Cooperative Zero Forcing in Multi-Pair Multi-Relay Networks

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***Abstract*—In this paper, unidirectional communication between K half-duplex node pairs is considered. The source nodes have N antennas each and the destination nodes have M antennas each. There is no direct link between the source and the destination nodes. Q half-duplex relays, each with R antennas, assist in the communication. It is assumed that the relays do not have enough antennas to spatially separate the data streams and hence, transceive zero forcing cannot be performed at the relays. In this paper, we propose a scheme in which the source nodes and the relays cooperate in choosing their precoding matrices and the filter coefficients, respectively, to perform a cooperative zero forcing. A closed form solution is proposed and the feasibility condition is derived. Simulation results show that the proposed cooperative zero forcing scheme achieves more degrees of freedom and hence, achieves higher sum rate as compared to reference schemes.**

***Index Terms*—zero forcing, one-way relaying, multiple relays, interference alignment**

I. Introduction

When there is no direct link between the source and the destination nodes, relays can be employed to aid the communication. In this paper, we focus on one-way relaying with amplify and forward [1] half-duplex relays. In [2], it has been shown that a single relay with $R \geq K$ antennas can support the communication between K source nodes and K destination nodes when the source and destination nodes have single antennas. The relay performs multiuser beamforming [2]. This beamforming is a combination of receive beamforming in the first time slot and transmit beamforming in the second time slot. The relay spatially separates the data streams based on the zero forcing (ZF) or minimum mean square error (MMSE) criterion. Therefore, the relay needs at least K antennas [2]. If multiple antennas at the source and the destination nodes are considered, then each source node can transmit d data streams to its destination node. In order to spatially separate these Kd data streams, at least $R \geq Kd$ antennas are required at the relay.

Multiuser beamforming with multiple relays is considered in [1]–[5]. In [1]–[4], the nodes and the relays have a single antenna each. In [1], [2], the relay coefficients are chosen such that at the destination nodes, the inter-pair interference is completely suppressed. At least $Q > K(K-1)$ relays are required for an interference-free communication. The relay coefficients in [3] are used for minimizing the mean squared error and in [4] to minimize the relay power subject to a signal to interference plus noise ratio (SINR) constraint. In [5], the nodes have N antennas each and transmit $d = N$ data streams to the destination. Each of the Q relays requires $R \geq Kd$ antennas to completely remove the inter-pair interference at the destinations.

In all the methods described in [1]–[5], a sufficient number of single antenna relays or sufficient antennas at each relay are necessary to spatially separate the data streams and, hence, completely suppress interference at the destination nodes. The source nodes do not help the relay in zero forcing the interference at the destination nodes. If the source nodes cooperate with the relays in choosing the transmit precoders and the relay processing matrices, then a smaller number $R < Kd$ of antennas as compared to the case without the cooperation is required at the relay. This means that for a given number of antennas at the relays and at the source nodes, now more users can be supported through the cooperation between the source and the relay nodes. In [6], a generalized iterative method to design the precoders, relay processing matrices and the receive filters jointly is given. These three matrices are designed one after another by setting the other two constant. In this case, $QR \geq Kd$ is sufficient to suppress the interference at the destinations if the source and destination nodes have multiple antennas to help the relay in suppressing the inter-pair interference. However, the algorithm in [6] is an iterative scheme which converges to a local optimum and during each iteration, a set of convex optimization problems has to be solved which involves large computational complexity. A closed form solution for zero forcing the interference at the destination nodes through the cooperation between the source and the relay nodes is not available in the literature.

In this paper, we propose a cooperative zero forcing scheme in which the source nodes help the relays to suppress inter-pair interference at the destination nodes. In other words, the source nodes help the relay to perform transceive zero forcing. We assume that the destination nodes have $M = d$ antennas, so that the destination nodes can spatially separate d data streams, but do not have additional dimensions to suppress inter-pair interference. A closed form solution is proposed to determine the source precoders and relay processing matrices. The feasibility condition is also derived in terms of $N,\ Q,\ R,\ K$ and d.

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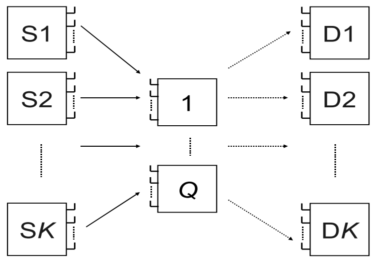

Fig. 1. K-pair one-way relay network

The organization of the paper is as follows. The system model is introduced in Section II. In Section III-A, the concept of transceive zero forcing is briefly explained. The proposed cooperative zero forcing scheme is then described in III-B. Section IV evaluates the performance of the proposed schemes in terms of the sum rate of the system. Section V concludes the paper.

We use lower case letters for scalars and lower case bold letters and upper case bold letters to denote column vectors and matrices, respectively. $(.)^*$ and $(.)^{\mathrm{H}}$ denote the complex conjugate and complex conjugate transpose of the element within the brackets, respectively.

