

# Regenerative Multi-Group Multi-Way Relaying

Aditya Umbu Tana Amah, *Student Member, IEEE*, and Anja Klein, *Member, IEEE*

**Abstract**—We consider regenerative multi-group multi-way (MGMW) relaying. A half-duplex regenerative multi-antenna relay station (RS) assists multiple communication groups. In each group, multiple half-duplex nodes communicate to each other. The number of communication phases is defined by the maximum number of nodes among all groups. In the first phase, the multiple access (MAC) phase, all nodes transmit simultaneously to the RS and the RS decodes the data streams of all nodes. In the following broadcast (BC) phases, the RS transmits to the nodes. We propose three BC strategies: unicasting, multicasting and hybrid uni/multicasting. For the multicasting strategy, either superposition or XOR network coding is applied. On the one hand, we propose a transmit beamforming algorithm minimising the RS's transmit power while guaranteeing that each node receives with a rate equal to the rate received at the RS for each particular data stream. The transmit beamforming algorithm is designed by coupling the MAC and BC phases. On the other hand, for the case that the RS's transmit power is fixed, we design low complexity transmit beamforming algorithms: matched filter, zero forcing, minimisation of mean square error and MGMW-aware transmit beamforming. It is shown that the multicasting strategy with XOR network coding improves the achievable sum rate and requires the least transmit power compared to the other strategies.

**Index Terms**—multi-way relaying, regenerative, network coding, transmit beamforming.

## I. INTRODUCTION

Two-way relaying defines how two half-duplex nodes communicate to each other through a half-duplex Relay Station (RS) in two communication phases [1], [2]. In the first phase, the multiple access (MAC) phase, the nodes transmit simultaneously to the RS. In the second phase, the broadcast (BC) phase, the RS transmits the superposition of the nodes' data streams to both nodes. Subsequently, each node performs self-interference cancellation to the received data stream to obtain the data stream of its partner.

Regarding the signal processing at the RS, it can be either regenerative, cf. [2], or non-regenerative, cf. [3], [4]. A regenerative RS regenerates (decodes and re-encodes) the data streams of the communicating nodes and forwards the regenerated data streams to the nodes. A non-regenerative RS only performs linear signal processing, e.g., pure amplification [1] or beamforming [4], to the received signals and forwards

the output to the nodes. The advantage of a regenerative RS is that the noise at the RS does not propagate to the nodes. Moreover, each node needs to know only its own channel to the RS for the decoding process.

It is well known that multiple antennas improve the reliability and/or the spectral efficiency of a communication system. A regenerative multi-antenna RS which supports one two-way pair using two-way relaying has been considered in [5], [6]. The extension for multiple two-way pairs has been considered in [7], [8], which is called multi-user two-way relaying [7]–[9].

In communication networks, it is natural that multiple nodes are communicating to each other. Video conference and multi-player gaming are examples of communication between multiple nodes. A multi-way relay channel where an RS assists multiple communication groups was considered in [10]. In each group, there are nodes who communicate to each other, but not to other nodes in other groups. In [10], full-duplex communication between full-duplex nodes via a full-duplex RS is assumed and time division is used to separate the groups.

While the full-duplex assumption is attractive from theoretical point of view, half-duplex nodes and a half-duplex RS are more into practical consideration [1]. Therefore, finding communication protocols and the corresponding signal processing for the multi-way relay channel are interesting open problems. Recently, protocols and the corresponding signal processing for single-group multi-way relaying were proposed in [11], [12] for a non-regenerative multi-antenna RS. The overall number of communication phases is equal to the number  $N$  of nodes in the group. They consist of one MAC phase and  $N - 1$  BC phases. In each BC phase, the RS transmits different data streams simultaneously to the nodes. This is to ensure that within  $N - 1$  BC phases, all nodes receive the data streams from the other  $N - 1$  nodes. The extension to non-regenerative multi-group multi-way relaying (MGMW) was considered in [13]. The number of communication phases is defined by the maximum number of nodes among the groups.

In this work, we consider regenerative MGMW relaying. Regenerative two-way relaying and regenerative multi-user two-way relaying are two special cases of the proposed regenerative MGMW relaying. In regenerative MGMW relaying, a half-duplex multi-antenna RS assists multiple communication groups. In each group, each member node communicates to all other member nodes. The number of communication phases is equal to the maximum number of nodes among all groups. We propose three broadcast strategies, namely, unicasting, hybrid uni/multicasting and multicasting.

Using unicasting strategy, in each BC phase, the RS transmits different data streams to different nodes and each data stream is intended exclusively only for one receiving node. Using hybrid uni/multicasting, the RS applies unicast and mul-

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A. U. T. Amah is with the Graduate School of Computational Engineering and the Communications Engineering Lab, Technische Universität Darmstadt, Darmstadt, 64283 Germany, e-mail: a.amah@nt.tu-darmstadt.de.

A. Klein is with the Communications Engineering Lab, Technische Universität Darmstadt, Darmstadt, 64283 Germany, e-mail: a.klein@nt.tu-darmstadt.de.

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ticast transmissions. For each served group, one data stream is transmitted to one node exclusively (unicast transmission) and one data stream is transmitted to the other nodes in the group (multicast transmission). Using multicasting strategy, the RS applies network coding [14] and transmits only one data stream for each served group in each BC phase. The transmitted data stream for each group is an output of a network coding operation on two data streams of two member nodes in the group. Consequently, each node has to perform a reverse operation to each of the received data streams. Hence, each node has to have side information about one of the two data streams contained in each received data stream. The side information can be its own transmitted data stream or a data stream which has been decoded in one of the previous BC phases. In this work, we consider two network coding operations for regenerative MGMW relaying, namely, modified superposition coding (mSPC) and exclusive-or (XOR).

We derive the achievable sum rate expression for regenerative MGMW relaying. Assuming perfect channel state information is available at the RS, we design generalised transmit beamforming algorithms which are suitable for all BC strategies. We use the term generalised to emphasize that each transmit beamforming algorithm is derived in a general way, such that it is valid for all BC strategies. First, we design a generalised transmit beamforming algorithm minimising the RS's transmit power while ensuring that each node receives with a rate equal to the rate received at the RS for each particular data stream. The transmit beamforming algorithm is designed by coupling the MAC and BC phases. We investigate the minimum transmit power for ensuring that each node receives with a rate equal to the rate received at the RS. Afterwards, for the case when the RS's transmit power is fixed, we design generalised low complexity transmit beamforming algorithms, namely, matched filter, zero forcing, minimisation of mean square error and MGMW-aware transmit beamforming. We investigate the sum rate performance of regenerative MGMW for fixed transmit power at the RS.

This paper is organised as follows. Section II explains the system model. The sum rate expression is derived in Section III. The transmit beamforming algorithms are explained in Section IV. The simulation results are given in Section V. Finally, Section VI provides the conclusion.

**Notations:** Boldface lower- and upper-case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts  $(\cdot)^T$ ,  $(\cdot)^*$  and  $(\cdot)^H$  stand for matrix or vector transpose, complex conjugate, and complex conjugate transpose, respectively. The operator  $\text{tr}\{\mathbf{X}\}$  denotes the trace of  $\mathbf{X}$  and  $\mathcal{CN}(0, \sigma^2)$  denotes the circularly symmetric zero-mean complex normal distribution with variance  $\sigma^2$ .

## II. SYSTEM MODEL

We consider  $L$  multi-way groups where in the  $l$ -th group,  $l \in \mathcal{L}, \mathcal{L} = \{1, \dots, L\}$ , there are  $N_l$  half-duplex single-antenna nodes that communicate to each other. For simplicity of notation, we assume the same number of nodes in each group, i.e.,  $N_l = N_{\text{mw}}, \forall l \in \mathcal{L}$ . However, the extension

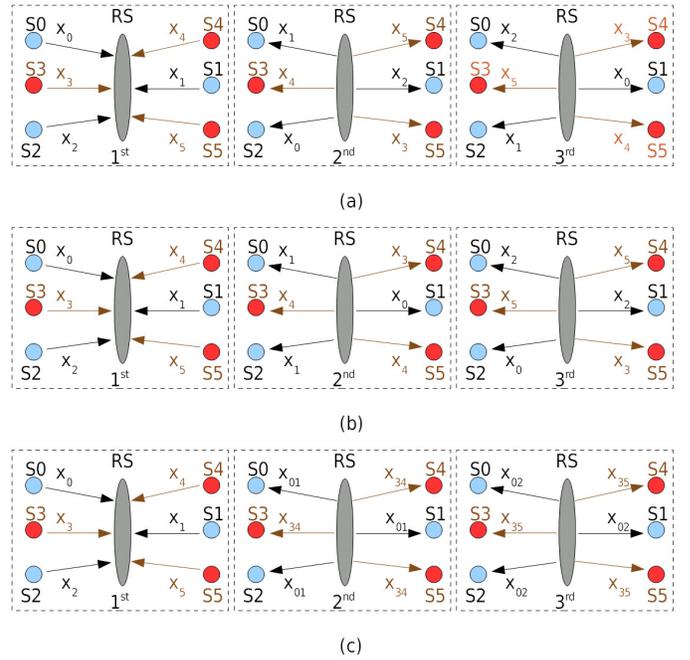


Fig. 1. Multi-group multi-way relaying with  $L=2, N_1=N_2=3$ : (a). Unicasting; (b). Hybrid uni/multicasting; (c). Multicasting.

to different numbers of nodes in each group is straightforward. The total number  $N$  of nodes in the network is  $N = \sum_{l \in \mathcal{L}} N_l = LN_{\text{mw}}$ . It is assumed that the MGMW communication can only be performed with the assistance of a half-duplex multi-antenna RS with  $M$  antenna elements.

The  $l$ -th group consists of nodes  $S_{i_l}, i_l \in \mathcal{I}_l$ , where  $\mathcal{I}_l$  is the set of nodes' indices in group  $l$  given by  $\mathcal{I}_l = \{a_l, \dots, b_l\}$ , with  $a_l = (l-1)N_{\text{mw}}, b_l = lN_{\text{mw}} - 1$ . Each node only communicates with the other nodes in its group and each node belongs only to one multi-way group, i.e.,  $\mathcal{I}_l \cap \mathcal{I}_k = \emptyset, \forall \{l \neq k\}$  and  $\mathcal{I} = \bigcup_{l=1}^L \mathcal{I}_l = \{0, \dots, N-1\}$ .

