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# On the Performance of Hybrid GPS/GSM Mobile Terminal Tracking

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*Abstract*—**The Global Positioning System (GPS) has become one of the state-of-the-art location systems that offers reliable mobile terminal (MT) location estimates. However, there exist situations where GPS is not available, e.g., when the MT is used indoors or when the MT is located close to high buildings. In these scenarios, a promising approach is to combine the GPS measured values with measured values from the Global System for Mobile Communication (GSM), which is known as hybrid localization method. In this paper, a hybrid MT tracking algorithm based on a Rao-Blackwellized Unscented Kalman filter (RBUKF) is proposed that combines pseudoranges from GPS with timing advance and received signal strengths from GSM. Simulation results show that the proposed hybrid method outperforms the GSM method. Furthermore, the performance of the RBUKF is compared to the extended Kalman filter and the corresponding posterior Cramér-Rao lower bounds.**

## I. Introduction

Wireless location systems offering reliable mobile terminal (MT) location estimates have become an important field for researchers and engineers over the past few years. The applications arise from emergency services and commercial services, such as intelligent transport systems, location sensitive billing and others, that rely on accurate MT estimates [1].

Several localization methods have been proposed to solve the problem of locating a MT [2]. The Global Positioning System (GPS) has become one of the state-of-the-art location system that offers precise MT location estimates [3]. In GPS, the MT location is determined from the propagation time of the satellite signals, which is known as time of arrival (ToA) method. If the MT receives satellite signals from at least four different satellites, a three-dimensional (3-D) MT location estimate can be found, where the fourth satellite signal is needed to resolve the unknown bias between the MT and satellite clock [3]. Similarly, a 2-D MT location estimate is obtained from signals of at least three different satellites. However, there exist situations where GPS is not available, e.g., when the MT is located inside a building or is surrounded by high buildings. In these scenarios, the number of satellites in view is often not sufficient to obtain a 3-D or even 2-D MT location estimate.

In the Global System for Mobile Communication (GSM), measurements such as the received signal strength (RSS), timing advance (TA), angle of arrival (AoA) or enhanced observed time difference (E-OTD) exist that give information on the MT location. Although these measurements cannot provide the same accuracy as GPS measurements, GSM measurements have the advantage that they are almost everywhere available. The fusion of measured values of GPS and GSM is, thus, a promising approach in order to obtain MT location estimates even if less than four or three satellites are in view [4]–[9].

In [4] and [5], a hybrid localization method combining pseudorange (PR) measured values from GPS and E-OTD measured values from GSM is investigated. In [6], a hybrid method is presented that is based on the fusion of PR measured values from GPS and round trip delay measured values from a cellular radio network that is synchronized to GPS time. However, [4]–[6], only provide general descriptions of their hybrid methods and no algorithms or theoretical performance bounds are given. In [7], a hybrid method based on the combination of PR measured values from GPS and time difference of arrival measured values from a cellular radio network using a least squares approach is introduced. In [9], we have developed an extended Kalman filter (EKF)-based MT tracking algorithm that is based on the fusion of TA and RSS measured values from GSM and PR measured values from GPS.

This paper is focussed on the combination of RSS, TA and PR measured values from GSM and GPS, as they can be easily obtained from off-the-shelf mobile handsets and conventional GPS receivers. In contrast to [9], a Rao-Blackwellized Unscented Kalman filter (RBUKF) instead of an EKF is used for the hybrid localization problem, in order to overcome the shortcomings of the EKF [10], [11]. The achievable performance of the RBUKF is then compared to the EKF and the posterior Cramér-Rao lower bound (PCRLB) by means of simulations. The PCRLB gives the theoretical best achievable performance of nonlinear filters [12] and serves here as an important tool for the design of a hybrid MT tracking system.

The rest of this paper is organized as follows: The statistical models for the RSS, TA and PR measured values are reviewed in Section II. The proposed RBUKF-based hybrid localization method and the corresponding PCRLBs are determined in Section III. The performance of the RBUKF based on simulations is evaluated in Section IV. Finally, conclusions are drawn in Section V.