II. System Model

Figure 1 shows a K-pair one-way relay network. Each of the source nodes Si wants to transmit d data streams to its destination node Di, for $i = 1, 2, \ldots, K$. There is no direct link between the source and the destination nodes. The source nodes have $N \geq d$ antennas each and the destination nodes have $M = d$ antennas and hence, the destination nodes can spatially separate d data streams, but cannot aid the relays in suppressing the interference. There are Q relays. All the nodes and the relays are assumed to be half-duplex. Each relay has R antennas where it is assumed that $R < Kd$. If $R \geq Kd$, transceive zero forcing could be performed at each relay [5]. In the first time slot, the source nodes transmit the signal to the relays and in the second time slot, after linear signal processing, the relays forward the signals to the destination nodes. The relays and the source nodes can cooperate in choosing their signal processing matrices and precoding matrices, respectively, but they do not share their signals. Let $\mathbf{d}_j$ and $\mathbf{V}_j$ denote the data symbols and the precoding matrix, respectively, of source node Sj. Let $\mathbf{H}_{qj}$ denote the Multiple Input Multiple Output (MIMO) channel between source node Sj and relay q. The noise at relay q is denoted by the vector $\mathbf{n}_{1q}$. The components of the noise vector $\mathbf{n}_{1q}$ are i.i.d. complex Gaussian random variables which follow $\mathcal{CN}(0, \sigma_1^2)$. The signal received at relay q is given by

$$\mathbf{x}_q = \sum_{j=1}^{K} \mathbf{H}_{qj} \mathbf{V}_j \mathbf{d}_j + \mathbf{n}_{1q}. \tag{1}$$

Amplify and Forward (AF) relaying is assumed. The relay q multiplies the received signal with the relay processing matrix $\mathbf{G}_q$ and forwards the resulting signal $\mathbf{s}_q = \mathbf{G}_q \mathbf{x}_q$. Let $\mathbf{s} = \begin{bmatrix} \mathbf{s}_1^{\mathrm{H}} & \ldots & \mathbf{s}_Q^{\mathrm{H}} \end{bmatrix}^{\mathrm{H}}$. The relays have a sum power constraint defined by trace $\left(\mathbf{s}\mathbf{s}^{\mathrm{H}}\right) \leq P_{\mathrm{Q}}$, where P_{Q} is the total transmit power available at the relays. Let $\mathbf{n}_{2k}$ denote the noise at the destination node Dk. The components of the noise vector $\mathbf{n}_{2k}$ are i.i.d. complex Gaussian random variables which follow $\mathcal{CN}(0, \sigma_2^2)$. The received signal at destination node Dk is denoted by

$$\hat{\mathbf{d}}_k = \sum_{q=1}^{Q} \mathbf{F}_{kq}^{\mathrm{H}} \mathbf{G}_q \left(\sum_{j=1}^{K} \mathbf{H}_{qj} \mathbf{V}_j \mathbf{d}_j + \mathbf{n}_{1q} \right) + \mathbf{n}_{2k}, \tag{2}$$

where $\mathbf{F}_{kq}^{\mathrm{H}}$ is the matrix denoting the channel between relay q and destination node Dk. Let $\widetilde{\mathbf{n}}_k = \sum_{q=1}^{Q} \mathbf{F}_{kq}^{\mathrm{H}} \mathbf{G}_q \mathbf{n}_{1q} + \mathbf{n}_{2k}$ denote the effective noise at destination node Dk. Equation (2) can be rewritten as

$$\hat{\mathbf{d}}_k = \sum_{q=1}^{Q} \mathbf{F}_{kq}^{\mathrm{H}} \mathbf{G}_q \mathbf{H}_{qk} \mathbf{V}_k \mathbf{d}_k \quad + \sum_{q=1}^{Q} \mathbf{F}_{kq}^{\mathrm{H}} \mathbf{G}_q \sum_{j=1, j\neq k}^{K} \mathbf{H}_{qj} \mathbf{V}_j \mathbf{d}_j + \widetilde{\mathbf{n}}_k. \tag{3}$$

In the above equation, the first and the second terms correspond to the useful and the interference signals, respectively. Let

$$\mathbf{H}_j = \begin{bmatrix} \mathbf{H}_{1j} \\ \vdots \\ \mathbf{H}_{\mathrm{Q}j} \end{bmatrix}, \; \mathbf{F}_k^{\mathrm{H}} = \begin{bmatrix} \mathbf{F}_{k1}^{\mathrm{H}} & \ldots & \mathbf{F}_{k\mathrm{Q}}^{\mathrm{H}} \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \ldots & \mathbf{0} \\ \vdots & & & \vdots \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{G}_{\mathrm{Q}} \end{bmatrix}$$

for $j = 1, 2 \ldots K$ and $k = 1, 2 \ldots K$. Then, (3) can be written as

$$\hat{\mathbf{d}}_k = \mathbf{F}_k^{\mathrm{H}} \mathbf{G} \mathbf{H}_k \mathbf{V}_k \mathbf{d}_k + \mathbf{F}_k^{\mathrm{H}} \mathbf{G} \sum_{j=1, j\neq k}^{K} \mathbf{H}_j \mathbf{V}_j \mathbf{d}_j + \widetilde{\mathbf{n}}_k. \tag{4}$$