When  $N_{\text{mw}}$  half-duplex nodes communicate to each other without an RS, since each node cannot transmit and receive simultaneously, each node has to transmit sequentially. When one node transmits, the other  $N_{\text{mw}} - 1$  nodes receive. Thus, the required number  $P$  of communication phases is equal to  $N_{\text{mw}}$ . In this work, we propose communication protocols for MGMW relaying where  $P = N_{\text{mw}}$ . Within  $P$  phases, there is only one MAC phase where all nodes transmit their data streams simultaneously to the RS. The remaining  $P-1$  phases are the BC phases where the RS transmits to the nodes. In order to ensure that the MGMW communication is completed within  $P$  phases, we propose three BC strategies namely, unicasting, hybrid uni/multicasting and multicasting strategy.

Figure 1 shows the proposed BC strategies for the case of  $L=2, N_1=N_2=N_{\text{mw}}=3$ . In the first phase, all nodes  $S_{i_1}, i_1 \in \mathcal{I}_1 = \{0, 1, 2\}$ , and  $S_{i_2}, i_2 \in \mathcal{I}_2 = \{3, 4, 5\}$ , transmit their data streams  $x_i, \forall i, i \in \bigcup_1^2 \mathcal{I}_l$ , simultaneously to the RS. Let  $Q$  denote the number of transmitted data streams from the RS to the nodes in each BC phase. For the unicasting strategy, the RS transmits  $Q = N$  data streams simultaneously to the nodes, one data stream for each intended node. For example in Figure 1.(a), in the second phase the RS transmits  $x_1$  to S0,

$x_2$  to S1,  $x_0$  to S2,  $x_4$  to S3,  $x_5$  to S4,  $x_3$  to S5, and  $Q = 6$ .

Using hybrid uni/multicasting strategy, for each group the RS transmits one unicast data stream to one node exclusively and one multicasted data stream to the other  $N_l - 1$  nodes, i.e.,  $Q = 2L$ . The unicast data stream is fixed and it is transmitted to a different node in the group in each BC phase. Consequently, the multicasted data stream has to be changed in each BC phase to ensure that each node receives all data streams of the other nodes in its group. For example in Figure 1.(b) for group 1, in the second phase, the RS transmits the unicast data stream  $x_0$  to node S1 and the multicasted data stream  $x_1$  to both nodes S0 and S2. In the third phase, the RS transmits the unicast data stream  $x_0$  to node S2 and the multicasted data stream  $x_2$  to both nodes S0 and S1. For the example in Figure 1.(b), the RS transmits  $Q = 4$  data streams in each BC phase.

Using multicasting strategy, the RS transmits only one data stream per group, i.e.,  $Q = L$ . The data stream for each group is an output of a linear operation on two data streams of two nodes in the group. The general rule for selecting the two data streams is that we have to ensure that each data stream from each node has to be selected at least once within  $P - 1$  BC phases. For example in Figure 1.(c), for group 1 the RS transmits  $x_{01}$  in the second phase and  $x_{02}$  in the third phase to the group member nodes, with  $x_{v_l w_l}$  the output of a linear operation of two data streams  $x_{v_l}$  and  $x_{w_l}$ . For the example in Figure 1.(c), the RS transmits  $Q = 2$  data streams in each BC phase.

In this work, we consider two linear operations for multicasting strategy, namely, mSPC and XOR. mSPC is a modification of superposition coding (SPC) for two-way relaying in [1], [15]. While for SPC as in [1], [15], each symbol is weighted differently and, afterwards, they are added, using mSPC, the RS adds two symbols and, afterwards, the output is weighted. Thus, both symbols in mSPC are equally weighted. Hence, mSPC is suboptimum compared to SPC, but it requires lower computational complexity. Moreover, the modification in mSPC suits well with MGMW relaying and it allows us to have a simpler system model for MGMW relaying. We consider also XOR network coding since it provides low complexity solutions in three different aspects [15]: implementation, encoding/decoding and the required information for self-interference cancellation. Moreover, the practicality of XOR network coding in wireless network has been shown in [16].

#### A. Multiple Access Phase

In this subsection, the system model for the MAC phase of MGMW is introduced. The overall channel matrix from the nodes to the RS is given by  $\mathbf{H} = [\mathbf{h}_0, \dots, \mathbf{h}_{N-1}] \in \mathbb{C}^{M \times N}$ , with  $\mathbf{h}_i = (h_{i,1}, \dots, h_{i,M})^T \in \mathbb{C}^{M \times 1}$ ,  $i \in \mathcal{I}$ , the channel vector between node  $S_i$  and the RS. The channel coefficient  $h_{i,m}$ ,  $m \in \mathcal{M}$ ,  $\mathcal{M} = \{1, \dots, M\}$ , follows  $\mathcal{CN}(0, \sigma_h^2)$ . The information bit sequence of node  $i$ , denoted by  $\mathbf{b}_i$ , is coded into the complex transmit symbol  $x_i$ , i.e.,  $\mathbf{b}_i \rightarrow x_i \in \mathbb{C}$ , cf. [15]. The vector  $\mathbf{x} \in \mathbb{C}^{N \times 1}$  is given by  $(x_0, \dots, x_{N-1})^T$ , with  $x_i$  the transmit symbol of node  $S_i$  that follows  $\mathcal{CN}(0, \sigma_x^2)$ .

TABLE I  
DATA PERMUTATION MATRIX  $\mathbf{\Pi}_d^p$  OF FIGURE 1

	$\mathbf{\Pi}_d^2$	$\mathbf{\Pi}_d^3$
UC	$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$
U/MC	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$
MC-mSPC	$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$
MC-XOR	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
UC: Unicasting, U/MC: Hybrid uni/multicasting, MC-mSPC: Multicasting with modified-SPC, MC-XOR: Multicasting with XOR		

The additive white Gaussian noise (AWGN) vector at the RS is denoted as  $\mathbf{z}_{RS} = (z_{RS1}, \dots, z_{RSM})^T \in \mathbb{C}^{M \times 1}$  with  $z_{RSm}$  following  $\mathcal{CN}(0, \sigma_{z_{RS}}^2)$ . The received signal at the RS is given by

$$\mathbf{y}_{RS} = \mathbf{H}\mathbf{x} + \mathbf{z}_{RS}. \quad (1)$$

The RS decodes all received data streams from all nodes. It is assumed that the RS decodes all data streams correctly. Hence, the RS has all the information bit sequences  $\mathbf{b}_i$  from all nodes  $i \in \mathcal{I}$ .

#### B. Broadcast Phase

In this subsection, the system model for the BC phase is given. Let  $p, p \in \mathcal{P}$ ,  $\mathcal{P} = \{2, \dots, P\}$ , denote the index of the BC phase. Assuming reciprocal and time-invariant channels in  $P$  phases, the downlink channel from the RS to the nodes is simply the transpose of the uplink channel  $\mathbf{H}$ . In the  $p$ -th phase, the RS transmits the corresponding data streams to the nodes according to the chosen BC strategy. For that, the RS determines the transmit data vector  $\mathbf{x}_{RS}^p$  and the data permutation matrix  $\mathbf{\Pi}_d^p$ . Afterwards, the RS computes the transmit beamforming matrix  $\mathbf{G}^p$ .

$\mathbf{x}_{RS}^p$  is not the same for all BC strategies. For unicasting, hybrid uni/multicasting and multicasting-mSPC (MC-mSPC) strategies, the RS re-encodes the bits into a complex transmit signal, i.e.,  $\mathbf{b}_i \rightarrow x_{RSi}^p \in \mathbb{C}, \forall i \in \mathcal{I}$ , and  $\mathbf{x}_{RS}^p = (x_{RS0}^p, \dots, x_{RS_{N-1}}^p)^T$ , with  $x_{RSi}^p$  following  $\mathcal{CN}(0, \sigma_x^2)$ . For the multicasting-XOR (MC-XOR) strategy, the RS first performs a bitwise-XOR operation to the information bits of nodes  $S_{v_l}$  and  $S_{w_l}$ , i.e.,  $\mathbf{b}_{v_l w_l} = \mathbf{b}_{v_l} \oplus \mathbf{b}_{w_l}$ . The RS re-encodes the XOR-ed bits into a complex transmit signal, i.e.,  $\mathbf{b}_{v_l w_l} \rightarrow x_{RS_{v_l w_l}}^p \in \mathbb{C}$ , and  $\mathbf{x}_{RS}^p = (x_{RS_{v_l w_l}}^p, \dots, x_{RS_{L w_L}}^p)^T$ , with  $x_{RS_{v_l w_l}}^p$  following  $\mathcal{CN}(0, \sigma_x^2)$ .

In the following, let  $\mathbf{d}_{RS}^p = (d_{RS1}^p, \dots, d_{RSQ}^p)^T = \mathbf{\Pi}_d^p \mathbf{x}_{RS}^p$  and  $\mathbf{z}_{nodes}^p = (z_0^p, \dots, z_{N-1}^p)^T$ , with  $Q$  the number of the chosen RS's transmitted data streams according to the BC strategy, and  $z_{r_l}^p$  the noise at receiving node  $r_l$  which follows  $\mathcal{CN}(0, \sigma_{z_{node}}^2)$ .  $\mathbf{\Pi}_d^p$  defines the data streams to be transmitted by the RS according to the chosen BC strategy. In Table I, we provide  $\mathbf{\Pi}_d^p$  of Figure 1. For unicasting strategy,  $\mathbf{\Pi}_d^p$  only permutes  $\mathbf{x}_{RS}^p$  and has a size of  $Q \times N = N \times N$ . For the hybrid uni/multicasting strategy,  $\mathbf{\Pi}_d^p$  chooses the data streams to be unicast and multicasted in each BC phase and has a size of

TABLE II  
RELATIONSHIP OF THE PARAMETERS

UC	$q = r_l,$ $t_l = a_l + \text{mod}_{N_l}(r_l + p - a_l - 1)$
U/MC	$q = \begin{cases} 2l-1, & \text{if } r_l = t_{l_m} \\ 2l, & \text{otherwise} \end{cases}$ $t_l = \begin{cases} t_{l_u} = a_l, & \text{if } r_l = t_{l_m} \\ t_{l_m} = (p + a_l) - 1, & \text{otherwise} \end{cases}$
MC	$q = l,$ $v_l = a_l, w_l = (p + a_l) - 1$ $t_l = \begin{cases} v_l, & \text{for } r_l = (p + a_l) - 1, \\ w_l, & \text{otherwise.} \end{cases}$
$t_{l_u}$ : index of transmitting node whose data stream is unicast, $t_{l_u} \in \mathcal{I}_l$ $t_{l_m}$ : index of transmitting node whose data stream is multicasted, $t_{l_m} \in \mathcal{I}_l \setminus \{t_{l_u}\}$	

$Q \times N = 2L \times N$ . For example for the first group in Figure 1, in the second phase, it chooses  $x_{\text{RS}_0}^p$  for unicast transmission and  $x_{\text{RS}_1}^p$  for multicast transmission. In the third phase, it chooses again  $x_{\text{RS}_0}^p$  for unicast transmission and  $x_{\text{RS}_2}^p$  for multicast transmission. As a result,  $d_{\text{RS}_q}^p, \forall q = 2l - 1, \forall l$ , are the unicasted data streams for all groups and  $d_{\text{RS}_q}^p, \forall q = 2l, \forall l$ , are the multicasted data streams for all groups. Using multicasting strategy, the RS transmits  $Q = L$  data streams in each BC phase. For MC-XOR,  $\mathbf{\Pi}_d^p$  is simply an identity matrix with a size  $Q \times L = L \times L$ . However, for MC-mSPC,  $\mathbf{\Pi}_d^p$  chooses and adds two data streams for each group and has a size of  $Q \times N = L \times N$ .

