bidirectional communications. Thus, significantly higher overall sum rates can be achieved using a multi-antenna relay station which spatially separates the pairwise communications than using direct communications between the nodes and applying a frequency or time division access scheme for interference mitigation.

The existing works on non-regenerative multi-pair two-way relaying [2]–[6] assume perfect channel state information (CSI) at the nodes and at RS to subtract the back-propagated self-interference and to determine the relay transceive filter, respectively. However, in realistic scenarios, the channels have to be estimated [7]. Thus, the available CSI is not perfect due to estimation errors caused by noisy measurements, quantization and / or outdated CSI.

The authors of [8]–[11] investigate robust filter design for non-regenerative single-pair multiple-input multiple-output (MIMO) one-way relaying with imperfect CSI. Robust filter design for one-way relaying with multiple source-destination pairs is investigated in [12], [13].

The authors of [14], [15] investigate single-pair two-way relaying with imperfect CSI. Channel estimation and training design for single-pair two-way relaying is considered in [16]–[19]. However, channel estimation and robust filter design for multi-pair two-way relaying have not been investigated, so far.

In this paper, non-regenerative multi-pair two-way relaying with imperfect CSI is investigated. The bidirectional communications between the single antenna nodes are supported by an intermediate non-regenerative multi-antenna relay station which performs linear signal processing based on the available imperfect CSI. To obtain CSI at the relay station and at the nodes, a novel pilot transmission scheme for multi-pair two-way relaying is proposed. Furthermore, it is assumed that the nodes can subtract the back-propagated self-interference and the cases of perfect and imperfect self-interference cancellation are considered. Additionally, a robust self-interference aware relay transceive filter is introduced which minimizes the mean square error between the estimated and the transmitted signals if the proposed pilot transmission scheme is applied.

The paper is organized as follows. In Section II, the system model is presented. A novel pilot transmission scheme for multi-pair two-way relaying is proposed in Section III. In Section IV, a robust self-interference aware transceive filter is introduced. Simulation results in Section V confirm the analytical investigations and Section VI concludes the paper.

Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{*}$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate and complex conjugate transpose, respectively. The operators $\mathrm{tr}(\cdot)$, $\otimes$ denote the sum of the main diagonal elements of a matrix and the Kronecker product of matrices, respectively. The operators $\Re[\cdot]$ and $||\cdot||_2$ denote the real part of a scalar and the Frobenius norm of a matrix, respectively. The matrix vectorization operator $\mathrm{vec}(Z)$ stacks the columns of matrix Z into a vector. The operator $\mathrm{vec}^{-1}_{M,N}(\cdot)$ is the revision of the operator $\mathrm{vec}(\cdot)$, i.e., a vector of length MN is sequentially divided into N vectors of length M which are combined to a matrix with M rows and N columns. The operator $\mathrm{mod}_y x$ returns the modulus of x after division by y and $\mathbf{I}_M$ denotes an identity matrix of size $M \times M$.

II. System Model

In this paper, K pairwise bidirectional communications via an intermediate non-regenerative multi-antenna relay station RS of $2K$ single-antenna half-duplex nodes are considered as shown in Figure 1. Nodes S_k and S_l form a bidirectional communication pair for $l = k - 1 + 2 \cdot \mathrm{mod}_2 k$, $k = 1, 2, ..., 2K$, i.e., S_1 and S_2, S_3 and S_4, ..., S_{2K-1} and S_{2K} form bidirectional communication pairs. It is assumed that all signals are transmitted via one single carrier and time division duplex is used. In the first time slot, all nodes are simultaneously transmitting to RS and within each pair, the two-way protocol of [1] is applied. In the second time slot, RS retransmits a linearly processed version of the received signals towards the nodes. This scheme is termed non-regenerative multi-pair two-way relaying. The transmit powers at each node and at RS are limited by P_{node} and P_{RS}, respectively. The number of antennas at RS is given by $L \geq 2K - 1$ to enable the suppression of inter-pair-interference and it is assumed that the nodes can subtract the back-propagated self-interference based on the available CSI.

Channel reciprocity is assumed and the SIMO Rayleigh fading channels $\mathbf{h}_k \in \mathbb{C}^{L \times 1}$ from S_k to RS are assumed to be constant during one transmission cycle of the two-way scheme. All signals are assumed to be statistically independent and the noise at RS and at the nodes is assumed to be additive white Gaussian with variances $\sigma^2_{\mathrm{n,RS}}$ and σ^2_{n}, respectively. Furthermore, it is assumed that the nodes transmit with maximum power and the transmitted symbol of S_k is described by s_k, $\mathrm{E}\{s_k s_k^*\} = P_{\mathrm{node}}$. Thus, the received baseband signal at RS for multi-pair two-way relaying [2]–[6] is given by

$$\mathbf{y}_{\mathrm{RS}} = \sum_{k=1}^{2K} \mathbf{h}_k s_k + \mathbf{n}_{\mathrm{RS}}, \tag{1}$$

where $\mathbf{n}_{\mathrm{RS}}$ represents the complex white Gaussian noise vector at RS. RS linearly processes the received superimposed signals using a relay transceive filter $\mathbf{G}$ as described in Section IV. To ensure that the power constraint at RS is fulfilled, the following definition for $\mathbf{G}$ is used:

