Non-Regenerative Multi-Way Relaying: Space-Time Analog Network Coding and Repetition

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***Abstract*—In this letter, we investigate non-regenerative multi-way relaying when channel state information is not available at the multi-antenna relay station (RS). For stationary channels, space-time analog network coding (STANC) transmission is applied at the RS. For non-stationary channels, the RS applies repetition transmission. For both transmission schemes, in order to obtain the data streams of the other nodes, each node has to perform three receive processing operations successively: Zero Forcing (ZF) detection, back-propagated self-interference cancellation and joint detection. We assess the achievable sum rate performance of non-regenerative multi-way relaying for the proposed transmission schemes. It is shown that STANC with joint detection outperforms both benchmark transmissions, namely, ZF and Maximisation of Signal to Noise Ratio (MSNR) transceive beamforming with single stream decoding performed at each node. When the channels are non-stationary, the repetition transmission enables the multi-way communication and in medium to high SNR region, it outperforms MSNR.**

***Index Terms*—Multi-way relaying; analog network coding; space-time block code.**

I. Introduction

RECENT communication applications such as voice or video conference allow multiple nodes to communicate with each other. If there are no direct links between the nodes, they can communicate with each other via a relay station (RS). Such a multi-way relay channel has been considered in [1]. In [1], a multi-way communication among full-duplex nodes via a full-duplex RS is considered. However, since half-duplex devices are more suitable for practical implementation [2], an efficient communication protocol for multi-way communication among half-duplex nodes via a half-duplex RS is needed.

Multi-way relaying protocols for N half-duplex nodes to communicate with each other via a half-duplex multi-antenna RS have been proposed in [3] for a non-regenerative RS and in [4] for a regenerative RS. The total number of communication phases is equal to the number of nodes N. Multi-way relaying is an extension of two-way relaying proposed in [2]. In the first phase, the multiple access (MAC) phase, all nodes transmit simultaneously to the RS. In the following $N-1$ broadcast (BC) phases, the RS transmits to all nodes.

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In [3] and [4], it is assumed that the channel coefficients are stationary within N communication phases and the channel state information (CSI) is available at the multi-antenna RS such that the RS is able to perform beamforming. In practice, however, there may be some cases where there is no CSI available at the RS to perform beamforming. Therefore, it is the aim of this letter to propose transmission schemes for non-regenerative multi-way relaying when CSI is not available at the RS for both stationary and non-stationary channels.

When the channels are stationary, the multi-antenna RS applies space-time analog network coding (STANC) transmission. The term analog network coding is used to differentiate with the conventional space-time block coding (STBC) due to the fact that the RS sends the received mixture signals of all nodes simultaneously to all nodes after power amplification subject to per-antenna power constraint at the RS. Different to conventional STBC, in order to obtain the data streams of the other $N-1$ nodes, each node has to perform three receive processing operations successively: Zero Forcing (ZF) detection, back-propagated self-interference cancellation and joint detection.

When the channels are not stationary, similar to STBC, STANC cannot be applied. Therefore, we propose repetition transmission for non-regenerative multi-way relaying to be applied at the RS. In order to obtain the data streams from the other $N-1$ nodes, each node also has to perform successively the three receive processing operations as for STANC.

This paper is organized as follows. Section II provides the system model. Section III explains the sum rate expression. Section IV provides the simulation results.

Notations: Boldface lower and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate, and complex conjugate transpose, respectively. The operators $\mathrm{E}\{\mathbf{X}\}$ and $\mathrm{tr}\{\mathbf{X}\}$ denote the expectation and the trace of $\mathbf{X}$, respectively. $\mathcal{CN}(0,\sigma^2)$ denotes the circularly symmetric zero-mean complex normal distribution with variance σ^2.

II. System Model

In this section, we provide the system model under consideration in this work. There are N single-antenna nodes which communicate with each other via a multi-antenna RS with R antennas. The required number of communication phases is equal to the number of nodes N. Within N phases, all nodes transmit to the RS in the first phase and in the remaining phases, the RS transmits to the nodes. Because the RS has $N-1$ time slots to transmit to the nodes, to ensure that each node is able to decode all data streams of the other $N-1$ nodes, it is assumed that the RS has $R = N-1$ antennas.