Let $\mathbf{W}_{kj} = \mathbf{F}_k^{\mathrm{H}} \mathbf{G} \mathbf{H}_j \mathbf{V}_j$. Assuming that the input symbols denoted by the elements of the vector $\mathbf{d}_k$ are independent and zero mean complex Gaussian distributed with variance one, the achievable rate with which node Dk can transmit is given by

$$R_k = \frac{1}{2} \log_2 \left| \mathbf{I} + \left(\sum_{j=1, j\neq k}^{K} \mathbf{W}_{kj} \mathbf{W}_{kj}^{\mathrm{H}} + \mathbf{R}_{\widetilde{\mathbf{n}}\widetilde{\mathbf{n}}} \right)^{-1} \mathbf{W}_{kk} \mathbf{W}_{kk}^{\mathrm{H}} \right| \tag{5}$$

where $\mathbf{R}_{\widetilde{\mathbf{n}}\widetilde{\mathbf{n}}}$ is the covariance matrix of the effective noise $\widetilde{\mathbf{n}}_k$.

III. Transceive Zero Forcing

A. Introduction to Transceive Zero Forcing

In this section the concept of transceive zero forcing [7] is explained. The *receive signatures* of the signal from the j^{th} source node at the relays are given by $\mathbf{H}_j \mathbf{V}_j$. In order to spatially separate the signals in the QR dimensional relays space, combined receive zero forcing has to be done at the relays. The receive zero forcing matrix $\mathbf{G}_{\text{Rx}}$ is given by

$$\mathbf{G}_{\text{Rx}} = \begin{bmatrix} \mathbf{H}_1\mathbf{V}_1 & \mathbf{H}_2\mathbf{V}_2 & \ldots & \mathbf{H}_K\mathbf{V}_K \end{bmatrix}^+. \quad (6)$$

Here $+$ denotes the pseudo-inverse. Then the relays need to perform transmit zero forcing to transmit the signals interference free to the destinations. The transmit zero forcing matrix $\mathbf{G}_{\text{Tx}}$ is given by

$$\mathbf{G}_{\text{Tx}} = \begin{bmatrix} \mathbf{F}_1^{\text{H}} \\ \mathbf{F}_2^{\text{H}} \\ \vdots \\ \mathbf{F}_K^{\text{H}} \end{bmatrix}^+. \quad (7)$$

The columns of the matrix $\mathbf{G}_{\text{Tx}}$ correspond to the *transmit signatures* of the relay signals. The receive zero forcing followed by the transmit zero forcing is called transceive zero forcing and is denoted by the matrix

$$\mathbf{G} = \mathbf{G}_{\text{Tx}}\mathbf{G}_{\text{Rx}}. \quad (8)$$

Note that, when the precoding vectors $\mathbf{V}_j$ are chosen arbitrarily, it cannot be guaranteed that the matrix $\mathbf{G}$ is block-diagonal. However, we can design the the precoding matrices $\mathbf{V}_j$ for $j = 1, 2, \ldots, K$ and hence, the receive signatures $\mathbf{H}_j\mathbf{V}_j$ such that $\mathbf{G}$ is block-diagonal.

B. Cooperative Zero Forcing Scheme

In this section, the proposed cooperative zero forcing scheme is described. The main idea is as follows: The relays alone cannot perform transceive zero forcing due to the block diagonal structure of the matrix $\mathbf{G}$. The nodes cooperate with the relays in choosing their precoding matrices to achieve zero interference at the destination nodes. Perfect channel knowledge is assumed at the source nodes and at the relays. For zero interference at the destination nodes, the relay should transmit the signal from source node $\text{D}j$ in a direction perpendicular to the channels of all the other $K-1$ destination nodes. These transmission directions also called the transmit zero forcing directions or the transmit signatures are given by the columns of the matrix $\mathbf{G}_{\text{Tx}}$. For any given choice of precoding matrices $\mathbf{V}_j$ for $j = 1, \ldots, K$, the signals received at the relays are in the directions given by the columns of the $\mathbf{W} = \begin{bmatrix} \mathbf{H}_1\mathbf{V}_1 & \ldots & \mathbf{H}_K\mathbf{V}_K \end{bmatrix}$. The relays with the block-diagonal matrix $\mathbf{G}$ should perform a linear transformation that maps the receive signatures to the transmit signature in a QR dimensional space. Due to the block-diagonal structure of the matrix G, the number of variable in the matrix $\mathbf{G}$ is QR^2. Hence, only R receive signatures can be mapped to their corresponding desired transmit zero forcing directions. Fortunately, matrix $\mathbf{G}$ is a full rank matrix. This means the other $Kd - R$ receive signatures map to some linearly independent directions, but not necessarily to the desired transmit signatures. By modifying the precoding matrices at the source nodes, the receive signatures can be altered. In the proposed scheme, the sources have sufficient number of antennas so that the precoding vectors can be chosen such that designing the linear transformation for the first R receive signatures also maps the other $Kd - R$ receive signatures to their desired transmit zero forcing directions. The number of antennas required at the source nodes is derived later. The cooperative zero forcing is performed in two steps. First, the precoders are designed. Second, the linear signal processing matrices $\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_Q$ are determined.

