14th International OFDM-Workshop (InOWo), Sep 2009, Hamburg, Germany

Consideration of signaling overhead in adaptive Multi-user OFDMA/TDD systems

Alexander Kühne*, Anja Klein*, Xinning Wei**, and Tobias Weber**
*Communications Engineering Lab, Technische Universität Darmstadt, 64283 Darmstadt, Germany
**Institute of Communications Engineering, University of Rostock, 18119 Rostock, Germany
{a.kuehne, a.klein}@nt.tu-darmstadt.de, {xinning.wei, tobias.weber}@uni-rostock.de

***Abstract*— In this paper, the overhead due to pilot transmission and signaling in an adaptive multi-user Orthogonal Frequency Division Multiple Access (OFDMA) Time Division Duplex (TDD) system is considered. Applying adaptive transmission schemes results in good performances. However, Channel State Information (CSI) is required at the transmitter, i.e., resources have to be spent for providing accurate CSI. These resources cannot be used to transmit data anymore, i.e., overhead is produced which can become prohibitive. Using less resources for providing CSI leads to imperfect CSI and performance degradations. Hence, a tradeoff between the achievable data rate corresponding to given CSI accuracy and the effort providing this CSI has to be found. In this work, analytical expressions for the user data rate and bit error rate are derived taking into account impact of imperfect CSI at the transmitter and the receiver. Furthermore, the effective system rate is derived taking into account the overhead due to signaling and pilot transmission.**

I. Introduction

The impact of imperfect Channel State Information (CSI) on the performance of adaptive OFDM systems has already been studied intensely in the literature, e.g. [1]-[6] and references therein, where the CSI imperfectness arises from channel estimation, time delays and quantization. In [7] an adaptive multi-user single carrier system which exploits multi-user diversity is investigated where the CSI is assumed to be outdated. In [8] an adaptive multi-user OFDMA system is analyzed assuming imperfect CSI due to channel estimation, time delays, quantization and imperfect feedback link. However, not only the impact of the imperfect channel knowledge on the system performance is important but also the effort that has to be spent in order to provide accurate CSI. On the one hand, by using resources for providing accurate channel information at the transmitter, high data rates applying adaptive transmission schemes can be achieved. On the other hand, these resources cannot be used to transmit data any more, i.e., signaling overhead is produced which can become prohibitive. Hence, when considering the performance of an adaptive transmission scheme, the signaling overhead also has to be taken into account, i.e., we have a tradeoff between the achievable the data rate corresponding to a given CSI accuracy and the signaling overhead required for providing the CSI. The present paper will contribute to this aspect. Assuming a Time Division Duplex (TDD) system, we provide a framework for the evaluation of the average system data rate applying adaptive multi-user OFDMA transmission schemes in the uplink (UL) and the downlink (DL) taking into account imperfect CSI *and* signaling overhead. The remainder of this paper is organized as follows. In Section II, the considered OFDMA/TDD system model is presented. In Section III, the modelling of imperfect CSI due to channel estimation and time delays are presented. In Section IV, closed form expressions for the data rate and Bit Error Rate (BER) are derived analytically taking into account the impact of imperfect CSI. Section V introduces the impact of signaling overhead on the system performance. In Section VI, numerical results illustrate the tradeoff between the system performance achievable with a given CSI accuracy and the required signaling overhead providing this CSI.

II. System model

We consider a single cell OFDMA scenario with one base station (BS) and U mobile stations (MSs) each having one antenna. The bandwidth in UL and DL is subdivided into N orthogonal subcarriers. We define a block of Q subcarriers, also called chunk, as radio resource. Hence, a total number of $N_{ch} = \lfloor N/Q \rfloor$ resources is assumed with $\lfloor . \rfloor$ the nearest integer lower than or equal the argument. We assume that Q is chosen in such a way that the channel does not vary significantly within a chunk. Since a TDD system is considered, we assume that the UL and DL channels are the same with $H_u(n,k)$ denoting the channel transfer factor of user u on chunk n. $H_u(n,k)$ is assumed to be constant for M_T OFDM symbols which corresponds to the duration of a time slot with index k. Further on, $H_u(n,k)$ is a i.i.d. complex Gaussian random variable with zero mean and variance one, where the channels of adjacent chunks are assumed to be statistical independent in frequency. We assume temporally correlated block fading [9], i.e., the channel $H_u(n,k)$ is constant for M_T OFDM symbols and is time correlated with the channel of the previous time slot $H_u(n,k-1)$.