The received signal vector of all nodes in the  $p$ -th phase can be written as

$$\mathbf{y}_{\text{nodes}}^p = \mathbf{H}^T \mathbf{G}^p \mathbf{d}_{\text{RS}}^p + \mathbf{z}_{\text{nodes}}^p, \quad (2)$$

and, accordingly, the received signal at node  $S_{r_l}, r_l \in \mathcal{I}_l$ , when receiving the data stream  $d_{\text{RS}_q}^p$  from the RS is given by

$$y_{r_l}^p = \underbrace{\mathbf{h}_{r_l}^T \mathbf{g}_q^p d_{\text{RS}_q}^p}_{\text{useful signal}} + \underbrace{\sum_{\substack{j \in \mathcal{Q} \\ j \neq q}} \mathbf{h}_{r_l}^T \mathbf{g}_j^p d_{\text{RS}_j}^p}_{\text{other stream interference signals}} + \underbrace{z_{r_l}^p}_{\text{node } r_l \text{'s noise}}, \quad (3)$$

where  $\mathbf{g}_q^p$  is the corresponding transmit beamforming vector for the RS's transmitted data stream  $d_{\text{RS}_q}^p$ , i.e., the  $q$ -th column of  $\mathbf{G}^p$ , and  $\mathcal{Q} = \{1, \dots, Q\}$  is the set of indices of the RS's transmitted data streams.

In the  $p$ -th phase, the receiving node  $r_l$  is intended to receive the data stream from a transmitting node  $t_l$ . Therefore, we need to have the relationship between the receiving index  $r_l$ , the transmitting index  $t_l$ , the RS transmitted data stream index  $q$  and the BC phase index  $p$ . Table II provides the relationship for all BC strategies for MGMW relaying for any number  $L$  of groups and any number  $N_l$  of nodes in each group. Table II can be read as follows: in the  $p$ -th phase, the node with receiving index  $r_l$  receives the RS's data stream with index  $q$  which corresponds to the data stream of transmitting node  $t_l$ .

Using MC-mSPC, for each group, the RS adds two data streams from two different nodes. Therefore, the useful signal in (3) contains the intended data stream and the self- and known-interference which has to be cancelled. In order to better understand, we provide an explanation using Figure 1c. Let  $x_{v_l w_l}$  denote the output of the linear operation of two

data streams of two nodes in group  $l, l = \{1, 2\}$ , namely,  $S_{v_l}$  and  $S_{w_l}$ . In the second phase, the RS sends  $x_{01}$  to all nodes in the first group and  $x_{34}$  to all nodes in the second group. Nodes S0 and S1 perform self-interference cancellation by cancelling their transmitted data stream from  $x_{01}$ . Hence, S0 obtains  $x_1$  and S1 obtains  $x_0$ . Node S2 cannot yet perform self-interference cancellation, since  $x_{01}$  does not contain its own data stream. Similarly, S3 and S4 perform self-interference cancellation but node S5 cannot yet perform self-interference cancellation. In the third phase, the RS transmits  $x_{02}$  to all nodes in the first group and  $x_{35}$  to all nodes in the second group. Nodes S0 and S2 perform self-interference cancellation so that S0 obtains  $x_2$  and S2 obtains  $x_0$ . Since S1 knows  $x_0$  from the second phase, it performs known-interference cancellation to obtain  $x_2$ . Since S2 knows  $x_0$  from the third phase, it obtains  $x_1$  by performing known-interference cancellation to the received data stream in the second phase, namely  $x_{01}$ . Similar process happens to nodes S3, S4 and S5 in the second group.

Regarding MC-XOR, the process of cancellation is similar to MC-mSPC. However, the cancellation is performed at bit level. After each node decodes the received data stream and obtains the bit sequence, it performs self- or known-interference cancellation by XORing the decoded bits with the a priori known own or known bits.

### III. SUM RATE EXPRESSION

In this section, we derive the achievable sum rate expression of regenerative MGMW relaying. The achievable sum rate is the sum of the rates at each receiving node. We first explain the MAC phase rate which is achieved at the RS. It is followed by the BC rate which can be achieved at each receiving node. Finally, the overall achievable sum rate is given.

#### A. MAC Phase

Different to previous works in regenerative two-way relaying which consider the achievable rate region, e.g., [5], [6], in this work, we consider the achievable sum rate of regenerative MGMW relaying. Therefore, we propose to apply a practical multi user detector for decoding at the RS which achieves the optimum rate of the MAC phase, i.e., one of the  $N!$  rate tuples. Hence, we consider a Minimum Mean Square Error (MMSE) with successive interference cancellation (SIC) multi-user detector since it is information theoretically optimal for the uplink MAC scenario [17].

First, we compute the signal to interference and noise ratio (SINR) of each node  $i$ , which is given by

$$\gamma_i = \sigma_x^2 \mathbf{h}_i^H \left( \sigma_x^2 \sum_{\substack{j=0 \\ j \neq i}}^{N-1} \mathbf{h}_j \mathbf{h}_j^H + \sigma_{z_{\text{RS}}}^2 \mathbf{I}_M \right)^{-1} \mathbf{h}_i \quad (4)$$

[17] and, afterwards, perform the SIC. In this work, we consider only one possible SIC based on SINR in (4). The data stream of the node with the highest SINR is decoded first and subtracted from the received data streams. The data streams of the other nodes with lower SINR are decoded successively

afterwards in a similar way. After SIC, the SINR of node  $i$  is given by

$$\gamma_i^{\text{MAC}} = \begin{cases} \gamma_i, & \text{if } i \text{ is decoded first} \\ \frac{\sigma_x^2 |\mathbf{h}_i|^2}{\sigma_{z_{\text{RS}}}^2}, & \text{if } i \text{ is decoded last} \\ \gamma_i^{\text{sic}}, & \text{otherwise} \end{cases} \quad (5)$$

where  $\gamma_i^{\text{sic}} = \sigma_x^2 \mathbf{h}_i^H \left( \sigma_x^2 \sum_{j \in \mathcal{B}_i} \mathbf{h}_j \mathbf{h}_j^H + \sigma_{z_{\text{RS}}}^2 \mathbf{I}_M \right)^{-1} \mathbf{h}_i$ , with  $\mathcal{B}_i$  the set of all nodes whose data streams have not been decoded in the previous SIC stage, excluding node  $i$ . The achievable rate of node  $i$  at the RS is defined by

$$R_i^{\text{MAC}} = \log_2 (1 + \gamma_i^{\text{MAC}}) \quad (6)$$

and the achievable sum rate for the MAC phase is given by

$$R^{\text{MAC}} = \sum_{i=0}^{N-1} R_i^{\text{MAC}}. \quad (7)$$

Different SIC ordering due to certain requirements, such as group priority, etc., can be applied. However, it is beyond the scope of this work.

### B. BC Phase

Given the received signal in (3), for unicasting, hybrid uni/multicasting and MC-XOR strategies, the SINR of node  $r_l$  when receiving data stream  $d_{\text{RS}_q}^p$  in the  $p$ -th phase is given by

$$\gamma_{r_l}^p = \frac{\sigma_x^2 |\mathbf{h}_{r_l}^T \mathbf{g}_q^p|^2}{\sigma_x^2 \sum_{j \in \mathcal{Q} \setminus \{q\}} |\mathbf{h}_{r_l}^T \mathbf{g}_j^p|^2 + \sigma_{\text{node}}^2}. \quad (8)$$

The SINR for MC-mSPC strategy is different, since the RS adds two complex signals for each group and the sum is transmitted to all group members. Therefore, for MC-mSPC,

$$\gamma_{r_l}^p = \frac{\sigma_x^2 |\mathbf{h}_{r_l}^T \mathbf{g}_q^p|^2}{\underbrace{\sigma_x^2 |\mathbf{h}_{r_l}^T \mathbf{g}_q^p|^2}_{\text{self- or known-interference}} + 2\sigma_x^2 \sum_{j \in \mathcal{Q} \setminus \{q\}} |\mathbf{h}_{r_l}^T \mathbf{g}_j^p|^2 + \sigma_{\text{node}}^2}. \quad (9)$$

For MC-mSPC strategy, each node has to perform self- or known-interference cancellation. After the self- or known-interference is perfectly cancelled, the SINR is given by

$$\gamma_{r_l}^p = \frac{\sigma_x^2 |\mathbf{h}_{r_l}^T \mathbf{g}_q^p|^2}{2\sigma_x^2 \sum_{j \in \mathcal{Q} \setminus \{q\}} |\mathbf{h}_{r_l}^T \mathbf{g}_j^p|^2 + \sigma_{\text{node}}^2}. \quad (10)$$

The achievable rate at a receiving node  $r_l$  in the  $p$ -th phase is given by

$$R_{r_l}^p = \log_2 (1 + \gamma_{r_l}^p). \quad (11)$$

### C. Overall Achievable Sum Rate

In regenerative two-way relaying, the rate at a receiving node  $r_l$  is defined by  $\min(R_{t_l}^{\text{MAC}}, R_{r_l}^{\text{BC}})$  with  $R_{t_l}^{\text{MAC}}$  the MAC rate that is achieved by the RS from node  $t_l$  in the MAC phase and  $R_{r_l}^{\text{BC}}$  is the possible rate that can be achieved at the nodes from the RS [1]. In the following, we provide the overall achievable rate of MGMW relaying.

