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Interference Alignment in Multi-User Two Way Relay Networks

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***Abstract*—In this paper, we consider a *bidirectional* communication between K node pairs, where each node is equipped with multiple antennas and has a message to be transmitted to its communication partner. There is no direct link between the $2K$ nodes and a relay is employed to enable the communication. It is assumed that the relay does not have enough antennas to perform receive and transmit zero forcing. As interferences are unavoidable at each receiver, we call this channel K *user pair symmetric relay interference channel*. We show that the Degrees of Freedom (DoF) achievable by the multiple input multiple output (MIMO) Interference Alignment in such a K user pair symmetric relay interference channel are the same as in a K user symmetric interference channel [1] without a relay. Besides this, the presence of a relay simplifies the algorithm for computing the alignment solution. We propose a two step closed form solution for the alignment problem based on the two way relaying protocol.**

I. Introduction

Recently, interference alignment has evolved as an efficient technique to handle interferences at high signal to noise ratio (SNR). In [1], interference alignment is introduced for a K user interference channel, where K nodes want to communicate with their corresponding partners. In case of interference alignment, the receiver subspace is divided into two halves: the useful subspace (USS) and the interference subspace (ISS). All transmitters choose their transmit signals such that at each receiver, all the interferences are within the interference subspace, thus leaving the desired signal subspace interference free [1]. The sum capacity per user is shown to be

$$SR = \frac{1}{2}\log(SNR) + o(\log(SNR)). \tag{1}$$

At high SNR, the second term becomes negligible and every user is able to achieve (almost surely) one half Degrees of Freedom (DoF), resulting in a total of $K/2$ degrees of freedom in the system. Interference alignment can be achieved in time, frequency and spatial dimensions. In this paper we consider interference alignment in the spatial domain.

In [2], iterative methods have been proposed for MIMO (spatial) interference alignment in a K user interference channel. The iterative algorithms can only converge if an alignment solution is feasible for the considered scenario. In [3], [4], feasibility conditions for the interference alignment are derived in terms of the number of antennas at each node and the number of nodes in the system. In [2], a K user single input single output (SISO) interference channel is also considered. Here, a single antenna relay is used to help interference alignment. By transmitting the signal directly from the source to the destination and additionally indirectly through a relay using altogether two time slots, a virtual MIMO scenario is created. Iterative algorithms can be applied to this virtual MIMO channel to achieve the alignment solution [2].

In [5], a MIMO Y channel with three multi-antenna nodes is considered. Each node wants to transmit independent messages to the other two nodes. There is no direct path between the nodes. A relay is used as a means to connect the nodes. In the first time slot, all the nodes transmit their signal to the relay such that for any given receiver, the self interferences and the useful signals are aligned at the relay [5]. In the second time slot, the relay performs physical layer network coding based selective interference nulling beamforming to transmit the signal to the receivers [5].

In [6], [7], [8], [9], [10] and the references therein, multi-user two-way and multi-way relaying has been considered from different perspectives, however, not utilizing interference alignment approaches. They consider users with single antenna where the pairs are separated in code dimension or signal level or spatial dimension assuming the relay has enough antennas to perform transmit and receive zero forcing.

In this paper, we consider a *bidirectional* communication between K node pairs. Each of the $2K$ nodes wants to transmit d data streams to its partner. Each node has multiple antennas and there is no direct path between any two nodes. A half-duplex relay with multiple antennas is used to enable their communication. It is assumed that the relay does not have enough antennas to perform receive and transmit zero forcing. The interferences at each receiver are of approximately the same average power as the useful signal and if not treated properly, they will limit the system capacity. Hence, we call this channel a K *user pair symmetric relay interference channel*. The term *symmetric* implies that each of the $2K$ node has the same number N of antennas. The nodes can use their antennas to perform interference alignment. We derive the feasibility conditions for interference alignment in terms of the number N of antennas at each node and the number $2K$ of nodes in the system. Then, the value of the achievable degrees of freedom in the system is obtained. We propose a two step algorithm based on two way relaying to achieve the interference alignment solution.