$$\mathbf{G} = \gamma \widetilde{\mathbf{G}}, \tag{2}$$

$$\gamma = \sqrt{\frac{P_{\mathrm{RS,max}}}{\sum_{k=1}^{2K} ||\widetilde{\mathbf{G}}\mathbf{h}_k||_2^2 P_{\mathrm{node}} + ||\widetilde{\mathbf{G}}||_2^2 \sigma^2_{\mathrm{n,RS}}}}, \tag{3}$$

where $\widetilde{\mathbf{G}}$ is the transceive filter at RS which does not implicitly fulfill the power constraint and γ is a scalar value to satisfy the relay power constraint [6]. The received signal y_k at node S_k is given by

$$y_k = \mathbf{h}_k^{\mathrm{T}} \mathbf{G} \mathbf{y}_{\mathrm{RS}} + n_k, \tag{4}$$

where n_k represents the complex white Gaussian noise at S_k. The compositions of the receive signals are also illustrated

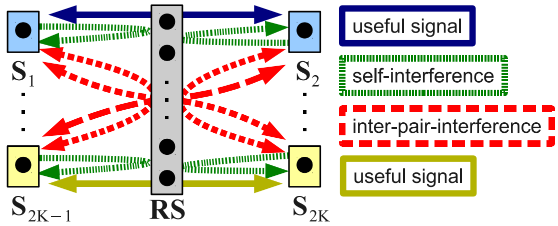

Fig. 1. Composition of useful signals and interferences in a bidirectional multi-pair two-way relaying scenario.

in Figure 1. Each node receives its intended useful signal, receives interference from the signals intended for the other node pairs termed inter-pair-interference, and receives back-propagated self-interference as well as noise. The inter-pair-interference has to be mitigated by the transceive filter at RS, but the back-propagated self-interference can be subtracted at each node [1] assuming that $h_{\mathrm{SI},k} = \mathbf{h}_k^{\mathrm{T}} \mathbf{G} \mathbf{h}_k$ is known at S_k. With $\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}$ the noise covariance matrix at RS, the signal, interference and noise powers for the transmission from S_k to S_l assuming perfect self-interference cancellation are given by

$$\begin{aligned} P_{\mathrm{S},k} &= \mathbf{h}_l^{\mathrm{T}} \mathbf{G} \mathbf{h}_k P_{\mathrm{node}} \mathbf{h}_k^{\mathrm{H}} \mathbf{G}^{\mathrm{H}} \mathbf{h}_l^*, \\ P_{\mathrm{I},k} &= \mathbf{h}_l^{\mathrm{T}} \mathbf{G} \left(\sum_{j=1, j \neq k,l}^{2K} \mathbf{h}_j P_{\mathrm{node}} \mathbf{h}_j^{\mathrm{H}} \right) \mathbf{G}^{\mathrm{H}} \mathbf{h}_l^*, \\ P_{\mathrm{n},k} &= \mathbf{h}_l^{\mathrm{T}} \mathbf{G} \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}} \mathbf{G}^{\mathrm{H}} \mathbf{h}_l^* + \sigma^2_{\mathrm{n}}, \end{aligned} \tag{5}$$

respectively. If Gaussian codebooks are used for each data stream, the achievable rate from S_k to S_l is given by

$$C_{\mathrm{S}_k} = \frac{1}{2} \log_2 (1 + P_{\mathrm{S},k} (P_{\mathrm{I},k} + P_{\mathrm{n},k})^{-1}), \tag{6}$$

and the sum rate C_{sum} is given by

$$C_{\mathrm{sum}} = \sum_{k=1}^{2K} C_{\mathrm{S}_k}. \tag{7}$$

III. Pilot Transmission Scheme for Multi-Pair Two-Way Relaying

In this section, a novel pilot transmission scheme for pilot assisted channel estimation (PACE) in non-regenerative multi-pair two-way relaying scenarios is introduced. As mentioned in Section II, the channels are assumed to be constant during one transmission cycle of the multi-pair two-way scheme and all transmissions are performed via one single carrier. Thus, the proposed PACE can be performed between the receive and the transmit phase of the multi-pair two-way scheme.

In multi-pair two-way relaying, the multi-antenna relay station requires knowledge of the channels $\mathbf{h}_k$, $k = 1, 2, ..., 2K$ of all nodes to determine a transceive filter which suppresses inter-pair interference and which maximizes the achievable sum rate. Furthermore, knowledge of the back-propagation channel $h_{\mathrm{SI},k}$ is required at S_k to subtract the back-propagated self-interference. Additionally, knowledge of the concatenated receive channel $h_{\mathrm{Rx},k} = \mathbf{h}_k^{\mathrm{T}} \mathbf{G} \mathbf{h}_l$ is required at S_k to estimate the symbols transmitted by its communication partner S_l.

To effectively estimate the channels $\mathbf{h}_k$ at RS, the nodes have to transmit pilot symbols on orthogonal resources [19].