## II. Statistical models of measured values

### A. Introduction

The statistical models for the measured values available from GPS and GSM are reviewed in the following [9]. The MT location $\mathbf{x}_{ms} = [x_{ms}, y_{ms}]^\mathsf{T}$ to be estimated and the known base station (BS) locations $\mathbf{x}_{bs}^{(n)} = [x_{bs}^{(n)}, y_{bs}^{(n)}]^\mathsf{T}$, $n = 1, ..., N_{bs}$

are assumed to lie in the $xy$-plane, where $[\cdot]^\mathsf{T}$ denotes the transpose of a vector or matrix. The known satellite locations are given by $\mathbf{x}_{sat}^{(l)} = [x_{sat}^{(l)}, y_{sat}^{(l)}, z_{sat}^{(l)}]^\mathsf{T}$, $l = 1, ..., N_{sat}$. The statistical models for 3-D BS and MT locations can be obtained in a similar way.

### B. Received Signal Strength

In GSM, the RSS value is an averaged value of the strength of a radio signal received by the MT. The attenuation of the signal strength through a mobile radio channel is caused by three factors, namely fast fading, slow fading and path loss. As the RSS measured values are averaged over several time-consecutive measurements, the error due to fast fading can be neglected. The path loss $L^{(n)}(\mathbf{x}_{ms}(k)) \triangleq \tilde{L}^{(n)}$ in dB at time index $k$ is given by the well known formula

$$\tilde{L}^{(n)} = A^{(n)} + 10 \cdot B^{(n)} \cdot \log_{10}\left(d_{bs}^{(n)}(\mathbf{x}_{ms}(k))/\mathrm{km}\right) \quad (1)$$

[2], where $d_{bs}^{(n)}(\mathbf{x}_{ms}(k))$ denotes the Euclidean distance between the MT and the $n$-th BS and $A^{(n)}$, $B^{(n)}$ are model parameters that can be determined empirically or from well known path loss models as, e.g., COST 231 Walfisch-Ikegami [13].

As done in [8], it is assumed that antenna gain models are a-priori available. Let $\varphi_{bs}^{(n)}(\mathbf{x}_{ms}(k)) \triangleq \tilde{\varphi}^{(n)}$ denote the azimuth angle between the MT and the $n$-th BS antenna, counted counterclockwise from the boresight direction of the BS antenna. Let further $A_m^{(n)}$ and $\varphi_{3dB}^{(n)}$ denote the minimum gain and 3 dB beamwidth of the BS antenna. Then, a model for the normalized antenna gain in dB scale is given by

$$g(\tilde{\varphi}^{(n)}) = -\min\left\{12\left(\tilde{\varphi}^{(n)}/\varphi_{3dB}^{(n)}\right)^2, A_m^{(n)}\right\} \quad (2)$$

[8], where $\min\{a, b\}$ denotes the smallest value of the set $\{a, b\}$. Let $\mathbf{y}_{rss}(k)$ denote the vector of $N_{bs}$ RSS measured values. Then, the statistical model of the RSS measured values in dB scale is given by

$$\mathbf{y}_{rss}(k) = \mathbf{h}_{rss}(\mathbf{x}_{ms}(k)) + \mathbf{v}_{rss}(k), \quad (3)$$

with $\mathbf{h}_{rss}(\mathbf{x}_{ms}(k)) = [h_{rss}^{(1)}(\mathbf{x}_{ms}(k)), \cdots, h_{rss}^{(N_{bs})}(\mathbf{x}_{ms}(k))]^\mathsf{T}$, where $h_{rss}^{(n)}(\mathbf{x}_{ms}(k)) = P_t^{(n)} - \{\tilde{L}^{(n)} - g(\tilde{\varphi}^{(n)})\}$ and $P_t^{(n)}$ denotes the $n$-th BS's equivalent isotropic radiated power. The random variable $\mathbf{v}_{rss}(k)$ describes the error in dB due to slow fading which is assumed to be zero-mean Gaussian distributed with covariance matrix $\mathbf{R}_{rss} = \mathrm{diag}((\sigma_{rss}^{(1)})^2, \cdots, (\sigma_{rss}^{(N_{bs})})^2)$.

