H. Degenhardt, T. Unger, A. Klein, ”Self-interference aware MMSE filter design for a cellular multi-antenna two-way relaying scenario,” in *Proc. of the Eighth International Symposium on Wireless Communication Systems (ISWCS'11), Aachen, Germany, November 2011.*

Self-interference aware MMSE filter design for a cellular multi-antenna two-way relaying scenario

Holger Degenhardt, Timo Unger and Anja Klein
Institute of Telecommunications, Communications Engineering Lab,
Technische Universität Darmstadt, Merckstr. 25, 64283 Darmstadt, Germany
Email: {h.degenhardt, t.unger, a.klein}@nt.tu-darmstadt.de

***Abstract*—A single cell scenario with different multi-antenna nodes, i.e., a base station and several mobile stations, is considered. An intermediate multi-antenna relay station is used to support the bidirectional communications. In such a scenario, the required data rates for up- and downlink are typically different which is considered by introducing asymmetric rate requirements. Non-regenerative two-way relaying is applied and the nodes can subtract the back-propagated self-interference. To fully utilize this capability of the nodes for the transceive filter design at the relay station, an MMSE based transceive filter is derived which does not suppress self-interference. Thus, the required number of antennas at the relay station is reduced. Additionally, the transceive filter is optimized for asymmetric rate requirements by introducing a weighting parameter. Furthermore, an alternating optimization between the transceive filter at the relay station and the transmit and receive filters at the nodes is introduced to fulfill the asymmetric rate requirements and to increase the achievable sum rates. Simulation results show that the proposed filter design at the nodes and at the relay station requires less antennas and achieves higher sum rates compared to conventional approaches.**

I. Introduction

One major challenge for future wireless communication systems is the ubiquitous demand for high transmission rates. Relays can be used to support the communication between nodes if a direct communication does not satisfy this high demand and a two-hop relaying scheme can be applied. In this paper, non-regenerative two-hop relaying is considered, i.e., linear signal processing is applied at RS. RS is assumed to be half-duplex and time-division duplex is used. To achieve high throughput, the well-known two-way relaying protocol from [1] is applied which is described in the following. In two-way relaying, all nodes transmit simultaneously to RS in the first time slot. After linear signal processing, RS retransmits the superimposed received signals in the second time slot. Afterwards, the desired signal at each node is recovered from the receive signal by subtracting the back-propagated self-interference. The capability of the nodes to perform self-interference cancellation can be exploited for the transceive filter design at RS, because self-interference does not have to be suppressed. This reduces the required number of antennas at RS for non-regenerative two-way relaying, because RS does not have to separate all receive signals and can just forward superimposed combinations.

Typically, more or less symmetric data rates for the up- and downlink are achieved in non-regenerative two-way relaying. However, many practical applications require asymmetric data rates. Therefore, asymmetric rate requirements are introduced and are considered for the transceive filter design. In this paper, an asymmetric rate requirement describes the required relation between the instantaneous data rate from BS to a mobile station and from the mobile station to BS.

In [2], non-regenerative single-pair multi-antenna two-way relaying is extensively studied and a minimum mean square error (MMSE) filter exploiting self-interference cancellation is derived. Non-regenerative multi-pair two-way relaying with single-antenna nodes and a multi-antenna relay has been considered in [3], [4], [5] and different transceive filter designs based on block-diagonalization are proposed to exploit self-interference cancellation. The authors of [6] investigate a pairwise communication between multi-antenna nodes. MMSE and zero-forcing (ZF) filter design methods which mitigate self-interference are introduced. Single cell scenarios with single antenna mobile stations and a multi-antenna base station have been considered in [7], [8] and [9], but an MMSE based transceive filter at RS for a multi-user multi-antenna scenario exploiting self-interference cancellation has not been derived so far. Also, asymmetric rate requirements have not been considered.

In this paper, a single cell scenario with two multi-antenna mobile stations S1 and S2 and a multi-antenna base station BS is considered and the capability of the nodes to perform self-interference cancellation is fully exploited for the design of the relay transceive filter. This reduces the required number of antennas at RS. To exploit self-interference cancellation in case of asymmetric rate requirements, a weighted self-interference aware MMSE transceive filter at RS is derived. The extension of the derived MMSE transceive filter to a single cell scenario with more than two mobile stations is straightforward. Additionally, an alternating optimization between the transceive filter at RS and the transmit and receive filters at the nodes is introduced to fulfill the asymmetric rate requirements and to increase the achievable sum rates.

The paper is organized as follows. In Section II, the system model is given. The self-interference aware transceive filter design at the relay station is presented in Section III. In Section IV, the filter design at the nodes is presented and an alternating filter optimization between the filters at the nodes and at the relay station is introduced. Simulation results in Section V confirm the analytical investigations and Section VI concludes the paper.

Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate and complex conjugate transpose, respectively. The operators $\mathrm{tr}(\cdot)$, $\mathrm{diag}[\cdot]$, $\otimes$ denote the sum of the main diagonal elements of a matrix, the construction of a diagonal matrix with the elements contained in the vector and the Kronecker product of matrices, respectively. The operators $\Re[\cdot]$, $||\cdot||_2$ denote the real part of a scalar or a matrix and the Frobenius norm of a matrix, respectively. The vectorization operator $\mathrm{vec}(Z)$ stacks the columns of matrix Z into a vector. The operator $\mathrm{vec}^{-1}_{M,N}(\cdot)$ is the revision of the operator $\mathrm{vec}(\cdot)$, i.e., a vector of length MN is sequentially divided into N smaller vectors of length M which are combined to a matrix with M rows and N columns. $\mathbf{I}_M$ denotes an identity matrix of size M.