1) Transmit Precoders: In this section, the precoders at the source nodes are designed such that when the relay performs transceive zero forcing for the first R data streams, all the other $Kd - R$ data streams are automatically transceive zero forced. Let the columns of the matrix

$$\mathbf{G}_{\text{Tx}} = \begin{bmatrix} \mathbf{Z}_1 & \ldots & \mathbf{Z}_K \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1^{\text{H}} \\ \mathbf{F}_2^{\text{H}} \\ \vdots \\ \mathbf{F}_K^{\text{H}} \end{bmatrix}^+ \quad (10)$$

denote the transmit signatures. It has to be noted that if we assume i.i.d. MIMO channel model [8] and independent destination nodes, then $\mathbf{Z}_j$ for $j = 1, 2, \ldots, K$ obtained from (10) has rank d with a probability of one. The receive signatures at the relay are given by $\mathbf{W} = \begin{bmatrix} \mathbf{H}_1\mathbf{V}_1 & \ldots & \mathbf{H}_K\mathbf{V}_K \end{bmatrix}$. The objective is to design the matrices $\mathbf{V}_j$ for $j = 1, 2, \ldots, K$ such that

$$\mathbf{G}_{\text{Tx}} = \mathbf{G}\mathbf{W} \quad (11)$$

is satisfied for a block-diagonal matrix $\mathbf{G}$. The transmit-receive signature pair corresponding to node-pair j can be written as

$$\mathbf{Z}_j = \mathbf{G}\mathbf{H}_j\mathbf{V}_j \quad (12)$$

for $j = 1, 2, \ldots, K$. Equation (12) means that the receive signatures $\mathbf{H}_j\mathbf{V}_j$ are mapped to the transmit signatures $\mathbf{Z}_j$. In general, only the subspace spanned by the receive signatures $\mathbf{H}_j\mathbf{V}_j$ needs to be equal to the subspace spanned by the transmit signatures $\mathbf{Z}_j$ to achieve zero interference at the destination nodes. Equation (12) can be written in terms of the linear signal processing matrices of each relay as

$$\begin{bmatrix} \mathbf{Z}_j^1 \\ \mathbf{Z}_j^2 \\ \vdots \\ \mathbf{Z}_j^Q \end{bmatrix} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \ldots & \mathbf{0} \\ \vdots & & & \vdots \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{G}_{\text{Q}} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{1j} \\ \mathbf{H}_{2j} \\ \vdots \\ \mathbf{H}_{Qj} \end{bmatrix} \mathbf{V}_j \quad (13)$$

for $j = 1, 2, \ldots, K$. Here, $\mathbf{Z}_j^q$ corresponds to the d transmit signatures of the q^{th} relay. As each of the relay spans an R dimensional signal space and $R < Kd$, the Kd transmit signatures $\begin{bmatrix} \mathbf{Z}_1^q & \ldots & \mathbf{Z}_K^q \end{bmatrix}$ of the q^{th} relay will be linearly dependent on each other. Let $n = R/d$. Then $R = nd$ transmit signatures will be linearly independent of each other and the other $Kd - R$ transmit signatures will be linearly dependent on the first nd transmit signatures. In general, $\lfloor n \rfloor d$ transmit signatures correspond to $\lfloor n \rfloor$ destination nodes and the other $(n - \lfloor n \rfloor)\, d$ transmit signatures correspond to the $(\lfloor n \rfloor + 1)^{\text{th}}$ destination node. For simplicity, throughout the rest of the paper we assume R to be an integer multiple of d. Then the transmit signatures corresponding to n destination

