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Projection based Space-Frequency Interference Alignment in a Multi-Carrier Multi-User Two-Way Relay Network

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***Abstract*—We consider a multi-carrier multi-user two-way relay network with multiple single antenna node pairs and a single non-regenerative relay with two antennas. In a conventional multi-user two-way relaying scheme where different node pairs transmit their data streams on different subcarriers, only one data stream per channel use can be achieved. Taking into account that the nodes can perform self interference cancellation, one antenna at the relay is sufficient to achieve one data stream per channel use. Hence, the two antennas at the relay are not fully utilized. In this paper, using the interference alignment technique in space-frequency dimensions, it is shown that when the number of users in the system is large, almost two data streams per channel use are achievable. The gain comes from the joint consideration of the spatial and frequency dimensions at the relay. A new projection based interference alignment algorithm is proposed. The ideas of signal alignment and channel alignment [1] are used to achieve the interference alignment solution. Furthermore, for the performance improvement of the proposed interference alignment scheme at moderate Signal to Noise Ratios (SNRs), a new gradient based method for optimization of the projection space at the relay is proposed.**

I. INTRODUCTION

Employing a relay helps in range extensions and capacity enhancements when there is no direct link between the source and the destination nodes. Two basic relaying schemes, namely, one way relaying and two-way relaying are well known in the literature. In this paper, we focus on non-regenerative two-way relaying. In two-way relaying [2], [3], the communicating nodes transmit their signal to the relay during the first time slot and the relay broadcasts the superposed signal to the nodes during the second time slot. If the nodes have the same number of antennas as the relay and if the nodes can perform self interference cancellation, the relay does not need to spatially separate the data streams of the nodes. In this case, the number of data streams that can be transmitted in each direction is the same as the number of antennas at the relay. Hence, the relay space is fully utilized. We define the relay utilization factor as

$$\eta = \frac{\text{number of data streams in each direction}}{\text{size of the relay space}}. \qquad (1)$$

When there are multiple users that want to communicate with their communication partners through a relay, besides the self interference, inter-pair interference occurs. If there are K single antenna node pairs, then the relay requires at least $2K$ antennas to spatially separate the node pairs and perform transceive zero forcing [4], [5]. In [6], assuming that each node can perfectly cancel its self interference, only $2K-1$ antennas are required at the relay. In [7], [8], [9], multi antennas nodes are considered. Each node wants to transmit d data streams to its communication partner, then the relay needs $2Kd$ antennas to perform transceive zero forcing. If self interference cancellation is taken into consideration, using the method proposed in [6], only $(2K-1)d$ antennas are required at the relay. All the methods discussed above can achieve only a maximum relay utilization factor of $\eta \approx 0.5$.

Recently, using the idea of interference alignment in [1], it has been shown that when each node has $M \geq \frac{(K+1)}{2}d$ antennas, Kd antennas at the relay are enough to make the transmission possible. In the MAC phase, the nodes align their signal with their communication partners' signal at the relay [1]. This process is called signal alignment. In the BC phase, each node chooses its receive filter such that the effective channel from the relay to the node including the receive filter spans the same subspace as that of its communication partner [1]. This process is called channel alignment. After the signal and the channel alignment, the relay performs transceive zero forcing to spatially separate the pairs. Here, the relay space is fully utilized as Kd data streams are transmitted in each direction through Kd antennas at the relay. Hence, a relay utilization factor $\eta = 1$ is achieved. However, in [1], nodes need multiple antennas to make this feasible.

In all the methods described above, a single subcarrier has been considered. These methods can be extended to the case of multiple subcarriers by applying them individually to each of the subcarriers, but still the relay utilization factor η remains to be 0.5 or multiple antennas are required at the nodes.

