Distributed Margin Optimization Using Spectrum Balancing in Multi-user DSL Systems

Dominique Würtz, Anja Klein
Communications Engineering Lab
Technische Universität Darmstadt
Darmstadt, Germany

Martin Kuipers
Nokia Siemens Networks
GmbH & Co. KG
Berlin, Germany

Abstract—**Optimizing the SNR margin in Dynamic Spectrum Management (DSM) systems can significantly improve stability of DSL networks in the presence of slowly varying noise. In this work, we present a novel distributed approach to the margin optimization problem where the network operator specifies the desired ratios of user margins relative to each other and, given this constraint, the Spectrum Balancing algorithm seeks the solution with the largest sum margin. By employing partial Lagrangian relaxation of a rate-adaptive problem each user optimizes its own transmit power spectrum while the Spectrum Management Center (SMC) iteratively updates the target margins of each user so that the system eventually converges to an optimal power allocation with desired margin ratios. Numerical simulations demonstrate fast convergence and show that significantly higher margins can be achieved with our approach compared to purely selfish optimization.**

I. Introduction

With constantly growing bandwidth demands of current Digital Subscriber Lines (DSLs) systems, the problem of far-end crosstalk (FEXT) between copper wires in telephone binders has become increasingly important [1]. Dynamic Spectrum Management (DSM) Level 1 and 2 attempt to minimize interference between loops (in the following also interchangeably referred to as users) by optimizing the transmit power spectra of modems employing Discrete Multitone (DMT) transmission [2]. Not only does this allow a significant improvement of the overall system performance, but it enables also accurate control of the quality of service (QoS) parameters per loop based on the actual measured line conditions rather than relying on overly pessimistic worst-case assumptions as done in traditional static spectrum management.

Centralized DSM Level 2 algorithms like Optimal Spectrum Balancing (OSB) [3] achieve optimal performance, but are impractical due the high complexity. Recently, low-complexity schemes such as Distributed Spectrum Balancing (DSB) [4] have been developed which typically achieve a performance close to OSB and allow a distributed optimization with low complexity similar to Iterative Water-Filling (IWF) [5] in DSM Level 1.

Stability in communication systems is characterized by the probability of outage which is the probability that a system cannot operate with a specified QoS anymore due to degrading channel conditions. In DSL networks, stability is affected by slowly time-varying noises originating from RFI ingress or other loops in the binder being switched on or off. These so-called *quasi-stationary* noises are typically dealt with by including a signal-to-noise ratio (SNR) safety margin in the bit-loading of DMT modems at initialization of a DSL session [1]. As such, the SNR margin is the key parameter to control long-term stability of DSLs.

Margin-adaptive DSM assigns a fixed data rate to each user, usually chosen according to some service level agreement, and uses the available power to maximize the SNR margins of each user. In practice, due to the vastly different noise conditions across lines, it is not desirable to assign equal margins to all users. How to exactly distribute margins across users in order to optimize overall system stability is an important topic and various works consider different approaches to margin optimization with different optimization goals.

In [6], a multi-user margin optimization algorithm based on IWF is proposed which aims at maximizing the minimum SNR margin among users in the system. The approaches presented in [7], [8] directly optimize the outage probabilities in DSM Level 1 and 2 systems, respectively, assuming statistics of quasi-stationary noise dynamics are available at the Spectrum Management Center (SMC). In [9], a multi-user margin maximization (MMM) algorithm is proposed which finds a power allocation with either DSM Level 1 or 2 coordination where the per-user margin values obey a ratio according to priorities specified by the provider. Unfortunately, this meta-algorithm is not well suited for distributed spectrum optimization as it requires repeated execution of a base DSM algorithm at the SMC. For the same reason, as admitted by the authors, it generally suffers from a high computational complexity.

In this work, we present a novel low-complexity, distributed approach to solving the margin optimization problem from [9] with DSM Level 2 coordination. Here, the network operator specifies the desired ratios of user margins relative to each other and, given this constraint, the Spectrum Balancing algorithm seeks the solution with the largest sum margin. By employing partial Lagrangian relaxation of a rate-adaptive problem each user optimizes its own transmit power spectrum and enforces its target rate while the SMC iteratively updates the target margins of each user so that the system eventually converges to an optimal power allocation with desired margin ratios.