### C. Timing Advance

In GSM, the TA is a parameter that is used to maintain frame alignment in the GSM system [1]. Basically, the TA is the round trip propagation delay, i.e., the time the radio signal needs to travel from the BS to the MT and back, quantized to finite precision. Let $\mathbf{y}_{ta}(k)$ denote the vector of $N_{bs}$ TA measured values multiplied by $c_0/2$, where $c_0$ is the speed of light. Then, the statistical model for the TA measured values is given by

$$\mathbf{y}_{ta}(k) = \mathbf{h}_{ta}(\mathbf{x}_{ms}(k)) + \mathbf{v}_{ta}(k), \quad (4)$$

where $\mathbf{h}_{ta}(\mathbf{x}_{ms}(k)) = [d_{bs}^{(1)}(\mathbf{x}_{ms}(k)), \cdots, d_{bs}^{(N_{bs})}(\mathbf{x}_{ms}(k))]^\mathsf{T}$. The random variable $\mathbf{v}_{ta}(k)$ accounts for the errors each TA measured value is affected by, which is assumed to be zero-mean Gaussian distributed with covariance matrix $\mathbf{R}_{ta} = \mathrm{diag}((\sigma_{ta}^{(1)})^2, \cdots, (\sigma_{ta}^{(N_{bs})})^2)$.

### D. Pseudorange

The GPS is based on the ToA principle, i.e., the MT is measuring the time the satellite signal requires to travel from the satellite to the MT [3]. The MT's clock is generally not time-synchronized to the clocks of the GPS satellites, resulting in an unknown receiver clock bias $\delta t(k)$. The satellite clocks, however, can be assumed to be mutually synchronized, so that for each time index $k$ the ToA measured values are affected by the same bias [3]. Let $\mathbf{y}_{pr}(k)$ denote the vector of $N_{sat}$ PR measured values obtained from multiplying the biased ToA measured values by $c_0$. Then, the statistical model of the PR measured values can be written as

$$\mathbf{y}_{pr}(k) = \mathbf{h}_{pr}(\mathbf{x}_{ms}(k), \delta t(k)) + \mathbf{v}_{pr}(k), \quad (5)$$

with $\mathbf{h}_{pr}(\mathbf{x}_{ms}(k), \delta t(k)) = [\tilde{h}_{pr}^{(1)}, \cdots, \tilde{h}_{pr}^{(N_{sat})}]^\mathsf{T}$, where $\tilde{h}_{pr}^{(l)} = d_{sat}^{(l)}(\mathbf{x}_{ms}(k)) + c_0 \cdot \delta t(k)$ and $d_{sat}^{(l)}(\mathbf{x}_{ms}(k))$ denotes the Euclidean distance between the MT and the $l$-th satellite. The random variable $\mathbf{v}_{pr}(k)$ describes the errors each PR measured value is affected by, which is assumed to be zero-mean Gaussian distributed with covariance matrix $\mathbf{R}_{pr} = \mathrm{diag}((\sigma_{pr}^{(1)})^2, \cdots, (\sigma_{pr}^{(N_{sat})})^2)$.

## III. Rao-Blackwellized Unscented Kalman Filter and Posterior Cramér-Rao Lower Bound for Hybrid Localization

### A. Introduction

After having described the nonlinear relationship between the RSS, TA, PR measured values and the MT location, the question is how one can efficiently estimate the MT location from these measured values. The EKF provides a solution to this problem, as it addresses the problem of recursively estimating the state of a discrete-time nonlinear dynamic system

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{f}(\mathbf{x}(k), \mathbf{w}(k)), \quad (6)\\ \mathbf{y}(k) &= \mathbf{h}(\mathbf{x}(k), \mathbf{v}(k)), \quad (7) \end{aligned}$$

where $\mathbf{x}(k)$ and $\mathbf{y}(k)$ are the state and measurement vectors, $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are process and measurement noise vectors and $\mathbf{f}(\cdot)$ and $\mathbf{h}(\cdot)$ are some vector valued mapping functions [14]. The EKF, however, is based on a first-order linearization of the nonlinear dynamic system. These approximations can lead to suboptimal performance or even divergence of the EKF. Additionally, due to the linearization of the system equations, Jacobian matrices have to be evaluated, which in some cases may become difficult. E.g., consider the case when antenna gain models, cf. (2), are available only from measurements and, consequently, no closed form expressions for these models exist. In these cases, it is much easier to