In this paper, we consider the case where the nodes have only one antenna each. The question addressed is whether it is possible to fully utilize the relay space in this case. It is well known that in most of the future communication systems, Orthogonal Frequency Divison Multiplexing (OFDM) will be employed and hence, multiple sub-carriers are available. Instead of multiple antennas, precoding over multiple subcarriers can be performed to maximize the relay utilization factor η. In contrast to [1] where the relay space is fixed by the number of antennas at the relay, when we replace the multiple antennas at the transmitter in [1] with multiple subcarriers, the relay

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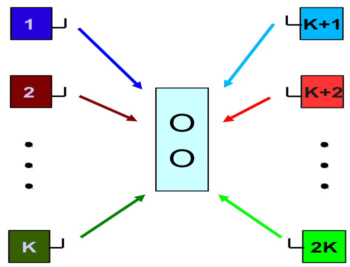

Fig. 1. Multi-user two-way relay network

space increases in proportion to the number of subcarriers and hence, the methods and feasibility conditions proposed in [1] cannot be applied directly to this scenario. It will be shown that the projection of the received signal at the relay to a subspace is necessary to make signal and channel alignment feasible. In this paper, we consider the case when the relay has two antennas and show that using the proposed projection based interference alignment scheme, the relay space can be almost fully utilized. Besides the consideration of the relay utilization factor η, the sum rate performance of the system is also considered. A gradient based optimization of the projection space under the interference alignment constraints is performed to enhance the sum rate of the system at moderate SNRs.

The organization of the paper is as follows. In Section II, the system model is introduced. In Section III-A, the ideas of signal alignment and channel alignment [1] in a multi-user two-way relay network are described. In Section III-B, the proposed projection based interference alignment scheme is discussed. The feasibility conditions are derived in Section III-C. The optimization of the signal spaces at the relay is described in Section IV. Simulation results comparing the performance of the projection based interference alignment scheme with that of the conventional multi-user two-way relaying without interference alignment are shown in the Section V. Section VI concludes the work. We use lower case letters for scalars and lower case bold letters and upper case bold letters to denote vectors and matrices, respectively. $(.)^*$ and $(.)^{\mathrm{H}}$ denote the complex conjugate and complex conjugate transpose of the element within the brackets, respectively.

II. System Model

The multi-user two-way relay network with $2K$ single antenna nodes and a half-duplex non-regenerative relay with two antennas is shown in the Figure 1. Each of the $2K$ nodes wants to transmit d data streams to its corresponding partner using N subcarriers, i.e., a bidirectional communication is assumed. Without loss of generality, assume node i and node j for $i = 1 \cdots K$ and $j = i + K$ are the communication partners. The channel coefficients corresponding to each of the N subcarriers are assumed to be independent of each other. In a real scenario, N subcarriers can be chosen among the available subcarriers such that the minimum distance between any two subcarriers is greater than the coherence bandwidth. Furthermore, the direct link between the nodes is assumed to be too weak for a direct communcation. In order to reduce the loss in spectral efficiency due to the half-duplex relay, the two-way relaying protocol [2] is considered. In the first phase, all the nodes transmit their signal to the relay. This is called multiple access (MAC) phase. In the second phase, the relay broadcasts the signals to the destination nodes. This is called broadcast (BC) phase. Let $\mathbf{d}_k$ denotes the data symbols of node k for $k = 1 \dots 2K$. Each node precodes the d data symbols over N subcarriers. $\mathbf{V}_k$ denotes the precoding matrix at node k. Let the channel between transmitter k and the relay be denoted by $\mathbf{H}_{\mathrm{r}k}$. Dimension of the matrix $\mathbf{H}_{\mathrm{r}k}$ is $2N \times N$, where 2 corresponds to the two antennas at the relay. The noise at the relay is denoted by the vector $\mathbf{n}_{\mathrm{r}}$. Amplify and Forward (AF) relaying is assumed. The linear signal processing matrix at the relay is denoted by $\mathbf{G}$. $\mathbf{H}_{k\mathrm{r}}$ is the matrix denoting the channel between the relay and node k. Let $\mathbf{n}_k$ denote the noise at node k. The components of the noise vectors $\mathbf{n}_r$ and $\mathbf{n}_k$ are i.i.d complex Gaussian random variables which follow $\mathcal{CN}(0, \sigma^2)$. The received signal at receiver k is given by

$$\mathbf{y}_k = \mathbf{H}_{k\mathrm{r}}\mathbf{G}\left[\sum_{l=1}^{2K}\mathbf{H}_{\mathrm{r}l}\mathbf{V}_l\mathbf{d}_l + \mathbf{n}_{\mathrm{r}}\right] + \mathbf{n}_k. \tag{2}$$

