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Hybrid GPS/GSM Localization of Mobile Terminals using the Extended Kalman Filter

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***Abstract*—In dense urban and indoor scenarios, the Global Positioning System (GPS) often cannot provide reliable mobile terminal (MT) location estimates, due to the attenuation or complete shadowing of the satellite signals. Cellular radio network-based localization methods, however, provide MT location estimates in almost every scenario, but they do not reach the accuracy of MT location estimates provided by GPS. Thus, a promising approach is to combine measured values from the cellular radio network and GPS, which is known as hybrid localization. In this paper, an extended Kalman filter (EKF)-based hybrid localization method is proposed that combines round trip time and received signal strength measured values available from the Global System for Mobile Communication (GSM) and pseudorange (PR) measured values from GPS, in order to track the MT's movement. In contrast to existing hybrid approaches, an EKF-based MT tracking algorithm is proposed that additionally takes into account sectorized BS antennas which are typically employed in existing GSM networks, and it takes into account PR instead of geometric range (or time of arrival) measured values from GPS as GSM is normally not time-synchronized to GPS. Simulation and experimental results show that, compared to an EKF that is based only on GSM measured values, the proposed EKF that additionally incorporates PR measured values yields improved MT location estimates.**

I. Introduction

In recent years, there is an increasing interest in localization methods offering reliable mobile terminal (MT) location estimates. On the one hand, this is due to commercial services such as, e.g., navigation, location sensitive billing and other promising applications that rely on accurate MT location estimates. On the other hand, the United States Federal Communications Commission (FCC) has mandated all wireless providers to report the location of all enhanced 911 (E-911) callers with specified accuracy, which has further pushed research and standardization activities in the field of MT localization [1].

Several localization methods have been proposed to solve the problem of locating a MT [2], [3]. In global navigation satellite systems (GNSS) such as, e.g., the Global Positioning System (GPS), the MT location is determined from the propagation time of the satellite signals, which is known as time of arrival (ToA) principle. A three dimensional (3-D) MT location estimate can be found if the MT receives satellite signals from at least four different satellites, where the fourth satellite signal is required to resolve the unknown clock bias between the satellite and MT clock [4]. In indoor and dense urban scenarios, however, the number of satellites in view is often insufficient to obtain a 3-D or even 2-D MT location estimate. Cellular radio network-based localization methods are based on, e.g., the round trip delay time (RTT), time difference of arrival (TDoA) or received signal strength (RSS) principle (an overview is given in [2], [3]). Although cellular radio network-based localization methods have the advantage that the MT location estimates are almost everywhere available, they do not reach the accuracy of MT location estimates provided by the GNSS. The combination of measured values of the cellular radio network and GNSS is, thus, a promising approach to obtain MT location estimates even if less than four satellites are in view [5]–[9]. The resulting hybrid localization methods are expected to enhance the accuracy and availability of MT location estimates.

In [5], a hybrid localization method is presented that combines pseudorange (PR) measured values from GPS with RTT measured values from a cellular radio network that is synchronized to GPS time. In this approach, the RTT measured value is used to synchronize the MT clock to GPS time, so that the unknown clock bias, incorporated in every PR measured value, can be resolved. In [6], a hybrid localization method is proposed that combines PR measured values from GPS and TDOA measured values from the Global System for Mobile Communication (GSM) that is assumed to be synchronized to GPS. Both [5] and [6], only briefly describe the concept of their hybrid localization methods and no algorithms are presented. In [7], a hybrid localization method is described that combines PR measured values from GPS and TDOA measured values from a synchronized cellular radio network using a least squares approach. In [8], a hybrid localization method is investigated in terms of the Cramér-Rao lower bound that combines RTT and RSS measured values from GSM and ToA measured values from GPS, where the receiver clock bias is assumed to be known. An extended Kalman filter (EKF)-based MT tracking algorithm that combines RTT and RSS measured values from GSM is introduced in [10].

In this paper, an EKF-based hybrid localization method is proposed that combines RTT and RSS measured values from GSM and PR measured values from GPS. Here, it is assumed that the GSM network is not time-synchronized to GPS, because a synchronization of the GSM network to GPS would lead to a substantial cost and implementation effort for the operators. As a result, the GSM network cannot provide GPS system time to resolve the unknown receiver clock bias included in the PR measured values. I.e., only the GPS PR measured values provide information about the unknown

receiver clock bias. In addition, it is assumed that information about sectorized base station (BS) antennas is a-priori available that can be incorporated into the EKF. Simulation and experimental results show that the proposed hybrid localization method yields improved MT location estimates compared to conventional GSM-based localization methods.
The remainder of this paper is organized as follows: In Section II, the statistical models for the PR, RTT and RSS measured values that are required for the EKF are presented. In Section III, the EKF for the hybrid localization method is introduced. In Section IV and V, simulation and experimental results are shown that demonstrate the advantage of the proposed hybrid localization method over existing methods. Section VI concludes the work.