II. System Model

As shown in Figure 1, the two-hop bidirectional communication over a single carrier between a half-duplex base station BS and two half-duplex mobile stations S1 and S2 via a half-duplex relay station RS is considered. The transmit power at BS, at each mobile station and at RS is limited by $P_{\mathrm{BS,max}}$, $P_{\mathrm{MS,max}}$ and $P_{\mathrm{RS,max}}$, respectively. Each mobile station is equipped with two antennas and simultaneously transmits two data streams, BS is equipped with four antennas and simultaneously transmits four data streams. In the considered scenario, the number of antennas at RS is given by $L \geq 4$ to support the multiplexing into a four dimensional subspace for each direction of transmission. The nodes are assumed to have perfect channel state information (CSI) and can perform power allocation and linear transmit and receive filtering as well as subtract the back-propagated self-interference. The required instantaneous data rates for up- and downlink via RS are assumed to be different: The required instantaneous data rates from BS to each mobile station shall be r times the instantaneous data rates from each mobile station to BS, $r > 0$. Additionally, the required instantaneous data rates from BS to each mobile station are assumed to be equal.

The channels $\mathbf{H}_{\mathrm{BS}} \in \mathbb{C}^{L\times 4}$ and $\mathbf{H}_k \in \mathbb{C}^{L\times 2}$ from BS to RS and from Sk to RS for $k = 1, 2$, respectively, are assumed to be constant during one transmission cycle of the two-way scheme and channel reciprocity is assumed. All signals are assumed to be statistically independent and the noise at RS and at the nodes is assumed to be additive white Gaussian with variances $\sigma^2_{\mathrm{n,RS}}$ and σ^2_n, respectively. The system equations for two-way relaying are presented in the following where all nodes are simultaneously transmitting to RS. The transmitted symbols of BS and Sk are contained in the vectors $\mathbf{x}_{\mathrm{BS}}$ and $\mathbf{x}_{\mathrm{S}k}$, respectively. Using the transmit filters $\mathbf{Q}_{\mathrm{BS}}$ and $\mathbf{Q}_k$, the received baseband signal at RS is given by

$$\mathbf{y}_{\mathrm{RS}} = \mathbf{H}_{\mathrm{BS}}\mathbf{Q}_{\mathrm{BS}}\mathbf{x}_{\mathrm{BS}} + \sum_{k=1}^{2} \mathbf{H}_k\mathbf{Q}_k\mathbf{x}_{\mathrm{S}k} + \mathbf{n}_{\mathrm{RS}}, \tag{1}$$

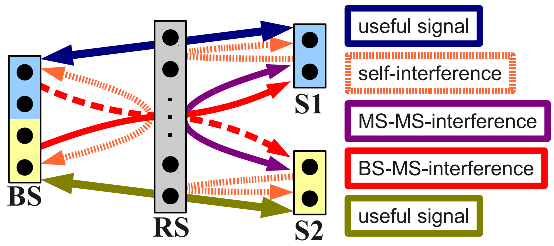

Fig. 1. Composition of useful signals and interferences in a bidirectional single cell two-way relaying scenario.

where $\mathbf{n}_{\mathrm{RS}}$ represents the complex white Gaussian noise vector at RS. RS linearly processes the received signal and the transceive filter at RS is given by

$$\mathbf{G} = \gamma\widetilde{\mathbf{G}}, \tag{2}$$

where $\widetilde{\mathbf{G}}$ is the transceive filter at RS which does not implicitly fulfill the power constraint and γ is a scalar value to satisfy the relay power constraint. It is given by

$$\gamma = \sqrt{\frac{P_{\mathrm{RS,max}}}{||\widetilde{\mathbf{G}}\mathbf{H}_{\mathrm{BS}}\mathbf{Q}_{\mathrm{BS}}||_2^2 + \sum_{k=1}^{2}||\widetilde{\mathbf{G}}\mathbf{H}_k\mathbf{Q}_k||_2^2 + ||\widetilde{\mathbf{G}}||_2^2\sigma^2_{\mathrm{n,RS}}}}. \tag{3}$$

The relay transmits the linearly processed version of $\mathbf{y}_{\mathrm{RS}}$ to all nodes. The received signals $\mathbf{y}_{\mathrm{BS}}$ and $\mathbf{y}_k$ using the receive filters $\mathbf{D}_{\mathrm{BS}}$ and $\mathbf{D}_k$ at BS and Sk, respectively, are given by

$$\mathbf{y}_{\mathrm{BS}} = \mathbf{D}_{\mathrm{BS}}(\mathbf{H}_{\mathrm{BS}}^{\mathrm{T}}\mathbf{G}\mathbf{y}_{\mathrm{RS}} + \mathbf{n}_{\mathrm{BS}}), \tag{4}$$

$$\mathbf{y}_{\mathrm{S}k} = \mathbf{D}_k(\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{y}_{\mathrm{RS}} + \mathbf{n}_k), \tag{5}$$