Let $\mathbf{U}_k^{\mathrm{H}}$ denote the zero forcing matrix at receiver k. Then the estimated data stream is given by

$$\hat{\mathbf{d}}_k = \mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{k\mathrm{r}}\mathbf{G}\left[\sum_{l=1}^{2K}\mathbf{H}_{\mathrm{r}l}\mathbf{V}_l\mathbf{d}_l + \mathbf{n}_{\mathrm{r}}\right] + \mathbf{U}_k^{\mathrm{H}}\mathbf{n}_k. \tag{3}$$

III. Relay Aided Interference Alignment

A. Interference Alignment in Multi-User Two-Way Relay Networks

In [1], interference alignment in a multi-user two-way relay network is achieved by performing precoding over multiple antennas at the nodes. In the MAC phase, the precoding vectors of each node are chosen such that the transmitted signals are pair-wise aligned at the relay. Consider the communication partner nodes i, j with

$$j = i + K, i = 1 \dots K. \tag{4}$$

The signal alignment is given by

$$\mathrm{span}\left(\mathbf{H}_{\mathrm{r}i}\mathbf{V}_i\right) = \mathrm{span}\left(\mathbf{H}_{\mathrm{r}j}\mathbf{V}_j\right) \tag{5}$$

[1]. In the BC phase, channel alignment followed by zero forcing is performed to achieve interference alignment at the destination nodes [1]. With channel alignment, the effective channels of the communication partners are made to span the same subspace [1]. The term effective channel refers to the product of the matrix corresponding to the channel from the relay to the receiver and the receive processing matrix. The channel alignment is given by

$$\mathrm{span}\left(\mathbf{U}_i^{\mathrm{H}}\mathbf{H}_{i r}\right) = \mathrm{span}\left(\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_{j\mathrm{r}}\right) \tag{6}$$

[1]. After channel alignment, there are only Kd effective channels in the system and the transmit space of the relay is of size Kd [1]. Hence, transmit zero forcing can be performed at the relay.

B. Projection based Interference Alignment

The proposed scheme is based on signal alignment and channel alignment. The idea is that if the data streams of the communication partners are pairwise aligned and if there are enough user pairs to fully span the relay space, a relay utilization factor η of one is achievable. However, in the considered scenario, the nodes have only one antenna each and hence, precoding over multiple antennas is not possible. Instead, a multi-dimensional space at the nodes and at the relay can be achieved by considering N subcarriers.

In the MAC phase, the nodes perform signal alignment by precoding their data symbols over multiple subcarriers. In the BC phase, channel alignment followed by transmit zero forcing at the relay is performed to achieve interference alignment at the destination nodes. The signal and channel alignment conditions for the current scenario are the same as in (5) and (6) except that the precoding and the receive zero forcing at the nodes are performed over multiple subcarriers. Here, $\mathbf{H}_{\mathrm{r}i}\mathbf{V}_i$ and $\mathbf{H}_{\mathrm{r}j}\mathbf{V}_j$ span an N-dimensional subspace S_i and S_j, respectively, in the $2N$ dimensional relay space. For the signal alignment between the communication partners, we need at least d intersections [1] between these two subspaces S_i and S_j. In [1], the intersection between these two planes is achieved by increasing the number of antennas at the nodes. However, in our considered scenario, when we increase the number of subcarriers, the size of the relay space also increases and hence, with a very high probability the subspace S_i and S_j will not intersect. d intersections can be achieved by projecting the received signal to a subspace of size