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approximate the models using interpolation than trying to evaluate the corresponding Jacobian matrices. Thus, in this paper, an unscented Kalman filter (UKF) is proposed for the hybrid localization problem [10]. The UKF is based on a deterministic sampling approach where no explicit calculation of Jacobian matrices is necessary and the computational complexity is the same order as that of the EKF. In this approach, a set of carefully chosen sample points (sigma points) is propagated through the true nonlinear system. The nonlinear transformed samples capture the posterior mean and covariance accurately to at least the second-order of the Taylor series expansion, whereas the EKF only achieves first-order accuracy [10].

### *B. Process and Measurement Model*

For the UKF-based hybrid localization method, the states of the process model include the MT location, velocity, clock bias and drift, i.e., $\mathbf{x} = [x_{ms},\, \dot{x}_{ms},\, y_{ms},\, \dot{y}_{ms},\, c_0 \cdot \delta t,\, c_0 \cdot \dot{\delta t}]^{\mathsf{T}}$. The MT's motion is approximated with a constant velocity (CV) model [14] and the receiver clock bias is modeled by a second-order Markov process whose input is white noise [14], [15]. The corresponding linear process model for the hybrid localization method is, thus, given by

$$\mathbf{x}(k+1) = \mathbf{\Phi} \cdot \mathbf{x}(k) + \mathbf{\Gamma} \cdot \mathbf{w}(k), \tag{8}$$

with $\mathbf{\Phi} = \mathbf{I}_3 \otimes \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}$, $\mathbf{\Gamma} = \mathrm{diag}(\mathbf{I}_2 \otimes \gamma, \mathbf{I}_2 \cdot c_0)$,where $\mathbf{I}_q$ is the identity matrix of size $q$, $\otimes$ denotes the Kronecker product, $\gamma = [T_s^2/2, T_s]^{\mathsf{T}}$ and $T_s$ is the sampling time. The process noise $\mathbf{w}(k) = [w_{a_x}(k),\, w_{a_y}(k),\, w_{\delta t}(k),\, w_{\dot{\delta t}}(k)]^{\mathsf{T}}$ is assumed to be a zero-mean white Gaussian noise sequence with block diagonal covariance matrix $\mathbf{Q} = \mathrm{diag}(\mathbf{Q}_{cv}, \mathbf{Q}_{\delta t})$, with $\mathbf{Q}_{cv} = \mathrm{diag}(\sigma_{a_x}^2, \sigma_{a_y}^2)$, where $\sigma_{a_x}^2$ and $\sigma_{a_y}^2$ denote the noise variances in the $x$- and $y$-direction. The elements of the symmetric $2 \times 2$ matrix $\mathbf{Q}_{\delta t}$ are given by $Q_{11} = h_0 T_s/2 + 2h_{-1}T_s^2 + 2/3\pi^2 h_{-2} T_s^3$, $Q_{12} = 2h_{-1}T_s + \pi^2 h_{-2} T_s^3$ and $Q_{22} = h_0/(2T_s) + 2h_{-1} + 8/3\pi^2 h_{-2} T_s$, where the parameters $h_0 = 9.4 \cdot 10^{-20}$, $h_{-1} = 1.8 \cdot 10^{-19}$ and $h_{-2} = 3.8 \cdot 10^{-21}$ correspond to values of a typical quartz standard [15].
In the following, the unknown state vector $\mathbf{x}(k)$ is estimated from the PR, TA and RSS measured values. These measured values can be combined into a single measurement vector $\mathbf{y}(k) = \left[\mathbf{y}_{pr}^{\mathsf{T}}(k), \mathbf{y}_{ta}^{\mathsf{T}}(k), \mathbf{y}_{rss}^{\mathsf{T}}(k)\right]^{\mathsf{T}}$, so that the corresponding nonlinear measurement model for the hybrid localization method can be written as