where $\mathbf{n}_{\mathrm{BS}}$ and $\mathbf{n}_k$ represent the complex white Gaussian noise vectors at BS and Sk, respectively. The compositions of the receive signals are also illustrated in Figure 1. BS receives the useful signals from S1 and S2, receives back-propagated self-interference and noise. Each mobile station receives its intended useful signals, receives interference from the signals intended for the other mobile station, termed "BS-MS-interference", receives interference from the signals transmitted by the other mobile station which are retransmitted by RS, termed "MS-MS-interference", and receives back-propagated self-interference as well as noise. The "MS-MS-interference" as well as the "BS-MS-interference" has to be suppressed by the transceive filter at RS, but the back-propagated self-interference can be subtracted at the nodes [1] assuming that $\mathbf{H}_{\mathrm{BS}}^{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{BS}}$ and $\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{H}_k$ are perfectly known at BS and Sk, respectively. After self-interference cancellation, the received signals $\mathbf{y}_{\mathrm{BS-SI}}$, $\mathbf{y}_{\mathrm{S}k-\mathrm{SI}}$ at BS and Sk, respectively, are given by

$$\mathbf{y}_{\mathrm{BS-SI}} = \mathbf{D}_{\mathrm{BS}}\left(\mathbf{H}_{\mathrm{BS}}^{\mathrm{T}}\mathbf{G}\left(\sum_{k=1}^{2}\mathbf{H}_k\mathbf{Q}_k\mathbf{x}_{\mathrm{S}k} + \mathbf{n}_{\mathrm{RS}}\right) + \mathbf{n}_{\mathrm{BS}}\right), \tag{6}$$

$$\mathbf{y}_{\mathrm{S}k-\mathrm{SI}} = \mathbf{D}_k(\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\left(\mathbf{y}_{\mathrm{RS}} - \mathbf{H}_k\mathbf{Q}_k\mathbf{x}_{\mathrm{S}k}\right) + \mathbf{n}_k). \tag{7}$$

The fact that the back-propagated self-interferences can be subtracted at the nodes can be exploited for the design of the transceive filter $\mathbf{G}$ at RS.

III. Transceive filter design at the relay station

In the following, a weighted minimum mean square error (MMSE) filter is derived which exploits the capability of the nodes to perform self-interference cancellation and which can be optimized to consider asymmetric rate requirements. This weighted self-interference aware MMSE filter is named WMMSE-SI. The capability of the nodes to perform self-interference cancellation is considered by the definition of the MSE equations. In these equations, the error caused by self-interference is removed so that back-propagated self-interference is only considered in the power constraint at RS and is not intentionally suppressed by the transceive filter design. To consider the asymmetric rate requirements, the errors for each direction of transmission are separated and weighting parameters are introduced. In these equations, the transmitted symbols of BS which are intended for Sk are described by the vector $\mathbf{x}_{\mathrm{BS}k}$. The weighting parameter is used to optimize the filter design with respect to the asymmetric rate requirements based on the results in [10] where it is shown that a weighted sum rate maximization problem can be solved using the ideas of weighted MMSE filtering. The general equation for the transceive filter design at RS is given by

$$\mathbf{G} = \arg\min_{\mathbf{G}} \mathrm{E}\left\{\alpha \sum_{k=1}^{2} \|\mathbf{x}_{\mathrm{BS}k} - \hat{\mathbf{x}}_{\mathrm{BS}k}\|_2^2\right\} + \mathrm{E}\left\{(2-\alpha) \sum_{k=1}^{2} \|\mathbf{x}_{\mathrm{S}k} - \hat{\mathbf{x}}_{\mathrm{S}k}\|_2^2\right\}, \tag{8}$$

where the parameter α, $0 \leq \alpha \leq 2$ is used to weight the MSE for each direction of transmission and where $\hat{\mathbf{x}}_{\mathrm{S}k}$ and $\hat{\mathbf{x}}_{\mathrm{BS}k}$ are the estimates of $\mathbf{x}_{\mathrm{S}k}$ and $\mathbf{x}_{\mathrm{BS}k}$ at BS and Sk, respectively. In the following the notation $\beta = (2-\alpha)$ is used. Only one weighting parameter is used, because the data rates from BS to each mobile station shall be equal. The introduction of additional weighting parameters for each source to destination error is straightforward, but is omitted to keep the notation simple. The MSE from BS to Sk with interference from Sl is given by

$$\begin{aligned}
&\mathrm{E}\left\{\|\mathbf{x}_{\mathrm{BS}k} - \hat{\mathbf{x}}_{\mathrm{BS}k}\|_2^2\right\} \\
&= \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}_{\mathrm{BS}k}}\right) - 2\Re\left[\mathrm{tr}\left(\mathbf{D}_k \mathbf{H}_k^{\mathrm{T}} \mathbf{G} \mathbf{H}_{\mathrm{BS}} \mathbf{Q}_{\mathrm{BS}k} \mathbf{R}_{\mathbf{x}_{\mathrm{BS}k}}\right)\right] \\
&+ \mathrm{tr}\left(\mathbf{D}_k \mathbf{H}_k^{\mathrm{T}} \mathbf{G} \mathbf{H}_{\mathrm{BS}} \mathbf{Q}_{\mathrm{BS}} \mathbf{R}_{\mathbf{x}_{\mathrm{BS}}} \mathbf{Q}_{\mathrm{BS}}^{\mathrm{H}} \mathbf{H}_{\mathrm{BS}}^{\mathrm{H}} \mathbf{G}^{\mathrm{H}} \mathbf{H}_k^{*} \mathbf{D}_k^{\mathrm{H}}\right) \\
&+ \mathrm{tr}\left(\mathbf{D}_k \mathbf{H}_k^{\mathrm{T}} \mathbf{G} \mathbf{H}_l \mathbf{Q}_l \mathbf{R}_{\mathbf{x}_{\mathrm{S}l}} \mathbf{Q}_l^{\mathrm{H}} \mathbf{H}_l^{\mathrm{H}} \mathbf{G}^{\mathrm{H}} \mathbf{H}_k^{*} \mathbf{D}_k^{\mathrm{H}}\right) \\
&+ \mathrm{tr}\left(\mathbf{D}_k \mathbf{H}_k^{\mathrm{T}} \mathbf{G} \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}} \mathbf{G}^{\mathrm{H}} \mathbf{H}_k^{*} \mathbf{D}_k^{\mathrm{H}} + \mathbf{D}_k \mathbf{R}_{\mathbf{n}_{\mathrm{S}k}} \mathbf{D}_k^{\mathrm{H}}\right),
\end{aligned} \tag{9}$$