II. Statistical models of measured values

A. Introduction

In the following, the statistical models of the measured values that can be obtained from the GNSS, namely PR, and from the cellular radio network, namely RTT and RSS, are presented. The BS locations $\mathbf{x}_{bs}^{(n)}=[x_{bs}^{(n)}, y_{bs}^{(n)}]^\mathsf{T}$, $n=1,...,N_{bs}$, as well as the MT location $\mathbf{x}_{ms}=[x_{ms}, y_{ms}]^\mathsf{T}$ to be estimated, are assumed to lie in the xy-plane, where $[\cdot]^\mathsf{T}$ denotes the transpose of a vector or matrix. The satellite locations are given by $\mathbf{x}_{sat}^{(l)}=[x_{sat}^{(l)}, y_{sat}^{(l)}, z_{sat}^{(l)}]^\mathsf{T}$, $l=1,...,N_{sat}$. For the case of 3-D BS and MT locations and, thus, an estimation of the 3-D MT location vector, the statistical models can be obtained in a similar way.

B. Pseudorange

The GNSS-based localization methods utilize the concept of ToA, i.e., the MT is measuring the time the satellite signal requires to travel from the satellite to the MT [4]. In general, the MT's clock is not synchronized to the clocks of the GPS satellites. The satellite clocks, however, can be assumed to be mutually synchronized, so that for each time step k the unknown receiver clock bias $\delta t(k)$ is equal for the ToA measured values from the different satellites [4]. Let $\mathbf{m}_{pr}(k)$ denote the vector of N_{sat} PR measured values that can be obtained from multiplying the biased ToA measured values by the speed of light c_0. Then, the statistical model of the PR measured values is given by

$$\mathbf{m}_{pr}(k)=\mathbf{h}_{pr}(\mathbf{x}_{ms}(k),\delta t(k))+\mathbf{v}_{pr}(k), \tag{1}$$

with

$$\begin{aligned}\mathbf{h}_{pr}(\mathbf{x}_{ms}(k),\delta t(k)) \quad = \quad & \Big[h_{pr}^{(1)}(\mathbf{x}_{ms}(k),\delta t(k)),\cdots, \\ & h_{pr}^{(N_{sat})}(\mathbf{x}_{ms}(k),\delta t(k))\Big]^\mathsf{T}, \end{aligned} \tag{2}$$

where $h_{pr}^{(l)}(\mathbf{x}_{ms}(k),\delta t(k)) = d_{sat}^{(l)}(\mathbf{x}_{ms}(k))+c_0\cdot\delta t(k)$ and $d_{sat}^{(l)}(\mathbf{x}_{ms}(k))$ denotes the Euclidean distance between the MT and the l-th satellite. The random variable $\mathbf{v}_{pr}(k)$ describes the errors each PR measured value is affected by, such as, delays as the signal propagates through the atmosphere, receiver noise as well as errors due to changing propagation conditions, i.e., line-of-sight (LOS) or non-line-of-sight (NLOS) situation. It is assumed that $\mathbf{v}_{pr}(k)$ is Gaussian distributed with mean vector $\boldsymbol{\mu}_{pr}=[\mu_{pr}^{(1)},\cdots,\mu_{pr}^{(N_{sat})}]^\mathsf{T}$ accounting for NLOS propagation and covariance matrix $\mathbf{R}_{pr}=\mathrm{diag}((\sigma_{pr}^{(1)})^2,\cdots,(\sigma_{pr}^{(N_{sat})})^2)$, where $\mu_{pr}^{(l)}$ and $\sigma_{pr}^{(l)}$ denote the mean and standard deviation of the PR measured value from the l-th satellite.

C. Round Trip Delay Time

In cellular radio networks, the RTT is the time the radio signal requires to travel from the BS to the MT and back. Let $\mathbf{m}_{rtt}(k)$ denote the vector of N_{bs} RTT measured values. Then, the statistical model for the RTT measured values is given by

$$\mathbf{m}_{rtt}(k)=\mathbf{h}_{rtt}(\mathbf{x}_{ms}(k))+\mathbf{v}_{rtt}(k), \tag{3}$$

with

$$\mathbf{h}_{rtt}(\mathbf{x}_{ms}(k))=[h_{rtt}^{(1)}(\mathbf{x}_{ms}(k)),\cdots,h_{rtt}^{(N_{bs})}(\mathbf{x}_{ms}(k))]^\mathsf{T}, \tag{4}$$