The remainder of the paper is structured as follows: Sec-

tion II defines the system model for the DSL binder. The spectrum management problem for margin maximization is introduced and analyzed in Section III. The proposed distributed Spectrum Balancing algorithm is presented in Section IV. Finally, Section V discusses performance results for the new scheme obtained from numerical simulations.

II. SYSTEM MODEL

An N-user DSL binder is considered where users $n \in \mathcal{N}$ employ DMT transmission over K subcarriers (tones). Assuming perfect synchronization of the transmitters and a sufficiently long cyclic prefix, the tones $k \in \mathcal{K}$ can be modeled as K parallel interference channels where crosstalk from other loops is treated as noise. Let $g_k^{n,n} = |h_k^{n,n}|^2$ denote the channel power gain of user n and $g_k^{n,m} = |h_k^{n,m}|^2$ ($n \neq m$) the FEXT power coupling gain from disturber m to victim loop n on tone k. Let s_k^n denote the allocated transmit power of user n on tone k and σ_k^n the sum power of received alien noise and interference not managed by the DSM system. Then the signal-to-interference-plus-noise ratio (SINR) at receiver n on tone k is obtained by

$$\mathrm{SINR}_k^n = \frac{g_k^{n,n} s_k^n}{\sum_{m \neq n} g_k^{n,m} s_k^m + \sigma_k^n}. \tag{1}$$

Protection against noise variation is achieved by incorporating an *SNR margin* $\gamma \geq 1$ (in linear scale) in the bit-loading at initialization of a DSL session. Using the Shannon-gap approximation [10], the number of bits per symbol that can be loaded on tone k of user n is given by

$$b_k^n(\gamma) = \log_2\left(1 + \frac{\mathrm{SINR}_k^n}{\gamma\Gamma}\right) \tag{2}$$

where $\Gamma > 1$ denotes the SNR gap to capacity [10] which is a function of the code and target bit error rate (BER).

The aggregate power P^n and data rate R^n of user n are given by

$$P^n = \sum_{k \in \mathcal{K}} s_k^n \quad \text{and} \quad R^n(\gamma) = f_s \sum_{k \in \mathcal{K}} b_k^n(\gamma), \tag{3}$$

respectively, where f_s is the symbol rate of the DMT system.

III. PROBLEM STATEMENT

Let $\boldsymbol{\mu} = (\mu^1, \ldots, \mu^N)$ ($\mu^n > 0 \; \forall n \in \mathcal{N}$) denote the vector of positive per-user priorities specified by the network operator and let $\boldsymbol{s} = \{s_k^n | k \in \mathcal{K}, n \in \mathcal{N}\}$ denote the set of transmit power levels to be optimized, respectively. A target data rate R_{target}^n is assigned to user n by the network operator. Furthermore, the power allocation of each user may not exceed a spectral mask $s_{k,\mathrm{mask}}^n$ and a maximum aggregate power P_{max}^n in order to comply with DSL standards. Given these constraints, this work considers the multi-user margin optimization problem

$$\max_{\boldsymbol{s},\bar{\gamma}} \bar{\gamma} \tag{4}$$
$$\text{subject to } R^n(\mu^n \bar{\gamma}) \geq R_{\mathrm{target}}^n \qquad \forall n \in \mathcal{N}$$
$$P^n \leq P_{\mathrm{max}}^n \qquad \forall n \in \mathcal{N}$$
$$0 \leq s_k^n \leq s_{k,\mathrm{mask}}^n \qquad \forall n \in \mathcal{N}, k \in \mathcal{K}$$
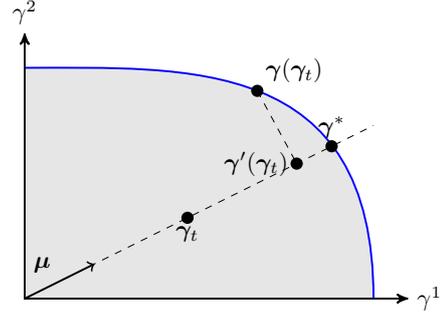
Fig. 1: Two-user margin region with optimal margin tuple $\boldsymbol{\gamma}^*$ for priority vector $\boldsymbol{\mu}$. Projection $\boldsymbol{\gamma}'(\boldsymbol{\gamma}_t)$ of operating point $\boldsymbol{\gamma}(\boldsymbol{\gamma}_t)$ on $\boldsymbol{\mu}$.