$$\mathbf{y}(k) = \mathbf{h}(\mathbf{x}(k)) + \mathbf{v}(k), \tag{9}$$

where $\mathbf{h}(\mathbf{x}(k)) = \left[\mathbf{h}_{pr}^{\mathsf{T}}(\mathbf{x}(k)), \mathbf{h}_{ta}^{\mathsf{T}}(\mathbf{x}(k)), \mathbf{h}_{rss}^{\mathsf{T}}(\mathbf{x}(k))\right]^{\mathsf{T}}$ and $\mathbf{v}(k) = \left[\mathbf{v}_{pr}^{\mathsf{T}}(k), \mathbf{v}_{ta}^{\mathsf{T}}(k), \mathbf{v}_{rss}^{\mathsf{T}}(k)\right]^{\mathsf{T}}$. The random variable $\mathbf{v}(k)$ is zero-mean Gaussian distributed with block diagonal covariance matrix $\mathbf{R} = \mathrm{diag}(\mathbf{R}_{pr}, \mathbf{R}_{ta}, \mathbf{R}_{rss})$.

### *C. Rao-Blackwellized Unscented Kalman Filter*

Due to the fact that the process model is linear, the measurement model is nonlinear and the process and measurement noise is additive Gaussian, cf. (8) and (9), a Rao-Blackwellized version of the UKF (RBUKF) instead of an UKF can be used. The idea of the Rao-Blackwellization is that one can conceptually use the Kalman filter for the linear part (time update) and the UKF for the nonlinear part (measurement update). As a result, the quasi Monte Carlo variance and computational complexity is reduced [11]. The RBUKF equations, adopted to the proposed hybrid localization method, are summarized in Table I.

TABLE I: Rao-Blackwellized Unscented Kalman Filter

1. Initialization

$\hat{\mathbf{x}}(0|0) = \mathbb{E}\{\mathbf{x}(0)\}$

$\mathbf{P}(0|0) = \mathbb{E}\{(\mathbf{x}(0) - \hat{\mathbf{x}}(0|0))(\mathbf{x}(0) - \hat{\mathbf{x}}(0|0))^{\mathsf{T}}\}$

2. For $k = 1, 2, \ldots$

(a) Time Update Equations

$\hat{\mathbf{x}}(k|k-1) = \mathbf{\Phi}\hat{\mathbf{x}}(k-1|k-1)$

$\mathbf{P}(k|k-1) = \mathbf{\Phi}\mathbf{P}(k-1|k-1)\mathbf{\Phi}^{\mathsf{T}} + \mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^{\mathsf{T}}$

Calculate matrix $\boldsymbol{\mathcal{X}}$ with $2L+1$ sigma vectors:

$\boldsymbol{\mathcal{X}}(k|k-1) = \left[\hat{\mathbf{x}}(k|k-1) \quad \hat{\mathbf{x}}(k|k-1) \pm \sqrt{(L+\lambda)\mathbf{P}(k|k-1)}\right]$

$\boldsymbol{\mathcal{Y}}(k|k-1) = \mathbf{h}(\boldsymbol{\mathcal{X}}(k|k-1))$

$\hat{\mathbf{y}}(k|k-1) = \sum_{i=0}^{2L} W_i^{(m)}\, \mathcal{Y}_i(k|k-1)$

(b) Measurement Update Equations

$\tilde{\mathcal{Y}}_i = \mathcal{Y}_i(k|k-1) - \hat{\mathbf{y}}(k|k-1),\ \tilde{\mathcal{X}}_i = \mathcal{X}_i(k|k-1) - \hat{\mathbf{x}}(k|k-1)$

$\mathbf{P}_{\tilde{\mathbf{y}}_k\tilde{\mathbf{y}}_k} = \sum_{i=0}^{2L} W_i^{(c)} \tilde{\mathcal{Y}}_i \tilde{\mathcal{Y}}_i^{\mathsf{T}} + \mathbf{R}, \quad \mathbf{P}_{\mathbf{x}_k\mathbf{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} \tilde{\mathcal{X}}_i \tilde{\mathcal{Y}}_i^{\mathsf{T}}$