For simplicity, it is assumed that each node transmits a single pilot symbol p_k for channel estimation and that the pilot transmissions of the nodes are separated in time as shown in Figure 2, but for the simulations the extension to t transmitted pilot symbols per node is also considered. In this case, the transmission scheme is repeated t times. In the considered scenario, $2K$ time slots are needed to obtain the required CSI at RS. After obtaining the required CSI at RS and computing the relay transceive filter $\mathbf{G}$ as described in Section IV, the pilot symbols can be retransmitted after transceive filtering by RS to obtain the required CSI at the nodes. However, if each pilot symbol is retransmitted separately, $2K$ additional time slots are required. Thus, in total $4K$ time slots would be required by such a scheme to obtain the required CSI at RS and at the nodes.

In the following, a novel pilot transmission scheme is proposed which only requires $2K + 2$ time slots to obtain the required CSI at RS and at the nodes. The proposed pilot transmission scheme consists of two phases which are shown in Figure 2. In the first phase, each node transmits one pilot symbol to RS as described above. The pilot transmissions of the nodes are performed sequentially and are separated in time. The receive signal at RS for receiving the known pilot symbol p_k from node S_k is given by

$$\mathbf{y}_{\mathrm{pilot},k} = \mathbf{h}_k p_k + \mathbf{n}_{\mathrm{RS},k}, \tag{8}$$

where $\mathbf{n}_{\mathrm{RS},k}$ represents the complex white Gaussian noise vector at RS during the reception of p_k. Thus, the least squares estimate of the channel $\mathbf{h}_k$ at RS is given by

$$\hat{\mathbf{h}}_k = \frac{\mathbf{y}_{\mathrm{pilot},k}}{p_k} = \mathbf{h}_k + \mathbf{e}_k, \tag{9}$$

with

$$\mathbf{e}_k = \frac{\mathbf{n}_{\mathrm{RS},k}}{p_k} \tag{10}$$

modeling the estimation error due to additive noise at RS. The elements of $\mathbf{e}_k$ are zero mean with variance

$$\sigma_e^2 = \frac{\sigma_{\mathrm{n,RS}}^2}{P_{\mathrm{node}}}. \tag{11}$$

Based on the estimated channels at RS, the relay transceive filter $\mathbf{G}$ is calculated. The relay transceive filter is designed to suppress inter-pair interferences with respect to a minimum mean square error (MMSE) criteria. This enables the simultaneous retransmission of pilot symbols of nodes which do not belong to the same pair, because the interferences between these pilot symbols are suppressed. The pilot symbols of the nodes S_k and S_l which belong to the same pair have to be retransmitted separately, because the channels $h_{\mathrm{SI},k}$ and $h_{\mathrm{Rx},k}$ cannot be estimated simultaneously at S_k and because the interferences between these pilot symbols are not suppressed by the relay transceive filter. Thus, the second phase requires two time slots and two retransmission groups are formed which contain one node of each pair as shown in Figure 2. In each time slot, the received pilot symbols of each group

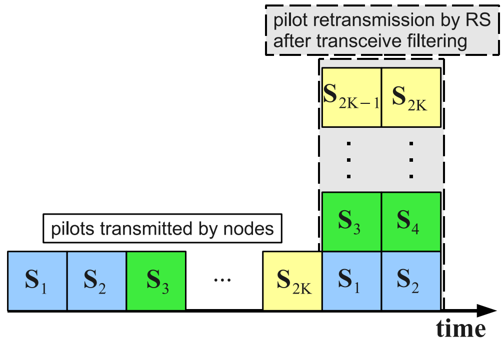

Fig. 2. Pilot transmission scheme for multi-pair two-way relaying.

are superimposed and retransmitted via $\mathbf{G}$. Thus, the transmit signals of RS in the first and second time slot are given by

$$\mathbf{x}_{\mathrm{RS},1} = \mathbf{G} \sum_{k=1}^{K} \mathbf{y}_{\mathrm{pilot},2k-1}, \tag{12}$$

$$\mathbf{x}_{\mathrm{RS},2} = \mathbf{G} \sum_{k=1}^{K} \mathbf{y}_{\mathrm{pilot},2k}, \tag{13}$$

respectively. Assuming that all nodes transmit the same pilot symbol $p_k = p$, the least squares estimates of $\hat{h}_{\mathrm{SI},k}$ and $\hat{h}_{\mathrm{Rx},k}$ for a node with an odd index k are given by

$$\hat{h}_{\mathrm{SI},k} = \mathbf{h}_k^{\mathrm{T}} \mathbf{G} \sum_{j=1}^{K} \hat{\mathbf{h}}_{2j-1} + f_{k,1}, \tag{14}$$

$$\hat{h}_{\mathrm{Rx},k} = \mathbf{h}_k^{\mathrm{T}} \mathbf{G} \sum_{j=1}^{K} \hat{\mathbf{h}}_{2j} + f_{k,2}, \tag{15}$$

respectively, where $f_{k,1/2}$ models the estimation error due to additive noise at S_k. $f_{k,1/2}$ has zero mean and variance

$$\sigma_f^2 = \frac{\sigma_{\mathrm{n}}^2}{P_{\mathrm{node}}}. \tag{16}$$

Based on the estimated channels, the nodes can perform receive processing and can subtract the back-propagated self-interference.