$$M \leq 2N - d. \tag{7}$$

Hence, the signal received at the relay has to be projected to an M dimensional subspace to make the signal alignment feasible. The signal alignment and channel alignment are problems of similar structure [1]. Hence, the condition for channel alignment is the same as (7). Let $\mathbf{T}^{\mathrm{H}}$ be the projection matrix that projects the received $2N$ dimensional signal at the relay to an M dimensional subspace. Let $\mathbf{G}_{\mathrm{s}}$ denote the transcieve zero forcing matrix at the relay. Let $\mathbf{Q}$ be the projection matrices that projects the M dimensional transmit signal at the relay to a $2N$ dimensional space. The relay processing matrix $\mathbf{G}$ can be written as

$$\mathbf{G} = \mathbf{Q}\mathbf{G}_{\mathrm{s}}\mathbf{T}^{\mathrm{H}}. \tag{8}$$

The transceive zero forcing matrix $\mathbf{G}_{\mathrm{s}}$ can be further represented in terms of the receive zero forcing matrix $\mathbf{G}_{\mathrm{r}}^{\mathrm{H}} = [\mathbf{g}_1 \cdots \mathbf{g}_K]^{\mathrm{H}}$, power allocation matrix $\mathbf{G}_{\mathrm{p}}$ and transmit zero forcing matrix $\mathbf{G}_{\mathrm{t}}$. We assume equal power allocation among all pairs at the relay, that is, $\mathbf{G}_{\mathrm{p}} = \mathbf{I}$. Hence, $\mathbf{G}_{\mathrm{s}} = \mathbf{G}_{\mathrm{t}}\mathbf{G}_{\mathrm{r}}^{\mathrm{H}}$. Eq. (8) implies that the received signal is projected to an M dimensional subspace and the signal and the channel alignment are made in this M dimensional subspace. Now, the condition for the signal and the channel alignment can be written as

$$\mathrm{span}\left(\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}i}\mathbf{V}_i\right) = \mathrm{span}\left(\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}\mathbf{V}_j\right) \tag{9}$$

and

$$\mathrm{span}\left(\mathbf{U}_i^{\mathrm{H}}\mathbf{H}_{i r}\mathbf{Q}\right) = \mathrm{span}\left(\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_{j \mathrm{r}}\mathbf{Q}\right), \tag{10}$$

respectively.

C. Feasiblity Condition

In this subsection, we derive the feasibility condition for the proposed scheme taking into account the need for projection and the size of the projected subspace as a limiting factor for the maximum number of users allowed in the system. The projection constraint in (7) ensures that the signal alignment and channel alignment between the communication partners are possible. In order to be able to decode the desired signal at each of the receive nodes, it is required to make sure that the signal alignment subspaces of each pair are independent of those of the other pairs. Two subspaces are said to be linearly independent if no vector in one subspace can be expressed as a linear combination of the basis vectors of the other subspace. Hence, the size M of the projected subspace should be greater than the sum of the sizes of all the signal alignment subspaces given by

$$M \geq Kd. \tag{11}$$

From (7) and (11), we get the following feasibility condition:

$$N \geq \left(\frac{K+1}{2}\right) d. \tag{12}$$

D. Relay Utilization Factor η

In this subsection, the expression for the achievable relay utilization factor η for the proposed scheme is derived. From (1), η is given by

$$\eta = \frac{Kd}{2N}. \tag{13}$$

Substituting N from the feasibility condition (12), we get

$$\eta \leq \frac{K}{K+1}. \tag{14}$$

When K is large, a relay utilization factor η close to one is achievable. Eq. (14) implies that η is independent of the number of data streams being transmitted by each node. This means that in order to reduce the required number of subcarriers, one can limit the number of data streams d to be one.