The time frame structure of the considered OFDMA/TDD system is depicted in Fig. 1. At the beginning of each time slot in the DL, a pilot transmission (PT) is performed in order to obtain an estimate of the DL channel at the MSs for channel equalization. Further on, signaling of the scheduling and modulation scheme information to the users is performed. We consider a Max-SNR Scheduling approach assigning chunk n in time slot k to user u which has the highest instantaneous SNR

$$\gamma_u(n,k) = \bar{\gamma} \cdot |H_u(n,k)|^2 \tag{1}$$

with $\bar{\gamma}$ denoting the average SNR. The remaining OFDM symbols are used for data transmission from the BS to the MSs according to the scheduling decisions which are constant for the entire time slot. In the UL, PT is performed as well in order to obtain an estimate of the complete UL channel of each user at the BS. Based on the scheduling decisions of the previous DL time slot made at the BS, each user u transmits data to the BS in the remaining OFDM symbols on the corresponding subcarriers. At the BS, a channel estimation (CE) is done in order to equalize the received UL data transmission and to perform a resource allocation and adaptive modulation for the next DL time slot.

III. Modelling imperfect CSI

In this section, the modelling of imperfect CSI arising from pilot-based channel estimation and time delays are presented. The following considerations are valid for each user, chunk and time slot. Hence, the indices u, n and k are skipped for simplicity.

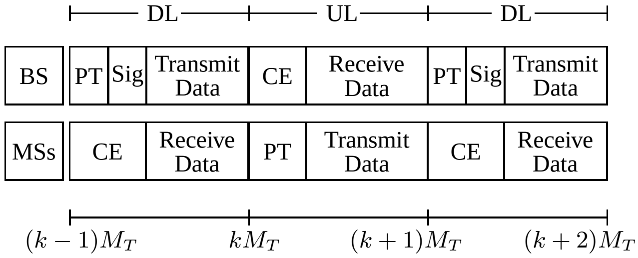

Fig. 1. Time frame structure

A. Noisy estimated CSI

Regarding channel estimation, there are two effects on the system performance which have to be considered. Firstly, the scheduling decisions are based on noisy estimated Transmitter CSI (TCSI). Secondly, the equalization is performed using noisy estimated Receiver CSI (RCSI).

Transmitting a pilot sequence $\mathbf{p} = [p_1, ..., p_{M_P}]^{\mathrm{T}}$ of length M_P with $\mathbf{p}^{\mathrm{H}}\mathbf{p} = M_P$ over a constant channel H, the received signal vector $\mathbf{y}$ of length M_P is given by $\mathbf{y} = H \cdot \mathbf{p} + \mathbf{n}$ with the complex normal distributed noise vector $\mathbf{n}$ of length M_P with zero mean and variance $\sigma_n^2 = \frac{1}{\bar{\gamma}}$. Using Least Squares (LS) channel estimation at the receiver, the estimated channel $\hat{H} = (\mathbf{p}^{\mathrm{H}}\mathbf{p})^{-1}\mathbf{p}^{\mathrm{H}}\mathbf{y}$ can be modeled as $\hat{H} = H + E$, where E is a complex normal distributed random variable with zero mean and variance

$$\sigma_E^2 = \frac{1}{\bar{\gamma} M_P}. \tag{2}$$

Equalizing the received data with the estimated channel $\hat{H}$ leads to an SNR degradation compared to the case of equalizing the received data with the perfect channel given by

$$l_{EQ} = \frac{1}{1 + \bar{\gamma}\sigma_E^2} = \frac{M_P}{1 + M_P}. \tag{3}$$

B. Outdated CSI

Until now, we only considered imperfect CSI due to channel estimation. However, as described in Section II, the estimated channel $\hat{H}$ and the corresponding scheduling decisions are already outdated when the data transmission is performed. With J_0 the 0-th order Bessel function, M_T the number of OFDM symbols per time slot, T_S the symbol duration, f_0 the carrier frequency, v the velocity of the MSs and c the speed of light, this is modeled by correlation, i.e. the outdated channel and the actual channel are time correlated by the correlation coefficient

$$\rho = J_0(2\pi T_S f_0 v/c \cdot M_T) \tag{4}$$

assuming Jake's spectrum.