A. MAC Phase

The first phase is the MAC phase where all nodes transmit simultaneously to the RS. Let the channel coefficient between node i and RS antenna r be denoted by $h_{ri}, r \in \{1, \cdots, R\}, i \in \{1, \cdots, N\}$, which follows $\mathcal{CN}(0, \sigma_h^2)$. The channel vector of RS antenna r is given by $\mathbf{h}_r = (h_{r1}, \cdots, h_{rN})^{\mathrm{T}}$ and the overall channel matrix is given by

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_R]^{\mathrm{T}}. \tag{1}$$

The transmit data vector of the nodes is given by $\mathbf{x} = (x_1, \cdots, x_N)^{\mathrm{T}}$, where x_i is the transmit signal from node i which follows $\mathcal{CN}(0, \sigma_x^2)$. The received signal at the RS is given by

$$\mathbf{y}_{\mathrm{RS}} = \mathbf{H}\mathbf{x} + \mathbf{n}_{\mathrm{RS}} \tag{2}$$

with $\mathbf{n}_{\mathrm{RS}} = (n_{\mathrm{RS}_1}, \cdots, n_{\mathrm{RS}_R})^{\mathrm{T}}$. n_{RS_r} is the AWGN noise at RS antenna r which follows $\mathcal{CN}(0, \sigma_n^2)$. Correspondingly, the received signal at RS antenna r is given by

$$y_{\mathrm{RS}_r} = \mathbf{h}_r^{\mathrm{T}}\mathbf{x} + n_{\mathrm{RS}_r}. \tag{3}$$

B. BC Phases for Stationary Channels: STANC

In this work, we consider a special case of multi-way relaying where $N = 3$ and $R = 2$. Thus, we modify the Alamouti code [5] to be applied for three-way relaying. Nevertheless, the extension to $N > 3$ is straight forward if only a full rate orthogonal space-time block code for more than two antennas is available.

Let $p, p \in \{2, 3\}$, denote the phase index for the BC phases. Assuming reciprocal and stationary channels, the received signal at the nodes in the p-th phase is given by

$$\mathbf{y}_{\mathrm{nodes}_p} = \mathbf{H}^{\mathrm{T}}\mathbf{t}_{\mathrm{BC}_p} + \mathbf{n}_{\mathrm{nodes}_p}. \tag{4}$$

where $\mathbf{t}_{\mathrm{BC}_p}$ is the transmitted signal vector from the RS in the p-th phase and $\mathbf{n}_{\mathrm{nodes}_p} = (n_{1_p}, n_{2_p}, n_{3_p})^{\mathrm{T}}$ with n_{i_p} the AWGN noise at node i in p-th phase which follows $\mathcal{CN}(0, \sigma_n^2)$.

Having stationary channels within two BC phases, the RS applies STANC. In the second phase, the transmitted signal vector from the RS is given by

$$\mathbf{t}_{\mathrm{BC}_2} = (t_1, t_2)^{\mathrm{T}} \tag{5}$$

with

$$t_r = g_r y_{\mathrm{RS}_r} \tag{6}$$

and g_r the scaling factor to meet the per-antenna power constraint at the RS, P_{a}, given by

$$g_r = \sqrt{\frac{P_{\mathrm{a}}}{\mathrm{E}\{y_{\mathrm{RS}_r} y_{\mathrm{RS}_r}^*\}}}. \tag{7}$$

The received signal at node i in the second phase is given by

$$y_{i_2} = \tilde{\mathbf{h}}_i \mathbf{t}_{\mathrm{BC}_2} + n_{i_2}, \tag{8}$$

with $\tilde{\mathbf{h}}_i$ the i-th row of $\mathbf{H}^{\mathrm{T}}$.

