Consideration of inaccurate outdating parameters in adaptive OFDMA

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***Abstract*— In this paper, an adaptive multi-user OFDMA scheme in the downlink is considered which is aware of outdated channel knowledge. Assuming that the base station has knowledge about the outdating parameters in terms of correlation coefficients between the outdated channels and the actual channels of the different users, it is possible to adjust the Signal-to-Noise Ratio (SNR) thresholds for the applied modulation schemes to guarantee a certain target Bit Error Rate (BER) in the presence of outdated Channel Quality Information (CQI). As these outdating parameters cannot be assumed to be perfectly known in a realistic scenario, the impact of inaccurate outdating parameters on the system performance is investigated. Furthermore, a safety margin approach is presented to reduce the loss in system performance.**

I. Introduction

The impact of imperfect channel knowledge on the performance of adaptive Orthogonal Frequency Division Multiplexing (OFDM)-based systems has already been studied intensely in the literature. For the single-user transmission case, imperfect Channel State Information (CSI) have been studied in [1]-[5] and references therein. In [1], adaptive OFDM with imperfect CSI for uncoded variable bit rates are studied, where the imperfect CSI arises from noisy channel estimates and the time delay of getting the CSI to the transmitter. The authors propose the use of multiple estimates to improve the performance. In [2], a subchannel loading algorithm is proposed combating the negative effects resulting from channel errors in coherent detection at the receiver. In [3], channel prediction is used to combat the impact of outdated CSI and in [4] a statistical adaptive modulation scheme based on long-term statistics is proposed. In [5], an optimal power loading algorithm for OFDM based on average and outage capacity criteria is presented assuming imperfect CSI at the transmitter. For the multi-user transmission case, analytical expressions for the system performance are derived in [6] and [7] assuming outdated, noisy estimated and quantized CSI. In [8], an adaptive Orthogonal Frequency Division Multiple Access (OFDMA) system is investigated which applies outdated Channel Quality Information (CQI) to allocate the different resources to the different users following a Proportional Fair Scheduling (PFS) approach. Assuming that the correlation coefficients ρ_u between the actual channel and the outdated channel of each user u are perfectly known to the base station (BS), it has been shown how to adjust the Signal-to-Noise Ratio (SNR) thresholds for the modulation scheme selection such that for each user, a given Bit Error Rate (BER) requirement is fulfilled taking into account the fact that the CQI is outdated. However, in a realistic scenario, these outdating parameters representing the reliability of the CQI cannot be assumed to be perfectly known as they have to be measured or estimated.

In this paper, the impact of inaccurate outdating parameters on the system performance of an adaptive OFDMA system with outdated CQI is investigated. As outdating parameters, the correlation coefficients ρ_u which depend on the velocity v_u of each user and on the time delay T between the time instant of measuring the CQI and transmitting data are applied. Furthermore, a safety margin approach is proposed to reduce the loss in system performance caused by the inaccurate outdating parameters. The remainder of this paper is organized as follows. In Section II, the OFDMA system model together with the channel model and system assumptions is provided. Furthermore, the modeling of outdated channel knowledge is presented. In Section III, an adaptive multi-user OFDMA scheme which is aware of outdated channel knowledge is introduced and it is shown how to adjust the SNR thresholds based on analytical derivations of the system performance. In Section IV, inaccurate outdating parameters are considered and a safety margin approach is presented to cope with the parameter inaccuracies. In Section V, the impact of inaccurate outdating parameters on the achievable data rate is illustrated with and without applying the proposed safety margin. Finally, conclusions are drawn in Section VI.

II. System model and assumptions

In this section, the considered system model is introduced. Furthermore, the modeling of outdated channel knowledge is discussed.