*Unicasting Strategy:* Considering the information flow from one transmitting node  $t_l$  to a receiving node  $r_l$  in the  $p$ -th phase, the information rate is defined by

$$R_{r_l, t_l} = \min(R_{t_l}^{\text{MAC}}, R_{r_l}^p), \quad (12)$$

since the RS cannot transmit to node  $r_l$  with higher rate than what it received from node  $t_l$  in the MAC phase. The sum of the rates received at all  $N_l - 1$  receiving nodes  $r_l \in \mathcal{I}_l \setminus \{t_l\}$  when they receive the data stream of node  $t_l$  is defined by

$$R_{t_l} = \sum_{r_l \in \mathcal{I}_l \setminus \{t_l\}} R_{r_l, t_l}. \quad (13)$$

Equation (13) is different to the sum of the rates received at  $N_l - 1$  nodes when they receive the data stream of node  $t_l$  for non-regenerative MGMW relaying as in [13]. In non-regenerative MGMW relaying, the RS is a transparent RS which does not decode and re-encode the received data streams. Since the nodes can transmit only once, each transmitting node  $t_l$  has to ensure that its data stream can be decoded correctly by  $N_l - 1$  receiving nodes. Consequently, node  $t_l$  has to transmit with a rate defined by  $\min_{r_l} (R_{r_l, t_l})$ .

In regenerative MGMW relaying, the RS decodes all the received data streams and the data streams of all nodes are available at the RS prior to transmission in the BC phases. It is the task of the RS to transmit the re-encoded data streams to the nodes with a rate that can be correctly decoded by each intended receiving node. Due to the decoding and re-encoding at RS, the MAC phase and the BC phases are decoupled. Therefore, for unicasting strategy, we have (12) since in each BC phase, each node receives a different data stream from the RS and for each data stream, the RS cannot transmit with higher rate than what it received in the MAC phase. Hence, since a data stream from node  $t_l$  is received by  $N_l - 1$  receiving nodes in different BC phases, the sum of the rates received at  $N_l - 1$  receiving nodes is given by (13).

*Hybrid Uni/Multicasting Strategy:* For hybrid uni/multicasting strategy, in each BC phase the RS transmits two data streams for each group. One is the unicasted data stream which is fixed in all BC phases and the other one is the multicasted data stream which is different in each BC phase. Therefore, we have two kinds of sums of the rates. One relates to the data stream of a fixed node whose data stream is unicasted, namely  $R_{r_l, t_{l_u}}$ , and the other one relates to the data streams of nodes whose data streams are multicasted sequentially in the BC phases, namely  $R_{r_l, t_{l_m}}, t_{l_m} \in \mathcal{I} \setminus \{t_{l_u}\}$ . In the following, we first explain  $R_{r_l, t_{l_u}}$  and, afterwards, we explain  $R_{r_l, t_{l_m}}$ .

Considering the information flow from a transmitting node  $t_{l_u}$  whose data stream is always unicasted in all BC phases, to a receiving node  $r_l$  in the  $p$ -th phase, the information rate

is defined by

$$R_{r_l, t_{l_u}} = \min \left( R_{t_{l_u}}^{\text{MAC}}, R_{r_l}^p \right), \quad (14)$$

with  $r_l = t_{l_m}$  since the node whose data stream is multicasted by the RS to  $N_l - 1$  nodes in its group receives the unicast data stream. The sum of the rates received at all nodes  $r_l \in \mathcal{I}_l \setminus \{t_{l_u}\}$  when they receive the data stream of node  $t_{l_u}$  is given by

$$R_{t_{l_u}} = \sum_{r_l \in \mathcal{I}_l \setminus \{t_{l_u}\}} R_{r_l, t_{l_u}}. \quad (15)$$

As can be seen, (14) and (15) for hybrid uni/multicasting are similar to (12) and (15) for unicasting, respectively.

Considering the information flow from a transmitting node  $t_{l_m}$  whose data stream is multicasted in the  $p$ -th phase to  $N_l - 1$  receiving nodes  $r_l$ ,  $r_l \in \mathcal{I}_l \setminus \{t_{l_m}\}$ , the information rate is given by

$$R_{r_l, t_{l_m}} = \min \left( R_{t_{l_m}}^{\text{MAC}}, \min_{r_l \in \mathcal{I}_l \setminus \{t_{l_m}\}} R_{r_l}^p \right), \quad (16)$$

since the RS has to ensure that all  $N_l - 1$  receiving nodes can decode the data stream from node  $t_{l_m}$  correctly and that the RS cannot transmit with higher rate than what it received from node  $t_{l_m}$  in the MAC phase. The sum of the rates received at all nodes  $r_l \in \mathcal{I}_l \setminus \{t_{l_m}\}$  when they receive from node  $t_{l_m}$  is given by

$$R_{t_{l_m}} = \sum_{r_l \in \mathcal{I}_l \setminus \{t_{l_m}\}} R_{r_l, t_{l_m}} = (N_l - 1) R_{r_l, t_{l_m}}. \quad (17)$$

*Multicasting-XOR Strategy:* The RS performs bitwise XOR operation of two information bits from two nodes  $S_{v_l}$  and  $S_{w_l}$ . The rate received at node  $r_l$  when receiving from node  $t_l$  is given by

$$R_{r_l, t_l} = \min \left( \min \left( R_{v_l}^{\text{MAC}}, R_{w_l}^{\text{MAC}} \right), \min_{r_l \in \mathcal{I}_l} R_{r_l}^p \right). \quad (18)$$

The second part in (18) is since the RS has to ensure that all  $N_l$  receiving nodes are able to decode the multicasted data stream correctly. The first part is since the RS has to perform a bitwise-XOR operation of two information bit sequences. Since both bit sequences can be of different length, the RS needs to take the minimum out of those two. However, if zero padding (ZP) is applied to the shorter bit sequence such that both bit sequences are in the same length, we can perform XOR operation to both sequences without losing any information, cf. [15]. Since there are known zeros that are added to the shorter bit sequence, if node  $r_l$  receives from node  $t_l$  with the longer bit sequence,

$$R_{r_l, t_l} = \min \left( \max \left( R_{v_l}^{\text{MAC}}, R_{w_l}^{\text{MAC}} \right), \min_{r_l \in \mathcal{I}_l} R_{r_l}^p \right), \quad (19)$$

and if node  $r_l$  receives from node  $t_l$  with the shorter bit sequence,

$$R_{r_l, t_l} = \min \left( \min \left( R_{v_l}^{\text{MAC}}, R_{w_l}^{\text{MAC}} \right), \min_{r_l \in \mathcal{I}_l} R_{r_l}^p \right). \quad (20)$$

The sum of the rates received at all  $N_l - 1$  receiving nodes  $r_l \in \mathcal{I}_l \setminus \{t_l\}$  when they receive from node  $t_l$  is defined by

$$R_{t_l} = \sum_{r_l \in \mathcal{I}_l \setminus \{t_l\}} R_{r_l, t_l}. \quad (21)$$

*Multicasting-mSPC Strategy:* The information rate at node  $r_l$  when receiving from node  $t_l$  is given by

$$R_{r_l, t_l} = \min \left( R_{t_l}^{\text{MAC}}, \min_{r_l \in \mathcal{I}_l} R_{r_l}^p \right), \quad (22)$$

where  $t_l$  can be either  $v_l$  or  $w_l$ , with their relationship given in Table II. Equation (22) is valid since the RS has to ensure that all receiving nodes can decode the corresponding data stream from either node  $v_l$  or  $w_l$  correctly and that the RS cannot transmit with higher rate than what it received from these nodes in the MAC phase. Note that it is important to ensure that all nodes are able to decode the received data streams correctly in each BC phase, since each node needs to perform self- and known-interference cancellation. The sum of the rates received at all  $N_l - 1$  receiving nodes  $r_l \in \mathcal{I}_l \setminus \{t_l\}$  when they receive from node  $t_l$  is defined by

$$R_{t_l} = \sum_{r_l \in \mathcal{I}_l \setminus \{t_l\}} R_{r_l, t_l}. \quad (23)$$

*Asymmetric Traffic:* In regenerative MGMW relaying, the nodes in each group can communicate with different rate. In such situation, we have asymmetric traffic and the overall achievable sum rate for unicasting and multicasting strategies is given by

$$R_{\text{asymm}} = \frac{1}{P} \sum_{l=1}^L \sum_{t_l \in \mathcal{I}_l} R_{t_l}, \quad (24)$$

while for hybrid uni/multicasting strategy, it is given by

$$R_{\text{asymm}} = \frac{1}{P} \sum_{l=1}^L \left( R_{t_{l_u}} + \sum_{t_{l_m} \in \mathcal{I}_l \setminus t_{l_u}} R_{t_{l_m}} \right). \quad (25)$$

The pre-log factor  $\frac{1}{P}$  factor is because of the half-duplex constraint which requires  $P$  channel uses to perform MGMW relaying.

Asymmetric traffic allows the  $N_l - 1$  receiving nodes to receive from a transmitting node  $t_l$  with different rates. This is suitable for some applications where the nodes may communicate with different rates. In the following, we consider the case where all nodes have to communicate with the same rate.

*Symmetric Traffic:* In certain applications, the nodes in each group should communicate with the same data rate. In such situation, we have symmetric traffic. The overall rate is now defined by the minimum rate among all the links in the group. The achievable sum rate for unicasting and multicasting strategies is given by

$$R_{\text{symm}} = \frac{1}{P} \sum_{l=1}^L N_l (N_l - 1) \min_{\{r_l, t_l\} \in \mathcal{I}_l, r_l \neq t_l} R_{r_l, t_l}, \quad (26)$$

while for hybrid uni/multicasting strategy, it is

$$R_{\text{symm}} = \frac{1}{P} \sum_{l=1}^L N_l (N_l - 1)$$

$$\min \left( \min_{r_l \in \mathcal{I}_l \setminus \{t_{lu}\}} R_{r_l, t_{lu}}, \min_{t_{lm} \in \mathcal{I}_l \setminus \{t_{lu}\}, r_l \in \mathcal{I}_l \setminus \{t_{lm}\}} R_{r_l, t_{lm}} \right). \quad (27)$$

The  $N_l - 1$  factor is due to for each transmitting node there are  $N_l - 1$  receiving nodes. The  $N_l$  factor is due to there are  $N_l$  transmitting nodes in each group.

#### IV. TRANSMIT BEAMFORMING

In this section, we explain the design of the transmit beamforming for regenerative MGMW relaying. We start by explaining the optimum transmit beamforming which ensures that the RS transmits with the achievable MAC rate to each of the receiving nodes. The low complexity transmit beamforming algorithms will be explained successively afterwards.