$$\underbrace{\begin{bmatrix} \left(\mathbf{C}_{1n+1}^{1\mathrm{T}} \otimes \mathbf{H}_{11}\right) & \dots & \left(\mathbf{C}_{nn+1}^{1\mathrm{T}} \otimes \mathbf{H}_{1n}\right) & -\left(\mathbf{I}^{\mathrm{T}} \otimes \mathbf{H}_{1n+1}\right) & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & & & & \vdots \\ \left(\mathbf{C}_{1n+1}^{Q\mathrm{T}} \otimes \mathbf{H}_{Q1}\right) & \dots & \left(\mathbf{C}_{nn+1}^{Q\mathrm{T}} \otimes \mathbf{H}_{Qn}\right) & -\left(\mathbf{I}^{\mathrm{T}} \otimes \mathbf{H}_{Qn+1}\right) & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & & & & \vdots \\ \left(\mathbf{C}_{1K}^{Q\mathrm{T}} \otimes \mathbf{H}_{Q1}\right) & \dots & \left(\mathbf{C}_{nK}^{Q\mathrm{T}} \otimes \mathbf{H}_{Qn}\right) & \mathbf{0} & \dots & \mathbf{0} & -\left(\mathbf{I}^{\mathrm{T}} \otimes \mathbf{H}_{QK}\right) \end{bmatrix}}_{\mathbf{S}} \begin{bmatrix} \mathrm{vec}\left(\mathbf{V}_1\right) \\ \mathrm{vec}\left(\mathbf{V}_2\right) \\ \vdots \\ \mathrm{vec}\left(\mathbf{V}_K\right) \end{bmatrix} = \mathbf{0} \quad (23)$$

nodes will be linearly independent of each other and the transmit signatures corresponding to the other $K-n$ destination nodes can be written as a linear combination of the transmit signatures corresponding to the first n destination nodes. This is represented by

$$\mathbf{Z}_i^q = \left[\begin{array}{ccc} \mathbf{Z}_1^q & \dots & \mathbf{Z}_n^q \end{array} \right] \mathbf{T}_{\mathrm{tx}i}^q \quad (14)$$

where $\mathbf{T}_{\mathrm{tx}i}^q$ for $q = 1, 2, \dots, Q$ and $i = n+1, n+2, \dots, K$ gives the linear dependence relation between $\mathbf{Z}_i^q$ and $\left[\begin{array}{ccc} \mathbf{Z}_1^q & \dots & \mathbf{Z}_n^q \end{array} \right]$. Similarly, the linear dependency of the receive signatures can be denoted by

$$\mathbf{H}_{qi}\mathbf{V}_i = \left[\begin{array}{ccc} \mathbf{H}_{q1}\mathbf{V}_1 & \dots & \mathbf{H}_{qn}\mathbf{V}_n \end{array} \right] \mathbf{T}_{\mathrm{rx}i}^q \quad (15)$$

for $i = n+1, \dots, K$. Without loss of generality, we will assume that the matrices $\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_Q$ will be chosen such that the first R columns of the matrix equality in Equation (11) will be satisfied. That is, the matrix $\mathbf{G}$ maps the first R receive signatures to the first R transmit signatures. This is given by

$$\mathbf{Z}_l^q = \mathbf{G}_{\mathrm{q}}\mathbf{H}_{ql}\mathbf{V}_l \quad (16)$$

for $q = 1, 2, \dots, Q$ and $l = 1, 2, \dots, n$. The other $Kd - R$ receive signatures will be mapped to the corresponding $Kd-R$ transmit signatures if and only if

$$\mathbf{Z}_i^q = \mathbf{G}_{\mathrm{q}}\mathbf{H}_{qi}\mathbf{V}_i \quad (17)$$

holds for $q = 1, 2, \dots, Q$ and $i = n+1, n+2, \dots, K$. The matrices $\mathbf{G}_{\mathrm{q}}$ for $q = 1, 2 \dots Q$ denote a set of linear transformations and hence, in order to satify (17),

$$\mathbf{T}_{\mathrm{tx}i}^q = \mathbf{T}_{\mathrm{rx}i}^q \quad (18)$$

need to hold for $q = 1, 2, \dots, Q$ and $i = n+1, n+2, \dots, K$. From (14), (15) and (18), we get

$$\mathbf{H}_{qi}\mathbf{V}_i = \left[\begin{array}{ccc} \mathbf{H}_{q1}\mathbf{V}_1 & \dots & \mathbf{H}_{qn}\mathbf{V}_n \end{array} \right] \left[\begin{array}{ccc} \mathbf{Z}_1^q & \dots & \mathbf{Z}_n^q \end{array} \right]^{-1} \mathbf{Z}_i^q. \quad (19)$$

Let

$$\begin{bmatrix} \mathbf{C}_{1i}^q \\ \vdots \\ \mathbf{C}_{ni}^q \end{bmatrix} = \left[\begin{array}{ccc} \mathbf{Z}_1^q & \dots & \mathbf{Z}_n^q \end{array} \right]^{-1} \mathbf{Z}_i^q. \quad (20)$$