A. System model

In this work, we consider a one cell downlink scenario with one BS and U Mobile Stations (MSs) with user index $u = 1, \cdots, U$ located in the cell. The BS and the MSs are equipped with one antenna each, where the MSs are assumed to be uniformly distributed inside the cell. OFDMA is used and the bandwidth is subdivided into N orthogonal subcarriers with frequency spacing Δf. We define a block of Q adjacent subcarriers, also called chunk [9], as a resource unit. Hence, a total number of $N_{\mathrm{ru}} = \lfloor N/Q \rfloor$ resource units is assumed with $\lfloor . \rfloor$ the nearest integer lower than or equal to the argument. We assume that Q is chosen in such a way that the channel does not vary significantly within a resource unit. Furthermore, the channels of adjacent resource units are assumed to be uncorrelated. The fast fading described by the transfer factor $H_u(n,k)$ of each user u on the resource unit with index $n = 1.., N_{\mathrm{ru}}$ in a time slot $k \in \mathbb{N}$ is then modeled as a complex Gaussian distributed random process with variance one. It is assumed that the BS transmits with power P_{T} where the transmit power is equally shared among the N subcarriers. With the noise power spectral density N_0, the average SNR $\bar{\gamma}_u$ at the MS of user u can be calculated by

$$\bar{\gamma}_u = \frac{P_{\mathrm{T}}}{N \cdot \Delta f \cdot N_0} \cdot \left(\frac{d_u}{d_0}\right)^{-\alpha}, \tag{1}$$

with d_u denoting the distance between the MS of user u and the BS, d_0 the minimum distance between any MS and the BS and α the pathloss coefficient. From this, it follows that the instantaneous SNR $\gamma_u(n,k)$ of user u on resource unit n in time slot k is given by

$$\gamma_u(n,k) = \bar{\gamma}_u \cdot |H_u(n,k)|^2 , \tag{2}$$

i.e., $\gamma_u(n,k)$ follows an exponential distribution. In this work, these instantaneous SNR values are applied as CQI for the adaptive OFDMA scheme.

B. Outdated CQI

Due to the time delay T between the time instant when measuring the SNR and the actual time of data transmissions, the CQI is outdated. From literature, e.g. [10], it is known that when the angles of arrival for the different propagation paths are assumed to be uniformly distributed and, thus, the distribution of the Doppler shifts corresponds to a Jake's spectrum, the correlation coefficient ρ_u between the outdated channel and the actual channel of user u only depends on the time delay T and the maximum Doppler shift $f_{D,u}$ of user u given by $\rho_u = J_0(2\pi f_{D,u} T)$ with $J_0(x)$ denoting the 0th-order Bessel function of the first kind. With the carrier frequency f_0 and the speed of light c, $f_{D,u}$ is given by $f_{D,u} = \frac{f_0 \cdot |v_u|}{c}$ with v_u the radial component of the velocity of user u along a line from user u to the BS. From this, it follows that the correlation coefficient ρ_u of the channel transfer factor of user u is given by

$$\rho_u = J_0\left(2\pi f_0 T c^{-1} \cdot |v_u|\right). \tag{3}$$

In the following, we assume that each user has a different velocity $\mathbf{v} = [v_x, v_y]^{\mathrm{T}}$, where the x- and y-components of $\mathbf{v}$ are independent from each other and normally distributed with zero mean and variance σ_v. From this, it follows that the velocity component v_ϕ in any direction with angle ϕ is normally distributed with zero mean and variance σ_v^2. Hence, the radial component of the velocity v_u of user u is also $\mathcal{N}(0, \sigma_v^2)$ distributed. The absolute value $|v_u|$ is then half-normally distributed with expectation value $\bar{v} = \sqrt{\frac{2}{\pi}} \cdot \sigma_v$. In the following, the dynamic of the user movements inside the cell is expressed by this average velocity $\bar{v}$.

III. Outdated CQI aware adaptive OFDMA

In the following section, the adaptive multi-user OFDMA transmission scheme which is aware of outdated CQI is introduced [8]. Firstly, the adaptive resource allocation and modulation is shortly discussed. Based on that, analytical expressions of the average user data rate and BER are derived taking into account outdated CQI. Finally, it is shown how to adjust the SNR thresholds for the modulation scheme selection based on the derived expressions.