Using Alamouti scheme, the transmitted signal vector from the RS in the third phase is given by

$$\mathbf{t}_{\mathrm{BC}_3} = (-t_2^*, t_1^*)^{\mathrm{T}}. \tag{9}$$

The received signal at node i in the third phase is given by

$$y_{i_3} = \tilde{\mathbf{h}}_i \mathbf{t}_{\mathrm{BC}_3} + n_{i_3}. \tag{10}$$

After receiving the signals from the RS in two BC phases, each node i arranges the received signals into a vector $\mathbf{y}_i = (y_{i_2}, -y_{i_3}{}^*)^{\mathrm{T}}$, which can be rewritten as

$$\mathbf{y}_i = \underbrace{\begin{bmatrix} h_{1i} & h_{2i} \\ -h_{2i}^* & h_{1i}^* \end{bmatrix}}_{\mathbf{F}_i} \mathbf{t}_{\mathrm{BC}_2} + \mathbf{n}_i \tag{11}$$

with $\mathbf{n}_i = (n_{i_2}, -n_{i_3}{}^*)^{\mathrm{T}}$. In order to detect the STANC symbols $\mathbf{t}_{\mathrm{BC}_2}$, in this work we consider a zero forcing detector as given by

$$\mathbf{z}_i = \left(\mathbf{F}_i^{\mathrm{H}}\mathbf{F}_i\right)^{-1}\mathbf{F}_i^{\mathrm{H}} \tag{12}$$

and the output of the detector is given by

$$\hat{\mathbf{t}}_{\mathrm{BC}_2} = \mathbf{t}_{\mathrm{BC}_2} + \mathbf{z}_i\mathbf{n}_i. \tag{13}$$

Afterwards, each node performs self-interference cancellation, which can be written as

$$\begin{aligned} \mathbf{u}_i &= \hat{\mathbf{t}}_{\mathrm{BC}_2} - \mathbf{s}_i + \mathbf{z}_i\mathbf{n}_i \\ &= \underbrace{\mathbf{G}\mathbf{H}_{\setminus i}\mathbf{x}_{\setminus i}}_{\text{useful signal}} + \underbrace{\begin{bmatrix} \mathbf{G} & \mathbf{z}_i \end{bmatrix}\begin{bmatrix} \mathbf{n}_{\mathrm{RS}} \\ \mathbf{n}_i \end{bmatrix}}_{\text{undesired noise}} \end{aligned} \tag{14}$$

with $\mathbf{s}_i = (g_1 h_{1i} x_i, g_2 h_{2i} x_i)^{\mathrm{T}}$, $\mathbf{G} = \left[\begin{smallmatrix} g_1 & 0 \\ 0 & g_2 \end{smallmatrix}\right]$ the RS amplification matrix, $\mathbf{H}_{\setminus i}$ the channel matrix $\mathbf{H}$ excluding its i-th column and $\mathbf{x}_{\setminus i}$ the data vector $\mathbf{x}$ excluding the data stream x_i of node i. For example, for node 1, $\mathbf{u}_1$ is given by

$$\mathbf{u}_1 = \underbrace{\begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix}}_{\mathbf{G}} \underbrace{\begin{bmatrix} h_{12} & h_{13} \\ h_{22} & h_{23} \end{bmatrix}}_{\mathbf{H}_{\setminus 1}} \underbrace{[x_2 x_3]^{\mathrm{T}}}_{\mathbf{x}_{\setminus 1}} + \begin{bmatrix} \mathbf{G} & \mathbf{z}_1 \end{bmatrix}\begin{bmatrix} \mathbf{n}_{\mathrm{RS}} \\ \mathbf{n}_1 \end{bmatrix} \tag{15}$$

C. BC Phases for Non-Stationary Channels: Repetition

The repetition transmission can be applied for non-regenerative multi-way relaying with any number N of nodes, when the RS has $R = N - 1$ antennas. Using repetition transmission, the RS amplifies the received signal in each antenna subject to a per-antenna power constraint and repeatedly transmits the amplified signals within $N - 1$ BC phases. The transmitted signal vector from the RS in all $N - 1$ BC phases is given by

$$\mathbf{d} = (d_1, \cdots, d_R)^{\mathrm{T}} \tag{16}$$

where

$$d_r = b_r y_{\mathrm{RS}_r} \tag{17}$$

and b_r is the scaling factor to meet the per-antenna power constraint at the RS, with $b_r = g_r$ and g_r given by (7).