Now, the relation between the outdated and noisy estimated channel $\hat{H}$ and the actual channel H_{EQ} seen by the receiver at the output of the equalizer is given by

$$\hat{H} = \frac{\rho}{\sqrt{l_{EQ}}} H_{EQ} + \sqrt{1-\rho^2} X + E \qquad (5)$$

where X is a complex normal distributed random variable with zero mean and variance one. Using (1) and [10, p. 43], the conditional probability density function (PDF) of the actual SNR γ_{EQ} seen by the receiver at the output of the equalizer and the outdated and noisy estimated SNR $\hat{\gamma}$ available at the BS is given by

$$p_{\gamma_{EQ}|\hat{\gamma}}(\gamma_{EQ}|\hat{\gamma}) = \frac{1}{\bar{\gamma}\sigma_r^2} e^{-\frac{\mu^2\hat{\gamma}+\gamma_{EQ}}{\bar{\gamma}\sigma_r^2}} I_0\left(\frac{2\mu\sqrt{\gamma_{EQ}\hat{\gamma}}}{\bar{\gamma}\sigma_r^2}\right) \qquad (6)$$

with $\mu = \sqrt{l_{EQ}}\frac{\rho}{1+\sigma_E^2}$, $\sigma_r^2 = l_{EQ}\frac{1+\sigma_E^2-\rho^2}{1+\sigma_E^2}$ and $I_0(x)$ denoting the 0th-order modified Bessel function of the first kind.

IV. Impact of imperfect CSI on the system performance

In the following, analytical expressions for the user data rate and bit error rate (BER) are derived taking into account imperfect TCSI and RCSI.

A. Data rate

The average user rate is formulated as the sum rate of the different modulation schemes weighted by their probability, where we assume that there are M modulation schemes available. Let $\gamma = [\gamma_0, \gamma_1, ..., \gamma_M]^{\mathrm{T}}$, with $\gamma_0 = 0$ and $\gamma_M = \infty$ denotes the threshold vector which contains the SNR threshold values determining the interval in which a particular modulation scheme is applied. Thus, the average user data rate $\bar{R}$ can be formulated as

$$\bar{R} = \sum_{m=1}^{M} \int_{\gamma_{m-1}}^{\gamma_m} b_m \cdot p_{\hat{\gamma}}(\hat{\gamma})\, d\hat{\gamma} \qquad (7)$$

with b_m denoting the number of bits per symbol corresponding to the applied modulation scheme and the PDF of the outdated and noisy estimated SNR $\hat{\gamma}$ of the scheduled resource given by

$$p_{\hat{\gamma}}(\hat{\gamma}) = \frac{U}{\bar{\gamma}_E} e^{-\frac{\hat{\gamma}}{\bar{\gamma}_E}} \left(1 - e^{-\frac{\hat{\gamma}}{\bar{\gamma}_E}}\right)^{U-1} \qquad (8)$$

with $\bar{\gamma}_E = \bar{\gamma}(1+\sigma_E^2)$. Inserting (8) in (7) results in

$$\bar{R} = \sum_{m=1}^{M} b_m \cdot \left[\left((1 - e^{-\frac{\gamma_m}{\bar{\gamma}_E}}\right)^U - \left((1 - e^{-\frac{\gamma_{m-1}}{\bar{\gamma}_E}}\right)^U\right]. \qquad (9)$$

B. Bit Error Rate

Approximating the instantaneous BER for M-QAM and M-PSK modulation as done in [11] leads to

$$BER_m(\gamma_{EQ}) = 0.2 \cdot \exp(-\beta_m \gamma_{EQ}) \qquad (10)$$

with $m = 1, .., M$, where $\beta_m = \frac{1.6}{2^{b_m}-1}$ using M-QAM modulation and $\beta_m = \frac{7}{2^{1.9 b_m}+1}$ using M-PSK modulation, respectively. The average BER is then defined as the sum of the average bit errors of the different modulation constellations divided by the average bit rate [7]. The average BER $\overline{BER}$ is then given by

$$\overline{BER} = \frac{1}{\bar{R}} \sum_{m=1}^{M} b_m \int_{\gamma_{m-1}}^{\gamma_m} p_{\hat{\gamma}}(\hat{\gamma}) \cdot \int_0^\infty BER_m(\gamma_{EQ}) \cdot p_{\gamma_{EQ}|\hat{\gamma}}(\gamma_{EQ}|\hat{\gamma})\, d\gamma_{EQ}\, d\hat{\gamma}. \qquad (11)$$