A. Adaptive resource allocation and modulation

The main advantage of adaptive OFDMA is to exploit multi-user diversity by capitalizing the variations in the channel of different users to transmit data only on the strongest channels [11]. In order to take into account the current SNR conditions of the resource units and introduce fairness among the users, a PFS approach is applied [12]. Hence, the subcarriers of resource unit n in time frame k are allocated to the user $u^\star(n,k)$ with the highest ratio between the instantaneous SNR and the average SNR $\bar{\gamma}_u$, leading to

$$u^\star(n,k) = \arg\max_u \left\{ \frac{\gamma_u(n,k)}{\bar{\gamma}_u} \right\}. \quad (4)$$

By doing so, each resource unit is allocated to one user exclusively while each user has the same probability of getting access to a resource unit. After resource allocation, the modulation scheme is selected for each allocated resource unit based on the instantaneous SNR values, i.e., for each subcarrier inside one resource unit the same modulation scheme is applied where the same transmit power per subcarrier is assumed. By doing so, the modulation is adapted to the pathloss and to the fast fading. In this work, uncoded M-ary Quadrature Amplitude Modulation (M-QAM) and M-ary Phase Shift Keying (M-PSK) are considered.

B. Performance analysis taking into account outdated CQI

To determine the proper SNR thresholds for the modulation scheme selection taking into account outdated CQI, analytical expressions of the user data rate and BER have to be derived. In [8], we performed this derivation for a more general case assuming multiple antennas and different user priorities concerning channel access. Hence, the following analytical expressions presented here can be considered as special cases of the expressions derived in [8] applying only a single antenna BS and single antenna MSs with each user having the same priority. On that account, the exact derivation steps are omitted and only the final closed form expressions are presented.

1) Average user data rate: The average data rate is defined as sum rate of the different number of bits per symbol according to the applied modulation schemes weighted by their probability. Assuming that there are M modulations schemes available, $\gamma^{(u)} = [\gamma_0^{(u)}, \gamma_1^{(u)}, ..., \gamma_M^{(u)}]^{\mathrm{T}}$ denotes the SNR threshold vector of user u which contains the SNR threshold values determining the interval in which a particular modulation scheme is applied, where $\gamma_0^{(u)} = 0$ and $\gamma_M^{(u)} = \infty$ for all users. Using (6), the average data rate $\bar{R}^{(u)}$ of user u can be formulated as

$$\bar{R}^{(u)} = \sum_{m=1}^{M} b_m \cdot \left(F_{\hat{\gamma}}^{(u)}(\gamma_m^{(u)}) - F_{\hat{\gamma}}^{(u)}(\gamma_{m-1}^{(u)}) \right) \quad (5)$$

with b_m denoting the number of bits per symbol corresponding to the applied modulation scheme and

$$F_{\hat{\gamma}}^{(u)}(\hat{\gamma}) = \left(1 - e^{-\frac{\hat{\gamma}}{\bar{\gamma}_u}}\right)^U. \quad (6)$$

2) Average user BER: Using the approximation for the instantaneous BER for M-QAM and M-PSK modulation introduced in [13] given by

$$BER_m(\gamma) = 0.2 \cdot \exp(-\beta_m \gamma) \quad (7)$$

with $m = 1,..,M$, where $\beta_m = \frac{1.6}{2^{b_m}-1}$ for M-QAM modulation and $\beta_m = \frac{7}{2^{1.9 b_m}+1}$ for M-PSK modulation, the average BER is defined as the sum of the number of bit errors of the different modulation constellations divided by the average bit rate [6] resulting in

$$\begin{aligned} \overline{BER}^{(u)} \; = \; & \frac{0.2 \cdot U}{\bar{R}^{(u)}} \cdot \sum_{m=1}^{M} b_m \sum_{v=0}^{U-1} \binom{U-1}{v} \\ & \cdot \frac{(-1)^v}{\bar{\gamma}_u(1+\beta_m\bar{\gamma}_u(1-\rho_u^2))} \\ & \cdot \left[e^{-\gamma_{m-1}^{(u)}\Psi(m,v)} - e^{-\gamma_m^{(u)}\Psi(m,v)} \right] \end{aligned} \quad (8)$$

with

$$\Psi(m,v) = \frac{(v+1) + \beta_m\bar{\gamma}_u((v+1) - \rho_u^2 v)}{\bar{\gamma}_u(1+\beta_m\bar{\gamma}_u(1-\rho_u^2))}. \quad (9)$$