where $\bar{\gamma}$ is the overall positive margin scaling factor to be maximized. The nonlinear dependency of $R^n(\mu^n \bar{\gamma})$ in the rate constraints on the variable $\bar{\gamma}$ to be maximized greatly complicates the problem compared to rate-adaptive spectrum management problems as it introduces an additional coupling both across tones and users.

For given SINR_k^n and target rate R_{target}^n, the effective margin γ^n of user n is found by solving

$$R^n(\gamma^n) - R_{\mathrm{target}}^n = 0 \tag{5}$$

for γ^n. From (2) and assuming b_k^n to be continuous, it follows that (5) establishes a strictly monotone one-to-one mapping between rate $R^n(\gamma^n)$ and margin γ^n of user n. This relation for a single user n can directly be extended to the multi-user case: given a tuple $(R_{\mathrm{target}}^n | n \in \mathcal{N})$ of target rates, a tuple of achieved rates $(R^n(\gamma^n) | n \in \mathcal{N})$ is uniquely mapped to a tuple $\boldsymbol{\gamma}$ of achieved margins. From this follows, as explained in [6], that a rate region which represents the set of all feasible operating points, i.e. rate tuples, achieved by some DSM scheme can be transformed via (5) into an equivalent margin region. Furthermore, due to monotonicity of (5), the Pareto optimal points on the boundary of the rate region are mapped to points on the boundary of the margin region.

The margin tuple $\boldsymbol{\gamma}^*$ corresponding to the unique optimal solution of Problem (4) is the intersection point of the boundary of the margin region and the straight line from the origin along the vector $\boldsymbol{\mu}$ [9], as depicted in Figure 1. Note that the special case $\mu^1 = \mu^2 = \cdots = \mu^N$ corresponds to the max-min problem from [6] where the minimum margin among users is to be maximized.

To further characterize $\boldsymbol{\gamma}^*$, consider a standard sum rate maximization problem

$$\max_{\boldsymbol{s}} \sum_{n \in \mathcal{N}} R^n(\gamma_t^n) \tag{6}$$
$$\text{subject to } R^n(\gamma_t^n) \geq R_{\mathrm{target}}^n \qquad \forall n \in \mathcal{N}$$
$$P^n \leq P_{\mathrm{max}}^n \qquad \forall n \in \mathcal{N}$$
$$0 \leq s_k^n \leq s_{k,\mathrm{mask}}^n \qquad \forall n \in \mathcal{N}, k \in \mathcal{K}$$

for fixed target margins $\boldsymbol{\gamma}_t = (\gamma_t^n | n \in \mathcal{N})$. Let $\boldsymbol{\gamma}(\boldsymbol{\gamma}_t) = (\gamma^n(\boldsymbol{\gamma}_t) | n \in \mathcal{N})$ denote the effective margins according to (5) using spectra $\boldsymbol{s}^*$ obtained by solving Problem (6) with given

γ_t. It can be easily verified that $\gamma_t = \gamma(\gamma_t)$, i.e. γ_t is a fixed point of the mapping $\gamma(\gamma_t)$, iff γ_t lies on the boundary of the margin region. Now define a new mapping

$$\gamma'(\gamma_t) = \mathrm{proj}_{\boldsymbol{\mu}}\, \gamma(\gamma_t) = \frac{\gamma(\gamma_t) \cdot \boldsymbol{\mu}}{||\boldsymbol{\mu}||^2} \boldsymbol{\mu} \qquad (7)$$

which is the orthogonal projection of $\gamma(\gamma_t)$ on $\boldsymbol{\mu}$, as illustrated in Figure 1. It is not hard to see that $\gamma'(\gamma_t)$ has γ^* as its unique fixed point. This observation is the key in our iterative algorithm to find the optimal value γ^* of Problem (4).