Inserting (6), (8) and (10) in (11), (11) can be rewritten to

$$\overline{BER} = \frac{U}{5 \cdot \bar{R}} \sum_{m=1}^{M} b_m \sum_{v=0}^{U-1} \binom{U-1}{v} \cdot \frac{(-1)^v}{\Lambda(m,v)} \cdot \left[e^{\frac{-\gamma_{m-1}\cdot\Lambda(m,v)}{\bar{\gamma}_E\cdot(1+\beta_m\bar{\gamma}\sigma_r^2)}} - e^{\frac{-\gamma_m\cdot\Lambda(m,v)}{\bar{\gamma}_E\cdot(1+\beta_m\bar{\gamma}\sigma_r^2)}}\right] \qquad (12)$$

with $\Lambda(m,v) = (v+1) + \beta_m\left((v+1)\bar{\gamma}\sigma_r^2 + \bar{\gamma}_E\mu^2\right)$.

C. Maximum user data rate

In the following, we are looking for the optimal modulation scheme threshold vector γ which maximizes the average data rate under the constraint of a target BER BER_T:

$$\bar{R}_{\mathrm{opt}}(M_T, M_P) = \max_{\gamma}\left\{\bar{R}(\gamma, M_T, M_P)\right\} \qquad (13)$$
$$\text{subject to} \quad \overline{BER}(\gamma, M_T, M_P) \le BER_T.$$

As shown in [12], this type of problem can be solved by a Lagrange multiplier approach, i.e., for each value of M_T and M_P we can determine the optimal SNR threshold vector γ_{opt} which maximizes the user data rate under the constraint of a target BER.

V. Consideration of pilot and signaling overhead

Until now, we only considered the impact of imperfect TCSI and RCSI on the system performance. In the following, also the overhead due to signaling and pilot transmission is taken into account.

A. Signaling in DL

After the scheduling and adaptive modulation are performed at the BS, the user index and modulation scheme index of each scheduled chunk have to be signalled to the users before data transmission. This results in

$$M_{sig} = \frac{\log_2(U) + \log_2(M)}{Q \cdot b_{sig}} \tag{14}$$

OFDM symbols signaling overhead with b_{sig} denoting the number of bits per symbol used for signaling.

B. Pilot transmission in DL

Assuming that the channel does not chance significantly within one chunk, it is sufficient to send pilots only on one subcarrier per chunk. The remaining subcarriers can be used for data transmission. This corresponds to

$$M_{P,\mathrm{DL}} = \frac{M_P}{Q} \tag{15}$$

OFDM symbols pilot overhead in the DL.

C. Pilot transmission in UL

In the UL, each MS has to send pilots on each chunk so that the BS is able to estimate the whole UL channel of each user in order to equalize the UL data and to perform the scheduling for the next DL time slot. As with the pilot transmission in the DL, it is sufficient for each MS to send pilots on only one subcarrier per chunk. This corresponds to

$$M_{P,\mathrm{UL}} = U \cdot \frac{M_P}{Q} \tag{16}$$

OFDM symbols pilot overhead in the UL.

D. Effective system data rate

As shown in Section IV, the maximum user data rate can be determined for each value of M_T and M_P solving (13). Hence, the maximum user data rate in the DL is given by $\bar{R}_{\mathrm{opt,DL}}(M_T, M_P)$. For the user data rate in the UL, we have to keep in mind that the scheduling decisions used for the UL data transmission in time slot k are based upon the TCSI estimated in time slot $k-2$. Hence, the user data rate in the UL is given by $\bar{R}_{\mathrm{opt,UL}}(2M_T, M_P)$.