where $h_{rtt}^{(n)}(\mathbf{x}_{ms}(k))=2\cdot d_{bs}^{(n)}(\mathbf{x}_{ms}(k))/c_0$ and $d_{bs}^{(n)}(\mathbf{x}_{ms}(k))$ denotes the Euclidean distance between the MT and the n-th BS. The random variable $\mathbf{v}_{rtt}(k)$ models the errors due to quantization, changing propagation conditions - LOS or NLOS situation - and due to measurement noise. These errors are assumed to be Gaussian distributed with mean vector $\boldsymbol{\mu}_{rtt}=[\mu_{rtt}^{(1)},\cdots,\mu_{rtt}^{(N_{bs})}]^\mathsf{T}$ accounting for NLOS propagation and covariance matrix $\mathbf{R}_{rtt}=\mathrm{diag}((\sigma_{rtt}^{(1)})^2,\cdots,(\sigma_{rtt}^{(N_{bs})})^2)$, where $\mu_{rtt}^{(n)}$ and $\sigma_{rtt}^{(n)}$ denote the mean and standard deviation of the RTT measured value from the n-th BS [11], [12].

D. Received Signal Strength

In cellular radio networks, the RSS value is an averaged value of the strength of a radio signal received by the MT. The attenuation of the signal strength through a mobile radio channel is caused by three factors, namely path loss, fast and slow fading. The errors due to fast fading are removed as the RSS measured values are averaged over several time-consecutive measurements. The path loss $PL^{(n)}(\mathbf{x}_{ms}(k)) \triangleq \tilde{PL}^{(n)}$ in dB is given by

$$\tilde{PL}^{(n)}=A^{(n)}+10\cdot B^{(n)}\cdot\log_{10}\left(\frac{d_{bs}^{(n)}(\mathbf{x}_{ms}(k))}{\mathrm{km}}\right) \tag{5}$$

[2], where $A^{(n)}$ denote the reference path loss at a BS to MT distance of $1\,\mathrm{km}$ and $B^{(n)}$ the path loss exponent of the n-th BS. Both parameters $A^{(n)}$ and $B^{(n)}$ strongly depend on the BS antenna settings and the propagation conditions and can be determined empirically or from well known path loss models as, e.g., COST 231 Walfisch-Ikegami [10].
In a real system, the BS may be equipped with directional antennas in order to enhance the cell's capacity. In the following, it is assumed that normalized 2-D antenna gain models are a-priori available. Let $\varphi_{bs}^{(n)}(\mathbf{x}_{ms}(k)) \triangleq \tilde{\varphi}^{(n)}$ denote the azimuth angle between the MT and the n-th BS, counted counterclockwise from the positive x-axis. Let further $\varphi_0^{(n)}$ denote the BS antenna's boresight direction, $\varphi_{3dB}^{(n)}$ the half-power beamwidth of the BS antenna in degrees and $A_m^{(n)}$

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the minimum normalized antenna gain. Then, a model for the normalized antenna gain in dB scale is given by

$$g(\tilde{\varphi}^{(n)})=\begin{cases} -12\left(\frac{\tilde{\varphi}^{(n)}-\varphi_0^{(n)}}{\varphi_{3dB}^{(n)}}\right)^2, & \left|\frac{\tilde{\varphi}^{(n)}-\varphi_0^{(n)}}{\varphi_{3dB}^{(n)}}\right|\leq\sqrt{\frac{A_m^{(n)}}{12}} \\ -A_m^{(n)}, & \text{else,} \end{cases} \tag{6}$$

[8]. Let $\mathbf{m}_{rss}(k)$ denote the vector of N_{bs} RSS measured values. Then, the statistical model of the RSS measured values in dB scale is given by

$$\mathbf{m}_{rss}(k) = \mathbf{h}_{rss}(\mathbf{x}_{ms}(k)) + \mathbf{v}_{rss}(k) \tag{7}$$

with

$$\mathbf{h}_{rss}(\mathbf{x}_{ms}(k))=[h_{rss}^{(1)}(\mathbf{x}_{ms}(k)),\cdots,h_{rss}^{(N_{bs})}(\mathbf{x}_{ms}(k))]^\mathsf{T} \tag{8}$$

where $h_{rss}^{(n)}(\mathbf{x}_{ms}(k)) = P_t^{(n)} - \{\tilde{PL}^{(n)} - g(\tilde{\varphi}^{(n)})\}$ and $P_t^{(n)}$ denotes the n-th BS's equivalent isotropic radiated power. The random variable $\mathbf{v}_{rss}(k)$ describes the errors in dB due to quantization and slow fading which is assumed to be zero-mean Gaussian distributed with covariance matrix $\mathbf{R}_{rss} = \mathrm{diag}((\sigma_{rss}^{(1)})^2,\cdots,(\sigma_{rss}^{(N_{bs})})^2)$, where $\sigma_{rss}^{(n)}$ denotes the standard deviation of the RSS measured value from the n-th BS antenna [2].