IV. ALGORITHM

In the following, we describe our novel Spectrum Balancing algorithm to solve Problem (4) in a distributed fashion. Based on the observations from Section III, we propose an iterative approach where in each iteration the rate-adaptive Problem (6) is solved for a fixed vector γ_t of target margins and γ_t is updated so that it eventually converges to the fixed point γ^* of the mapping $\gamma'(\gamma_t)$.

Although the rate-adaptive Problem (6) is non-convex, known low-complexity Spectrum Balancing algorithms can be employed to yield near-optimal spectra. Since we are aiming at a distributed solution, schemes like DSB seem attractive. Like other sub-optimal spectrum optimization schemes, DSB tackles the non-convex problem by iterative convex approximation in a way that each user updates its own transmit power spectrum based only on locally available information as well as limited information infrequently exchanged between users. Unfortunately, DSB as proposed in [4] cannot be directly applied to Problem (6) because DSB only attempts to maximize a weighted sum rate without accounting for the target rate constraints of each user. Therefore, we now develop an extended version of DSB based on partial Lagrangian relaxation of the rate constraints where each user enforces its own target rate by locally tuning its weight factor in the Lagrangian dual of Problem (6).

Following a similar approach to [11], [12] which focus on distributed power minimization subject to data rate constraints, we obtain a convex approximation of Problem (6) which can be solved locally by user n to update its own spectrum by carrying out the following steps:

1) Partial Lagrangian relaxation of the rate constraints of other users $p \neq n$, resulting in a new objective

$$\sum_{p \in \mathcal{N}} R^p(\gamma_t^n) + \sum_{p \neq n} \omega^p \big[R^p(\gamma_t^n) - R_{\mathrm{target}}^p \big].$$

2) Fixing variables s_k^p ($p \neq n$, $k \in \mathcal{K}$) and Lagrange multipliers ω^p associated with the rate constraints of other users $p \neq n$.

3) Approximation of the non-concave parts of the objective resulting from the above steps by an affine function $-\sum_k \sum_m W_k^m s_k^n + U^n$ in the variables s_k^n ($k \in \mathcal{K}$). The approximation point is chosen as the optimal values of s_k^n from a previous iteration.

4) Dropping any additive terms of the objective that are constant in s_k^n ($k \in \mathcal{K}$), such as U^n, as these do not influence the optimal solution.

Applying all steps results in the convex sub-problem for user n given by

$$\max_{\{s_k^n | k \in \mathcal{K}\}} \tilde{R}^n(\gamma_t^n) - \sum_{k \in \mathcal{K}} W_k^n s_k^n \qquad (8)$$
$$\text{subject to } \tilde{R}^n(\gamma_t^n) \geq R_{\mathrm{target}}^n$$
$$P^n \leq P_{\max}^n$$
$$0 \leq s_k^n \leq s_{k,\mathrm{mask}}^n \qquad \forall k \in \mathcal{K}$$

where

$$\tilde{R}^n(\gamma_t^n) = f_s \sum_{k \in \mathcal{K}} \log_2 \left(1 + \frac{1}{\gamma_t^n \Gamma} \frac{g_k^{n,n} s_k^n}{\mathrm{int}_k^n} \right) \qquad (9)$$

$$W_k^n = \sum_{m \notin n} \frac{f_s g_k^{m,n}}{\log(2)} V_k^m \qquad (10)$$

$$V_k^n = (1 + \omega^n) \left(\frac{1}{\mathrm{int}_k^n} - \frac{1}{\mathrm{rec}_k^n} \right) \qquad (11)$$