##### A. Optimum Transmit Beamforming

The information rate at node  $r_l$  when it receives from node  $t_l$  is defined by the minimum between the MAC rate and the BC rate. Since before the transmission in each BC phase, the RS knows already  $R_i^{\text{MAC}}, \forall i \in \mathcal{I}$ , the optimum transmit strategy at the RS is to ensure that each receiving node  $r_l$  receives the data streams from node  $t_l$  with the rate equal to  $R_{t_l}^{\text{MAC}}$ . However, since transmit power at the RS is a limited resource, we have to minimise the use of it. Therefore, in this work, we propose optimum transmit beamforming which achieves the aim while minimising the transmit power at the RS.

The optimisation problem can be written as

$$\min_{\mathbf{G}^p} E\{\|\mathbf{G}^p \mathbf{\Pi}_d^p \mathbf{x}_{\text{RS}}^p\|^2\} \quad \text{s.t.} \quad \gamma_{r_l}^p \geq \gamma_{t_l}^{\text{MAC}}, \quad (28)$$

with  $\gamma_{r_l}^p$  of (8) or (10) depending on the chosen BC strategy, and  $\gamma_{t_l}^{\text{MAC}}$  given in (5). For multicasting strategy, since the RS transmits a common message which is an output of a linear operation of two data streams,  $x_{v_l}$  and  $x_{w_l}$ ,  $\gamma_{t_l}^{\text{MAC}} = \max(\gamma_{v_l}^{\text{MAC}}, \gamma_{w_l}^{\text{MAC}})$  has to be used to achieve the MAC rate.

Different to previous works in regenerative (two-way) relaying, which decouple the MAC and BC phase and treat each of them independently, by (28) we couple the MAC and BC phases. The idea comes from the fact that the RS knows already the MAC rate prior to BC transmission, and it can use this information for optimising the transmission in the BC phases. The constraint in (28) shows the coupling of MAC and BC phases, where  $\gamma_{r_l}^p$  and  $\gamma_{t_l}^{\text{MAC}}$  are the MAC-BC coupling parameters. This constraint ensures the transmission of rate  $R_{t_l}^{\text{MAC}}$  for each corresponding receiving node  $r_l$  in each  $p$ -th BC phase. Even though in this work we consider MMSE-SIC detector to achieve the optimum MAC rate, our proposal can be used for any multi-user detector.

Assuming mutually uncorrelated symbols in  $\mathbf{x}_{\text{RS}}^p$  with  $\sigma_x^2 = 1$ , (28) can be written as

$$\min_{\mathbf{g}_q^p \forall q \in \mathcal{Q}} \alpha \cdot \sum_{q \in \mathcal{Q}} \|\mathbf{g}_q^p\|_2^2 \quad \text{s.t.} \quad \gamma_{r_l}^p \geq \gamma_{t_l}^{\text{MAC}}. \quad (29)$$

The optimisation problem in (29) is valid for all BC strategies by relating the index variables  $r_l$ ,  $t_l$ ,  $q$  and  $p$ . The scalar factor  $\alpha$  depends on the BC strategy. For unicasting, hybrid uni/multicasting and MC-XOR  $\alpha = 1$ , while for MC-mSPC  $\alpha = 2$  due to the superposition of two symbols in  $\mathbf{x}_{\text{RS}}^p$ .

Since  $\alpha$  is only a scalar factor, it may be omitted from (29). Equation (29) is similar to the optimisation problem treated in [18] for downlink unicast beamforming with Quality of Service (QoS) constraint, in [19] for single-group multicast beamforming and in [20] for multi-group multicast beamforming with QoS constraint. It can be treated as the problem in [18] if  $|\mathcal{Q}| = N$ , i.e., unicasting strategy, or as the problem in [19] if  $|\mathcal{Q}| = 1$ , i.e., multicasting strategy with  $L = 1$ , or as the problem in [20] if  $|\mathcal{Q}| = L$  and  $L > 1$ , i.e., multicasting strategies with  $L > 1$ , and if  $|\mathcal{Q}| = 2L$ , i.e., hybrid uni/multicasting, with  $|\cdot|$ , in this case, the cardinality of a set. Since if we have multicasting strategy with  $L = 1$  the problem associates to [19], (29) is NP-hard [20].

By defining,  $\mathbf{X}_q^p = \mathbf{g}_q^p \mathbf{g}_q^{pH}$ ,  $\mathbf{W}_{r_l} = \mathbf{h}_{r_l}^* \mathbf{h}_{r_l}^T$ , and by dropping the rank-one constraint, we can rewrite (28) into

$$\min_{\mathbf{X}_q^p \forall q \in \mathcal{Q}} \sum_{q \in \mathcal{Q}} \text{tr}(\mathbf{X}_q^p)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{W}_{r_l} \mathbf{X}_q^p) - \alpha \gamma_{t_l}^{\text{MAC}} \sum_{j \in \mathcal{Q} \setminus \{q\}} \text{tr}(\mathbf{W}_{r_l} \mathbf{X}_j^p) \geq \gamma_{t_l}^{\text{MAC}} \sigma_{\text{node}}^2$$

$$\mathbf{X}_q^p \succeq \mathbf{0}. \quad (30)$$

Let us define  $\boldsymbol{\varkappa} = [\text{vec}(\mathbf{X}_1^p), \dots, \text{vec}(\mathbf{X}_Q^p)]$  with  $\text{vec}(\cdot)$  the vectorisation of a matrix, and a  $Q \times 1$  vector  $\mathbf{a}_{r_l} = [(\alpha \gamma_{t_l}^{\text{MAC}} + 1) \mathbf{e}_{r_l} - \alpha \gamma_{t_l}^{\text{MAC}} \mathbf{1}_Q]$  where  $\mathbf{e}_{r_l}$  is a  $Q \times 1$  vector with all zeros elements except for its  $q$ -th element which has a value of one, and  $\mathbf{1}_Q$  is a  $Q \times 1$  vector of ones. We can rewrite (30) as

$$\min_{\boldsymbol{\varkappa}} [\mathbf{1}_Q \otimes \text{vec}(\mathbf{I}_M)]^T \text{vec}(\boldsymbol{\varkappa})$$

$$\text{s.t.} \quad [\mathbf{a}_{r_l} \otimes \text{vec}(\mathbf{W})^T]^T - s_{r_l} = \gamma_{t_l}^{\text{MAC}} \sigma_{\text{node}}^2,$$

$$\mathbf{X}_q^p \succeq \mathbf{0}, \quad (31)$$

with  $s_{r_l}, \forall r_l$ , slack variables to convert the inequality constraints into equality ones. Equation (31) is a semidefinite program which can be solved using SeDuMi solver [21].

Note that for the unicasting case, since the problem associates to the problem in [18], (30) is not a relaxation, but indeed equivalent to (29). However for hybrid uni/multicasting and multicasting strategies, by dropping the rank-one constraint, the solution may be of higher rank. Therefore, one of the randomisation techniques as given in [19] needs to be performed. After finding the optimum transmit beamforming, the required transmit power at the RS is given by

$$P_{\text{RSmin}} = \text{tr}(\mathbf{G}^p \mathbf{G}^{pH}). \quad (32)$$

##### B. Low Complexity Transmit Beamforming Algorithms

Given the system model of Section II, we intend to design generalised low complexity transmit beamforming algorithms for all BC strategies. Consequently, in this subsection, it is assumed that  $M > N$ . For the design of generalised

TABLE III  
PERMUTATION MATRICES  $\Pi_G^2$  AND  $\Pi_G^3$  OF FIGURE 1

	$\Pi_G^2$	$\Pi_G^3$
UC	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
U/MC	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
MC	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$

transmit beamforming algorithms for all BC strategies based on matched filter (MF), zero forcing (ZF) and minimisation of mean square error (MMSE), another permutation matrix  $\Pi_G^p$  is needed.  $\Pi_G^p$  is needed to select which data stream  $d_{RS_q}^p$  needs to be sent to which receiving node  $Sr_l$ . Note that  $\Pi_G^p$  consists of  $e_{r_l}^T$  in its  $r_l$ -th row. Table III provides  $\Pi_G^p$  for Figure 1. The MGMW-aware transmit beamforming is designed to directly suit the system model.

1) *Matched Filter*: In [22], an MF optimisation problem is formulated using a different expression of signal to noise ratio (SNR) compared to the well known SNR of the standard MF optimisation. The equivalence of both SNR formulations is proven in Appendix A6 of [22]. In this work, we use the formulation of SNR as in [22]. For regenerative MGMW relaying, the MF optimisation problem can be written as

$$\mathbf{G}_{MF}^p = \underset{\mathbf{G}^p}{\operatorname{argmax}} \frac{|\mathbb{E}\{\mathbf{s}^p \mathbf{H} \mathbf{y}_{\text{nodes}}^p\}|^2}{\mathbb{E}\{\|\mathbf{s}^p\|^2\} \mathbb{E}\{\|\mathbf{z}_{\text{nodes}}^p\|^2\}} \quad (33)$$

s.t.  $\mathbb{E}\{\|\mathbf{G}^p \mathbf{s}^p\|^2\} \leq P_{RS}$ ,

with  $\mathbf{s}^p = \Pi_G^p \mathbf{d}_{RS}^p$  and  $\mathbf{d}_{RS}^p = \Pi_d^p \mathbf{x}_{RS}^p$ . The desired signal portion in  $\mathbf{y}_{\text{nodes}}^p$  is obtained by correlation with the desired signal  $\mathbf{s}$  and  $\mathbb{E}\{\|\mathbf{s}^p\|^2\}$  in the denominator is needed to end up with a ratio of powers [22]. Equation (33) can be rewritten as

$$\mathbf{G}_{MF}^p = \underset{\mathbf{G}^p}{\operatorname{argmax}} \frac{|\mathbb{E}\{\{\Pi_G^p \Pi_d^p \mathbf{x}_{RS}^p\}^H \mathbf{y}^p\}|^2}{\mathbb{E}\{\|\Pi_G^p \Pi_d^p \mathbf{x}_{RS}^p\|^2\} \mathbb{E}\{\|\mathbf{z}_{\text{nodes}}^p\|^2\}} \quad (34)$$

s.t.  $\mathbb{E}\{\|\mathbf{G}^p \Pi_G^p \Pi_d^p \mathbf{x}_{RS}^p\|^2\} \leq P_{RS}$ .