Then, (19) can be written as

$$\mathbf{H}_{qi}\mathbf{V}_i = \left[\begin{array}{ccc} \mathbf{H}_{q1}\mathbf{V}_1 & \dots & \mathbf{H}_{qn}\mathbf{V}_n \end{array} \right] \begin{bmatrix} \mathbf{C}_{1i}^q \\ \vdots \\ \mathbf{C}_{ni}^q \end{bmatrix}. \quad (21)$$

Vectorizing the matrices on both sides of (21) and applying the property $\mathrm{vec}\left(\mathbf{AXB}\right) = \left(\mathbf{B}^{\mathrm{T}} \otimes \mathbf{A}\right) \mathrm{vec}\left(\mathbf{X}\right)$, we get

$$\left(\mathbf{I}^{\mathrm{T}} \otimes \mathbf{H}_{qi}\right) \mathrm{vec}\left(\mathbf{V}_i\right) = \left(\mathbf{C}_{1i}^{q\mathrm{T}} \otimes \mathbf{H}_{q1}\right) \mathrm{vec}\left(\mathbf{V}_1\right) + \dots + \left(\mathbf{C}_{ni}^{q\mathrm{T}} \otimes \mathbf{H}_{qn}\right) \mathrm{vec}\left(\mathbf{V}_n\right) \quad (22)$$

for $q = 1, 2, \dots, Q$ and $i = n+1, n+2, \dots, K$. Here, $\otimes$ denotes the Kronecker product. Equation (22) is a system of linear homogeneous equations which can be written as given in (23). The dimension of matrix $\mathbf{S}$ is $(Kd-R)QR \times KNd$. The number of rows and columns denotes the number of equations and variables, respectively, in (23). A non-trivial solution for (23) exists when the number of variables is greater than the number of equations. This is given by

$$KNd > (Kd - R)QR. \quad (24)$$

Equation (24) gives the feasibility condition for the proposed cooperative zero forcing scheme. It has to be noted $\mathbf{Z}_j$ for $j = 1, 2, \dots, K$ obtained from (10) has rank d with a probability of one. Hence, (13) guarantees that $\mathbf{V}_j$ for $j = 1, 2, \dots, K$ is of rank d. Also, note that the number N of antennas required at each source node is directly proportional to the number of relays. This is due to the fact that when the number of relays increases, the dimension of the space where the linear transformation is performed also increases. Hence, more variables are required at the source nodes to satisfy (18). In this paper, we assume that Q is equal to the minimum number of relays required to satisfy the condition $QR \geq Kd$. That is $QR \geq Kd > (Q-1)R$. If there are $Q' > Q$ relays available in the system, the extra $Q' - Q$ relays will be switched off. For particular integer values of R, N, d, K and Q, (24) can be satisfied only with strict inequality. In this case, there is a possibility to choose the transmit precoder matrices from the solution space of (23). Let the span of the columns of the matrix

$$\mathbf{A} = \left[\begin{array}{cccccc} \mathbf{A}_{11}^{\mathrm{T}} & \mathbf{A}_{12}^{\mathrm{T}} & \cdots & \mathbf{A}_{1d}^{\mathrm{T}} & \cdots & \mathbf{A}_{Kd}^{\mathrm{T}} \end{array} \right]^{\mathrm{T}} = \mathrm{null}(\mathbf{S}) \quad (25)$$

denote the solution space, then the m^{th} column of the precoding matrix $\mathbf{V}_j$ is obtained as

$$\mathbf{v}_{jm} = \mathbf{A}_{jm}\mathbf{t}, \quad (26)$$

where vector $\mathbf{v}_{jm}$ is a linear combination of the columns of the matrix $\mathbf{A}_j$ defined by the vector $\mathbf{t}$. Any arbitrary choice of $\mathbf{t}$

is a solution for (23). The vector $\mathbf{t}$ can be chosen to maximize a given objective function. For example, $\mathbf{t}$ can be chosen such that the total signal power received at the relays is maximized. That is

$$\mathbf{t}_{\text{opt}} = \underset{\mathbf{t}}{\text{argmax}} \quad \sum_{j=1}^{K} \sum_{m=1}^{d} \frac{\mathbf{t}^{\mathrm{H}} \mathbf{A}_{jm}^{\mathrm{H}} \mathbf{H}_{j}^{\mathrm{H}} \mathbf{H}_{j} \mathbf{A}_{jm} \mathbf{t}}{\mathbf{t}^{\mathrm{H}} \mathbf{A}_{jm}^{\mathrm{H}} \mathbf{A}_{jm} \mathbf{t}}. \tag{27}$$

The optimization problem described in (27) is non-convex [9] and gradient based methods described in [10] can be used to find the local maxima. In [10], it has been shown that the local maxima provides a significant gain in terms of the sum rate when compared to an arbitrary choice of $\mathbf{t}$.