III. The Extended Kalman Filter for Hybrid Localization

After having determined the non-linear relationship between the MT location and the PR, RTT and RSS measured values, the problem is how to efficiently estimate the MT location from these measured values. A solution to this problem is provided by the discrete extended Kalman Filter (EKF) that addresses the problem of recursively estimating the state of a discrete-time controlled process from measured values that provide information about the state [13]. In the following, the process model and the measurement model for the hybrid localization method are introduced that are required to utilize the EKF.

For the problem of estimating the MT location with the hybrid localization method, the states of the process model include the MT location, the MT velocity, receiver clock bias and drift, i.e., $\mathbf{x}(k) = [x_{ms}(k), \dot{x}_{ms}(k), y_{ms}(k), \dot{y}_{ms}(k), c_0\cdot\delta t(k), c_0\cdot\delta\dot{t}(k)]^\mathsf{T}$. The movement of the MT is assumed to follow a constant velocity (CV) model, with an appropriate level of process noise to deal with possible MT maneuvers [14]. The receiver clock bias is modeled by a second-order Markov process whose input is white noise as described in [14], [15]. The corresponding process model for the hybrid localization method is, thus, given by

$$\mathbf{x}(k+1) = \mathbf{\Phi}\cdot\mathbf{x}(k) + \mathbf{\Gamma}\cdot\mathbf{w}(k), \tag{9}$$

with

$$\mathbf{\Phi} = \mathbf{I}_3\otimes\begin{bmatrix} 1 & T_s \\ 0 & 1\end{bmatrix}, \mathbf{\Gamma} = \mathrm{diag}(\mathbf{I}_2\otimes[T_s^2/2, T_s]^\mathsf{T}, \mathbf{I}_2\cdot c_0), \tag{10}$$

where $\mathbf{I}_q$ is the identity matrix of size q, $\otimes$ the Kronecker product and T_s is the sampling time. The process noise $\mathbf{w}(k)=[w_{a_x}(k), w_{a_y}(k), w_{\delta t}(k), w_{\delta\dot{t}}(k)]^\mathsf{T}$ is assumed to be a zero-mean white Gaussian noise sequence with block diagonal covariance matrix $\mathbf{Q} = \mathrm{diag}(\mathbf{Q}_{cv}, \mathbf{Q}_{\delta t})$. $\mathbf{Q}_{cv}$ is given by $\mathbf{Q}_{cv} = \mathrm{diag}(\sigma_{a_x}^2, \sigma_{a_y}^2)$, where $\sigma_{a_x}^2$ and $\sigma_{a_y}^2$ are the noise variances in the x- and y-direction. $\mathbf{Q}_{\delta t}$ is a symmetric matrix of size 2×2 whose elements are given by

$$\begin{aligned} Q_{11} &= \frac{h_0}{2}T_s + 2h_{-1}T_s^2 + \frac{2}{3}\pi^2 h_{-2}T_s^3, \\ Q_{12} &= 2h_{-1}T_s + \pi^2 h_{-2}T_s^3, \\ Q_{22} &= \frac{h_0}{2T_s} + 2h_{-1} + \frac{8}{3}\pi^2 h_{-2}T_s, \end{aligned} \tag{11}$$

where the parameters $h_0 = 9.4\cdot 10^{-20}$, $h_{-1} = 1.8\cdot 10^{-19}$ and $h_{-2} = 3.8\cdot 10^{-21}$ correspond to values of a typical quartz standard [15].

In the following, the PR, RTT and RSS measured values are used to estimate the unknown state vector $\mathbf{x}(k)$. These measured values can be concatenated into a measurement vector $\mathbf{m}(k) = \left[\mathbf{m}_{pr}^\mathsf{T}(k), \mathbf{m}_{rtt}^\mathsf{T}(k), \mathbf{m}_{rss}^\mathsf{T}(k)\right]^\mathsf{T}$. The corresponding measurement model for the hybrid localization method is, thus, given by