Using Lagrangian multiplier method, i.e., following the same procedure as in [22], [23], we obtain

$$\mathbf{G}_{MF}^p = \beta^p \underline{\mathbf{G}}_{MF}^p, \quad (35)$$

with  $\underline{\mathbf{G}}_{MF}^p = \mathbf{H}^* \Pi_G^p$  and  $\beta^p$  a normalisation factor to fulfill the transmit power constraint at the RS given by

$$\beta^p = \sqrt{\frac{P_{RS}}{\sigma_x^2 \operatorname{tr}\left(\underline{\mathbf{G}}_{MF}^p \Pi_d^p \Pi_d^p \underline{\mathbf{G}}_{MF}^{pH}\right)}}. \quad (36)$$

2) *Zero Forcing*: The optimisation problem of an MMSE with ZF constraint can be written as

$$\mathbf{G}_{ZF}^p = \underset{\mathbf{G}^p}{\operatorname{argmin}} \mathbb{E}\{\|\hat{\mathbf{s}}^p - \mathbf{s}^p\|^2\}$$

s.t.  $\mathbb{E}\{\|\mathbf{G}^p \Pi_G^p \Pi_d^p \mathbf{x}_{RS}^p\|^2\} \leq P_{RS}$ , (37)

$\hat{\mathbf{s}}^p = \mathbf{s}^p$  iff  $\mathbf{z}_{\text{nodes}}^p = 0$ ,

where the second constraint is the ZF constraint and  $\hat{\mathbf{s}}^p$  is a vector of the corresponding detected symbols at the nodes, with  $\mathbf{s}^p = \Pi_G^p \mathbf{d}_{RS}^p$  and  $\mathbf{d}_{RS}^p = \Pi_d^p \mathbf{x}_{RS}^p$ .

The ZF constraint leads to  $\mathbf{H}^T \mathbf{G}^p \Pi_G^p \Pi_d^p \mathbf{x}_{RS}^p = \Pi_G^p \Pi_d^p \mathbf{x}_{RS}^p$  which requires  $\mathbf{H}^T \mathbf{G}^p = \mathbf{I}$ . Finally, we obtain

$$\mathbf{G}_{ZF}^p = \beta^p \underline{\mathbf{G}}_{ZF}^p, \quad (38)$$

where  $\underline{\mathbf{G}}_{ZF}^p = \mathbf{H}^* (\mathbf{H}^T \mathbf{H}^*)^{-1} \Pi_G^p$  and  $\beta^p$  is solved for the first constraint and given by (36) by replacing  $\underline{\mathbf{G}}_{MF}^p$  with  $\underline{\mathbf{G}}_{ZF}^p$ .

3) *Minimisation of Mean Square Error*: The MMSE optimisation problem can be written as

$$\mathbf{G}_{MMSE}^p = \underset{\mathbf{G}^p}{\operatorname{argmin}} \mathbb{E}\{\|\hat{\mathbf{s}}^p - \Pi_G^p \Pi_d^p \mathbf{x}_{RS}^p\|^2\} \quad (39)$$

s.t.  $\mathbb{E}\{\|\mathbf{G}^p \Pi_G^p \Pi_d^p \mathbf{x}_{RS}^p\|^2\} \leq P_{RS}$ .

Using Lagrangian multiplier method, i.e., following the same procedure as in [22], [23], we obtain

$$\mathbf{G}_{MMSE}^p = \beta^p \underline{\mathbf{G}}_{MMSE}^p, \quad (40)$$

with  $\underline{\mathbf{G}}_{MMSE}^p = \mathbf{H}^* \left( \mathbf{H}^T \mathbf{H}^* + \frac{N \sigma_{\text{node}}^2}{P_{RS}} \mathbf{I}_N \right)^{-1} \Pi_G^p$  and  $\beta^p$  given by (36) by replacing  $\underline{\mathbf{G}}_{MF}^p$  with  $\underline{\mathbf{G}}_{MMSE}^p$ .

4) *MGMW-aware Transmit Beamforming*: MGMW-aware transmit beamforming is decomposed into two steps. The first step is to separate the nodes according to the transmitted data streams from the RS. The nodes who are receiving the same data stream from the RS are considered as one exclusive stream-group. Therefore, the number of stream-groups is equal to the cardinality of  $\mathcal{Q}$ . We make stream-group separation by using block diagonalisation (BD) proposed in [24] and regularised BD (RBD) proposed in [25]. We consider RBD since using BD results in a quite poor performance if the row subspaces of the users channel matrices overlap significantly [25].

The results of the first step are the equivalent channels of each stream-group which are free of interference of the other stream-groups. Therefore, in the second step, we can perform single-group transmit beamforming for each stream-group. In this work, we consider MF and semidefinite relaxation (SDR) of maximisation of minimum signal to noise ratio (MMSNR). We consider MF due to its low complexity and we consider SDR-MMSNR to balance the rates in each group since, in each group, multiple nodes want to communicate to each other.

*Equivalent Channel*: Given the channel matrix of a stream-group who receive the data stream  $d_{RS_q}^p$  from the RS as  $\mathbf{H}_q^T \in \mathbb{C}^{\eta_q \times M}$ , with  $\eta_q$  the number of nodes who receives the data stream  $d_{RS_q}^p$ , and the channel matrix of all other nodes in other stream-groups,  $\tilde{\mathbf{H}}_q^T \in \mathbb{C}^{(N-\eta_q) \times M}$ , we compute the equivalent downlink channel matrix of stream-group  $q$ ,  $\mathbf{H}_q^{\text{eq}} = \mathbf{H}_q^T \mathbf{F}_q$ . In order to obtain the null-space matrix  $\mathbf{F}_q$  of stream-group  $q$ , we perform singular value decomposition of  $\tilde{\mathbf{H}}_q^T$  given by

$$\tilde{\mathbf{H}}_q^T = \mathbf{U}_q \Sigma_q \underbrace{[\mathbf{V}_q^1 \mathbf{V}_q^0]}_{\mathbf{V}_q}, \quad (41)$$

where  $\mathbf{V}_q^0$  contains the last  $(M - \tilde{f}_q)$  right singular vectors of  $\mathbf{V}_q$  with  $\tilde{f}_q$  the rank of  $\tilde{\mathbf{H}}_q^T$ . For BD,  $\mathbf{F}_q = \mathbf{V}_q^0$ , and for RBD,  $\mathbf{F}_q = \mathbf{V}_q \left( \Sigma_q^T \Sigma_q + \frac{N \sigma_{\text{node}}^2}{P_{RS}} \mathbf{I}_M \right)$ .

TABLE IV  
MINIMUM  $P_{RS}$  FOR OPTIMUM TRANSMIT BEAMFORMING FOR  
DIFFERENT VALUES OF SNR: 1<sup>st</sup> SCENARIO

	SNR in dB						
	0	5	10	15	20	25	30
UC <sub>2</sub>	4.328	4.561	4.647	4.773	4.642	4.744	4.665
UC <sub>3</sub>	4.319	4.531	4.667	4.802	4.649	4.735	4.667
U/MC <sub>2</sub>	2.325	2.277	2.144	2.161	2.250	2.360	2.148
U/MC <sub>3</sub>	2.397	2.217	2.175	2.295	2.304	2.358	2.411
MC-XOR <sub>2</sub>	1.225	1.096	0.978	0.955	0.959	0.922	0.940
MC-XOR <sub>3</sub>	1.230	1.090	1.016	0.958	0.930	0.948	0.936
MC-SPC <sub>2</sub>	5.278	5.377	5.252	5.182	5.114	5.280	5.325
MC-SPC <sub>3</sub>	5.228	5.306	5.174	5.136	5.224	5.166	5.278

UC<sub>p</sub>: unicasting at  $p$ -th phase; U/MC<sub>p</sub>: hybrid uni/multicasting at  $p$ -th phase  
MC-XOR<sub>p</sub>: multicasting with XOR at  $p$ -th phase; MC-SPC<sub>p</sub>: multicasting with modified SPC at  $p$ -th phase

TABLE V  
MINIMUM  $P_{RS}$  FOR OPTIMUM TRANSMIT BEAMFORMING FOR  
DIFFERENT VALUES OF SNR: 2<sup>nd</sup> SCENARIO

	SNR in dB						
	0	5	10	15	20	25	30
UC <sub>2</sub>	12.84	14.61	15.19	15.12	15.42	15.82	15.28
UC <sub>3</sub>	12.86	14.65	15.21	15.13	15.39	15.79	15.41
U/MC <sub>2</sub>	6.899	7.472	8.042	8.340	8.579	9.229	8.909
U/MC <sub>3</sub>	6.824	7.272	7.832	8.366	8.785	8.925	9.021
MC-XOR <sub>2</sub>	3.773	3.772	3.808	3.916	3.862	3.870	4.095
MC-XOR <sub>3</sub>	3.781	3.811	3.831	3.922	3.825	3.875	3.981
MC-SPC <sub>2</sub>	17.88	18.03	18.69	18.26	18.55	18.53	18.55
MC-SPC <sub>3</sub>	17.80	18.27	18.40	18.14	18.67	18.54	18.20

UC<sub>p</sub>: unicasting at  $p$ -th phase; U/MC<sub>p</sub>: hybrid uni/multicasting at  $p$ -th phase  
MC-XOR<sub>p</sub>: multicasting with XOR at  $p$ -th phase; MC-SPC<sub>p</sub>: multicasting with modified SPC at  $p$ -th phase

After having the equivalent channel of each stream-group, we compute the transmit beamforming per stream-group. In this work, we consider matched filter and SDR-MMSNR.

*Matched Filter:* The transmit beamforming vector of stream-group  $q$  is given by

$$\mathbf{m}_q = \mathbf{F}_q \mathbf{H}_q^{\text{eqH}}. \quad (42)$$

*SDR-MMSNR:* In MGMW relaying, multiple nodes in each group communicate with each other. Thus, for each stream-group  $q$ , we intend to balance the BC rate at the intended receiving nodes. Therefore, we consider a fair algorithm for the transmit beamforming which aims at balancing the SNRs at the receiving nodes who are intended to receive stream-group  $q$ . Hence, we maximise the minimum SNR among the receiving nodes of stream-group  $q$ .