The organization of the paper is as follows. In section II, the system model is introduced. In section III, the feasibility condi-

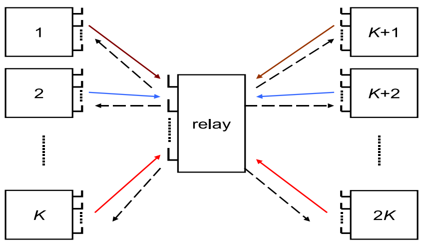

Fig. 1. K user pair symmetric relay interference channel

tions for the interference alignment in a K user pair symmetric relay interference channel are discussed. Section IV describes a two step closed form solution for the interference alignment problem. Simulation results comparing the performance of the proposed two step interference alignment algorithm with that of other state of the art beamforming methods without interference alignment are shown in the section V. Section VI concludes the work. We use lower case letters for scalars and lower case bold letters and upper case bold letters to denote vectors and matrices, respectively. We define two subspaces to be linearly independent if no vector in one subspace can be expressed as a linear combination of the basis vectors of the other subspace.

II. System Model

Figure 1 shows the K user pair symmetric relay interference channel. Each of the $2K$ nodes wants to transmit d data streams to its corresponding partner, that is, a bidirectional communication is assumed. There is no direct link between the nodes. All the nodes have the same number N of antennas and the relay has R antennas. The relay is assumed to be half-duplex. The two way relaying protocol [11] is considered in order to reduce the loss in spectral efficiency due to the half-duplex relay. In the first phase called multiple access phase (MAC phase), the transmitters transmit the signal to the relay and in the second phase called broadcast phase (BC phase), the relay broadcasts the signals to the destination. Let $\mathbf{d}_j$ and $\mathbf{V}_j$ denote the data symbols and the transmit beamforming matrix, respectively, of node j. Let $\mathbf{H}_{\mathrm{r}j}$ denote the MIMO channel between the transmitter j to the relay. The noise at the relay is denoted by the vector $\mathbf{n}_{\mathrm{r}}$. The components of the noise vector $\mathbf{n}_{\mathrm{r}}$ are i.i.d complex Gaussian random variables which follow $\mathcal{CN}(0, \sigma_{\mathrm{r}}^2)$. The signal received at the relay is given by

$$\mathbf{r} = \sum_{j=1}^{2K} \mathbf{H}_{\mathrm{r}j} \mathbf{V}_j \mathbf{d}_j + \mathbf{n}_{\mathrm{r}}. \tag{2}$$

Amplify and Forward (AF) relaying is assumed. The relay multiplies the received signal with the relay processing matrix $\mathbf{G}$ and broadcasts the resulting signal $\mathbf{s}$ defined by $\mathbf{s} = \mathbf{G}\mathbf{r}$. The relay has a transmit power constraint defined by $\mathrm{trace}\left(\mathbf{s}\mathbf{s}^{\mathrm{H}}\right) \leq P_{\mathrm{r}}$ where P_{r} is the total transmit power available at the relay. Let $\mathbf{n}_k$ denote the noise at node k. The components of the noise vector $\mathbf{n}_k$ are i.i.d complex Gaussian random variables which follow $\mathcal{CN}(0, \sigma_k^2)$. The received signal at receiver k is denoted by

$$\mathbf{y}_k = \mathbf{H}_{k\mathrm{r}} \mathbf{G} \left[\sum_{j=1}^{2K} \mathbf{H}_{\mathrm{r}j} \mathbf{V}_j \mathbf{d}_j + \mathbf{n}_{\mathrm{r}} \right] + \mathbf{n}_k, \tag{3}$$

where $\mathbf{H}_{k\mathrm{r}}$ is the matrix denoting the channel between the relay and node k. Without loss of generality, we can assume that node i and node k are pairs which want to communicate with each other for $i = 1, 2, \cdots K$ and $k = i+K$. Let $\widetilde{\mathbf{n}}_k = \mathbf{H}_{k\mathrm{r}} \mathbf{G} \mathbf{n}_{\mathrm{r}} + \mathbf{n}_k$ denote the effective noise at receiver k. Equation (3) can be rewritten as

$$\mathbf{y}_k = \mathbf{H}_{k\mathrm{r}} \mathbf{G} \left[\mathbf{H}_{\mathrm{r}i} \mathbf{V}_i \mathbf{d}_i + \mathbf{H}_{\mathrm{r}k} \mathbf{V}_k \mathbf{d}_k + \sum_{j=1, j \neq i,k}^{2K} \mathbf{H}_{\mathrm{r}j} \mathbf{V}_j \mathbf{d}_j \right] + \widetilde{\mathbf{n}}_k. \tag{4}$$