In order to satisfy the transmit power constraint at each node, the precoder matrices need to be normalized based on the total power available at each node. When the precoders are multipled by some scalars, the corresponding transmit signatures also have to be multiplied by the corresponding scalars so that (18) holds. Let P_{t} denote the transmit power available at each node. Assume uniform power allocation across the d data streams. Then the precoders are normalized as follows:

$$\mathbf{V}_{j}^{\text{norm}} = \sqrt{\frac{P_{\mathrm{t}}}{d}} \mathbf{V}_{j} \left(\text{Diag}\left(\mathbf{V}_{j}^{\mathrm{H}} \mathbf{V}_{j}\right)\right)^{-\frac{1}{2}} \tag{28}$$

where Diag(.) replaces all off-diagonal elements of the matrix within the brackets by zeros. For (18) to hold, the transmit signatures have to be normalized as follows:

$$\mathbf{Z}_{j}^{\text{norm}} = \sqrt{\frac{P_{\mathrm{t}}}{d}} \mathbf{Z}_{j} \left(\text{Diag}\left(\mathbf{V}_{j}^{\mathrm{H}} \mathbf{V}_{j}\right)\right)^{-\frac{1}{2}} \tag{29}$$

for $j = 1, 2, \ldots, K$.

2) Relay Processing Matrices: In this section, the relay processing matrices $\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_Q$ are determined. In Section III-B1, the precoders have been designed such that if the relays map the first R receive signatures to the first R transmit zero forcing directions, then the other $Kd - R$ receive signatures will be automatically mapped to their corresponding $Kd - R$ transmit zero forcing directions. In order for the relays to map the first R receive signatures to the first R transmit signatures, the following equation should hold:

$$\left[\begin{array}{ccc} \mathbf{Z}_{1}^{q} & \ldots & \mathbf{Z}_{n}^{q} \end{array}\right] = \mathbf{G}_{q} \left[\begin{array}{ccc} \mathbf{H}_{q1} \mathbf{V}_{1} & \ldots & \mathbf{H}_{qn} \mathbf{V}_{n} \end{array}\right] \tag{30}$$

for $q = 1, 2, \ldots, Q$. Therefore

$$\mathbf{G}_{q} = \left[\begin{array}{ccc} \mathbf{Z}_{1}^{q} & \ldots & \mathbf{Z}_{n}^{q} \end{array}\right] \left[\begin{array}{ccc} \mathbf{H}_{q1} \mathbf{V}_{1} & \ldots & \mathbf{H}_{qn} \mathbf{V}_{n} \end{array}\right]^{-1}. \tag{31}$$

The relays have a total transmit power constraint. The matrix $\mathbf{G}$ can be scaled to satisfy this power constraint without disturbing the zero forcing solutions. Note that the optimization problem described in (27) is used only to optimize over the many possible solutions. For the cooperative zero forcing problem, we need only one arbitrary zero forcing solution and the method described above provides a closed form solution. Hence, the computational complexity is very low compared to [6].

Interference alignment in the reciprocal network: Also for the case that the transmitters have $M = d$ antennas and the receivers have N antennas, the proposed method can be used to achieve interference-free transmission. In this case, the relays cooperate with the destination nodes to perform interference alignment at the destination nodes. It has to be noted that the source nodes do not need any channel knowledge and the relays receive the linear combination of the signals from the source nodes. Each of the relays cannot decode the signals itself, but the relays can cooperate with each other to perform interference alignment at the destination nodes, making it possible for the destination nodes to decode the useful signal. To obtain the interference alignment solution, first the network is converted into its reciprocal network with N antennas at the transmitter and M antennas at the receivers, with the channel between the source nodes and the relay nodes given by the Hermitian of the corresponding channel between the relays and the destination nodes in the original network. The channel between the relays and the destination nodes in the reciprocal network is given by the Hermitian of the corresponding channel between the source and the relay nodes in the original network. After this conversion, the proposed cooperative zero forcing method can be used to obtain the source precoders and the relay processing matrices of the reciprocal network. The relay processing matrices and the receive zero forcing matrices of the original network are then given by the Hermitian of the corresponding relay processing matrices and the source precoders, respectively, in the reciprocal network.

IV. Performance Analysis

In this section, the sum rate performance of the proposed cooperative zero forcing (CZF) scheme is investigated. For the simulation, we consider a $K = 3$ node pairs scenario. Each source node wants to transmit $d = 1$ data stream to its destination node. There are $Q = 2$ relays that assist in the communication. Each of the source and relay nodes has $N = R = 2$ antennas. The destination nodes have $M = d = 1$ antenna. According to (24), this scenario is feasible. In total, three data streams are transmitted in two time slots. Two reference methods are considered to compare with the sum rate performance of the proposed CZF scheme. The first reference method (SVD_ZF_TDMA) is based on transceive zero forcing [5] for a one way relaying scenario. The source nodes transmit their data streams in the directions corresponding to the largest singular values of the channel from the source node to the relays. The relays spatially separate the data streams and perform transceive zero forcing. In this method, only two node pairs can be supported at a time, additional node pairs are separated by time division multiple access (TDMA). The main difference between the CZF and SVD_ZF_TDMA scheme is that in CZF, the transmit precoders are used for increasing the number of degrees of freedom defined as the number of data streams in the system while in SVD_ZF_TDMA, the transmit precoders are used for a beamforming gain.