with $\mathbf{R}_{\mathbf{x}_{\mathrm{BS}k}}$, $\mathbf{R}_{\mathbf{x}_{\mathrm{S}k}}$ the signal covariance matrices of $\mathbf{x}_{\mathrm{BS}k}$, $\mathbf{x}_{\mathrm{S}k}$, respectively, and $\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}$, $\mathbf{R}_{\mathbf{n}_{\mathrm{S}k}}$ the noise covariance matrices at RS and Sk, respectively. With $\mathbf{D}_{\mathrm{BS}k}$ containing the $(2k-1)$th and the $2k$th row vector of $\mathbf{D}_{\mathrm{BS}}$ and with $\mathbf{Q}_{\mathrm{BS}k}$ containing the $(2k-1)$th and the $2k$th column vector of $\mathbf{Q}_{\mathrm{BS}}$, the MSE from Sk to BS is given by

$$\begin{aligned}
&\mathrm{E}\left\{\|\mathbf{x}_{\mathrm{S}k} - \hat{\mathbf{x}}_{\mathrm{S}k}\|_2^2\right\} \\
&= \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}_{\mathrm{S}k}}\right) - 2\Re\left[\mathrm{tr}\left(\mathbf{D}_{\mathrm{BS}k} \mathbf{H}_{\mathrm{BS}}^{\mathrm{T}} \mathbf{G} \mathbf{H}_k \mathbf{Q}_k \mathbf{R}_{\mathbf{x}_{\mathrm{S}k}}\right)\right] \\
&+ \sum_{l=1}^{2} \mathrm{tr}\left(\mathbf{D}_{\mathrm{BS}k} \mathbf{H}_{\mathrm{BS}}^{\mathrm{T}} \mathbf{G} \mathbf{H}_l \mathbf{Q}_l \mathbf{R}_{\mathbf{x}_{\mathrm{S}l}} \mathbf{Q}_l^{\mathrm{H}} \mathbf{H}_l^{\mathrm{H}} \mathbf{G}^{\mathrm{H}} \mathbf{H}_{\mathrm{BS}}^{*} \mathbf{D}_{\mathrm{BS}k}^{\mathrm{H}}\right) \\
&+ \mathrm{tr}\left(\mathbf{D}_{\mathrm{BS}k} \mathbf{H}_{\mathrm{BS}}^{\mathrm{T}} \mathbf{G} \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}} \mathbf{G}^{\mathrm{H}} \mathbf{H}_{\mathrm{BS}}^{*} \mathbf{D}_{\mathrm{BS}k}^{\mathrm{H}} + \mathbf{D}_{\mathrm{BS}k} \mathbf{R}_{\mathbf{n}_{\mathrm{BS}}} \mathbf{D}_{\mathrm{BS}k}^{\mathrm{H}}\right).
\end{aligned} \tag{10}$$

The MSE of (8) in combination with the power constraint of RS results in a convex problem with respect to $\mathbf{G}$ for fixed transmit and receive filters at the nodes. This problem can be solved by using Lagrangian optimization. Let matrices $\boldsymbol{\Upsilon}^{(k)}$, $\boldsymbol{\Upsilon}^{(\mathrm{BS})}$ and $\boldsymbol{\Upsilon}$ be given by

$$\boldsymbol{\Upsilon}^{(k)} = \mathbf{H}_k \mathbf{Q}_k \mathbf{R}_{\mathbf{x}_{\mathrm{S}k}} \mathbf{Q}_k^{\mathrm{H}} \mathbf{H}_k^{\mathrm{H}} + \frac{1}{2}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}, \tag{11a}$$

$$\boldsymbol{\Upsilon}^{(\mathrm{BS})} = \mathbf{H}_{\mathrm{BS}} \mathbf{Q}_{\mathrm{BS}} \mathbf{R}_{\mathbf{x}_{\mathrm{BS}}} \mathbf{Q}_{\mathrm{BS}}^{\mathrm{H}} \mathbf{H}_{\mathrm{BS}}^{\mathrm{H}} + \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}, \tag{11b}$$

$$\boldsymbol{\Upsilon} = \mathbf{H}_{\mathrm{BS}} \mathbf{Q}_{\mathrm{BS}} \mathbf{R}_{\mathbf{x}_{\mathrm{BS}}} \mathbf{Q}_{\mathrm{BS}}^{\mathrm{H}} \mathbf{H}_{\mathrm{BS}}^{\mathrm{H}} + \sum_{k=1}^{2} \mathbf{H}_k \mathbf{Q}_k \mathbf{R}_{\mathbf{x}_{\mathrm{S}k}} \mathbf{Q}_k^{\mathrm{H}} \mathbf{H}_k^{\mathrm{H}} + \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}. \tag{11c}$$