$$\mathrm{int}_k^n = \sum_{m \neq n} g_k^{n,m} s_k^m + \sigma_k^n \qquad (12)$$

$$\mathrm{rec}_k^n = \Gamma^{-1} g_k^{n,n} s_k^n + \mathrm{int}_k^n. \qquad (13)$$

int_k^n and rec_k^n denote the measured interference power and total received power of user n, respectively, and are both available in standard compliant DSL modems. The term W_k^n can be interpreted as the price of increasing power on tone k and hence increasing interference on other lines. Problem (8) can be efficiently solved via dual optimization. Based on the Karush-Kuhn-Tucker (KKT) stationarity condition, it can be shown that for fixed dual variables (ω^n, λ^n) which correspond to rate and power constraint, respectively, the values of s_k^n that maximize the dual objective are found analytically according to

$$s_k^n = \left[\frac{\frac{f_s}{\log(2)}(1 + \omega^n)}{\lambda^n + W_k^n} - \frac{\mathrm{int}_k^n}{\Gamma^{-1} g_k^{n,n}} \right]_0^{s_{k,\mathrm{mask}}^n} \qquad (14)$$

where $[x]_0^a$ means $\min(\max(x,0),a)$ [4].

Algorithm 1 provides a possible realization of the local update scheme where the optimal value for ω^n is found using projected subgradient method with step rule

$$\omega^n \leftarrow \left[\omega^n + \kappa \left(R_{\mathrm{target}}^n - \tilde{R}^n(\gamma_t^n) \right) \right]^+ \qquad (15)$$

where κ is the search step size and $[\cdot]^+$ is the projection on $\mathbb{R}_0^+$. Note that depending on the current interference int_k^n and γ_t, a feasible solution might not exist due to the data rate constraint. This has to be considered in the choice of a convergence criterion for $\{s_k^n\}$ in line 17 by quitting the loop if increasing ω^n does not change s_k^n. However, the user power constraint can and must always be satisfied which is guaranteed by the bisection search which finds the optimal value of λ^n for fixed ω^n. Here, $\lambda_{\max}$ should be chosen such that $\lambda_{\max} \gg f_s(1 + \omega^n)$.

Algorithm 1 Local algorithm of user n

1: **repeat**
2:     Receive messages W_k^n ($k \in \mathcal{K}$) and γ_t^n from SMC
3:     **repeat**
4:         Initialize $\lambda^n \leftarrow \lambda_{\max}/2$ and $\Delta\lambda \leftarrow \lambda^n/2$
5:         **repeat**
6:             Update $\{s_k^n\}$ using (14)
7:             Calculate power P^n using (3)
8:             **if** $P^n > P_{\max}^n$ **then**
9:                 $\lambda^n \leftarrow \lambda^n + \Delta\lambda$
10:            **else**
11:                $\lambda^n \leftarrow \lambda^n - \Delta\lambda$
12:            **end if**
13:            $\Delta\lambda \leftarrow \Delta\lambda/2$
14:         **until** λ^n converged for given ω^n
15:         Calculate rate $\tilde{R}^n$ using (9)
16:         Update ω^n using (15)
17:     **until** spectrum $\{s_k^n\}$ converged
18:     Calculate messages V_k^n ($k \in \mathcal{K}$) using (11), effective margin γ^n using (5) and send to SMC
19: **until** global convergence of spectra s

Algorithm 2 Central steering loop of SMC

1: $\gamma_t^n \leftarrow 1 \; \forall n \in \mathcal{N}$
2: **repeat**
3:     Receive V_k^n ($k \in \mathcal{K}$, $n \in \mathcal{N}$) along with $\gamma(\gamma_t)$
4:     Compute W_k^n ($k \in \mathcal{K}$, $n \in \mathcal{N}$) using (10)
5:     Update $\gamma_t \leftarrow \mathrm{proj}_{\mu}\, \gamma(\gamma_t)$
6:     Transmit W_k^n ($k \in \mathcal{K}$) along with γ_t^n to line $n \in \mathcal{N}$
7: **until** global convergence of spectra s