$\mathbf{K} = \mathbf{P}_{\mathbf{x}_k\mathbf{y}_k}\mathbf{P}_{\tilde{\mathbf{y}}_k\tilde{\mathbf{y}}_k}^{-1}$

$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}\left[\mathbf{y}(k) - \hat{\mathbf{y}}(k|k-1)\right]$

$\mathbf{P}(k|k) = \mathbf{P}(k|k-1) - \mathbf{K}\mathbf{P}_{\tilde{\mathbf{y}}_k\tilde{\mathbf{y}}_k}\mathbf{K}^{\mathsf{T}}$

where $L =$ dimension of state vector, $\lambda =$ composite scaling parameter, $\mathcal{X}_i$, $\mathcal{Y}_i = i$-th column of the matrices $\boldsymbol{\mathcal{X}}$ and $\boldsymbol{\mathcal{Y}}$, and $W_i^{(m)}, W_i^{(c)} =$ weights calculated according to [10].

### *D. Posterior Cramér-Rao Lower Bound*

After having introduced the RBUKF for the hybrid localization method, its performance should be compared to a theoretical performance bound. In the following, the PCRLB is determined that gives the best achievable performance for nonlinear filtering [12]. Let $\hat{\mathbf{x}}(k|k)$ be an unbiased estimate of the state vector $\mathbf{x}(k)$. Then, the covariance matrix of the estimation error satisfies the inequality

$$\mathbb{E}\{(\hat{\mathbf{x}}(k|k) - \mathbf{x}(k))(\hat{\mathbf{x}}(k|k) - \mathbf{x}(k))^{\mathsf{T}}\} \geq \mathbf{J}(k)^{-1}, \tag{10}$$

where $\mathbb{E}\{\cdot\}$ is the expectation, $\mathbf{J}(k)$ denotes the filtering information matrix and its inverse is the PCRLB matrix. The matrix inequality $\mathbf{A} \geq \mathbf{B}$ should be interpreted as the matrix $\mathbf{A} - \mathbf{B}$ being positive semidefinite. The aim is now to calculate $\mathbf{J}(k)$. In [16], an elegant method is presented, where $\mathbf{J}(k)$ can be determined recursively. This recursion, adapted to the hybrid localization problem involving additive Gaussian noise, cf. (8) and (9), can be written as

$$\mathbf{J}(k+1) = (\mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^{\mathsf{T}} + \mathbf{\Phi}\mathbf{J}(k)^{-1}\mathbf{\Phi}^{\mathsf{T}})^{-1} + \mathbb{E}\{\tilde{\mathbf{H}}^{\mathsf{T}}\mathbf{R}^{-1}\tilde{\mathbf{H}}\}, \tag{11}$$

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where $\tilde{\mathbf{H}} \triangleq \mathbf{H}(k+1)$ denotes the Jacobian matrix of the nonlinear measurement function $\mathbf{h}(\cdot)$, cf. (9), evaluated at the true value of the state $\mathbf{x}(k+1)$.

## IV. Simulation Results

It is assumed that a car is equipped with a MT that is capable of providing TA and RSS measured values from GSM and PR measured values from GPS. The car travels with a constant speed of 45 km/h on a straight line in a dense urban scenario of size $3\,\mathrm{km} \times 3\,\mathrm{km}$ as it is shown in Fig. 1. The GSM network is composed of $N_{bs} = 7$ BSs equipped with directional antennas. The BS locations as well as the BS antenna parameters are a-priori known. The satellite locations are assumed to be known and are chosen from the real GPS satellite constellation taking into account realistic satellite elevation masks. The simulation parameters are given in Table II and are assumed to be equal for all BSs and all satellites for the sake of simplicity. The following combinations of measured values are investigated:

- GSM method: One TA measured value from the serving BS and a total of seven RSS measured values from serving and neighbouring BS antennas,
- Hybrid 1 method: Measured values of GSM method and, in addition, one PR measured value from one satellite,
- Hybrid 2 method: Measured values of GSM method and, in addition, two PR measured values from two different satellites.