$$\mathbf{m}(k) = \mathbf{h}(\mathbf{x}(k)) + \mathbf{v}(k), \tag{12}$$

where $\mathbf{h}(\mathbf{x}(k)) = \left[\mathbf{h}_{pr}^\mathsf{T}(\mathbf{x}(k)), \mathbf{h}_{rtt}^\mathsf{T}(\mathbf{x}(k)), \mathbf{h}_{rss}^\mathsf{T}(\mathbf{x}(k))\right]^\mathsf{T}$ and $\mathbf{v}(k) = \left[\mathbf{v}_{pr}^\mathsf{T}(k), \mathbf{v}_{rtt}^\mathsf{T}(k), \mathbf{v}_{rss}^\mathsf{T}(k)\right]^\mathsf{T}$. The random variable $\mathbf{v}(k)$ is Gaussian distributed with mean vector $\boldsymbol{\mu} = [\boldsymbol{\mu}_{pr}^\mathsf{T}, \boldsymbol{\mu}_{rtt}^\mathsf{T}, \mathbf{0}]^\mathsf{T}$, where $\mathbf{0}$ denotes the zero vector of size $1\times N_{bs}$, and block diagonal covariance matrix $\mathbf{R} = \mathrm{diag}(\mathbf{R}_{pr}, \mathbf{R}_{rtt}, \mathbf{R}_{rss})$.

After having described the process model, cf. (9), and the measurement model, cf. (12), for the hybrid localization method, the EKF can be utilized to estimate the unknown state vector. The well-known EKF equations, adopted to the proposed hybrid localization method, are summarized in Table I.

TABLE I
EXTENDED KALMAN FILTER EQUATIONS FOR HYBRID LOCALIZATION

1. Initialization

$$\hat{\mathbf{x}}(0|0) = E\{\mathbf{x}(0)\}$$
$$\mathbf{P}(0|0) = E\{(\mathbf{x}(0)-\hat{\mathbf{x}}(0|0))(\mathbf{x}(0)-\hat{\mathbf{x}}(0|0))^\mathsf{T}\}$$

2. For $k = 1, 2, ...$

(a) Time Update Equations

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{\Phi}\hat{\mathbf{x}}(k-1|k-1)$$
$$\mathbf{P}(k|k-1) = \mathbf{\Phi}\mathbf{P}(k-1|k-1)\mathbf{\Phi}^\mathsf{T} + \mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^\mathsf{T}$$

(b) Measurement Update Equations

$$\begin{aligned} \mathbf{e}(k) &= \mathbf{m}(k) - \boldsymbol{\mu} - \mathbf{h}(\hat{\mathbf{x}}(k|k-1)) \\ \mathbf{S}(k) &= \mathbf{H}(k)\mathbf{P}(k|k-1)\mathbf{H}^\mathsf{T}(k) + \mathbf{R} \\ \mathbf{H}(k) &= \left.\frac{\partial\mathbf{h}(\mathbf{x}(k))}{\partial\mathbf{x}(k)}\right|_{\mathbf{x}(k)=\hat{\mathbf{x}}(k|k-1)} \\ \mathbf{K}(k) &= \mathbf{P}(k|k-1)\mathbf{H}^\mathsf{T}(k)\mathbf{S}^{-1}(k) \\ \hat{\mathbf{x}}(k|k) &= \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)\mathbf{e}(k) \\ \mathbf{P}(k|k) &= \mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{H}(k)\mathbf{P}(k|k-1) \end{aligned}$$

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IV. Simulation Scenario and Results

In the following, the simulation scenario is explained. It is assumed that a car is equipped with a MT that is capable of providing PR measured values from GPS and RTT and RSS measured values from GSM. The car moves clockwise on a trapezoidal route, divided into 4 sections, in an urban scenario of size $5\,\mathrm{km} \times 5\,\mathrm{km}$ as it is shown in Fig. 1. In each section, the car moves with a different velocity, in order to reflect a more realistic car movement as depicted in Fig. 2. The cellular radio network is composed of $N_{bs} = 12$ BSs, with known BS locations, where each BS is equipped with three directional antennas as shown in Fig. 1. The GPS satellite locations are assumed to be known since they can be calculated from the satellite's navigation message. The simulation parameters are given in Table II and are assumed to be equal for all BSs and all satellites for the sake of simplicity. The following combinations of measured values are investigated:

- One RTT and one RSS measured value from the serving BS antenna and one RSS measured value from each of six neighboring BS antennas (GSM method),
- Measured values of GSM method and, in addition, one PR measured value from one GPS satellite (Hybrid 1 method),
- Measured values of GSM method and, in addition, two PR measured values from two different GPS satellites (Hybrid 2 method).
- Measured values of GSM method and, in addition, three PR measured values from three different GPS satellites (Hybrid 3 method).
- Three PR measured values from three different GPS satellites (GPS 3 method)