The second reference method is based on iterative minimization of the sum mean square error (IterativeMMSE) proposed in [6]. In this method, the transmit precoders, relay filters and receive filters are optimized iteratively. In this method, all the three node pairs transmit at the same time. 50 iterations are

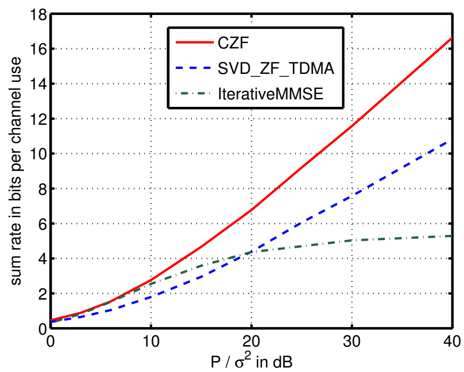

Fig. 2. Sum rate performance for a 3 node pairs scenario with $N = 2,\ R = 2,\ Q = 2$ and $M = d = 1$

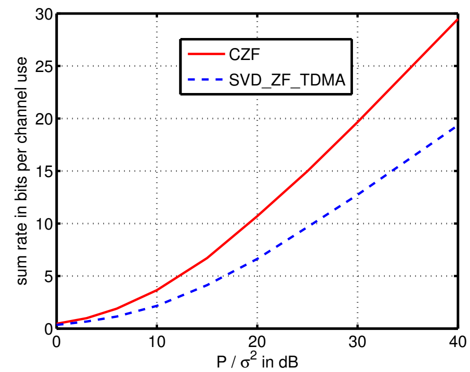

Fig. 3. Sum rate performance for a 3 node pairs scenario with $N = 3,\ R = 4,\ Q = 2$ and $M = d = 2$

considered for the simulation. Simulation results show that typically after 50 iterations the residual MMSE converges to a small value and remains almost constant.

Figure 2 shows the sum rate performance of each method as a function of P/σ^2. P is the transmit power at each node in the CZF scheme and in the IterativeMMSE scheme. In the SVD_ZF_TDMA scheme, as the nodes are sometimes silent due the TDMA, the power available at each node is scaled to $3P/2$ in order to have a fair comparison. The noise power at each node is assumed to be the same and is denoted by $\sigma^2 = \sigma_1^2 = \sigma_2^2$. The relays have a total transmit power $3P$ in all three cases. The MIMO channel matrices between the nodes and the relay are normalized such that in the CZF scheme, on an average, the transmitted signal power is the same as the received signal power. The sum rate is calculated as an average value from 10^3 channel realizations generated randomly using the i.i.d. frequency-flat Rayleigh fading MIMO channel model [8]. The red curve in Figure 2 shows the performance of the proposed scheme. The vector $\mathbf{t}$ is arbitrarily chosen. The blue and the green dashed lines correspond to the SVD_ZF_TDMA and IterativeMMSE schemes, respectively. It can be clearly seen that the proposed cooperative zero forcing scheme outperforms both the reference schemes. At high SNR values, the slopes of CZF and SVD_ZF_TDMA correspond to the total number of data streams transmitted over two time slots but the sum rate of IterativeMMSE scheme converges to a finite value. This is due to the fact that the iterativeMMSE method obtains only a local minimum and hence, there is an interference leakage. Furthermore, the correlations of the interfering signals coming through different relays are not considered in [6].

Figure 3 shows the sum rate performance when $K = 3,\ N = 3,\ R = 4,\ Q = 2$ and $M = d = 2$. From (24), it can be seen that this scenario is feasible. Note that for doubling the number of data streams, it is not necessary to double the number of antennas at each node. From Figure 3 it can be seen that CZF outperforms SVD_ZF_TDMA since with CZF, 6 degrees of freedom are achieved while with SVD_ZF_TDMA, only 4 degrees of freedom are achieved.

V. Conclusion

In this paper, one way relaying in a K node pairs multiple relay scenario is considered. A cooperative zero forcing scheme, where the source nodes and the relay nodes cooperate to perform zero forcing at the destination nodes, is proposed. Through this cooperative method, zero interference at each of the destination nodes can be achieved. Less antennas are required at the relays compared to the case where the nodes and relays do not cooperate to perform zero forcing. This means that for a given number of antennas at the nodes and the relays, more users can be supported compared to the case without the cooperation. The feasibility condition $KNd > (Kd - R)QR$ is derived. The simulation results show that the proposed scheme achieves more degrees of freedom and hence, achieves higher sum rate in the system as compared to reference schemes.

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