For simplicity, the serving BS is assumed to be the BS located at $[750\,\mathrm{m}, 1000\,\mathrm{m}]^\mathsf{T}$. The PR, TA and RSS measured values are updated every $T_s = 0.48\,\mathrm{s}$, which corresponds to the reporting period of measured values in GSM networks. The performance of the proposed RBUKF-based hybrid localization method is evaluated in terms of the root mean square error (RMSE) determined from $N_{mc} = 500$ Monte Carlo trials [12]. For each trial, the MT trajectory is initialized with $x(0) = [-200\,\mathrm{m}, 8.84\,\mathrm{m/s}, -200\,\mathrm{m}, 8.84\,\mathrm{m/s}, 0\,\mathrm{m}, 0\,\mathrm{m}]^\mathsf{T}$. The trajectory then is generated by (8), where the elements of the process noise covariance sub-matrix $\mathbf{Q}_{cv}$ are chosen to have very small values, cf. Table II. The initial state vector $\hat{\mathbf{x}}(0|0)$ for the RBUKF is obtained from random initialization [14], and the error covariance matrix is set to $\mathbf{P}(0|0) = \mathrm{diag}((200\,\mathrm{m})^2, (10\,\mathrm{m/s})^2, (200\,\mathrm{m})^2, (10\,\mathrm{m/s})^2, (300\,\mathrm{km})^2/3,\ (10\,\mathrm{m})^2/3)$. In order to account for possible MT maneuvers and receiver clock uncertainties, the covariance matrix $\mathbf{Q}$ for the filter is chosen to be $\mathbf{Q} = \mathrm{diag}(100 \cdot \mathbf{Q}_{cv}, 5 \cdot \mathbf{Q}_{\delta t})$. The measurement covariance matrix $\mathbf{R}$ for the simulations and the filter are assumed to be the same.

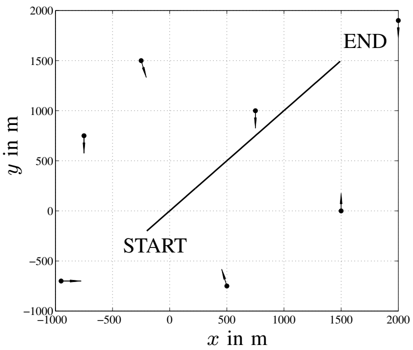


Fig. 1: Simulation scenario with $N_{\mathrm{BS}} = 7$ BSs (•). The arrows (→) indicate the BS antenna boresight direction.

TABLE II: Simulation parameters

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $A$ in dB | 132.8 | $P_t$ in dBm | 50 |
| $B$ in dB | 3.8 | $\sigma_{a_x}$ in $\mathrm{m/s^2}$ | $10^{-2}$ |
| $\sigma_{rss}$ in dB | 8 | $\sigma_{a_y}$ in $\mathrm{m/s^2}$ | $10^{-2}$ |
| $A_m$ in dB | 20 | $\sigma_{ta}$ in m | 300 |
| $\varphi_{3dB}$ in ° | 60 | $\sigma_{pr}$ in m | 15 |

In Fig. 2, the RMSE in dependence of the time index $k$ for the GSM, Hybrid 1 and Hybrid 2 method for the MT location, velocity, clock bias and drift are compared to the corresponding PCRLBs. From Fig. 2, it can be seen that the GSM method yields the worst results in terms of RMSE which can be explained by the fact that the RSS and TA measured values do not provide the same level of accuracy than the bias corrected GPS measured values. The RMSE can be improved by the Hybrid 1 and Hybrid 2 methods that additionally take into account one or two PR measured values from GPS. The improvement of the MT location RMSE, however, is marginal for the Hybrid 1 method which can be explained by the fact that the RBUKF is not able to accurately estimate the unknown receiver clock bias, cf. 2(c).