With non-stationary channels, we mean that the channel coefficients are constant during phase p, but they are different in different phases. Let $p, p \in \{2, \cdots, N\}$, denote the BC phase index and $\mathbf{c}_{\mathrm{BC}_i}^p = \left(c_{\mathrm{BC}_{1i}}^p, \cdots, c_{\mathrm{BC}_{Ri}}^p\right)^{\mathrm{T}}$ with $c_{\mathrm{BC}_{ri}}^p$ the channel coefficients between node i and RS antenna r in the p-th phase which follows $\mathcal{CN}(0, \sigma_h^2)$.

Let in the following $\mathbf{C}_{\mathrm{BC}_i} = \left[\mathbf{c}_{\mathrm{BC}_i}^2, \cdots, \mathbf{c}_{\mathrm{BC}_i}^N\right]^{\mathrm{T}}$ denote the BC channel matrix at node i, which is the stack of $N-1$ channel vectors from $N-1$ BC phases, and $\mathbf{B} = \mathrm{diag}\,(b_1, \cdots, b_R)$ denote the RS's amplification channel matrix, with $\mathrm{diag}(\mathbf{b})$ a diagonal matrix with vector $\mathbf{b}$ in its diagonal.

The received signal at node i after $N-1$ BC phases can be written as

$$\mathbf{y}_i = \mathbf{C}_{\mathrm{BC}_i} \underbrace{(\mathbf{BHx} + \mathbf{Bn}_{\mathrm{RS}})}_{\mathbf{d}} + \mathbf{n}_i^{\mathrm{BC}}, \tag{18}$$

with $\mathbf{n}_i^{\mathrm{BC}} = \left(n_{i_2}^{\mathrm{BC}}, \cdots, n_{i_N}^{\mathrm{BC}}\right)^{\mathrm{T}}$ and $n_{i_p}^{\mathrm{BC}}$ the AWGN noise at node i in the p-th phase which follows $\mathcal{CN}(0, \sigma_n^2)$. In order to detect the RS's transmitted symbols $\mathbf{d}$, in this work we consider a zero forcing detector given by

$$\mathbf{q}_i = \left(\mathbf{C}_{\mathrm{BC}_i}^{\mathrm{H}} \mathbf{C}_{\mathrm{BC}_i}\right)^{-1} \mathbf{C}_{\mathrm{BC}_i}^{\mathrm{H}} \tag{19}$$

and the output of the detector is given by

$$\hat{\mathbf{d}} = \mathbf{d} + \mathbf{q}_i \mathbf{n}_i^{\mathrm{BC}}. \tag{20}$$

Afterwards, each node performs self-interference cancellation, which can be written as

$$\begin{aligned} \mathbf{v}_i &= \hat{\mathbf{d}} - \mathbf{w}_i + \mathbf{q}_i \mathbf{n}_i^{\mathrm{BC}} \\ &= \underbrace{\mathbf{BH}_{\backslash i}\mathbf{x}_{\backslash i}}_{\text{useful signal}} + \underbrace{\begin{bmatrix} \mathbf{B} & \mathbf{q}_i \end{bmatrix} \begin{bmatrix} \mathbf{n}_{\mathrm{RS}} \\ \mathbf{n}_i^{\mathrm{BC}} \end{bmatrix}}_{\text{undesired noise}} \end{aligned} \tag{21}$$

with $\mathbf{w}_i = (b_1 c_{1i} x_i, \cdots, b_R c_{Ri} x_i)^{\mathrm{T}}$.