IV. Robust Self-Interference Aware Relay Transceive Filter

The authors of [2]–[6] propose different relay transceive filters assuming perfect CSI at RS. However, only imperfect CSI estimates can be obtained in a realistic scenario which decreases the performance of the proposed transceive filters. To recover part of this performance loss, a robust relay transceive filter design for non-regenerative multi-pair two-way relaying is introduced in this section.

The achievable sum rate C_{sum} of (7) for the considered multi-user multi-antenna scenario under the given transmit power constraints for the proposed PACE scheme shall be maximized. The sum rate maximization is a non-convex problem and an analytical solution cannot be obtained. To tackle this problem an MMSE filter design is proposed. In the following, a robust self-interference aware MMSE relay transceive filter termed RMMSE-SI is derived which is an extension of the non-robust MMSE-SI transceive filter presented in [6]. For the derivation of the RMMSE-SI transceive filter, the variances σ_e^2 and σ_f^2 are assumed to be known. Furthermore, the receive filter coefficients at the nodes are assumed to be one. The general equation for the transceive filter design at RS is given by

$$\mathbf{G} = \arg\min_{\mathbf{G}} \mathrm{E}\left\{\sum_{k=1}^{2K} \|s_k - \hat{s}_k\|_2^2\right\}, \tag{17}$$

where the mean square error (MSE) for the transmission from S_k to S_l is given by

$$\begin{aligned}\mathrm{E}\left\{\|s_k - \hat{s}_k\|_2^2\right\} &= P_{\mathrm{node}} - 2\Re\left[\mathbf{h}_l^{\mathrm{T}}\mathbf{G}\mathbf{h}_{\mathrm{k}}P_{\mathrm{node}}\right] \\ &+ \sum_{j=1,j\neq l}^{2K} \mathbf{h}_l^{\mathrm{T}}\mathbf{G}\mathbf{h}_j P_{\mathrm{node}}\mathbf{h}_j^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{h}_l^* \\ &+ \left(h_{\mathrm{SI},l} - \hat{h}_{\mathrm{SI},l}\right)\left(h_{\mathrm{SI},l} - \hat{h}_{\mathrm{SI},l}\right)^{\mathrm{H}} P_{\mathrm{node}} \\ &+ \mathbf{h}_l^{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^{\mathrm{H}}\mathbf{h}_l^* + \sigma_n^2. \end{aligned} \tag{18}$$

The true channels are not known at RS and the robust relay transceive filter is designed based on the estimated channels $\hat{\mathbf{h}}_k$ and based on the knowledge of the error variances σ_e^2 and σ_f^2. Thus, the MSE for the transmission from a node with odd index k to a node with even index $l = k - 1 + 2 \cdot \mod {}_2 k$ in terms of the estimated channels and the error variances is given by

$$\begin{aligned}\mathrm{E}\left\{\|s_k - \hat{s}_k\|_2^2\right\} &= P_{\mathrm{node}} - 2\Re\left[\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\hat{\mathbf{h}}_{\mathrm{k}}P_{\mathrm{node}}\right] \\ &+ \sum_{j=1,j\neq l}^{2K}\left(\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\hat{\mathbf{h}}_j\hat{\mathbf{h}}_j^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_l^* + \sigma_e^2\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_l^*\right)P_{\mathrm{node}} \\ &+ \sum_{j=1,j\neq l}^{2K}\left(\sigma_e^2\mathrm{tr}(\mathbf{G}\hat{\mathbf{h}}_j\hat{\mathbf{h}}_j^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}) + \sigma_e^4\mathrm{tr}(\mathbf{G}\mathbf{G}^{\mathrm{H}})\right)P_{\mathrm{node}} \\ &+ \left(\sigma_e^2\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_l^* + \sigma_e^4\mathrm{tr}(\mathbf{G}\mathbf{G}^{\mathrm{H}})\right)P_{\mathrm{node}} + \sigma_f^2 \\ &+ \sum_{j=1,j\neq l/2}^{K}\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\hat{\mathbf{h}}_{2j}\hat{\mathbf{h}}_{2j}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_l^*P_{\mathrm{node}} \\ &+ \sigma_e^2\sum_{j=1,j\neq l/2}^{K}\mathrm{tr}(\mathbf{G}\hat{\mathbf{h}}_{2j}\hat{\mathbf{h}}_{2j}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}})P_{\mathrm{node}} \\ &+ \hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_l^* + \sigma_e^2\mathrm{tr}(\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^{\mathrm{H}}) + \sigma_n^2. \end{aligned} \tag{19}$$

For the transmission from a node with even index l to a node with odd index k, the MSE equations are similar. The MSE of (17) using (19) in combination with the power constraint P_{RS} of RS results in a convex problem with respect to $\mathbf{G}$. This problem can be solved by using Lagrangian optimization. Let matrices $\boldsymbol{\Upsilon}^{(k)}$, $\boldsymbol{\Omega}^{(k)}$ and $\boldsymbol{\Upsilon}$ be given by

$$\boldsymbol{\Upsilon}^{(k)} = \hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^{\mathrm{H}}P_{\mathrm{node}} + \sigma_e^2\mathbf{I}_L P_{\mathrm{node}} + \frac{1}{2K-1}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}, \tag{20a}$$

$$\boldsymbol{\Omega}^{(k)} = \hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^{\mathrm{H}}P_{\mathrm{node}}, \tag{20b}$$

$$\boldsymbol{\Upsilon} = \sum_{k=1}^{2K}\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^{\mathrm{H}}P_{\mathrm{node}} + \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}. \tag{20c}$$