For simplicity, it is assumed that the PR, RTT and RSS measured values are updated every $T_s = 480\,\mathrm{ms}$, which corresponds to the reporting period of measured values in GSM networks. The initial state vector $\hat{\mathbf{x}}(0|0)$ and error covariance matrix $\mathbf{P}(0|0)$ are determined from a priori information. It is assumed that the MT location and velocity is uniformly distributed in the interval $0\,\mathrm{m}$ and $5\,\mathrm{km}$ and $0\,\mathrm{m/s}$ and $50\,\mathrm{m/s}$ in both x- and y- directions. In [4], it is stated that the clock bias $\delta t(k)$ can be maintained within $\pm 1\,\mathrm{ms}$ of GPS time. Thus, it is assumed that the corresponding bias $c_0 \cdot \delta t(k)$ is uniformly distributed between $\pm 300\,\mathrm{km}$. The drift $c_0 \cdot \delta \dot{t}(k)$ is assumed to be uniformly distributed between $\pm 10\,\mathrm{m}$. Consequently, the initial state vector and error covariance matrix are set to $\hat{\mathbf{x}}(0|0) = [2.5\,\mathrm{km}, 25\,\mathrm{m/s}, 2.5\,\mathrm{km}, 25\,\mathrm{m/s}, 0\,\mathrm{m}, 0\,\mathrm{m}]^\mathsf{T}$ and $\mathbf{P}(0|0) = \mathrm{diag}(\,(5\,\mathrm{km})^2/12, (50\,\mathrm{m/s})^2/12, (5\,\mathrm{km})^2/12, (50\,\mathrm{m/s})^2/12, (300\,\mathrm{km})^2/3, (10\,\mathrm{m})^2/3)$. The localization accuracy of the proposed EKF-based hybrid localization method is evaluated in terms of the position root mean square error (RMSE) that can be determined from N_{mc} Monte Carlo trials [10]. In the simulations, $N_{mc} = 100$ is chosen.

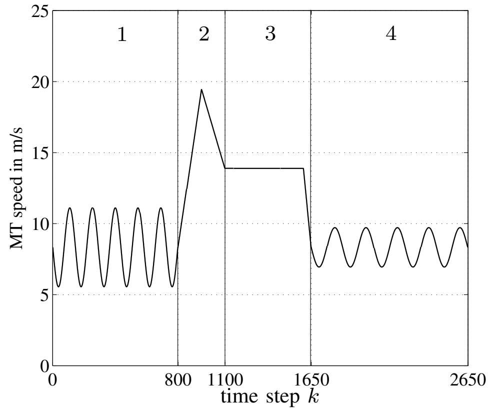

Fig. 2. MT speed vs. time step k for the simulation scenario.

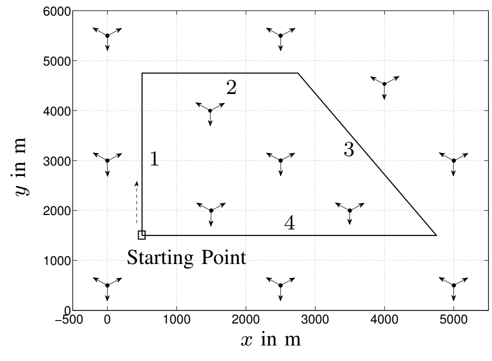

Fig. 1. Simulation scenario with $N_{\mathrm{BS}} = 12$ BSs ($\bullet$). Each BS is equipped with three directional antennas. The arrows ($\rightarrow$) indicate the BS antenna boresight direction. The MT starts at $\mathbf{x}_{ms}(0) = [500, 1500]^\mathsf{T}$ and moves clockwise on the trapezoidal route.

In Fig. 3(a), the simulation results for the RMSE in dependence of the time step k for the proposed GSM, Hybrid 1 and Hybrid 2 method are shown. The GSM method provides the worst results in terms of RMSE, which can be explained by the fact that the RSS and RTT measured values cannot provide the same level of accuracy than the receiver clock bias corrected GPS measured values. The Hybrid 1 and Hybrid 2 method additionally take into account one or two PR measured values from GPS, resulting in a further improvement of the RMSE. The improvement is marginal for the Hybrid 1 method and significant for the Hybrid 2 method. In contrast to the GSM method, the Hybrid 1 and Hybrid 2

TABLE II
Simulation Parameters

Parameter	Value	Parameter	Value
A in dB	132.8	σ_{a_x} in $\mathrm{m/s^2}$	1
B in dB	3.8	σ_{a_y} in $\mathrm{m/s^2}$	1
σ_{rss} in dB	6	μ_{rtt} in μs	0
A_m in dB	20	σ_{rtt} in μs	1
φ_{3dB} in $^\circ$	70	μ_{pr} in m	0
P_t in dBm	50	σ_{pr} in m	15