Considering the overhead in UL and DL, the effective system data rate $\bar{R}_{\mathrm{eff}}$ under the constraint of a target BER as function of M_T and M_P is then given by

$$\begin{aligned} &\bar{R}_{\mathrm{eff}}(M_T, M_P) = \\ &\frac{1}{2M_T} \cdot \left[(M_T - M_{P,\mathrm{UL}}) \cdot \bar{R}_{\mathrm{opt,UL}}(2M_T, M_P) + \right. \\ &\quad \left. (M_T - M_{P,\mathrm{DL}} - M_{sig}) \cdot \bar{R}_{\mathrm{opt,DL}}(M_T, M_P) \right] . \end{aligned} \tag{17}$$

As the DL and UL data rates depend on the quality of the available CSI and hence on the number M_P of pilots and the number M_T of OFDM symbols per time slot, we have a tradeoff between CSI accuracy and signaling overhead, i.e., if M_P is chosen large to assure a good channel estimate or M_T is chosen rather small to guarantee an updated CSI, the estimated CSI becomes closer to perfect CSI and the data rate $\bar{R}_{\mathrm{opt,UL}}(2M_T, M_P)$ in the UL and the data rate $\bar{R}_{\mathrm{opt,DL}}(M_T, M_P)$ in the DL will be high. Nonetheless, the effective system data rate $\bar{R}_{sys}$ will be small due to large overhead. Applying only a small number M_P of pilots or using a large number M_T of OFDM symbols per time slot will decrease the overhead but results in a poor CSI accuracy.

VI. Numerical results

In the following, we assume an OFDMA scenario with the parameters given in Table I. In Fig. 2, the effective system data rate $\bar{R}_{\mathrm{eff}}$ is depicted as a function of the number M_T of OFDM symbols for different pilot sequence length M_P for a MS velocity of $v = 10$ km/h and an average SNR $\bar{\gamma} = 12$ dB. M_T is limited to $M_T \leq \frac{T_C}{5T_S}$ with the coherence time $T_C = \frac{c}{2vf_0}$ in order to fulfill the block fading assumption, i.e., the duration of the time slot shall not exceed 20 % of the coherence time. As one can see, there exists an optimal M_T where an optimal tradeoff between CSI accuracy and overhead can be found, and thus, an maximum effective system data rate can be achieved. In this example, the effective system data rate is the highest with $M_{P,\mathrm{opt}} = 2$ and $M_T = 50$, i.e., the CSI has to be updated after 9.5 % of the coherence time using a pilot sequence of length 2. Increasing the MS velocity v,

TABLE I

System parameters

Bandwidth	10 MHz
Number N of subcarriers	512
Chunk size Q	4
Number U of users	32
Carrier frequency f_0	2 GHz
Target BER BER_T	10^{-3}
Number b_{sig} of bits per symbol (signaling)	2

the maximum system data rate decreases, since we have to update the CSI more often in order to cope with the channel variations. For $v = 30$ km/h, the optimal $M_{T,\text{opt}} = 19$ which corresponds to 10.8 % of the coherence time T_C, as shown in Fig. 3. In this case, the optimal number $M_{P,\text{opt}} = 1$, i.e., for an increasing v it is beneficial to use less pilots in order to optimize the effective system data rate.

In Fig. 4, the optimized effective system data rate $\bar{R}_{\text{eff,opt}}$ is depicted as a function of the MS velocity v for different values of average SNR where a degradation of the achievable system data rate for increasing MS velocity v can be observed.

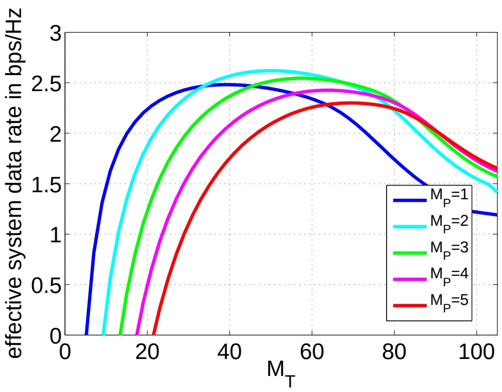

Fig. 2. Effective system data rate vs. number M_T of OFDM symbols with $v = 10$ km/h