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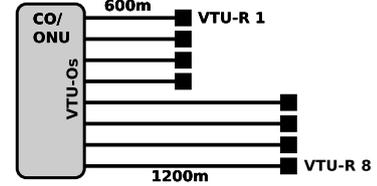
Fig. 2: 8-user VDSL2 upstream scenario

V. SIMULATION RESULTS

In this section, the performance of the proposed distributed algorithm which we refer to here as proportional margin (PM) DSB is studied and compared to other state-of-the-art schemes. For this, MATLAB simulations in an 8-user VDSL2 upstream scenario as depicted in Figure 2 have been carried out. Due to the near-far problem, this scenario offers significant potential gains from Spectrum Balancing while IWF is known to perform poorly due to the selfish optimization of each user. Table I summarizes the relevant system parameters used for the simulations.

Figure 3 shows the achieved operating points for different algorithms and priority vectors $\mu = (\mu_{\mathrm{short}}, \mu_{\mathrm{long}})$ in the 2-dimensional margin space. Note that the ratios are specified for margins in dB rather than linear margins. Target data rates 10 Mbps and 1 Mbps are assigned to short (600 m) and long (1200 m) loops, respectively. MMM+IWF and MMM+ISB are the schemes from [9] using DSM Level 1 and 2 coordination, respectively, whereas MHM is the DSM Level 1 algorithm from [6] which provides a max-min solution to the multi-user margin optimization problem. It shows that both MMM+ISB and our PM-DSB scheme achieve the same operating points on the Pareto boundary and as such clearly outperform MMM+IWF and MHM. Unlike MMM+ISB, PM-DSB allows a low-complexity distributed optimization which does not rely on repeated execution of computationally expensive feasibility checks.

In order to improve the local approximation coefficients W_k^n and steer the system to a solution that satisfies the margin ratios specified by μ, assistance of the SMC is required according to Algorithm 2. The SMC receives messages V_k^n from users $n \in \mathcal{N}$ which contain local information and are computed using (11). The prices W_k^n are then updated using (10) and distributed to the respective user. For further details about the derivation of update rules for W_k^n and V_k^n, see [4].

Having obtained a vector of effective margins $\gamma(\gamma_t)$ resulting from rate-adaptive spectrum optimization as described above, new target margins for the next iterations are found by setting $\gamma_t \leftarrow \gamma'(\gamma_t)$ with $\gamma'(\gamma_t)$ given by (7). A crucial advantage of our proposed scheme is that update of all quantities in the system such as target margins γ_t, spectra s_k^n and prices W_k^n can be carried out totally asynchronously without affecting convergence. This in particular means that the distributed rate maximization according to Problem (6) does not need to have fully converged before updating γ_t. While a general proof of convergence is beyond the scope of this paper, extensive numerical simulations asserted reliable and often very fast convergence in all considered scenarios. Uniqueness of the fixed point γ^* of $\gamma'(\gamma_t)$ assures that, once converged, we actually achieve the optimal margin tuple γ^* with desired priorities μ.

In the above considerations, the margins γ_t and γ were assumed in linear scale. In case the provider wishes to define the ratios of margins in logarithmic scale, i.e. dB, our scheme can be trivially modified by applying the above update rule for γ_t to the margin values in dB, naturally leading to a different solution than optimization of linear margins.