III. Sum Rate

In this section, the sum rate of the proposed transmission schemes is given. It is assumed that all $x_i, \forall i$, $n_{\mathrm{RS}_r}, \forall r$, and $n_{i_p}, \forall \{i, p\}$, are statistically independent. For stationary channels, given (14), the SNR of non-regenerative multi-way relaying with STANC when jointly detecting $\mathbf{x}_{\backslash i}$ at node i is given by

$$\gamma_i^{\mathrm{STANC}} = \frac{\sigma_x^2 \|\mathbf{GH}_{\backslash i}\|^2}{\sigma_r^2 \|\mathbf{G}\|^2 + \sigma_n^2 \|\mathbf{z}_i\|^2}. \tag{22}$$

For non-stationary channels, given (21), the SNR of non-regenerative multi-way relaying with repetition when jointly detecting $\mathbf{x}_{\backslash i}$ at node i is given by

$$\gamma_i^{\mathrm{Rep}} = \frac{\sigma_x^2 \|\mathbf{BH}_{\backslash i}\|^2}{\sigma_r^2 \|\mathbf{B}\|^2 + \sigma_n^2 \|\mathbf{q}_i\|^2}. \tag{23}$$

The sum rate at node i when decoding the $N-1$ data streams of the other $N-1$ nodes is given by

$$SR_i = \log_2(\det(\mathbf{I}_{N-1} + \gamma_i)), \tag{24}$$

where γ_i can be either $\gamma_i^{\mathrm{STANC}}$ or γ_i^{Rep} depending on the transmission scheme that is used.

The overall sum rate is given by

$$SR = \frac{1}{N} \sum_i SR_i \tag{25}$$

with the factor $\frac{1}{N}$ due to the N orthogonal resources that are used for multi-way relaying.

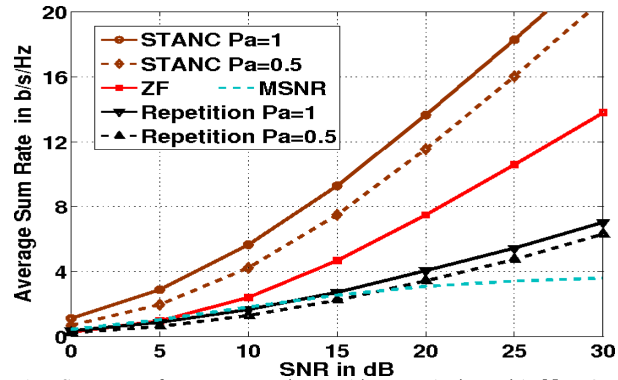

Fig. 1. Sum rate of non-regenerative multi-way relaying with $N = 3$.

IV. Simulation Results

For simulation, we have a scenario with $N = 3$ and $R = 2$ and we set $\sigma_x^2 = 1$ and $\sigma_n^2 = 1$. The SNR is defined by $\frac{\sigma_x^2}{\sigma_n^2}\sigma_h^2$. The channel gain is a realisation of i.i.d. Rayleigh channel. For both transmission schemes, we consider two different values of P_{a}, namely, 1 and 0.5. Figure 1 shows the simulation results of non-regenerative three-way relaying. We plot also the average sum rate of three-way relaying when CSI is available and the RS performs either ZF or Maximisation of SNR (MSNR) transceive beamforming while each node performs single stream detection as in [3]. For ZF and MSNR transceive beamforming as in [3], we set $R = N$ with unit transmit power at RS. When CSI is available and the RS performs transceive beamforming, the average sum rate for both stationary and non stationary cases is similar. Thus, both ZF and MSNR transceive beamforming are used for comparison to the proposed transmission schemes.

For stationary channels, STANC with joint detection outperforms both the ZF and MSNR transceive beamforming. Using ZF and MSNR, the nodes perform single stream detection. For each decoded data stream, each node sees the other data stream as interference. Using STANC, the nodes perform joint detection. Each node detects the data streams from the other nodes jointly such that all data streams are treated as useful data streams. For non-stationary channels, in medium to high SNR region, repetition transmission is able to outperform MSNR. This shows that repetition transmission with joint detection is able to manage the interference in the network better than MSNR. MSNR does not cancel the interference and thus it performs worse in interference-limited environment.

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