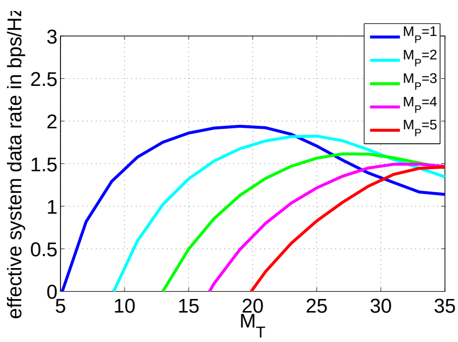

Fig. 3. Effective system data rate vs. number M_T of OFDM symbols with $v = 30$ km/h

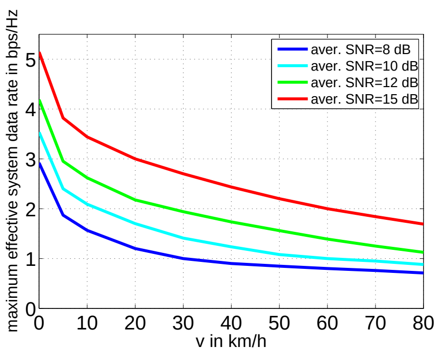

Fig. 4. Maximum effective system data rate vs. MS velocity v

VII. Conclusions

In this paper, the tradeoff between CSI accuracy and signaling overhead in an adaptive multi-user OFDMA/TDD system has been investigated. Analytical expressions for the average user data rate and BER are derived taking into account imperfect CSI at the transmitter and the receiver. Considering additionally the overhead due to signaling and pilot transmission, the effective system data rate is introduced. For increasing MS velocity, it appears that the CSI has to be updated more frequently. This leads to an increased overhead resulting in a degradation of the effective system data rate. Furthermore, for an increasing v it is beneficial to use less pilots to optimize the effective system data rate.

References

[1] S. Ye, R. S. Blum, and L. J. Cimini, ”Adaptive OFDM Systems with imperfect Channel State Information,” *IEEE Trans. on Wireless Communications*, vol. 5, no. 11, pp. 3255-3264, November 2006

[2] Q. Su and S. Schwartz, ”Effects of imperfect channel information on adaptive loading gain of OFDM,” in Proc. *IEEE VTC-Fall'01*, vol. 1, pp. 475-478, October 2001

[3] A. Leke and J. M. Chioffi, ”Multicarrier systems with imperfect channel knowledge,” in Proc. *PIMRC'98*, pp. 549-553, September 1998

[4] M. R. Souryal and R. L. Pickholtz, ”Adaptive modulation with imperfect channel information in OFDM,” in Proc. *ICC'01*, pp. 1861-1865, June 2001

[5] Z. Song, K. Zhang, and Y. L. Guan, ”Statistical adaptive modulation for QAM-OFDM systems,” in Proc. *GLOBECOM'02*, Birmingham, pp. 706-710, November 2002

[6] Y. Yao and G. B. Giannakis, ”Rate-maximizing power allocation in OFDM based on partial channel knowledge,” *IEEE Trans. Wireless Comm.*, vol. 4, pp. 1073-1083, May 2005

[7] Q. Ma and C. Tepedelenlioglu, ”Practical Multi-user diversity with outdated channel feedback,” *IEEE Trans. on Vehicular Technology*, vol. 54, no. 4, pp. 1334-1345, July 2005

[8] A. Kühne, A. Klein, ”Throughput analysis of Multi-user OFDMA-Systems using imperfect CQI feedback and diversity techniques,” *IEEE Jour. Select. Areas in Comm.*, vol. 26, no. 8, pp. 1440-1451, October 2008

[9] M. Castaneda, A. Mezghani, and J. A. Nossek, ”Bounds on the capacity of MISO channels with different types of imperfect CSI,” in Proc. *WSA'09*, February 2009, Berlin, Germany

[10] J. Proakis *Digital Communications*, 3rd edition,McGraw Hill, 1995

[11] S. T. Chunk and A. Goldsmith, ”Degrees of freedom in adaptive modulation: A unified view,” *IEEE Trans. Commun.*, pp. 1561-1571, Sep. 2001.

[12] A. Kühne and A. Klein, “Adaptive multi-user OFDMA systems with high priority users in the presence of imperfect CQI,” in Proc. *MC-SS'09*, Herrsching, Germany, May 2009.