Using matrices $\boldsymbol{\Upsilon}^{(k)}$ and $\boldsymbol{\Upsilon}^{(\mathrm{BS})}$ and $\boldsymbol{\Upsilon}$ of (11) in (8), and rewriting the RS transmit power constraint $||\mathbf{G}\mathbf{H}_{\mathrm{BS}}\mathbf{Q}_{\mathrm{BS}}||_2^2 + \sum_{k=1}^{2} ||\mathbf{G}\mathbf{H}_k\mathbf{Q}_k||_2^2 + ||\mathbf{G}||_2^2 \sigma_{\mathrm{n,RS}}^2 \leq P_{\mathrm{RS,max}}$ by considering $\boldsymbol{\Upsilon}$, the Lagrangian function with the Lagrangian multiplier η results in

$$L(\mathbf{G}, \eta) = \alpha \sum_{k=1}^{2} F_1(\mathbf{G}, k) + \beta \sum_{k=1}^{2} F_2(\mathbf{G}, k) - \eta\left(\mathrm{tr}\left(\mathbf{G}\boldsymbol{\Upsilon}\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{RS,max}}\right), \tag{12}$$

with

$$\begin{aligned}
F_1(\mathbf{G}, k) &= \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}_{\mathrm{BS}k}}\right) - 2\Re\left[\mathrm{tr}\left(\mathbf{D}_k \mathbf{H}_k^{\mathrm{T}} \mathbf{G} \mathbf{H}_{\mathrm{BS}} \mathbf{Q}_{\mathrm{BS}k} \mathbf{R}_{\mathbf{x}_{\mathrm{BS}k}}\right)\right] \\
&+ \mathrm{tr}\left(\mathbf{D}_k \left(\mathbf{H}_k^{\mathrm{T}} \mathbf{G} \boldsymbol{\Upsilon}^{(\mathrm{BS})} \mathbf{G}^{\mathrm{H}} \mathbf{H}_k^{*} + \mathbf{R}_{\mathbf{n}_{\mathrm{S}k}}\right) \mathbf{D}_k^{\mathrm{H}}\right) \\
&+ \sum_{l=1, l\neq k}^{2} \mathrm{tr}\left(\mathbf{D}_k \mathbf{H}_k^{\mathrm{T}} \mathbf{G} \boldsymbol{\Upsilon}^{(l)} \mathbf{G}^{\mathrm{H}} \mathbf{H}_k^{*} \mathbf{D}_k^{\mathrm{H}}\right),
\end{aligned} \tag{13a}$$

$$\begin{aligned}
F_2(\mathbf{G}, k) &= \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}_{\mathrm{S}k}}\right) - 2\Re\left[\mathrm{tr}\left(\mathbf{D}_{\mathrm{BS}k} \mathbf{H}_{\mathrm{BS}}^{\mathrm{T}} \mathbf{G} \mathbf{H}_k \mathbf{Q}_k \mathbf{R}_{\mathbf{x}_{\mathrm{S}k}}\right)\right] \\
&+ \mathrm{tr}\left(\mathbf{D}_{\mathrm{BS}k} \left(\sum_{l=1}^{2} \mathbf{H}_{\mathrm{BS}}^{\mathrm{T}} \mathbf{G} \boldsymbol{\Upsilon}^{(l)} \mathbf{G}^{\mathrm{H}} \mathbf{H}_{\mathrm{BS}}^{*}\right) \mathbf{D}_{\mathrm{BS}k}^{\mathrm{H}}\right) \\
&+ \mathrm{tr}\left(\mathbf{D}_{\mathrm{BS}k} \mathbf{R}_{\mathbf{n}_{\mathrm{BS}}} \mathbf{D}_{\mathrm{BS}k}^{\mathrm{H}}\right).
\end{aligned} \tag{13b}$$

From the Lagrangian function, the Karush-Kuhn-Tucker (KKT) conditions can be derived, which are necessary conditions for a global optimum:

$$\frac{\partial L}{\partial \mathbf{G}} = \alpha \sum_{k=1}^{2} f_1(\mathbf{G}, k) + \beta \sum_{k=1}^{2} f_2(\mathbf{G}, k) - \eta\, \mathbf{G}^{*} \boldsymbol{\Upsilon}^{\mathrm{T}} = \mathbf{0} \tag{14a}$$

$$\eta\left(\mathrm{tr}\left(\mathbf{G}\boldsymbol{\Upsilon}\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{RS,max}}\right) = 0, \tag{14b}$$