Using matrices $\boldsymbol{\Upsilon}^{(k)}$, $\boldsymbol{\Omega}^{(k)}$ and $\boldsymbol{\Upsilon}$ of (20) in (17) and (19) and considering the power constraint at RS, the Lagrangian function with the Lagrangian multiplier η results in

$$\begin{aligned} L(\mathbf{G},\eta) = &\sum_{k=1}^{2K}F_0(\mathbf{G},k) + \sum_{k=1}^{K}F_1(\mathbf{G},2k) + F_2(\mathbf{G},2k-1) \\ &- \eta\left(\mathrm{tr}\left(\mathbf{G}\boldsymbol{\Upsilon}\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{RS,max}}\right), \end{aligned} \tag{21}$$

with

$$\begin{aligned} F_0(\mathbf{G},k) &= P_{\mathrm{node}} - 2\Re\left[\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\hat{\mathbf{h}}_{\mathrm{k}}P_{\mathrm{node}}\right] \\ &+ \mathrm{tr}\left(\sum_{j=1,j\neq l}^{2K}\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\boldsymbol{\Upsilon}^{(j)}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_l^*\right) + \sigma_n^2 \\ &+ \sigma_e^2\mathrm{tr}\left(\sum_{j=1,j\neq l}^{2K}\mathbf{G}\boldsymbol{\Upsilon}^{(j)}\mathbf{G}^{\mathrm{H}}\right) \\ &+ \left(\sigma_e^2\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_l^* + \sigma_e^4\mathrm{tr}(\mathbf{G}\mathbf{G}^{\mathrm{H}})\right)P_{\mathrm{node}} + \sigma_f^2 \end{aligned} \tag{22a}$$

using $l = k - 1 + 2\cdot\mathrm{mod}_2 k,$ and with

$$\begin{aligned} F_1(\mathbf{G},l) &= \sum_{j=1,j\neq l/2}^{K}\hat{\mathbf{h}}_l^{\mathrm{T}}\mathbf{G}\boldsymbol{\Omega}^{(2j)}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_l^* \\ &+ \sigma_e^2\sum_{j=1,j\neq l/2}^{K}\left(\mathrm{tr}(\mathbf{G}\hat{\mathbf{h}}_{2j}\hat{\mathbf{h}}_{2j}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}})\right)P_{\mathrm{node}} \end{aligned} \tag{22b}$$

$$\begin{aligned} F_2(\mathbf{G},k) &= \sum_{j=1,j\neq(k+1)/2}^{K}\hat{\mathbf{h}}_k^{\mathrm{T}}\mathbf{G}\boldsymbol{\Omega}^{(2j-1)}\mathbf{G}^{\mathrm{H}}\hat{\mathbf{h}}_k^* \\ &+ \sigma_e^2\sum_{j=1,j\neq(k+1)/2}^{K}\left(\mathrm{tr}(\mathbf{G}\hat{\mathbf{h}}_{2j-1}\hat{\mathbf{h}}_{2j-1}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}})\right)P_{\mathrm{node}}. \end{aligned} \tag{22c}$$

From the Lagrangian function, the Karush-Kuhn-Tucker (KKT) conditions can be derived:

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{G}} = &\sum_{k=1}^{2K}f_0(\mathbf{G},k) + \sum_{k=1}^{K}f_1(\mathbf{G},2k) + f_2(\mathbf{G},2k-1) \\ &- \eta\,\mathbf{G}^*\boldsymbol{\Upsilon}^{\mathrm{T}} = \mathbf{0}, \end{aligned} \tag{23a}$$

$$\eta\left(\mathrm{tr}\left(\mathbf{G}\boldsymbol{\Upsilon}\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{RS,max}}\right) = 0, \tag{23b}$$

with

$$\begin{aligned} f_0(\mathbf{G},k) = &-\hat{\mathbf{h}}_l\hat{\mathbf{h}}_k^{\mathrm{T}}P_{\mathrm{node}} + \sum_{j=1,j\neq l}^{2K}\hat{\mathbf{h}}_l\hat{\mathbf{h}}_l^{\mathrm{H}}\mathbf{G}^*{\boldsymbol{\Upsilon}^{(j)}}^{\mathrm{T}} \\ &+ \sigma_e^2\sum_{j=1,j\neq l}^{2K}\mathbf{G}^*{\boldsymbol{\Upsilon}^{(j)}}^{\mathrm{T}} + \sigma_e^4\mathbf{G}^*P_{\mathrm{node}} \\ &+ \sigma_e^2\hat{\mathbf{h}}_l\hat{\mathbf{h}}_l^{\mathrm{H}}\mathbf{G}^*P_{\mathrm{node}} \end{aligned} \tag{24a}$$