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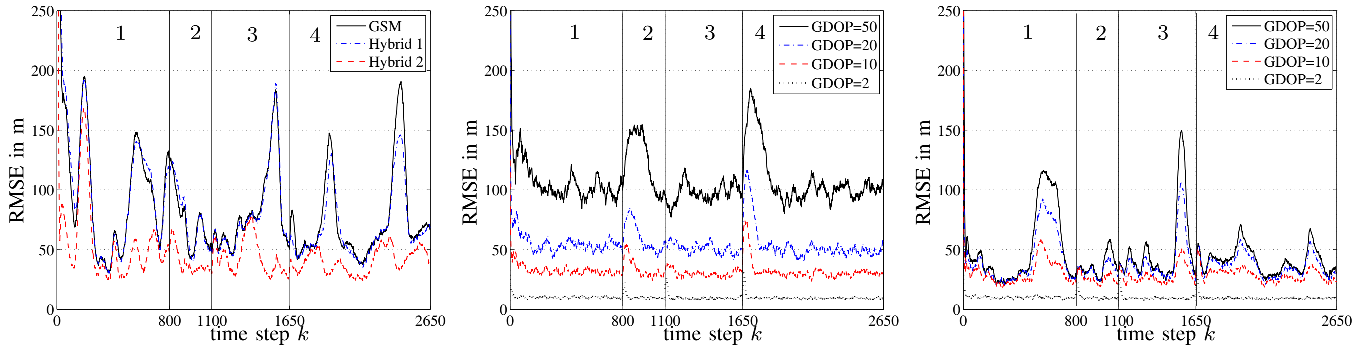

(a) (b) (c)

Fig. 3. Position RMSE vs. time step k for (a) GSM, Hybrid 1 and Hybrid 2 method, (b) GPS 3 method for different GDOP values, (c) Hybrid 3 method for different GDOP values.

method additionally have to estimate the unknown clock bias and drift states of the EKF. Thus, for the Hybrid 1 method only one PR measured value is available to estimate the clock bias and drift states of the EKF, resulting in a marginal improvement of the RMSE. For the Hybrid 2 method, the RMSE is further improved as two PR measured values are available to estimate the clock bias and drift states of the EKF. The variations in the localization accuracy for the three methods can be explained by the fact that the achievable localization accuracy strongly depends on the geometric constellation of the BSs and satellites relative to the MT location.

The impact of the geometric constellation of the satellites relative to the MT location on the achievable localization accuracy can be expressed by the well-known geometric dilution of precision (GDOP) value for 2-D scenarios [16]. Large GDOP values indicate a bad satellite to MT geometry and thus a poor localization accuracy, whereas small GDOP values imply a high accuracy. In Figs. 3(b) and 3(c), the RMSE versus time step k for different average GDOP values for the GPS 3 and Hybrid 3 method is shown. Note that the GDOP is defined and calculated only for the GPS 3 method and then the performance is compared to the Hybrid 3 method. From both figures it can be clearly seen that, the smaller the GDOP, the better the corresponding localization accuracy. It can be also seen that for small GDOP values the performance of the GPS and Hybrid 3 method is almost equal, whereas for large GDOP values, e.g. in situations where the MT is located in an urban street canyon, the Hybrid 3 method outperforms the GPS 3 method. In Fig. 3(b), it can be observed that at $k = 800$ and $k = 1650$ the RMSE increases which can be explained by the abrupt change of the MT's direction and velocity. For the Hybrid 3 method, however, this effect becomes noticeable only for small GDOP values. For large GDOP values, the BS to MT geometry dominates the achievable localization accuracy.

V. Experimental Results

In this section, the performance of the proposed hybrid localization method is verified with experimental data available from a field trial. The field trial was conducted in an operating GSM network in the city center of a German city, with a test area of approximately $3\,\text{km} \times 3\,\text{km}$. During the field trial, every $T_s = 480\,\text{ms}$ a car equipped with a standard cellular phone collected RXLEV (quantized RSS) and TA (quantized RTT) measured values from GSM. Note that in GSM the RXLEV measured values are available from the serving BS and between one and six strongest RXLEVs from the neighboring BS, whereas the TA measured value is only available from the serving BS. The investigated GSM network is composed of $N_{bs} = 13$ fixed BSs with known locations. The BSs are either equipped with a single omni-directional antenna or with directional antennas. The antenna boresight directions, half-power beamwidths and equivalent isotropic radiated powers are known, and the unknown antenna gain patterns are approximated with (6). The remaining parameters of the RTT and RSS statistical models, cf. (3) and (7), are estimated from the available field trial data. Note that in order to not overfit the EKF for this single trajectory the standard deviations of the RTT and RSS measured values were chosen to be $\sigma_{rss}^{(n)} \geq 3\,\text{dB}$ and $\sigma_{rtt}^{(n)} \geq 1\,\mu\text{s}$. For the path loss model, cf. (5), the parameters are in the range of $110\,\text{dB} \leq A^{(n)} \leq 150\,\text{dB}$ and $2\,\text{dB} \leq B^{(n)} \leq 5\,\text{dB}$.