C. SNR threshold calculation

In the following, we search for the optimal SNR threshold vector $\gamma^{(u)}$ of user u which maximizes the average user data rate subject to a target BER BER_T for any given correlation coefficient ρ_u, i.e., the following optimization problem has to be solved:

$$\begin{aligned} \bar{R}_{opt}^{(u)} \; = \; & \max_{\gamma^{(u)}} \left(\bar{R}^{(u)}(\gamma^{(u)}) \right) \\ & \text{subject to} \\ & \overline{BER}^{(u)}(\gamma^{(u)}, \rho_u) \leq BER_T. \end{aligned} \quad (10)$$

As shown in [8], (10) can be solved performing a Lagrange multiplier approach. Each element $\gamma_m^{(u)}$ of the optimal threshold vector $\gamma_{opt}^{(u)}$ can be calculated using an initial value $\gamma_1^{(u)}$. Thus, each threshold vector $\gamma^{(u)}$ is a function of the initial value $\gamma_1^{(u)}$, i.e., $\gamma^{(u)} = f(\gamma_1^{(u)})$. Determining the maximum average data rate subject to the target BER, we have to find the optimal initial value $\gamma_{1,opt}^{(u)}$ which fulfills

$$\overline{BER}^{(u)}(f(\gamma_{1,opt}^{(u)}), \rho_u) \leq BER_T, \quad (11)$$

which can be done numerically using for example the *fzero* function in MATLAB™.

IV. Inaccurate outdating parameters

In this section, the modeling of inaccurate outdating parameters is introduced. Furthermore, a safety margin is proposed to reduce the impact of parameter inaccuracies on the system performance.

A. Modeling inaccurate outdating parameters

For the calculation of the SNR thresholds shown in Section III-C, it has been assumed that the outdating parameters ρ_u of each user u and, hence, the user velocities v_u are perfectly known. Due to the fact that these parameters have to measured or estimated, it is assumed that there exists a deviation between the estimated MS velocity $\hat{v}_u$ and the actual MS velocity v_u, i.e., $\hat{v}_u$ is modeled by

$$\hat{v}_u = v_u \cdot (1 + v_{\mathrm{E},u}) \quad (12)$$

assuming that $v_{\mathrm{E},u}$ is normally distributed with zero mean and variance σ_{E}^2. In the following, σ_{E}^2 is chosen such that $\hat{v}_u$ deviates from v_u by p_{dev} percent on average, i.e., $E\{|v_{\mathrm{E},u}|\} = \frac{p_{\mathrm{dev}}}{100}$ resulting in

$$\sigma_{\mathrm{E}}^2 = \frac{\pi}{2} \cdot \left(\frac{p_{\mathrm{dev}}}{100}\right)^2. \quad (13)$$

The inaccuracy of the outdating parameters results in system performance degradations due to two reasons. In case that $\hat{v}_u$ is larger than v_u, the SNR thresholds and, thus, the applied modulation schemes are chosen too conservatively leading to a loss in data rate. However, the target BER is always fulfilled. In case that $\hat{v}_u$ is smaller than v_u, the modulation schemes are chosen too optimistically, i.e., the target BER can no longer be fulfilled. In this case, the user data rate is defined to be zero.

B. Safety margin

To reduce the loss in system performance due to inaccurate outdating parameters, the MS velocity applied for the SNR threshold calculations is always assumed to be larger than the estimated MS velocity $\hat{v}_u$. For this purpose, a safety margin is introduced, i.e., the assumed MS velocity is given by

$$\hat{v}_{\mathrm{sm},u} = \hat{v}_u \cdot (1 + \frac{p_{\mathrm{sm}}}{100}) \quad (14)$$

with p_{sm} denoting the safety margin in percent. In case that v_u is higher than $\hat{v}_u$, the safety margin decreases the deviation between $\hat{v}_{\mathrm{sm},u}$ and v_u leading to a better performance as the target BER is more likely to be fulfilled. In case that v_u is smaller than $\hat{v}_u$, the safety margin leads to a decrease in data rate due to the too conservatively chosen modulation schemes. However, the target BER is always fulfilled.