TABLE I: Simulation parameters

cable type	26-AWG [13]
FEXT model	ETSI 1% worst-case without FSAN sum [13]
background noise level	-140 dBm/Hz
alien noise model	ETSI MD_EX [13]
VDSL2 band profile	998-ADE17M2x-B without US0 [14]
symbol rate f_s	4 kHz
number of tones K	4096
tone spacing Δf	4.3125 kHz
SNR gap (for BER$= 10^{-7}$)	9.8 dB
coding gain	3 dB
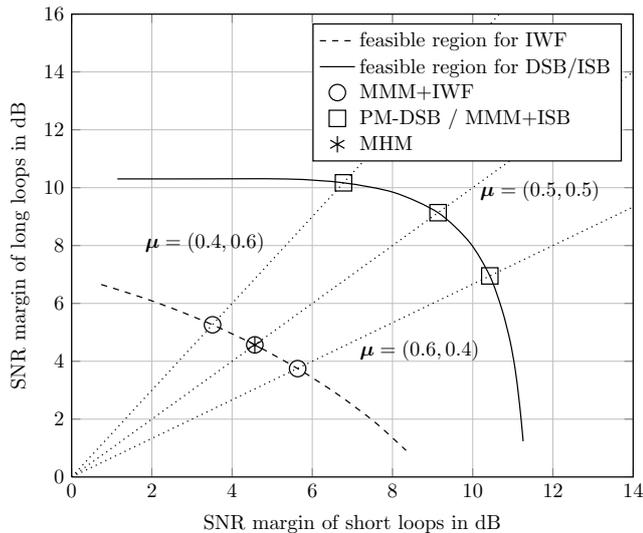
Fig. 3: Achieved operating points using various margin optimization techniques.

Fig. 4: Effective margins and target margins at iteration steps $i = 3, \ldots, 6$ of PM-DSB.

Figure 4 illustrates the convergence behavior of our proposed iterative PM-DSB when approaching the equal margin solution. The algorithm starts at iteration $i = 0$ with all users utilizing the highest power levels allowed by the spectral masks. At each iteration step $i = 1, 2, \ldots$, all users sequentially update their transmit spectrum using the local algorithm and the SMC afterwards computes new W_k^n values along with new target margins γ_t for the next iteration. The resulting effective margins $\gamma(i)$ and target margins $\gamma_t(i)$ at each iteration step i are marked by the squares and diamond shapes, respectively. Margins for iterations 0–2 are not shown since the points lie too far away from the optimal point to allow appropriate visualization. For reference, using the initial spectra, the resulting effective margins of short and long loops, respectively, are given by $\gamma(0) = (11.5\,\mathrm{dB}, -10.1\,\mathrm{dB})$. As can be seen in the figure, PM-DSB requires only about five iterations to find an operating on the Pareto boundary that satisfies the desired margin ratio given by μ.

Finally, more simulations have been carried out in up- and downstream scenarios for various choices of μ target rates R^n. Although with ratio vectors μ pointing near the corners of the margin region, the number of required iterations increases in some cases, convergence has been observed in all cases, asserting the robustness of our method.

VI. CONCLUSION

In this work, we presented a novel low-complexity, distributed Spectrum Balancing scheme for multi-user margin optimization with specified ratios of user margins. As such, our approach overcomes some of the main drawbacks of state-of-the-art algorithms like MMM+ISB. Simulation results attested optimal performance in problematic DSL deployment scenarios as well as fast convergence to the optimal solution. Also, it showed that Spectrum Balancing clearly outperforms DSM Level 1 coordination in terms of achievable margins in
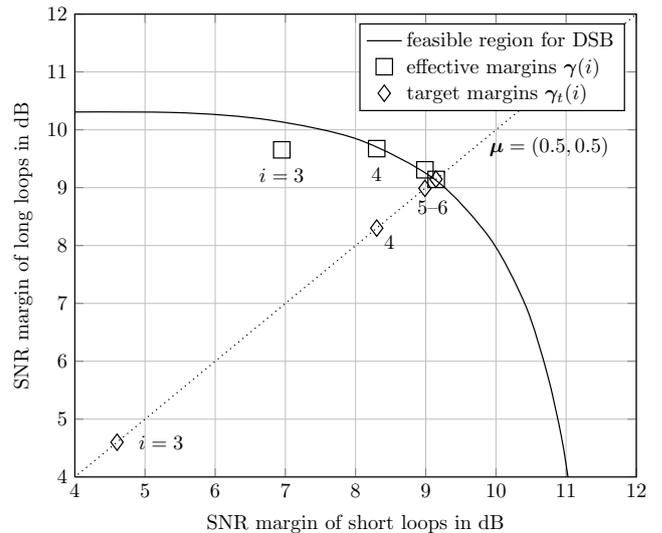
such scenarios.

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