For the Hybrid 2 method, the improvements are significant. Due to the fact that two PR measured values are available, the RBUKF can much more accurately estimate the receiver clock bias and drift states, cf. 2(c) and (d), which has a direct impact on the achievable MT location RMSE. For the RMSE of the MT velocity, cf. 2(b), similar conclusions can be drawn. Again, the poor performance of the GSM method can be further improved by the Hybrid 1 and Hybrid 2 methods. In Table III, the performance of the RBUKF is compared to the EKF [9] in terms of the RMSE averaged over the whole time period. From Table III, it can be seen that for this specific scenario the RBUKF slightly outperforms the EKF in terms of average RMSE.

TABLE III: Average RMSE performance of RBUKF and EKF

| Algorithm | Method | Location | Velocity | Bias | Drift |
|---|---|---|---|---|---|
| RBUKF | GSM | 74.5 m | 3.7 m/s | - | - |
| | Hybrid 1 | 71.9 m | 3.4 m/s | 33.6 m | 1.9 m/s |
| | Hybrid 2 | 44.4 m | 2.7 m/s | 4.6 m | 1.2 m/s |
| EKF [9] | GSM | 74.8 m | 3.7 m/s | - | - |
| | Hybrid 1 | 73.5 m | 3.5 m/s | 34.2 m | 1.9 m/s |
| | Hybrid 2 | 45.3 m | 2.8 m/s | 4.6 m | 1.2 m/s |

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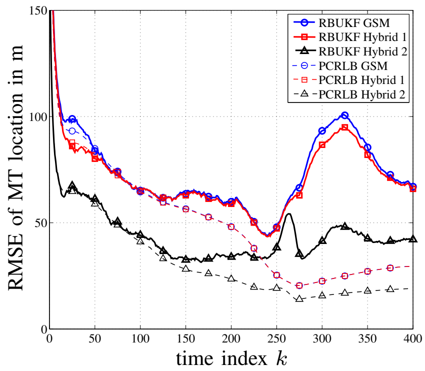
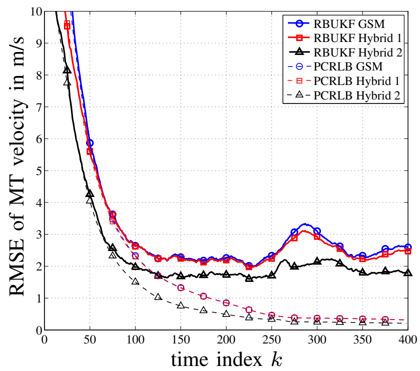
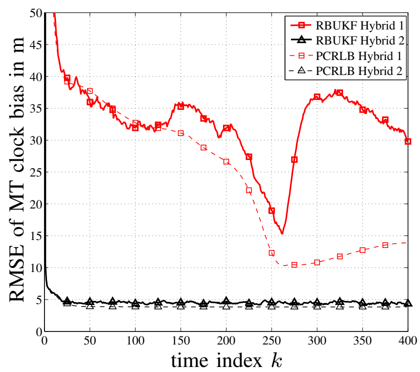


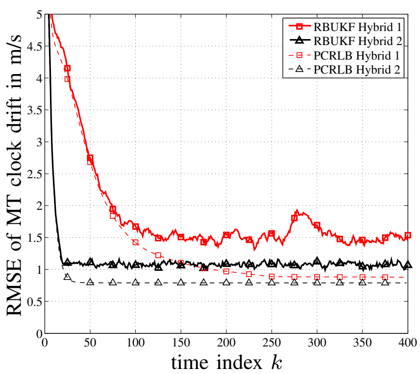

(a)




(b)




(c)




(d)

Fig. 2: RMSE of the RBUKF and corresponding PCRLB for the GSM, Hybrid 1 and Hybrid 2 method for MT (a) location, (b) velocity, (c) clock bias and (d) clock drift.

## V. Conclusion

In this paper, an RBUKF-based hybrid localization method is presented that combines RSS and TA measured values from GSM and PR measured values from GPS in order to track a MT. The performance of the RBUKF is evaluated in terms of the RMSE which is then compared to the PCRLB and the EKF. Simulations results have shown that compared to combining only measured values from GSM, the RMSE of the RBUKF and the corresponding PCRLBs can be improved by additionally taking into account measured values from GPS. In the future, it should be investigated how additional measurements, e.g., E-OTD measured values or road-information, can further improve the performance of the hybrid localization method.

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