with

$$
\begin{aligned}
f_1(\mathbf{G},k) =& -\mathbf{H}_k\mathbf{D}_k^{\mathrm{T}}\mathbf{R}_{\mathbf{x}_{\mathrm{BS}k}}^{\mathrm{T}}\mathbf{Q}_{\mathrm{BS}k}^{\mathrm{T}}\mathbf{H}_{\mathrm{BS}}^{\mathrm{T}} \\
&+\mathbf{H}_k\mathbf{D}_k^{\mathrm{T}}\mathbf{D}_k^{*}\mathbf{H}_k^{\mathrm{H}}\mathbf{G}^{*}\boldsymbol{\Upsilon}^{(\mathrm{BS})^{\mathrm{T}}} \\
&+\sum_{l=1,l\neq k}^{2}\mathbf{H}_k\mathbf{D}_k^{\mathrm{T}}\mathbf{D}_k^{*}\mathbf{H}_k^{\mathrm{H}}\mathbf{G}^{*}\boldsymbol{\Upsilon}^{(l)^{\mathrm{T}}}, \qquad (15a)\\
f_2(\mathbf{G},k) =& -\mathbf{H}_{\mathrm{BS}}\mathbf{D}_{\mathrm{BS}k}^{\mathrm{T}}\mathbf{R}_{\mathbf{x}_{\mathrm{S}k}}^{\mathrm{T}}\mathbf{Q}_k^{\mathrm{T}}\mathbf{H}_k^{\mathrm{T}} \\
&+\sum_{l=1}^{2}\mathbf{H}_{\mathrm{BS}}\mathbf{D}_{\mathrm{BS}k}^{\mathrm{T}}\mathbf{D}_{\mathrm{BS}k}^{*}\mathbf{H}_{\mathrm{BS}}^{\mathrm{H}}\mathbf{G}^{*}\boldsymbol{\Upsilon}^{(l)^{\mathrm{T}}}. \qquad (15b)
\end{aligned}
$$

In the following, matrix $\mathbf{K}$ is defined as

$$
\begin{aligned}
\mathbf{K} =& \alpha\sum_{k=1}^{2}\left[\boldsymbol{\Upsilon}^{(\mathrm{BS})^{\mathrm{T}}}\otimes\left(\mathbf{H}_k^{*}\mathbf{D}_k^{\mathrm{H}}\mathbf{D}_k\mathbf{H}_k^{\mathrm{T}}\right)\right] \\
&+\alpha\sum_{k=1}^{2}\sum_{l=1,l\neq k}^{2}\left[\boldsymbol{\Upsilon}^{(\mathrm{l})^{\mathrm{T}}}\otimes\left(\mathbf{H}_k^{*}\mathbf{D}_k^{\mathrm{H}}\mathbf{D}_k\mathbf{H}_k^{\mathbf{T}}\right)\right] \\
&+\beta\sum_{k=1}^{2}\sum_{l=1}^{2}\left[\boldsymbol{\Upsilon}^{(l)^{\mathrm{T}}}\otimes\left(\mathbf{H}_{\mathrm{BS}}^{*}\mathbf{D}_{\mathrm{BS}k}^{\mathrm{H}}\mathbf{D}_{\mathrm{BS}k}\mathbf{H}_{\mathrm{BS}}^{\mathrm{T}}\right)\right] \\
&+\left[\boldsymbol{\Upsilon}^{\mathrm{T}}\otimes\frac{1}{P_{\mathrm{RS,max}}}\mathrm{tr}\left(\sum_{k=1}^{2}\mathbf{R}_{\mathbf{n}_{\mathrm{S}k}}+\mathbf{R}_{\mathbf{n}_{\mathrm{BS}}}\right)\mathbf{I}_L\right]. \qquad (16)
\end{aligned}
$$

Using Eqs. (2), (3) and (16), the WMMSE-SI filter at RS which solves problem (8) is given by

$$
\begin{aligned}
\widetilde{\mathbf{G}} = \mathrm{vec}_{L,L}^{-1}&\left(\mathbf{K}^{-1}\ \mathrm{vec}\left(\alpha\sum_{k=1}^{2}\mathbf{H}_k^{*}\mathbf{D}_k^{\mathrm{H}}\mathbf{R}_{\mathbf{x}_{\mathrm{BS}k}}\mathbf{Q}_{\mathrm{BS}k}^{\mathrm{H}}\mathbf{H}_{\mathrm{BS}}^{\mathrm{H}}\right.\right. \\
&\left.\left.+\beta\sum_{k=1}^{2}\mathbf{H}_{\mathrm{BS}}^{*}\mathbf{D}_{\mathrm{BS}k}^{\mathrm{H}}\mathbf{R}_{\mathbf{x}_{\mathrm{S}k}}\mathbf{Q}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\right)\right). \qquad (17)
\end{aligned}
$$

The derived WMMSE-SI transceive filter at RS minimizes the weighted MSE for given transmit and receive filters at the nodes. The optimal weighting factor α is determined by numerical optimization with respect to the asymmetric rate requirement. The extension of the WMMSE-SI transceive filter to a single cell scenario with K mobile stations each equipped with M antennas is straightforward as long as the number of antennas at BS is equal to or greater than KM. If the number of antennas at BS is smaller than KM, schemes like time division multiple access (TDMA) or frequency division multiple access (FDMA) can be applied to form smaller groups of mobile stations.

IV. Filter Design at the Nodes and Overall Alternating Optimization

In this section, three different cases of filter design at the nodes are introduced and an alternating optimization between $\mathbf{G}$ and the filters at the nodes as shown in Figure 2 is proposed. In the first case (case Diag), only the node powers are optimized to fulfill the asymmetric rate requirements. In this case, the transmit filters are given by

$$
\begin{aligned}
\mathbf{Q}_{\mathrm{BS}} =& \rho\mathbf{W}, \\
\mathbf{Q}_k =& \sqrt{P_{\mathrm{S}k}/2}\cdot\mathbf{I}_2, \qquad (18)
\end{aligned}
$$