IV. Optimization of the Projection Subspaces

In this section, we optimize the subspace into which the received signal is projected at the relay. For any given choice of $\mathbf{T}$ and $\mathbf{Q}$, the interference alignment solution can be obtained if the condition in (12) holds. This gives us the opportunity to optimize the projection subspace. The basic idea is to decouple the MAC and BC phases to decouple the optimization of the matrices $\mathbf{T}$ and $\mathbf{Q}$. Eventhough the relay does not decode the received data streams, we define the virtual sum rate in the MAC phase as the sum of data rates achievable by each node assuming that the relay knows the data stream of this nodes' communication partner. Note that the decoupling into MAC and BC phase is only used to optimize $\mathbf{T}$ and $\mathbf{Q}$. For determining the sum rate performance in the numerical results, of course the overall sum rate over both the phases is considered. The $\mathbf{T}$ matrix is designed such that the virtual sum rate in the MAC phase is maximized subject to the signal alignment constraints. Similarly, the $\mathbf{Q}$ matrix is designed such that the sum rate in the BC phase is maximized subject to the channel alignment constraints. Eventhough the relay does not decode the received signal, we consider the sum rates in the MAC and BC phases separately in order to decouple the optimization of the matrices $\mathbf{T}$ and $\mathbf{Q}$. The optimization of the $\mathbf{Q}$ matrix is similar to that of the $\mathbf{T}$ matrix and hence, only the optimization of the $\mathbf{T}$ matrix is described in the following. As the relay utilization factor η is independent of the number of data streams transmitted by each node, we consider the case where each node transmits only one data stream. Hence, in the following the matrix $\mathbf{V}_k$ is replaced by the vector $\mathbf{v}_k$. $\mathbf{g}_i = \mathbf{g}_j$ denotes the receive zero forcing direction corresponding to the aligned data streams from the communication partners i and j for $i = 1 \ldots K, j = K + i$. P denotes the transmit power of each node. The signal to noise ratio of the signal from node k at the relay after receive zero forcing is given by

$$\mathrm{SNR}_k = \frac{P}{\sigma^2} \frac{\mathbf{v}_k^{\mathrm{H}} \mathbf{H}_{rk}^{\mathrm{H}} \mathbf{T} \mathbf{g}_k \mathbf{g}_k^{\mathrm{H}} \mathbf{T}^{\mathrm{H}} \mathbf{H}_{rk} \mathbf{v}_k}{\mathbf{v}_k^{\mathrm{H}} \mathbf{v}_k} \tag{15}$$

for $k = 1 \ldots 2K$. The rate achieved by the k^{th} node during the MAC phase is given by

$$R_k = \log_2 \left(1 + \mathrm{SNR}_k\right). \tag{16}$$

The optimization problem is as follows:

$$\begin{aligned} &\underset{\mathbf{v}_1 \cdots \mathbf{v}_{2K}, \mathbf{T}}{\text{maximize}} && \sum_{k=1}^{2K} R_k \\ &\text{subject to} && \mathrm{span}\left(\mathbf{T}^{\mathrm{H}} \mathbf{H}_{\mathrm{r}i} \mathbf{v}_i\right) = \mathrm{span}\left(\mathbf{T}^{\mathrm{H}} \mathbf{H}_{\mathrm{r}j} \mathbf{v}_j\right) \\ & && i = 1, 2 \ldots K, j = K + i. \end{aligned} \tag{17}$$

The signal alignment constraint in (17) can always be satisfied for any given choice of matrix $\mathbf{T}$ as long as (12) holds. Hence, we can write the precoder vector $\mathbf{v}$ as a function of the projection matrix $\mathbf{T}$. As the number of data streams transmitted per node is one, without loss of generality we can reformulate the signal alignment constraint as follows:

$$\mathbf{T}^{\mathrm{H}} \mathbf{H}_{\mathrm{r}i} \mathbf{v}_i = \mathbf{T}^{\mathrm{H}} \mathbf{H}_{\mathrm{r}j} \mathbf{v}_j. \tag{18}$$

This equation can be rewritten as

$$\mathbf{T}^{\mathrm{H}} \left[\mathbf{H}_{\mathrm{r}i} \ \ -\mathbf{H}_{\mathrm{r}j}\right] \begin{bmatrix} \mathbf{v}_i \\ \mathbf{v}_j \end{bmatrix} = 0. \tag{19}$$