$$f_1(\mathbf{G},l) = \sum_{j=1, j\neq l/2}^{K} \hat{\mathbf{h}}_l \hat{\mathbf{h}}_l^{\mathrm{H}} \mathbf{G}^* \mathbf{\Omega}^{(2j)^{\mathrm{T}}} + \sigma_e^2 \sum_{j=1, j\neq l/2}^{K} \mathbf{G}^* (\hat{\mathbf{h}}_{2j}\hat{\mathbf{h}}_{2j}^{\mathrm{H}})^{\mathrm{T}} P_{\mathrm{node}} \tag{24b}$$

$$f_2(\mathbf{G},k) = \sum_{j=1, j\neq (k+1)/2}^{K} \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^{\mathrm{H}} \mathbf{G}^* \mathbf{\Omega}^{(2j-1)^{\mathrm{T}}} + \sigma_e^2 \sum_{j=1, j\neq (k+1)/2}^{K} \mathbf{G}^* (\hat{\mathbf{h}}_{2j-1}\hat{\mathbf{h}}_{2j-1}^{\mathrm{H}})^{\mathrm{T}} P_{\mathrm{node}} \tag{24c}$$

The KKT conditions can be used to determine the transceive filter according to (17), because the optimization problem is convex. In the following, matrix $\mathbf{K}$ is defined as

$$\begin{aligned}
\mathbf{K} = & \left[\mathbf{\Upsilon}^{\mathrm{T}} \otimes \frac{2K\sigma_n^2}{P_{\mathrm{RS,max}}}\mathbf{I}_L\right] + \sum_{k=1}^{2K} \sigma_e^2 P_{\mathrm{node}} \left[\mathbf{I}_L \otimes \left(\hat{\mathbf{h}}_k^* \hat{\mathbf{h}}_k^{\mathrm{T}}\right)\right] \\
& + \sum_{k=1}^{2K} \sum_{j=1, j\neq k}^{2K} \left[\mathbf{\Upsilon}^{(j)^{\mathrm{T}}} \otimes \left(\hat{\mathbf{h}}_k^* \hat{\mathbf{h}}_k^{\mathrm{T}}\right)\right] \\
& + \sum_{k=1}^{2K} \left(\sum_{j=1, j\neq k}^{2K} \sigma_e^2 \left[\mathbf{\Upsilon}^{(j)^{\mathrm{T}}} \otimes \mathbf{I}_L\right] + \sigma_e^4 P_{\mathrm{node}} \mathbf{I}_{L^2} \right) \\
& + \sum_{k=1}^{K} \sum_{j=1, j\neq k}^{K} \left[\mathbf{\Omega}^{(2j)^{\mathrm{T}}} \otimes \left(\hat{\mathbf{h}}_{2k}^* \hat{\mathbf{h}}_{2k}^{\mathrm{T}}\right)\right] \\
& + \sigma_e^2 \sum_{k=1}^{K} \sum_{j=1, j\neq k}^{K} P_{\mathrm{node}} \left[\left(\hat{\mathbf{h}}_{2k}^* \hat{\mathbf{h}}_{2k}^{\mathrm{T}} \otimes \mathbf{I}_L\right)\right] \\
& + \sum_{k=1}^{K} \sum_{j=1, j\neq k}^{K} \left[\mathbf{\Omega}^{(2j-1)^{\mathrm{T}}} \otimes \left(\hat{\mathbf{h}}_{2k-1}^* \hat{\mathbf{h}}_{2k-1}^{\mathrm{T}}\right)\right] \\
& + \sigma_e^2 \sum_{k=1}^{K} \sum_{j=1, j\neq k}^{K} P_{\mathrm{node}} \left[\left(\hat{\mathbf{h}}_{2k-1}^* \hat{\mathbf{h}}_{2k-1}^{\mathrm{T}} \otimes \mathbf{I}_L\right)\right].
\end{aligned} \tag{25}$$

Using Eqs. (3) and (25), the RMMSE-SI transceive filter at RS which solves problem (17) based on the available CSI is given by

$$\mathbf{G} = \gamma \cdot \mathrm{vec}_{L,L}^{-1} \left(\mathbf{K}^{-1} \ \mathrm{vec}\left(\sum_{k=1}^{2K} \hat{\mathbf{h}}_k^* P_{\mathrm{node}}, \hat{\mathbf{h}}_l^{\mathrm{H}} \right) \right) \tag{26}$$

where $l = k - 1 + 2 \cdot \mathrm{mod}_2 k$. The derived robust RMMSE-SI relay transceive filter minimizes the MSE based on the proposed pilot transmission scheme. Modifying the equations for a PACE scheme which does not perform the superposition of pilot symbols in the second phase is straightforward.

V. Performance results

In this section, the achievable sum rates of the proposed approaches are investigated by numerical results. The channels between the nodes and RS are assumed to be i.i.d. Rayleigh fading channels with an average channel gain of one. It is assumed that $P_{\mathrm{RS,max}} = 4P_{\mathrm{MS,max}}$, $\sigma_{\mathrm{n,RS}}^2 = \sigma_{\mathrm{n}}^2$. Furthermore, it is assumed that $K = 4$ pairs are simultaneously transmitting to RS which is equipped with $L = 8$ antennas. The ratio $P_{\mathrm{MS,max}}/\sigma_{\mathrm{n,RS}}^2$ between the maximum transmit power at the nodes and the noise power at RS is termed average receive signal to noise ratio (SNR) at RS.