In the above equation, the first and the second terms correspond to the useful and the self-interference signals, respectively. The third term corresponds to the unknown interference. Self-interference can be perfectly cancelled at the receiver. Zero forcing is performed at the receiver in order to suppress the unknown interference. Let $\mathbf{U}_k^{\mathrm{H}}$ denote the zero forcing matrix at receiver k. Then the estimated data stream is given by

$$\hat{\mathbf{d}}_k = \mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{k\mathrm{r}} \mathbf{G} \left[\mathbf{H}_{\mathrm{r}i} \mathbf{V}_i \mathbf{d}_i + \sum_{j=1, j \neq i,k}^{2K} \mathbf{H}_{\mathrm{r}j} \mathbf{V}_j \mathbf{d}_j \right] + \mathbf{U}_k^{\mathrm{H}} \widetilde{\mathbf{n}}_k. \tag{5}$$

In order to decode the useful signal successfully, the unknown interference should be within the ISS and the USS should be linearly independent of the ISS and should be larger than or equal to the dimension of the data vector $\mathbf{d}_j$. This means that in equation (5), the following conditions need to be satisfied:

$$\mathrm{rank}\left(\mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{k\mathrm{r}} \mathbf{G} \mathbf{H}_{\mathrm{r}i} \mathbf{V}_i\right) = d, \tag{6}$$

$$\mathbf{U}_k^{\mathrm{H}} \mathbf{H}_{k\mathrm{r}} \mathbf{G} \mathbf{H}_{\mathrm{r}j} \mathbf{V}_j = 0 \qquad \forall j \neq i, k. \tag{7}$$

The number N of antennas at the nodes and the number R of antennas at the relay required for the alignment conditions to be feasible are derived in the following section in three steps. First, the number of antennas necessary at the relay station for the transmission to be possible in the system is derived. Secondly, the signal alignment at the relay is shown to be the only possible solution for achieving the interference alignment at the receivers. At last, the number of antennas necessary at each node in order to achieve the alignment is derived.

III. Interference Alignment: Feasibility Conditions

A. Constraint on the Number of Relay Antennas

In this section, the minimum required number of antennas at the relay is derived so that Kd streams can be transmitted in each direction in the system. The relay interference channel is a multiple keyhole channel. This can be clearly seen by inspecting the effective channel matrix from the transmitters to the receivers. For this purpose, consider one direction of transmission through the relay. Let us consider the channels from left to right in Figure 1 given by

$$\widetilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_{K+1,r} \\ \vdots \\ \mathbf{H}_{2K,r} \end{bmatrix} \mathbf{G} \left[\mathbf{H}_{\mathrm{r},1} \cdots \mathbf{H}_{\mathrm{r},K} \right]. \tag{8}$$

The channel matrices $\mathbf{H}_{k,r}$, $\mathbf{H}_{\mathrm{r},k}$ are assumed to be generic without any special structure and of full rank. The relay processing matrix $\mathbf{G}$ is assumed to be of full rank R. We also assume that $R < KN$. For $R \geq KN$, each node could transmit $\frac{N}{2}$ streams and the relay could perform transmit and receive zero forcing. From the properties of the matrix rank, it is clear that $\operatorname{rank}(\widetilde{\mathbf{H}}) = R$. If each node wants to transmit d data streams, Kd data streams need to be transmitted through the keyhole channel $\widetilde{\mathbf{H}}$ in total. This leads to the constraint

$$R \geq Kd. \tag{9}$$

The case $R = Kd$ is considered in the present paper.

B. Signal Alignment at the Relay

In this section, assuming $R = Kd$, it will be shown that in order to align the interferences at the receivers, the USS of any transmitter needs to be aligned with the USS of its communication partner at the relay. We call this *signal alignment*. The signal space of any given receiver k is of dimension N. In order to satisfy the interference alignment conditions given by equations (6) and (7), a USS of size d needs to be reserved for the useful signal and all the interferences should be within the ISS of size $N-d$. The self interference is assumed to be known and can easily be perfectly cancelled at the receiver. Hence, there are $2(K-1)d$ interfering data streams at the receiver. The channel matrix between the relay and receiver k is a linear map if $R \leq N$ and is a linear projection if $R > N$. Hence, the ISS at the receiver k corresponds to a subspace of maximum dimension $R-d = Kd-d = (K-1)d$ at the relay. All the $2(K-1)d$ interfering streams should lie in this $(K-1)d$ dimensional ISS at the relay and the d dimensional USS should be linearly independent of ISS at the relay. Similar conditions hold for each of the receiver $k = 1, 2 \cdots 2K$ and lead to the condition that at the relay, the USS of each of the nodes must align with the USS of its communication partner. Equivalently, we can say that at the relay, the useful signals from each of the nodes should span the same d dimensional subspace as its communication partner. This is expressed by the equation:

$$\operatorname{span}(\mathbf{H}_{\mathrm{r}i}\mathbf{V}_i) = \operatorname{span}(\mathbf{H}_{\mathrm{r}k}\mathbf{V}_k) \tag{10}$$

where $i = 1, 2, \cdots K$ and $k = i + K$.

C. Constraint on the Number of Transmit Antennas

In this section, the condition on the number N of antennas at each of the nodes is derived. Knowing that the interfering signals need to be pairwise aligned at the relay, we can derive the condition on N based on two constraints. The first constraint is that during the MAC phase, each node should be able to align its USS with its corresponding partner's USS at the relay. The second constraint is that during the BC phase, the receiver space has to be sufficiently large such that the relay can align the interferences in the ISS at each of the receivers. In the following subsections, these two constraints are considered and the conditions on N are derived.

1) MAC phase: In the MAC phase, each node needs to choose its transmit beamforming matrix $\mathbf{V}_j$ for $j = 1, 2 \cdots 2K$ such that the useful signal, when transmitted through the channel, spans the same subspace as its partner's signal at the relay. Any vector transmitted from node j to the relay will lie in the subspace spanned by the columns of the matrix $\mathbf{H}_{\mathrm{r}j}$ at the relay. This subspace can be viewed as an N dimensional subspace S_j^{r} in the R dimensional space. The columns of the matrix $\mathbf{H}_{\mathrm{r}j}$ form a basis for this subspace. Let the columns of $\widetilde{\mathbf{V}}_j$ denote the receive signatures at the relay defined by

$$\widetilde{\mathbf{V}}_j = \mathbf{H}_{\mathrm{r}j}\mathbf{V}_j. \tag{11}$$

Then $\widetilde{\mathbf{V}}_j$ is a d-dimensional subspace within S_j^{r}. In order to satisfy equation (10), we need to find a d-dimensional subspace that lies in S_j^{r} and S_k^{r}. In other words, the intersection of S_j^{r} and S_k^{r} should be at least d dimensional. Equivalently, the columns of the matrix $\mathbf{H}_{\mathrm{r}jk} = [\mathbf{H}_{\mathrm{r}j}\mathbf{H}_{\mathrm{r}k}]$ should have at least d column vectors that are linearly dependent on the other column vectors. As all channel matrices are assumed to be of full rank, the columns of $\mathbf{H}_{\mathrm{r}j}$ and $\mathbf{H}_{\mathrm{r}k}$ are each linearly independent. The rank of the matrix $\mathbf{H}_{\mathrm{r}jk}$ is given by $\min(R, 2N)$. If $2N > R$, then the rank of the matrix is R. This implies that $2N - R$ columns of $\mathbf{H}_{\mathrm{r}jk}$ are linearly dependent on the other R columns. For d dependent columns, $2N - R \geq d$. Hence

$$N \geq \frac{(K+1)}{2}d. \tag{12}$$

A similar derivation for the 3-user MIMO Y channel MAC phase is also available in [5].

2) BC Phase: In the BC phase, the relay processing matrix $\mathbf{G}$ has to be designed such that at each receiver all the interference signals lie in an $N-d$ dimensional ISS. Let $\mathbf{V}'_j$ be the transmit beamforming matrix corresponding to the aligned signal of the pair (j,k) at the relay. Then, $\mathbf{V}'_j$ is defined by

$$\mathbf{V}'_j = \mathbf{G}\widetilde{\mathbf{V}}_j \tag{13}$$

where $j = 1, 2, \cdots K$ and $\operatorname{trace}\left(\mathbf{V}'_j\mathbf{V}'^{\mathrm{H}}_j\right) \leq \frac{P_{\mathrm{r}}}{K}$. P_{r} is the total transmit power available at the relay. At each receiver we need to align $(K-1)d$ interferences in an $N-d$ dimensional subspace. Counting the number of constraints similar to the procedure described in [3], [4], we get the number of constraints in the system to be $N_c = 2Kd(Kd-N)$. There are $d(R-d))$ free variables in each matrix $\mathbf{V}'_j$, leading to $N_v = Kd(R-d)$ variables in the system. For the system to have a solution, $N_v \geq N_c$ must hold. From the above two equations we get the same constraint as in equation (12).