Let $\mathbf{t}$ denote the orthonormal vector corresponding to the subspace spanned by the columns of matrix $\mathbf{T}$ such that $\mathbf{t}\mathbf{t}^{\mathrm{H}} = \mathbf{I} - \mathbf{T}\mathbf{T}^{\mathrm{H}}$. Then, (19) is equivalent to

$$\left[\mathbf{H}_{\mathrm{r}i} \ \ -\mathbf{H}_{\mathrm{r}j}\right] \begin{bmatrix} \mathbf{v}_i \\ \mathbf{v}_j \end{bmatrix} = \mathbf{t}. \tag{20}$$

Let $\mathbf{H}_{ij} = [\mathbf{H}_{\mathrm{r}i} \ \ -\mathbf{H}_{\mathrm{r}j}]$. $\mathbf{H}_{ik}$ is an invertible matrix. The inverse of the matrix $\mathbf{H}_{ik}$ is denoted by

$$\mathbf{H}_{ij}^{-1} = \mathrm{inv}\left(\mathbf{H}_{ij}\right) = \begin{bmatrix} \mathbf{A}_i \\ \mathbf{A}_j \end{bmatrix}. \tag{21}$$

From (19) and (21) we get

$$\begin{aligned} \mathbf{v}_i &= \mathbf{A}_i \mathbf{t}, \\ \mathbf{v}_j &= \mathbf{A}_j \mathbf{t} \end{aligned} \tag{22}$$

with $i = 1, 2 \ldots K$ and $j = K + i$. For the general case when more than one stream is transmitted per node, precoding vectors for each data stream can be chosen from the subspace spanned by the columns of matrix $\mathbf{V}_k$ for $k = 1 \ldots 2K$. Substituting (22) in (15) yields

$$\mathrm{SNR}_k = \frac{P}{\sigma^2} \frac{\mathbf{t}^{\mathrm{H}} \mathbf{A}_k^{\mathrm{H}} \mathbf{H}_{rk}^{\mathrm{H}} \mathbf{T} \mathbf{g}_k \mathbf{g}_k^{\mathrm{H}} \mathbf{T}^{\mathrm{H}} \mathbf{H}_{rk} \mathbf{A}_k \mathbf{t}}{\mathbf{t}^{\mathrm{H}} \mathbf{A}_k^{\mathrm{H}} \mathbf{A}_k \mathbf{t}}. \tag{23}$$

Let SR denote the achievable sum rate in the system. Now the optimization problem becomes

$$\underset{\mathbf{t}}{\text{maximize}} \quad SR = \sum_{k=1}^{2K} \log(1 + \mathrm{SNR}_k). \tag{24}$$

The above equation is non-convex in $\mathbf{t}$ [10] and we obtain the local solution iteratively using the gradient approach [10]. The initial choice of $\mathbf{t}$ is chosen randomly. Then, $\mathbf{t}$ is updated iteratively based on the gradient using the following relation:

$$\mathbf{t} \to \mathbf{t} + \alpha \frac{\partial SR}{\partial \mathbf{t}^*}. \tag{25}$$

Here, the variable α controls the step size. Simulations show that the algorithm converges after few iterations when $\mathbf{t}$ is normalized after each iteration such that $\mathbf{t}^{\mathrm{H}}\mathbf{t} = 1$. Due to the limitation of space, the calculation of the partial derivative is not shown here. The partial derivative can be obtained by calculating the derivative [11], [12] with respect to each element of $\mathbf{t}^*$ and using the product and quotient rules [13] for the derivative.

Remark 1: Besides small scale fading, large scale fading can also be considered in the optimization of the signal space. This can be achieved by introducing a constant in the SNR term in (23).

Remark 2: The projection based interference alignment scheme, feasibility conditions and the solutions are in general valid for scenarios where the relay has twice the number of antennas compared to the nodes. It can also be utilized for other scenarios with a relay having more than twice the number of antennas at the nodes, but in this case, a relay utilization factor η less than one will be achieved. Hence, in this case, in order to achieve η close to one, the precoding and the projection matrices have to be designed jointly which is left for the future work.