The following approaches are compared:

- MMSE-SI (perfect CSI): self-interference aware MMSE-SI transceive filter of [6] using perfect CSI.
- RMMSE-SI (PACE, case 1 / 2): robust RMMSE-SI relay transceive filter proposed in this paper using the CSI which is obtained via the proposed PACE scheme. In case 1, perfect self-interference cancellation at the nodes is assumed and in case 2, the nodes perform self-interference cancellation based on the CSI obtained via the proposed PACE scheme.
- MMSE-SI (PACE, case 1 / 2): self-interference aware MMSE-SI transceive filter of [6] using the CSI which is obtained via the proposed PACE scheme. In case 1, perfect self-interference cancellation at the nodes is assumed and in case 2, the nodes perform self-interference cancellation based on the CSI obtained via the proposed PACE scheme.
- conv. MMSE (PACE, case 2): conventional MMSE relay transceive filter which suppresses the back propagation of self-interference and uses the CSI obtained via the proposed PACE scheme. The nodes subtract the remaining self-interference based on the CSI obtained via the proposed PACE scheme.

The average achievable sum rates versus the average receive SNR at RS are shown in Figure 3. The achievable sum rate assuming perfect CSI at all nodes and at RS is significantly higher than the achievable sum rate of the approaches which are based on the CSI obtained via the proposed PACE scheme. However, having perfect CSI at all nodes and at RS is not a realistic assumption. The achievable sum rates applying the proposed RMMSE-SI filter are approximately $18 - 20\%$ higher compared to the achievable sum rates of the MMSE-SI filter for an average receive SNR at RS of 10dB. The gain between the robust and non-robust filters decreases for higher SNR values, because the estimation error decreases. The performance of using a conventional MMSE relay transceive filter is worse than using the proposed RMMSE-SI transceive filter and the gap increases with increasing SNR values. The gain of using a robust relay transceive filter at RS compared to a non-robust transceive filter is higher in case of imperfect self-interference cancellation compared to assuming perfect self-interference cancellation.

In Figure 4, the average achievable sum rates versus the average receive SNR at RS are shown for using one (1 pilot) or two (2 pilots) transmitted pilot symbols per node for channel estimation. Using two pilot symbols doubles the required resources for channel estimation, but the achievable sum rate is increased, because the estimation error decreases and sum rates which are closer to the transceive filter design with perfect CSI can be achieved. In this figure, only the achievable sum rates

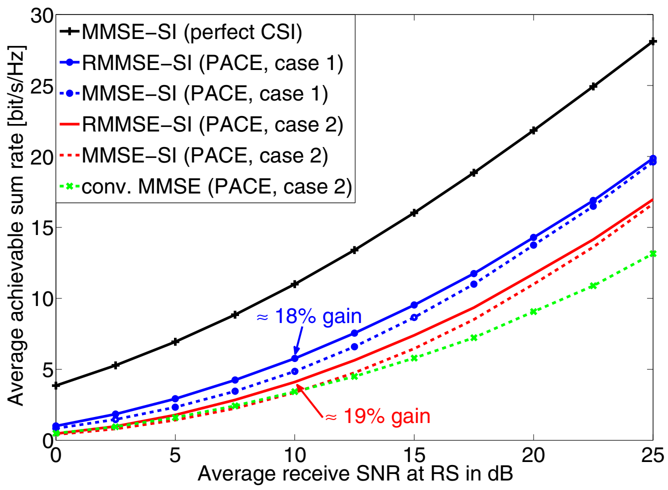

Fig. 3. Average achievable sum rates versus average receive SNR at RS for $L = 8$ considering one transmitted pilot symbol per node.

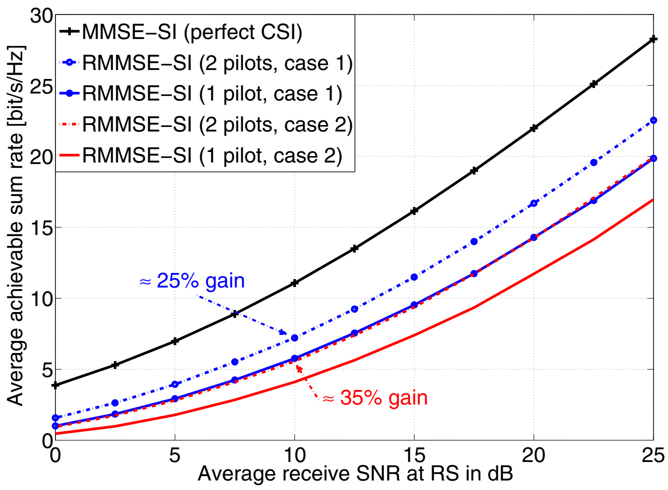

Fig. 4. Average achievable sum rates versus average receive SNR at RS for $L = 8$ considering one and two transmitted pilot symbols per node.

of the RMMSE-SI filter are compared, because the other relay transceive filters perform worse. The gain of using a second training sequence for an average receive SNR at RS of 10dB is approximately 25% if perfect self-interference cancellation is assumed and approximately 35% in case of imperfect self-interference cancellation.