For transmit beamforming of stream-group  $q$ , the SNR balancing problem can be written as

$$\begin{aligned} \operatorname{argmax}_{\mathbf{m}_{\text{SDR}_q}} \min_{i_q \in \Phi_q} & \left| \frac{\mathbf{m}_{\text{SDR}_q} \mathbf{h}_{i_q}^{\text{eq}}}{\sigma_{z_{\text{node}}}^2} \right|^2 \\ \text{s.t. } & \|\mathbf{m}_{\text{SDR}_q}\|_2^2 \leq 1, \end{aligned} \quad (43)$$

with  $\Phi_q$  the set of the nodes who are intended to receive the data stream  $d_{\text{RS}_q}^p$  and  $|\Phi_q| = \eta_q$ . Equation (43) is a non-convex quadratically constrained quadratic program and is proved to be NP-hard in [19]. Nonetheless, it can be approximately solved using SDR techniques [19], [20]. It can be rewritten into a semidefinite programming as in [12], [19] and, thus, it can be solved using SEDUMI [21]. The bounds on the approximation error of the SDR techniques have been developed in [26]. The transmit beamforming vector of stream-group  $q$  is given by

$$\mathbf{m}_q = \mathbf{F}_q \mathbf{m}_{\text{SDR}_q}, \quad (44)$$

The transmit beamforming matrix  $\mathbf{G}_{\text{Tx}}^p$  is given by

$$\mathbf{G}_{\text{Tx}}^p = [\mathbf{m}_1, \dots, \mathbf{m}_Q]. \quad (45)$$

In order to satisfy the transmit power constraint at the RS, a normalisation factor

$$\beta^p = \sqrt{\frac{P_{\text{RS}}}{\sigma_x^2 \operatorname{tr} \left\{ \mathbf{G}_{\text{Tx}}^p \mathbf{\Pi}^p \mathbf{\Pi}^{pH} \mathbf{G}_{\text{Tx}}^{pH} \right\}}} \quad (46)$$

is needed. Finally, the MGMW-aware transmit beamforming  $\mathbf{G}^p$  is given by

$$\mathbf{G}^p = \beta^p \mathbf{G}_{\text{Tx}}^p. \quad (47)$$

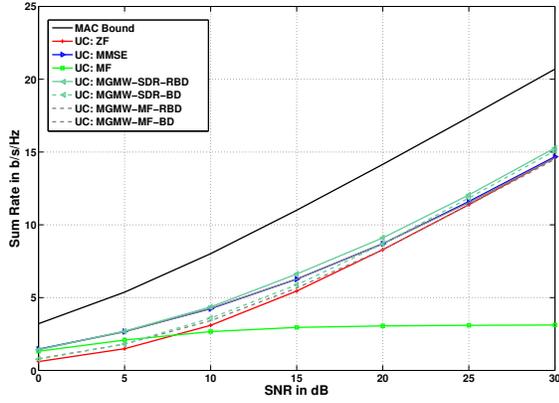
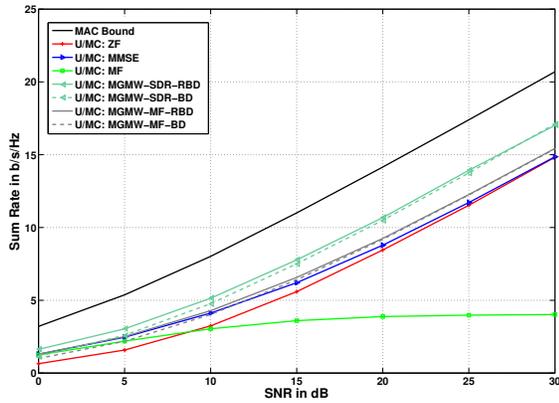
## V. SIMULATION RESULTS

In this section, we provide the simulation results to analyse the sum rate performance. In the first scenario, we consider single-group multi-way relaying with  $L = 1$ ,  $N = 3$  and  $M = N = 3$ . In the second scenario, we consider MGMW relaying as depicted in Figure 1, i.e.,  $L = 2$ ,  $N_1 = N_2 = 3$  and  $M = N = N_1 + N_2 = 6$ . We set  $\sigma_{z_{\text{RS}}}^2 = \sigma_{z_{\text{node}}}^2 = 1$ ,  $\sigma_x^2 = 1$ . The channel coefficients are i.i.d.  $\mathcal{CN}(0, \sigma_h^2)$ , i.e., Rayleigh fading, and the SNR is defined by  $\sigma_h^2$ .

Tables IV and V show the average minimum required power at the RS using the optimum transmit beamforming which guarantees the transmission in the BC phases with the achievable MAC rate for the first and second scenario, respectively, taken from 300 channel realisations where RandC as in [19] was used as the randomisation technique. Multicasting-XOR requires the least power followed by the hybrid uni/multicasting, unicasting and MC-mSPC strategies. The optimum transmit beamforming ensures that the MAC rate is achieved, which denoted as the MAC bound in the following simulation figures, i.e., Figures 2-9.

In order to assess the BC strategies for MGMW relaying with low complexity transmit beamforming, we perform 10000 channel realisations and we set  $P_{\text{RS}} = 1$ . Since asymmetric traffic leads to a higher sum rate performance compared to symmetric traffic, in order to reduce the number of lines in the figures, in the following, we only consider the asymmetric traffic case. Each of the following figures shows the performance comparison of a respective BC strategy with different transmit beamforming algorithms, namely, MF, ZF, MMSE, MGMW-aware MF and MGMW-aware SDR-MMSNR.

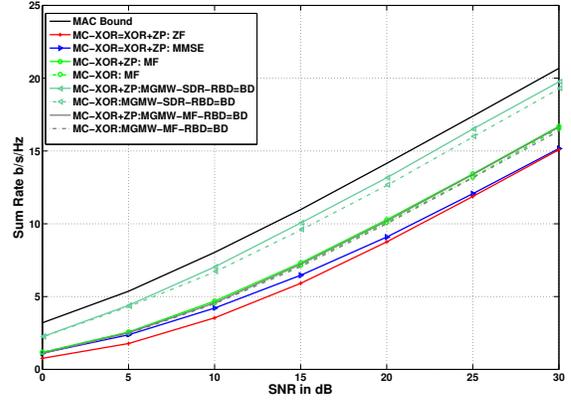
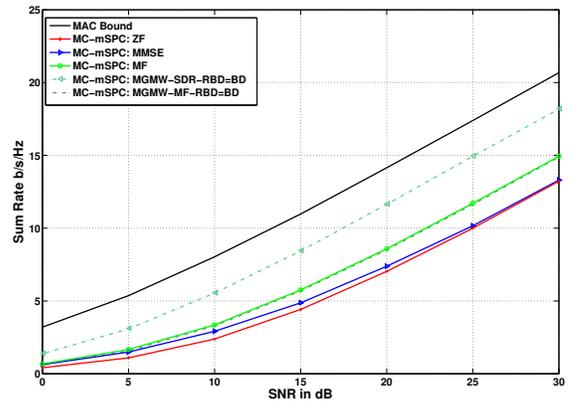
Figures 2, 3, 4 and 5 show the sum rate performances for the first scenario of the unicasting, hybrid uni/multicasting, multicasting XOR and multicasting mSPC strategies, respectively. For unicasting strategy in Fig. 2, MMSE and MGMW-aware with RBD perform similar, while ZF performs similar to MGMW-aware with BD. ZF and MGMW-aware with BD, which only suppress the interference without taking into consideration the perturbation due to noise, perform worse in low SNR region. In high SNR region, as expected, ZF converges


 Fig. 2. Sum rate performance of unicasting (UC) strategy:  $L = 1, N = 3$ 

 Fig. 3. Sum rate performance of hybrid uni/multicasting (U/MC) strategy:  $L = 1, N = 3$ 

to MMSE and MGMW-aware with BD converges to MGMW-aware with RBD. MF transmit beamforming performs worse in high SNR region since it does not manage the interference in the network. In low SNR region, MF performs better than ZF and MGMW-aware with BD, and it converges to MMSE.

As shown in Fig. 3 for hybrid uni/multicasting strategy, MGMW-aware transmit beamforming is able to outperform MMSE, ZF and MF. As for unicasting strategy in Fig. 2, MF performs worst since it does not manage the interference in the network. MGMW-aware SDR-MMSNR performs best followed by MGMW-aware MF. Since for hybrid uni/multicasting, the rate of the multicasted data stream is defined by the minimum BC rate of the  $N_l - 1$  receiving nodes, if one can improve the lowest BC rate, the sum rate performance will increase. MGMW-aware SDR-MMSNR aims at balancing the rate at the receiving nodes and, thus, it improve the worst SNR which leads to a higher BC rate of the minimum BC rate among the receiving nodes. Therefore, MGMW-aware SDR-MMSNR outperforms MGMW-aware MF and performs best compared to the other transmit beamforming algorithms.

For multicasting strategy, for both multicasting XOR in Fig. 4 and multicasting mSPC in Fig. 5, MF is able to outperform MMSE and ZF. For single-group multi-way relaying, by using


 Fig. 4. Sum rate performance of multicasting XOR strategy with Zero Padding (MC-XOR+ZP) and without Zero Padding (MC-XOR):  $L = 1, N = 3$ 

 Fig. 5. Sum rate performance of multicasting mSPC (MC-mSPC) strategy:  $L = 1, N = 3$ 

multicasting strategy there is no interstream interference. Thus, MF performs better than MMSE and ZF, and it has the same performance as MGMW-aware MF. Once again MGMW-aware SDR-MMSNR performs best followed by MF and MGMW-aware MF. The reason is the same as explained for hybrid uni/multicasting. Since MGMW-aware SDR-MMSNR increases the lowest BC rate among the receiving nodes by maximising the minimum SNR, the sum rate performance of multicasting strategy becomes better. Since in the first scenario, there is no interstream interference when using multicasting strategy, MGMW-aware with RBD performs the same as MGMW-aware with BD. Regarding multicasting XOR, MGMW-aware and MF transmit beamforming algorithms are able to get the benefit of ZP while MMSE and ZF are not. Comparing both Fig. 4 and 5, XOR network coding outperforms mSPC. mSPC adds two symbols, consequently, the power is divided for both symbols, while for XOR, there is only one transmitted symbols, since network coding is performed at bit level.

For the first scenario, comparing Figs. 2-5, in general, MC-XOR with MGMW-aware SDR-MMSNR performs best.