D. Degrees of Freedom

In this section, we derive the number of data streams per time slot which can be transmitted interference free in a K user pair symmetric relay interference channel. Equation (12) can be rewritten as $K \leq \left(\frac{2N}{d} - 1\right)$. Therefore, the maximum number of data streams per time slot that can be transmitted free of interference in the system is given by

$$D_{\mathrm{RIC}} = 2N - d. \tag{14}$$

Now consider a K user symmetric interference channel [1] without a relay where each node has N antennas and wants to transmit d data streams. From [3], [4], the total number of data streams per time slot which can be transmitted in the system is given by $2N - d$ which is the same as equation (14).

IV. Proposed Interference Alignment Algorithm

A two step interference alignment algorithm to achieve the alignment solution in a K user pair relay interference channel is described in this section. In the MAC phase, signal alignment is done. In the BC phase, based on the alignment made in the first step, the channels are forced to be aligned pair-wise resulting in a single effective channel per pair. We call this *channel alignment*. After the channel alignment, transmit zero forcing can be used to achieve the alignment solution at the receivers.

A. MAC Phase: Signal Alignment

In this section, a method to achieve the signal alignment at the relay is described. The alignment of the USSs of one pair is independent of the alignment of the USSs of other pairs. Hence, at the relay the alignment subspace of one pair will be almost surely linearly independent of the alignment subspace of the other pairs. Consider the pair (j, k). In order to satisfy equation (10), we need to find the intersection subspace between the subspaces corresponding to $\mathbf{H}_{\mathrm{r}j}$ and $\mathbf{H}_{\mathrm{r}k}$. Assuming that there exist intersections between these two subspaces, the following will be satisfied:

$$\begin{bmatrix}\mathbf{H}_{\mathrm{r},j} & -\mathbf{H}_{\mathrm{r},k}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{C}_j \\ \mathbf{C}_k\end{bmatrix} = \mathbf{0} \tag{15}$$

where $\mathbf{C}_j$ and $\mathbf{C}_k$ are complex coefficient matrices of size $N \times d$. Let $\mathbf{H}'_{\mathrm{r}jk} = [\mathbf{H}_{\mathrm{r},j} \quad -\mathbf{H}_{\mathrm{r},k}]$. Then

$$\begin{bmatrix}\mathbf{C}_j \\ \mathbf{C}_k\end{bmatrix} = \mathrm{null}\left(\mathbf{H}'_{\mathrm{r}jk}\right) \tag{16}$$

here $\mathrm{null}(.)$ denotes the null space of the matrix within the brackets. Choosing $\mathbf{V}_j$ and $\mathbf{V}_k$ from $\mathrm{span}\{\mathbf{C}_j\}$ and $\mathrm{span}\{\mathbf{C}_k\}$, respectively, we get the beamforming matrices such that at the relay, the USS of each node aligns with the USS of its partner. After the signal alignment, there are only Kd effective data streams at the relay.

B. BC Phase: Channel Alignment

In this section, a method to choose the transmit beamforming matrices at the relay and the receive zero forcing matrices at the receivers during the broadcast phase is described. In the broadcast phase, the transmit beamforming matrices $\mathbf{V}'_j$ for $j = 1, 2 \cdots K$ need to be chosen such that at each of the receivers, all the interferences are within the ISS and the useful signal is within the USS. This is achieved through channel alignment followed by transmit zero forcing. Consider the pair (j, k). If we can design the receive beamforming matrices such that

$$\mathrm{span}\left(\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_{jr}\right) = \mathrm{span}\left(\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{k\mathrm{r}}\right) \tag{17}$$

is guaranteed, then zero forcing the effective channel defined by $\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_{jr}$ of one node will also zero force the effective channel of its partner. Hence, we have only K effective channels. This constraint is similar to the signal alignment constraint in the MAC phase. The method described in section IV-A can be used to determine the receive zero forcing matrices. As the relay has $R = Kd$ antennas and wants to transmit d effective data streams to each pair, transmit zero forcing can be done at the relay to nullify the interferences. Let $\widetilde{\mathbf{U}}_k^{\mathrm{H}} = \mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{k\mathrm{r}}$. Then the transmit zero forcing matrix is given by