VI. Conclusions

Non-regenerative multi-pair two-way relaying with imperfect CSI has been investigated. A novel pilot transmission scheme is proposed which requires less resources than a conventional pilot transmission scheme to obtain CSI at the nodes and at the relay station due to the superposition of pilot symbols. Furthermore, a robust self-interference aware relay transceive filter termed RMMSE-SI is introduced which significantly increases the achievable sum rate in case of imperfect CSI at the relay station. Additionally, the impact of imperfect self-interference cancellation has been investigated.

Acknowledgment

The work of Holger Degenhardt is funded by the Deutsche Forschungsgesellschaft (DFG) under Grant No. Kl907/2-2 and is supported by the LOEWE Priority Program Cocoon and by the Graduate School of Computational Engineering at Technische Universität Darmstadt.

References

[1] B. Rankov and A. Wittneben, “Spectral efficient protocols for half-duplex fading relay channels,” *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 379–389, 2007.

[2] E. Yilmaz, R. Zakhour, D. Gesbert, and R. Knopp, “Multi-Pair Two-Way Relay Channel with Multiple Antenna Relay Station,” in *Proc. IEEE International Conference on Communications (ICC)*, 2010, pp. 1–5.

[3] A. Amah and A. Klein, “Pair-aware transceive beamforming for non-regenerative multi-user two-way relaying,” in *Proc. IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, 2010, pp. 2506–2509.

[4] C. Y. Leow, Z. Ding, K. Leung, and D. Goeckel, “On the Study of Analogue Network Coding for Multi-Pair, Bidirectional Relay Channels,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 2, pp. 670–681, 2011.

[5] J. Joung and A. Sayed, “Multiuser Two-Way Amplify-and-Forward Relay Processing and Power Control Methods for Beamforming Systems,” *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1833–1846, 2010.

[6] H. Degenhardt and A. Klein, “Self-interference aware MIMO filter design for non-regenerative multi-pair two-way relaying,” in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, 2012.

[7] B. Hassibi and B. Hochwald, “How much training is needed in multiple-antenna wireless links?” *IEEE Transactions on Information Theory*, vol. 49, no. 4, pp. 951–963, 2003.

[8] C. Xing, S. Ma, Y.-C. Wu, T.-S. Ng, and H. V. Poor, “Linear Transceiver Design for Amplify-And-Forward MIMO Relay Systems under Channel Uncertainties,” in *IEEE Wireless Communications and Networking Conference (WCNC)*, 2010, pp. 1–6.

[9] C. Xing, S. Ma, Y.-C. Wu, and T.-S. Ng, “Transceiver Design for Dual-Hop Nonregenerative MIMO-OFDM Relay Systems Under Channel Uncertainties,” *IEEE Transactions on Signal Processing*, vol. 58, no. 12, pp. 6325–6339, 2010.

[10] Y. Rong, “Robust Design for Linear Non-Regenerative MIMO Relays With Imperfect Channel State Information,” *IEEE Transactions on Signal Processing*, vol. 59, no. 5, pp. 2455–2460, 2011.

[11] B. K. Chalise and L. Vandendorpe, “Joint linear processing for an amplify-and-forward MIMO relay channel with imperfect channel state information,” *EURASIP J. Adv. Signal Process.*, 2010.

[12] B. Chalise and L. Vandendorpe, “MIMO Relay Design for Multipoint-to-Multipoint Communications With Imperfect Channel State Information,” *IEEE Transactions on Signal Processing*, vol. 57, no. 7, pp. 2785–2796, 2009.

[13] P. Ubaidulla and A. Chockalingam, “Relay Precoder Optimization in MIMO-Relay Networks With Imperfect CSI,” *IEEE Transactions on Signal Processing*, vol. 59, no. 11, pp. 5473–5484, 2011.

[14] A. Panah and R. Heath, “MIMO Two-Way Amplify-and-Forward Relaying With Imperfect Receiver CSI,” *IEEE Transactions on Vehicular Technology*, vol. 59, no. 9, pp. 4377–4387, 2010.

[15] J. Zou, W. Liu, M. Ding, H. Luo, and H. Yu, “Transceiver Design for AF MIMO Two-Way Relay Systems with Imperfect Channel Estimation,” in *IEEE Global Telecommunications Conference (GLOBECOM 2011)*, 2011, pp. 1–5.

[16] F. Gao, R. Zhang, and Y.-C. Liang, “Optimal channel estimation and training design for two-way relay networks,” *IEEE Transactions on Communications*, vol. 57, no. 10, pp. 3024–3033, 2009.

[17] B. Jiang, F. Gao, X. Gao, and A. Nallanathan, “Channel estimation and training design for two-way relay networks with power allocation,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 6, pp. 2022–2032, 2010.

[18] G. Wang, F. Gao, W. Chen, and C. Tellambura, “Channel Estimation and Training Design for Two-Way Relay Networks in Time-Selective Fading Environments,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 8, pp. 2681–2691, 2011.

[19] T. Unger, “Multi-antenna two-hop relaying for bi-directional transmission in wireless communication systems,” Ph.D. dissertation, TU Darmstadt, June 2009.