For the GPS network, PR measured values collected from a field trial are not available, so that synthetic PR measurement data have been generated with the parameters given in Table II. For simplicity, it is assumed that PR measured values are available every $T_s = 480\,\text{ms}$. The constellation of the GPS satellites during the field trial is reconstructed by taking true satellite locations from the real satellite constellation, as it can be provided from GPS almanac data. The visibility status of the satellites during the field trial cannot be reproduced subsequently, so that it is assumed that either $N_{sat} = 1$ or $N_{sat} = 2$ satellites are visible. Note that this assumption is only made in order to demonstrate the improvements that can be achieved by the proposed hybrid localization algorithm. In reality, however, the number of visible satellites changes with time, so that there will be points in time where GPS (i.e. $N_{sat} \geq 3$) is available.

In Fig. 4, the true MT trajectory and the trajectories estimated with the EKF of the GSM, Hybrid 1 and Hybrid 2 method for the field trial are shown. The true location of the MT in the field trial was obtained from detailed maps and from GPS, where GPS was available. From Fig. 4 it can be clearly seen that all three methods are able to moderately track the MT. In order to compare the three methods with each other,

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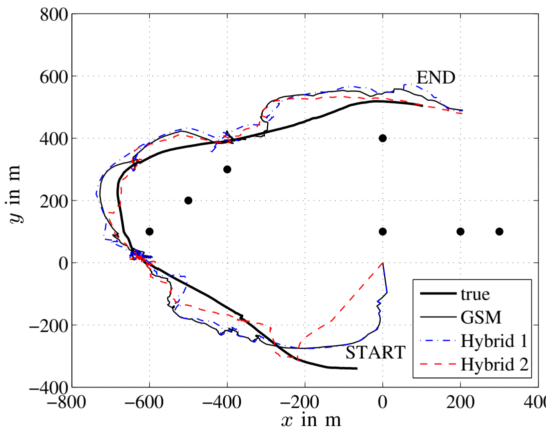

Fig. 4. Field trial scenario with true MT trajectory, approximate BS locations (●) and estimated trajectories from the GSM, Hybrid 1 and Hybrid 2 method.

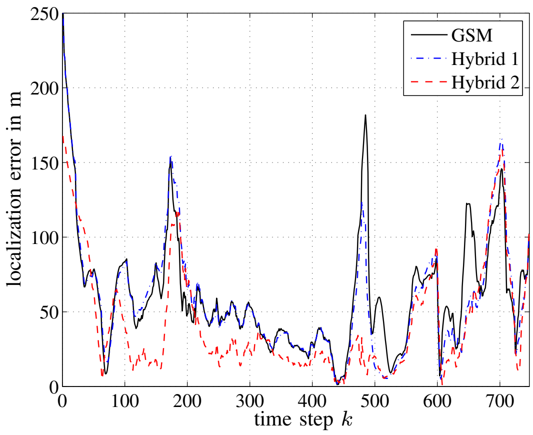

Fig. 5. Localization error vs. time step k for the GSM, Hybrid 1 and Hybrid 2 method of the field trial.

the localization error is introduced which is the Euclidean distance between the true and estimated MT location. In Fig. 5, the localization error in dependence of the time step k is shown for the GSM, Hybrid 1 and Hybrid 2 method. It can be seen that the GSM method provides the worst performance. The localization accuracy can be marginally improved with the Hybrid 1 method and significantly improved with the Hybrid 2 method. The peak values in the localization error can be explained by the geometric constellation of the BSs and satellites relative to the MT location and the change of the MT velocity during the field trial.

VI. Conclusion

In this paper, an EKF-based hybrid localization method is proposed that combines RSS and RTT measured values from GSM and PR measured values from GPS. It has been shown by simulations and experiments that, compared to combining only RSS and RTT measured values from GSM, the localization accuracy can be improved by additionally taking into account one or two PR measured values from GPS. In scenarios where three PR measured values are available and the corresponding GDOP value is low, it has been shown that it is sufficient to combine only the satellite measurements. However, in scenarios with high GDOP values, as e.g. when a MT is located in an urban street canyon, the proposed hybrid localization method should be used.

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