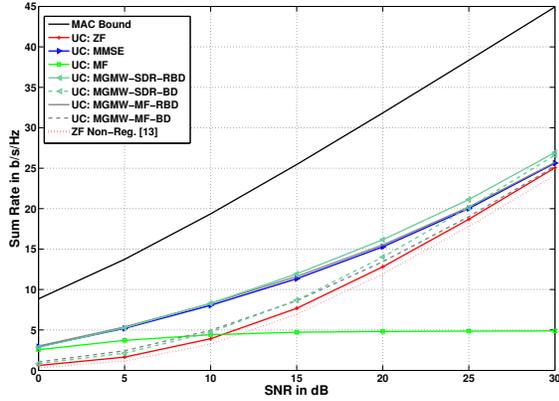


Fig. 6. Sum rate performance of unicasting (UC) strategy:  $L = 2, N_1 = N_2 = 3$

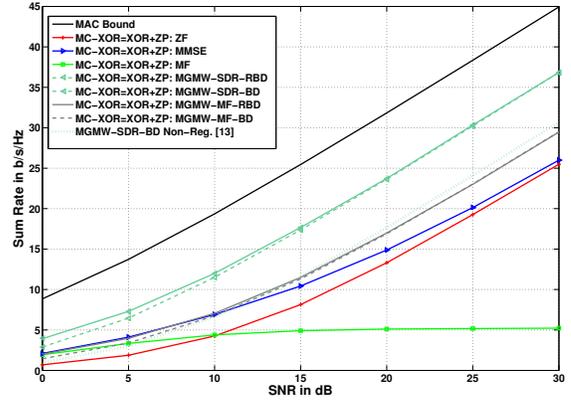


Fig. 8. Sum rate performance of multicasting XOR strategy with Zero Padding (MC-XOR+ZP) and without Zero Padding (MC-XOR):  $L = 2, N_1 = N_2 = 3$

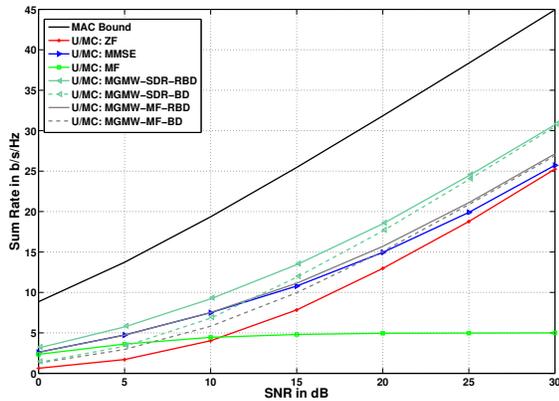


Fig. 7. Sum rate performance of hybrid uni/multicasting (U/MC) strategy:  $L = 2, N_1 = N_2 = 3$

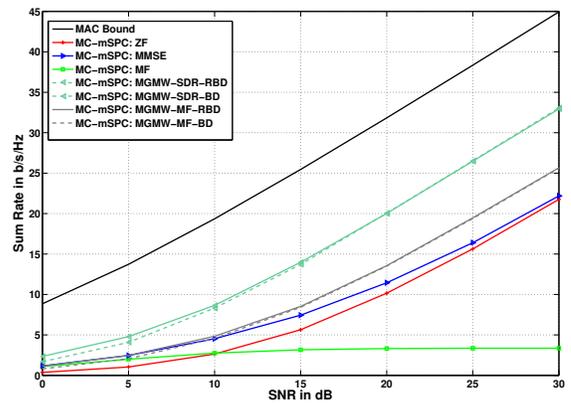


Fig. 9. Sum rate performance of multicasting mSPC (MC-mSPC) strategy:  $L = 2, N_1 = N_2 = 3$

Using MGMW-aware SDR-MMSNR, one can clearly see that multicasting strategy performs best followed by hybrid uni/multicasting and unicasting strategies. The performance improvement of MGMW-aware SDR-MMSNR compared to the other transmit beamforming algorithms is obtained most in case of multicasting strategy. The lower the number of interstream interferences and the higher the number of nodes who receive the multicasted data stream, the higher the performance gain of MGMW-aware SDR-MMSNR compared to the other algorithms. Another important analysis is that RBD outperforms BD. The higher the value of  $Q$  one strategy has, the higher the gain of RBD against BD. This is also the reason why for multicasting strategy there is no improvement when using RBD, since in this scenario, the RS transmits only one data stream to all nodes. Even though for unicasting and hybrid uni/multicasting strategies there is a performance improvement using RBD, in high SNR, BD converges to RBD. The reason is the same as why ZF converges to MMSE in high SNR region. Both BD and ZF only consider the interference and try to suppress it at the expense of a noise enhancement. On the other hand, both RBD and MMSE find a trade off between interference suppression and noise enhancement.

The sum rate performances of the second scenario are shown in Figs. 6-9 for unicasting, hybrid uni/multicasting, multicasting XOR and multicasting mSPC strategy, respectively. Each figure compares different transmit beamforming algorithms for a respective BC strategy. For unicasting strategy in Fig. 6, the trends that are seen in the first scenario in Fig. 2 also appear. As expected, the sum rate for the second scenario is higher than for the first scenario. In Fig. 6, for performance comparison, the sum rate performance of unicasting strategy for non-regenerative MGMW relaying with ZF transceiver (transmit-receive) beamforming as in [13] is plotted. It can be seen that the sum rate performance of regenerative MGMW relaying outperforms non-regenerative MGMW relaying with ZF. Only MF which does not manage the interference in the network performs worse than non-regenerative MGMW relaying with ZF.

For hybrid uni/multicasting strategies in Fig. 7, as in the first scenario, MGMW-aware SDR-MMSNR performs best followed by MGMW-aware MF. As in the case of unicasting strategy, MF is outperformed by MMSE and ZF.

Figures 8 and 9 show the sum rate performances of multi-

casting XOR and multicasting mSPC strategies, respectively. In this scenario, the gain when using RBD also is perceived for the multicasting strategy, since now there are other group interferences that have to be separated by the RS. The higher the value of  $Q$  one BC strategy has, the higher the RBD gain. As expected, MGMW-aware SDR-MMSNR outperforms the other transmit beamforming algorithms. Different to the first scenario, in the second scenario MF performs worse and does not perform as good as MGMW-aware MF. This is due to the appearance of other group interferences which are not cancelled by MF but by MGMW-aware MF. In Fig. 8, for performance comparison, the sum rate performance of non-regenerative MGMW relaying with MGMW-aware SDR-MMSNR transceive beamforming with BD as in [13] is plotted. It can be seen that regenerative MGMW relaying with MGMW-aware SDR-MMSNR outperforms non-regenerative MGMW relaying with MGMW-aware SDR-MMSNR transceive beamforming. Non-regenerative MGMW relaying with MGMW-aware SDR-MMSNR, however, is able to outperform regenerative MGMW relaying with multicasting strategy applying other transmit beamforming algorithms. Regarding multicasting XOR, in this scenario, there is no improvement when using ZP. This shows that the minimum BC rate is lower than the maximum between the two linearly processed data streams. However, if only the RS transmits with higher power, such that the minimum BC rate can be improved, one can see again the gain of using ZP.

In summary, the sum rate performance of the proposed BC strategies depends on the applied transmit beamforming. In general, MC-XOR always shows its superiority compared to the other strategies especially in high SNR. In low SNR, other strategies may perform better depending on the applied transmit beamforming. However, one can conclude that when interference defines the performance more than the noise, i.e., in high SNR region, BC strategies which have smaller number of  $Q$  in each BC phase perform better than ones with higher number of data streams.

*Related Open Issues:* In this work, we consider linear transmit beamforming algorithms for the BC phases. It has been shown in [27] that dirty paper coding achieves the capacity region of the BC channel. It is an interesting open issue to consider dirty paper coding for regenerative MGMW relaying.

In the simulation results, we have seen that MGMW-aware SDR-MMSNR performs best. In MGMW-aware SDR-MMSNR, we suppress the interference signal using BD or RBD and, afterwards, we balance the SNR. Another interesting issue is to directly balance the nodes' SINR. SINR balancing problem has been treated, for example, in [28], [29] where it is solved in an iterative way by exploiting the uplink-downlink duality.

In this work we assume that there are no direct links between the communicating nodes. We propose communication protocols where the number of communication phases is equal to the number of phases when the nodes communicate directly without an RS. If the nodes communicate with each other using both direct links and RS-nodes links, due to the half-duplex constraint, the number of communication phases will

be higher than the number of communicating nodes. The work of [30] which considers single-group multi-way relaying with direct links requires four phases for three-way relaying. If both direct links and RS-nodes links are available and multi-carrier transmission is used, the subcarrier pairing as proposed in [31] can be applied to regenerative MGMW relaying by pairing the subcarriers for direct links and the subcarriers for RS-nodes links.

## VI. CONCLUSION

We consider regenerative MGMW relaying. A half-duplex regenerative multi-antenna RS assists multiple communication groups. In each group, multiple half-duplex nodes communicate to each other. We propose three BC strategies: unicasting, hybrid uni/multicasting and multicasting. We propose a transmit beamforming algorithm minimising the RS's transmit power while guaranteeing that each node receives with a rate equal to the rate received at the RS for each particular data stream. The transmit beamforming algorithm is designed by coupling the MAC and BC phases. We also design low complexity transmit beamforming algorithms: MF, ZF, MMSE and MGMW-aware. In general, MC-XOR performs best and the performance of the BC strategies depends strongly on the applied transmit beamforming. It is also shown that MC-XOR requires less transmit power followed by hybrid uni/multicasting, unicasting and MC-mSPC.

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**Aditya Umbu Tana Amah (S'09)** received his BSc. in Electronics Engineering majoring in Communication Engineering from Satya Wacana Christian University, Salatiga, Indonesia, in 2000. After worked for Schlumberger in 2001, he joined Petra Christian University, Surabaya, Indonesia, in 2002-2005 as an academic staff at the Department of Electrical Engineering. He received his MSc. in Information and Communication Engineering from Technische Universität (TU) Darmstadt, Darmstadt, Germany, in 2007. Since 2008, he is with the Graduate School of Computational Engineering, TU Darmstadt, as a Ph.D. student and he conducts his research at the Communication Engineering Labs, TU Darmstadt.



**Anja Klein (S93-M96)** received the diploma and Dr.-Ing. (Ph.D.) degrees in electrical engineering from University of Kaiserslautern, Germany, in 1991 and 1996, respectively. From 1991 to 1996, she was a member of the staff of the Research Group for RF Communications at University of Kaiserslautern. In 1996, she joined Siemens AG, Mobile Networks Division, Munich and Berlin. She was active in the standardization of third generation mobile radio in ETSI and in 3GPP, for instance leading the TDD group in RAN1 of 3GPP. She was vice president, heading a development department and a systems engineering department. In May 2004, she joined Darmstadt University of Technology, Germany, as full professor, heading the Communications Engineering Lab. Her main research interests are in mobile radio, including multiple access, single and multi carrier transmission schemes, multi antenna systems, radio resource management, network planning and dimensioning, relaying and cooperative communication. Dr. Klein has published over 150 refereed papers and has contributed to five books. She is inventor and co-inventor of more than 45 patents in the field of mobile radio. In 1999, she was inventor of the year of Siemens AG. Dr. Klein is a member of IEEE and of the Verband Deutscher Elektrotechniker - Informationstechnische Gesellschaft (VDE-ITG).