$$\mathbf{V}' = \widetilde{\mathbf{U}}^{-1} \tag{18}$$

where $\widetilde{\mathbf{U}} = \left[\widetilde{\mathbf{U}}_1, \cdots, \widetilde{\mathbf{U}}_K\right]^H$ and $\mathbf{V}' = [\mathbf{V}'_1, \cdots, \mathbf{V}'_K]$. After channel alignment followed by transmit zero forcing, if we look at the signal space of any receiver k, all the $(K-1)d$ effective interfering streams will be in an ISS of maximum dimension $N - d$. This subspace will be the orthogonal complement of the subspace spanned by the corresponding zero forcing matrix $\mathbf{U}_k^{\mathrm{H}}$. In case of $d > 1$, each of the beamforming matrices can be chosen to be a unitary matrix, optimization of which is left for future work.

V. Performance Analysis

In this section, we compare the sum rate performance of the interference alignment scheme proposed in this paper with two reference schemes which are described below. For the simulation, we consider a 3 user pair scenario. Each node wants to transmit one data stream to its communication partner. From the feasibility conditions derived in section III, we need at least 3 antennas at the relay and 2 antennas at each node. In total 6 data streams are transmitted in two time slots. Two reference methods are considered to assess the sum rate performance of the interference alignment scheme. The first reference method is based on ANOMAX proposed in [12] for a two way relaying scenario. As this method is for a single pair of nodes, different pairs are separated by time division multiple access (TDMA). The second reference method is based on pair-wise zero forcing. As the relay has 3 antennas, 4 data streams can be pair-wise transmit and receive zero forced by the relay. Consequently, in two time slots two pairs can transmit their signals. Each node utilizes its antennas to transmit its respective data stream in the direction of the largest singular value. At the relay the data streams of each pair are pair-wise receive and transmit zero forced. Again TDMA is used to separate different sets of pairs. In the two reference methods described above, 4 data streams are transmitted in two time slots. The antennas in the system could be used either to increase the DoF or to improve the diversity gain in the system. Interference alignment aims at maximizing the degrees of freedom (DoF) in the system, while the two reference schemes aim at improving the signal quality.

Figure 2 shows the sum rate performance of each method as a function of P/σ^2. P is the transmit power at each node in the IA method. In the ANOMAX and ZF schemes, as the nodes are sometimes silent due the TDMA, the powers available at each node are scaled to $3P$ and $3P/2$, respectively, in order to have a fair comparison. The noise power at each node is assumed to be

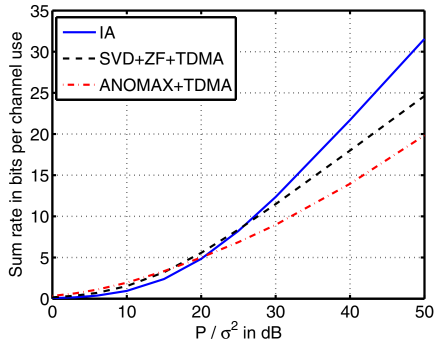

Fig. 2. Comparison of the sum rate performance for a 3 user pairs scenario with $N = 2$, $R = 3$ and $d = 1$

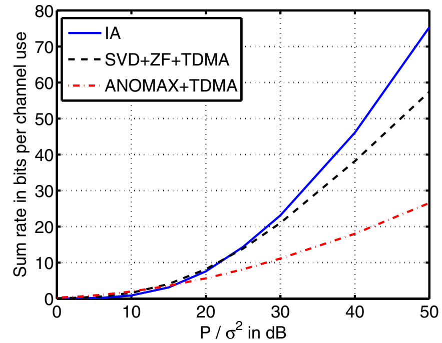

Fig. 3. Comparison of the sum rate performance for a 5 user pairs scenario with $N = 6$, $R = 10$ and $d = 2$

same and is denoted by $\sigma^2 = \sigma_r^2 = \sigma_k^2$. The relay has a transmit power P in all three cases. The MIMO channel matrices between the nodes and the relay are normalized such that in the IA scheme, on an average, the transmitted signal power is the same as the received signal power. The sum rate is calculated as an average value from 10^5 channel realizations generated randomly using the i.i.d frequency-flat Rayleigh fading MIMO channel model [13].