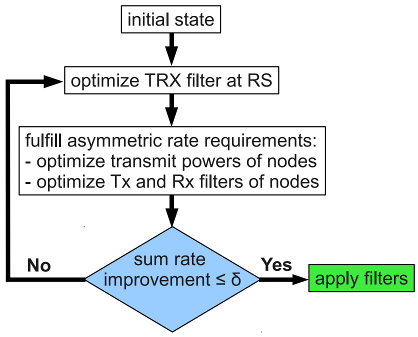

Fig. 2. Alternating filter optimization.

where $\mathbf{W} = P_{\mathrm{BS}}/2\cdot\mathrm{diag}[w,w,(1-w),(1-w)]$ with w a weighting parameter for the power distribution at BS, $0\leq w\leq 1$, and with ρ a scalar value to fulfill the power constraint at BS. The receive filters are given by

$$
\begin{aligned}
\mathbf{D}_{\mathrm{BS}} =& \mathbf{I}_4, \qquad (19a)\\
\mathbf{D}_k =& \mathbf{I}_2. \qquad (19b)
\end{aligned}
$$

To further improve the sum rate, the transmit (Tx) and receive (Rx) filters at the nodes have to be optimized based on the overall channel and, therewith, dependent on $\mathbf{G}$. In the second case (case Rx), the Rx filters of the nodes are optimized dependent on $\mathbf{G}$ to minimize the MSE on the overall channel and the Tx filters are given by (18). The Rx filters at the nodes are based on the overall channels $\mathbf{H}_{\mathrm{BS,ov}} = [\mathbf{H}_{\mathrm{BS}}^{\mathrm{T}}\mathbf{G}\mathbf{H}_1\mathbf{Q}_1, \mathbf{H}_{\mathrm{BS}}^{\mathrm{T}}\mathbf{G}\mathbf{H}_2\mathbf{Q}_2]$ and $\mathbf{H}_{\mathrm{S}k,\mathrm{ov}} = \mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{BS}}\mathbf{Q}_{\mathrm{BS}k}$ and are given by

$$
\begin{aligned}
\mathbf{D}_{\mathrm{BS}} =& \mathbf{H}_{\mathrm{BS,ov}}^{\mathrm{H}}(\mathbf{H}_{\mathrm{BS,ov}}\mathbf{H}_{\mathrm{BS,ov}}^{\mathrm{H}}+\mathbf{N}_{\mathrm{BS}})^{-1}, \qquad (20a)\\
\mathbf{D}_k =& \mathbf{H}_{\mathrm{S}k,\mathrm{ov}}^{\mathrm{H}}(\mathbf{H}_{\mathrm{S}k,\mathrm{ov}}\mathbf{H}_{\mathrm{S}k,\mathrm{ov}}^{\mathrm{H}}+\mathbf{N}_{\mathrm{S}k})^{-1}, \qquad (20b)
\end{aligned}
$$

with $\mathbf{N}_{\mathrm{BS}} = \mathbf{R}_{\mathbf{n}_{\mathrm{BS}}} + \mathbf{H}_{\mathrm{BS}}^{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_{\mathrm{BS}}^{*}$ and $\mathbf{N}_{\mathrm{S}k} = \mathbf{R}_{\mathbf{n}_{\mathrm{S}k}} + \mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_{\mathrm{k}}^{*}$ the noise matrices. In the third case (case Tx&Rx), the Rx and Tx filters at BS are optimized. The Rx filters are given by Eq. (19b) and Eq. (20a), and the Tx filters based on the overall channel $\mathbf{H}_{\mathrm{ov}} = [(\mathbf{H}_1^{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{BS}})^{\mathrm{T}}, (\mathbf{H}_2^{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{BS}})^{\mathrm{T}}]^{\mathrm{T}}$ are given by

$$
\begin{aligned}
\mathbf{Q}_{\mathrm{BS}} =& \rho(\mathbf{H}_{\mathrm{ov}}^{\mathrm{H}}\mathbf{W}\mathbf{H}_{\mathrm{ov}}+4\sigma_n^2\mathbf{I}_4)^{-1}\mathbf{H}_{\mathrm{ov}}^{\mathrm{H}}\mathbf{W}, \\
\mathbf{Q}_k =& \sqrt{P_{\mathrm{S}k}/2}\cdot\mathbf{I}_2, \qquad (21)
\end{aligned}
$$

For all cases, the transmit powers $P_{\mathrm{BS}} \leq P_{\mathrm{BS,max}}$, $P_{\mathrm{S}k} \leq P_{\mathrm{MS,max}}$ of the nodes are numerically optimized to fulfill the asymmetric rate requirement and the alternating optimization shown in Figure 2 is performed.

In a conventional transceive filter approach at RS, all signals and interferences are separated at RS, which requires at least one antenna at RS per simultaneously received signal. Due to exploiting the capability of the nodes to perform self-interference cancellation in the WMMSE-SI filter at RS, the required number of antennas at RS for spatial separation of useful signals and interferences at the receivers is reduced compared to a conventional approach. In case Diag and case Rx, the required number of antennas at RS is reduced from

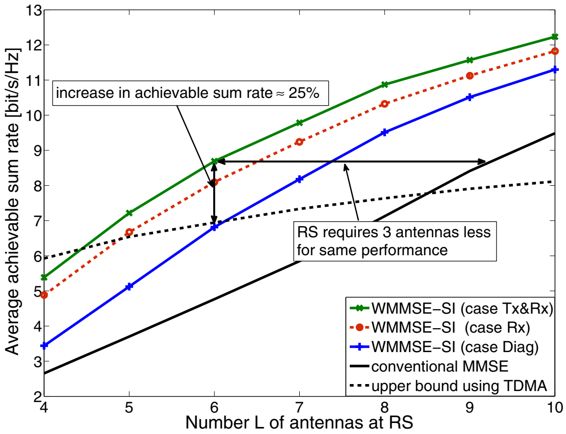

Fig. 3. Average achievable sum rates over number L of antennas at RS for an asymmetric rate requirement of $r = 3$, SNR1=SNR2=15dB.