V. Numerical results

In the following, an OFDMA scenario with the parameters given in Table I is assumed. The applied

TABLE I

System parameters

Bandwidth	10 MHz
Number N of subcarriers	500
Frequency block size Q	4
Number U of users	25
Carrier frequency f_0	2 GHz
Time delay T	2 ms
Target BER BER_{T}	10^{-3}
Cell radius R	300 m
Minimum distance BS-MS d_0	10 m
Pathloss coefficient α	2.6

modulation schemes range from QPSK for users at the cell edge up to 128-QAM for users near the BS.

In Fig. 1, the average system data rate is depicted as a function of the average MS velocity of the different users in the cell. The black dashed curve

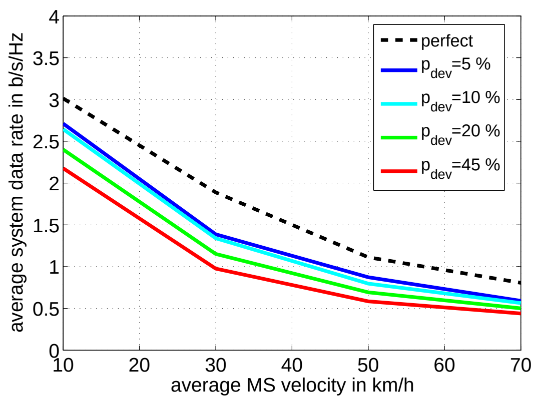

Fig. 1. System data rate vs. average MS velocity in km/h

represents the system performance assuming that the correlation coefficients are perfectly known. It can be seen that the system data rate decreases for an increasing MS velocity as the modulation schemes have to be chosen more conservatively to cope with the outdated CQI in order to fulfill the target BER. The solid lines represent the system performances assuming that there exists a deviation between the estimated MS velocity $\hat{v}_u$ and the actual MS velocity v_u. The blue curve, e.g., represents the system data rate assuming that $\hat{v}_u$ deviates from v_u by $p_{\text{dev}} = 5$ % on average. It can be seen that the system performance decreases with increasing p_{dev} as stated in Section IV-A.

In Fig. 2, the system data rate is depicted for different safety margins assuming that $\hat{v}_u$ differs from v_u by $p_{\text{dev}} = 10$ % on average. It can be

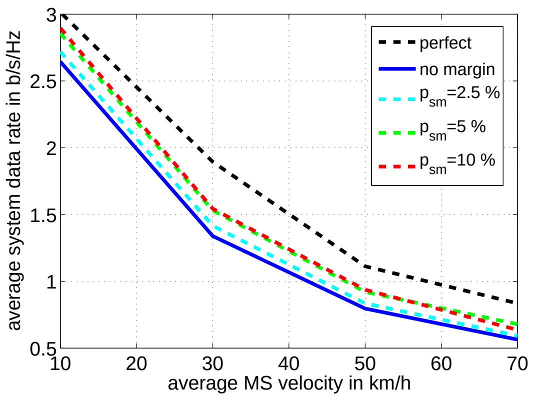

Fig. 2. System data rate vs. average MS velocity in km/h with $p_{\text{dev}} = 10$ %

seen that applying a safety margin of $p_{\text{sm}} = 5$ % to $p_{\text{sm}} = 10$ % results in a performance recovery of up to 50 % compared to the cases without safety margin and perfect outdating parameters. When further increasing the safety margin, the system performance decreases again due to the fact that the modulation scheme selection becomes too conservative.

VI. Conclusions

In this paper, an adaptive multi-user OFDMA transmission scheme applying outdated CQI is considered. Assuming that the BS has knowledge about the outdating parameters in terms of correlation coefficients between the outdated channels and the actual channels of the different users, it is possible to adjust the SNR thresholds for the applied modulation schemes to guarantee a certain target BER in the presence of outdated CQI. However, in a realistic scenario, these outdating parameters which depend on the MS velocities cannot be assumed to be perfectly known resulting in a system performance degradation as the SNR thresholds are either chosen too optimistically or too conservatively, respectively. To reduce this loss in system performance, a safety margin is introduced. By doing so, the MS velocity applied for the SNR threshold calculations is always assumed to be larger than the estimated MS velocity. Simulations show that if the estimated MS velocities do not derivate more than 10 % from the actual MS velocities, a performance recovery of up to 50 % compared to the cases without safety margin and perfect outdating parameters is possible.

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