In case of the IA scheme, 3/2 DoF are achieved in the system while when using the other schemes, only one DoF is achieved. However, the other two schemes utilize the antennas to improve the signal quality. This can be clearly seen in Figure 2. ANOMAX and the pair-wise zero forcing based methods perform better at low SNR and the IA scheme performs better at high SNR. The gain of the IA scheme is much higher if the number of nodes and the number of data streams per node increases in the system. This can be observed in Figure 3 where 5 user pairs want to transmit 2 data streams to each of their partners.

Remark: For a fair comparison we have scaled the powers of each node in the reference schemes to a higher value. In reality, it may occur that each of the nodes has limited transmit power. In this case, the sum rate performances of the reference schemes will shift towards the right and the gain of the IA scheme will further increase.

VI. Conclusion

In this paper, the feasibility conditions for the interference alignment in a K user pair symmetric relay interference channel are derived. If the relay has $R = Kd$ antennas, each of the nodes needs at least $N = \frac{(K+1)}{2}d$ antennas in order to transmit d data streams to its communication partner. Through the interference alignment scheme described in the paper, the number of data streams per time slot which can be transmitted interference free in the system is given by $D_{\text{RIC}} = 2N - d$. This is the same number as that in a K user symmetric interference channel. It is shown that in order to align the interferences within the ISS at each of the receivers, signal alignment has to be done at the relay. A two step algorithm with signal alignment in the first step and channel alignment followed by transmit zero forcing in the second step is proposed. The sum rate performance of the proposed method is better than that of the considered reference methods without interference alignment at high SNR.

References

[1] V. Cadambe and S. Jafar, "Interference Alignment and Degrees of Freedom for the K User Interference Channel," in *IEEE Transactions on Information Theory*, vol. 54, no. 8, Aug. 2008, pp. 3425 – 3441.

[2] K. Gomadam, V. Cadambe, and S. Jafar, "Approaching the Capacity of Wireless Networks through Distributed Interference Alignment," in *IEEE Global Telecommunications Conference*, Nov. 2008.

[3] F. Negro, S. Shenoy, I. Ghauri, and D. Slock, "Interference Alignment Limits for K-user Frequency-Flat MIMO Interference Channels," in *European Signal Processing Conference*, Aug. 2009.

[4] C. Yetis, T. Gou, S. Jafar, and A. Kayran, "Feasibility Conditions for Interference Alignment," in *IEEE Global Telecommunications Conference*, Nov. 2009.

[5] N. Lee and J.-B. Lim, "A Novel Signaling for Communication on MIMO Y Channel: Signal Space Alignment for Network Coding," in *IEEE International Symposium on Information Theory*, Jun. 2009, pp. 2892 –2896.

[6] M. Chen and A. Yener, "Multiuser Two-Way Relaying for Interference Limited Systems," in *IEEE International Conference on Communications*, May 2008, pp. 3883 –3887.

[7] Y. Mohasseb, H. Ghozlan, G. Kramer, and H. El Gamal, "The MIMO Wireless Switch: Relaying Can Increase the Multiplexing Gain," in *IEEE International Symposium on Information Theory ISIT*, July 2009, pp. 1448 –1452.

[8] A. Avestimehr, M. Khajehnejad, A. Sezgin, and B. Hassibi, "Capacity region of the deterministic multi-pair bi-directional relay network," in *IEEE Information Theory Workshop on Networking and Information Theory*, June 2009, pp. 57 –61.

[9] B. Hassibi, A. Sezgin, M. Khajehnejad, and A. Avestimehr, "Approximate capacity region of the two-pair bidirectional Gaussian relay network," in *IEEE International Symposium on Information Theory ISIT*, July 2009, pp. 2018 –2022.

[10] D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, "The Multi-way Relay Channel," in *IEEE International Symposium on Information Theory ISIT*, July 2009, pp. 339 –343.

[11] B. Rankov and A. Wittneben, "Spectral Efficient Signaling for Half-duplex Relay Channels," in *Conference Record of the Thirty-Ninth Asilomar Conference on Signals, Systems and Computers*, Oct. 2005, pp. 1066 – 1071.

[12] F. Roemer and M. Haardt, "Algebraic Norm-Maximizing (ANOMAX) Transmit Strategy for Two-Way Relaying With MIMO Amplify and Forward Relays," *IEEE Signal Processing Letters*, vol. 16, no. 10, pp. 909 –912, Oct. 2009.

[13] A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communications*. Cambridge, UK: Cambridge, 2007.