$L \geq 8$ for a conventional approach to $L \geq 6$ for the WMMSE-SI approach. In case Rx, interferences between the different useful signals are additionally mitigated. This does not reduce the required number of antennas at RS, but by accounting for these interferences at the nodes the "MS-MS-interference" suppression of $\mathbf{G}$ is improved. In case Tx&Rx, the Tx filter at BS performs an alignment of the BS to RS signals intended for Sk with the signals from Sk to RS. In case of perfect alignment, the required number of antennas at RS for spatial separation of useful signals and interferences at the receivers is further reduced to $L \geq 4$.

V. Simulation results

In this section, numerical results on the achievable sum rates for the different cases of filter design at the nodes in conjunction with the WMMSE-SI transceive filter at RS are compared. It is assumed that $P_{\text{BS,max}} = P_{\text{MS,max}} = P_{\text{RS,max}}$ and $\sigma^2_{\text{RS}} = \sigma^2_{\text{n}}$. The path-loss on the i.i.d. Rayleigh fading channels results in an average receive signal to noise ratio at RS for each link of 15dB if the nodes transmit with maximum power. The average achievable sum rates over the number L of antennas at RS for an asymmetric rate requirement between up- and downlink of $r = 3$ are shown in Figure 3. For comparison, a conventional MMSE approach is used, which does not exploit self-interference cancellation for the design of $\mathbf{G}$. Since all presented approaches are based on an MMSE and not on a zero-forcing design, these approaches can also be applied if RS has not enough antennas to spatially separate useful signals and interferences at the receivers. Additionally, the upper bound for a TDMA approach given in [11] is used for comparison. This TDMA approach performs two-way relaying between BS and S1 in the first two time slots and between BS and S2 in the following two time slots. For $L > 6$, the alternating optimization using the filters of case Diag performs better than the TDMA bound. If the filter design at the nodes is improved from case Diag over case Rx up to case Tx&Rx, the achievable sum rate increases and the performance is better compared to the TDMA bound for $L \geq 5$. Compared to the conventional MMSE approach, a sum rate of, e.g., 8.7 bit/s/Hz can be achieved with three antennas less at RS using the WMMSE-SI filter for case Tx&Rx. In case of six antennas at RS, the achievable sum rate for case Tx&Rx is increased by 25% compared to the TDMA bound.

VI. Conclusions

Non-regenerative two-way relaying for a multi-user multi-antenna single cell scenario has been investigated considering asymmetric rate requirements. A weighted MMSE filter has been derived exploiting self-interference cancellation and mitigating the interferences between the signals intended for and transmitted by the mobile stations. Thus, the required number of antennas at the relay station is reduced. Furthermore, the achievable sum rate is increased by an alternating filter design between the transmit and receive filters at the nodes and the transceive filter at the relay station.

Acknowledgment

The work of Holger Degenhardt is funded by the Deutsche Forschungsgesellschaft (DFG) under Grant No. Kl907/2-1 and is supported by the LOEWE Priority Program Cocoon and by the Graduate School of Computational Engineering at Technische Universität Darmstadt.

References

[1] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 379–389, 2007.

[2] T. Unger, "Multi-antenna two-hop relaying for bi-directional transmission in wireless communication systems," Ph.D. dissertation, TU Darmstadt, June 2009.

[3] E. Yilmaz, R. Zakhour, D. Gesbert, and R. Knopp, "Multi-Pair Two-Way Relay Channel with Multiple Antenna Relay Station," in *Proc. IEEE International Conference on Communications (ICC)*, 2010, pp. 1–5.

[4] A. Amah and A. Klein, "Pair-aware transceive beamforming for non-regenerative multi-user two-way relaying," in *Proc. IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, 2010, pp. 2506–2509.

[5] C. Y. Leow, Z. Ding, K. Leung, and D. Goeckel, "On the Study of Analogue Network Coding for Multi-Pair, Bidirectional Relay Channels," *IEEE Transactions on Wireless Communications*, vol. 10, no. 2, pp. 670–681, 2011.

[6] J. Joung and A. Sayed, "Multiuser Two-Way Amplify-and-Forward Relay Processing and Power Control Methods for Beamforming Systems," *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1833–1846, 2010.

[7] S. Toh and D. Slock, "A linear beamforming scheme for multi-user MIMO AF two-phase two-way relaying," in *Proc. IEEE 20th International Symposium onPersonal, Indoor and Mobile Radio Communications*, 2009, pp. 1003–1007.

[8] J. Zhang, F. Roemer, and M. Haardt, "Beamforming design for multi-user two-way relaying with MIMO amplify and forward relays," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2011, pp. 2824–2827.

[9] C. Sun, C. Yang, Y. Li, and B. Vucetic, "Transceiver optimization for multi-user multi-antenna two-way relay channels," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2011, pp. 3064–3067.

[10] S. Christensen, R. Agarwal, E. Carvalho, and J. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 4792–4799, 2008.

[11] H. Degenhardt and A. Klein, "Non-regenerative multi-antenna two-hop relaying under an asymmetric rate constraint," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, 2011, pp. 1454–1459.