V. Performance Analysis

In this section, we compare the performance of the proposed projection based interference alignment scheme with a conventional multi-user two-way relaying scheme. We consider a $K = 3$ user scenario where each node wants to transmit $d = 1$ data stream to its communication partner. From the feasibility conditions (12) derived in section III-C, precoding at each node has to be performed over at least $N = 2$ subcarriers.

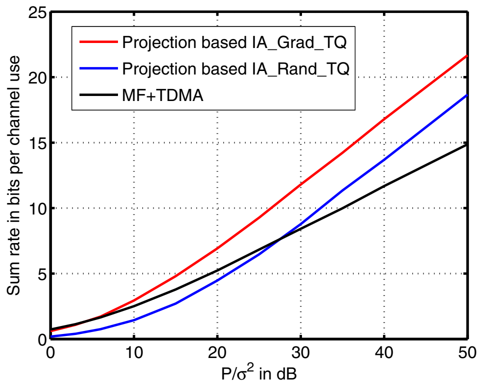

Fig. 2. Sum rate performance for a 3 user pairs scenario with $N = 2$ and $d = 1$

So we consider two subcarriers for precoding the data stream. In the reference scheme based on two-way relaying without interference alignment, only two user pairs can be active at the same time because the relay does not have enough antennas to separate all the 3 pairs. The first pair transmits on subcarrier one and the second pair transmits on subcarrier two. A matched filter is applied at the relay for obtaining a diversity gain due to the multiple antennas. Time Divison Multiple Access (TDMA) is assumed between the set of node pairs so that all the 3 node pairs are able to transmit their signals.

Figure 2 shows the sum rate performance of the proposed scheme with and without the optimization of the projection subspace and the reference scheme as a function of P/σ^2. P is the average transmit power at each node. We assume that the relay has an average transmit power $KP = 3P$. σ^2 is the noise power at each node and at the relay. The channel matrices corresponding to the channels between the nodes and the relay are generated randomly using the i.i.d. frequency-flat Rayleigh MIMO channel [14]. The channel matrices are normalized such that the average received power is the same as the average transmit power. It has to be noted that P is the average transmit power of each node. This means the average transmit power of each node when they are active is larger in the reference scheme than that in the proposed scheme. This is because in the reference scheme, the nodes are active only during a certain duration due to the underlying TDMA protocol.

The sum rate is calculated as an average value from 10^5 channel realizations. The black curve shows the sum rate performance of the reference method. The blue curve shows the sum rate performance of the proposed projection based interference alignment scheme when the matrices $\mathbf{T}$ and $\mathbf{Q}$ are chosen randomly. Figure 2 shows that the reference scheme performs better at low SNR while the proposed scheme performs better at high SNR. The red curve shows the sum rate performance of the proposed scheme when the matrices $\mathbf{T}$ and $\mathbf{Q}$ are optimized based on the gradient. For this optimization, 10 iterations are made to obtain $\mathbf{T}$ and $\mathbf{Q}$. The proposed scheme has a good performance already at 8 dB.

In the considered scenario, the relay utilization factor η for the reference scheme is 0.5 while that for the proposed scheme it is 0.75. There is a 50 % gain in η. This gain can be directly observed in the sum rate. For example at 50 dB, the proposed scheme achieves 22 bits per channel use which is approximately 50% more than that of the reference scheme which achieves 15 bits per channel use.

VI. Conclusion

In this paper, a projection based interference alignment scheme to maximize the utilization of the relay space has been proposed especially for the case of having single antenna nodes. Each node has to precode its data streams over $N \geq \left(\frac{K+1}{2}\right) d$ subcarriers for the interference alignment solution to be feasible. It is shown that when the number of user pairs in the system is large, a relay utilization factor η close to one is achievable. Furthermore, a gradient based method to improve the sum rate performance by optimizing the projection subspace at the relay has been proposed. Simulation results show that the algorithm converges after few iterations and has a better sum rate performance even at moderate signal to noise ratios compared to the reference scheme.

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