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Non-Regenerative Multi-Way Relaying with Linear Beamforming

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Abstract—In this paper, we introduce non-regenerative multi-way relaying. A half-duplex multi-antenna relay station (RS) assists multiple nodes which want to communicate to each other. Each node has a message and wants to decode the messages from all other nodes. The number of communication phases is equal to the number of nodes, \( N \), such that when \( N = 2 \), we have the well known two-way relaying. In the first phase, all nodes transmit simultaneously to the RS and in the following \( (N-1) \) phases the RS applies transceive beamforming and transmits to all nodes. The achievable sum rate for asymmetric traffic and symmetric traffic cases are derived for \( N \)-phase multi-way relaying. Three low complexity linear transceive beamformers based on Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Maximisation of Signal to Noise Ratio (MSNR) criteria are designed for \( N \)-phase multi-way relaying. From sum rate analysis, MMSE outperforms the other beamformers at the expense of using feedback channel to obtain the noise variance of the nodes. If interference cancellation is performed at all nodes, MSNR achieves the highest performance gain.

I. INTRODUCTION

The two-way (bidirectional) communication channel between two nodes which want to communicate to each other is introduced in [1]. Recently, as relay communication becomes an interesting topic of research, the work of [1] is extended by other works, such as [2]–[6], to the case of bidirectional communication using a relay station (RS).

Bidirectional communication using an RS can be realised by the one-way relaying or the two-way relaying protocol. In [2], it is shown that two-way relaying outperforms one-way relaying in terms of sum rate performance as it uses the resources more efficiently. Two-way relaying adopts the idea of network coding [7] where the RS uses either analog network coding [2], [3] or digital network coding [2], [4]–[6].

An RS applying analog network coding can be classified as a non-regenerative RS since the RS does not try to regenerate (decode and reencode) the data streams of the nodes. A non-regenerative RS has three advantages: no decoding error propagation, no delay due to decoding and deinterleaving and transparency to the modulation and coding schemes being used at the nodes [8].

A multi-antenna non-regenerative RS which serves one bidirectional node pair using two-way relaying is considered in [8]–[10]. While [8], [9] assume multi-antenna nodes, [10] assumes single antenna nodes. Their works consider optimal beamforming to maximise the sum rate as well as linear transceive beamforming based on Zero Forcing (ZF) and Minimum Mean Square Error (MMSE), and in [8] also maximisation of Signal to Noise Ratio (MSNR) criteria.

Multi-user two-way relaying is treated in [11]–[13] where a regenerative RS serves more than one bidirectional pair. In [11], all bidirectional pairs are separated using Code Division Multiple Access. Every two nodes in a bidirectional pair have their own code which is different from other pairs’ codes. In contrast to [11], the separation in [12] and [13] is done spatially through the use of multiple antennas at the RS.

A different scenario where more than two nodes are in the same communication group may appear, such as in video conference or multi-player gaming. In such applications, multiple nodes want to communicate to each other. Until now, there are only few works on such multi-way channel, e.g., the work in [14], [15], in which [1] is a special case when the number of nodes is \( N = 2 \).

There is different notion in [14] and [15] about a multi-way channel. Here, we are using the same notion as in [15], an \( N \)-node multi-way channel is one in which each node has a message and wants to decode the messages from all other nodes. An example scenario is given in [15] where members of an emergency response team, each equipped with a wireless enabled device, wants to transmit and receive data to and from all other responders at a disaster site. In a disaster site where the communication infrastructures are down, an alternative multi-way communication using a newly installed RS may help the emergency response team members to communicate to each other.

In multi-way communication, if all \( N \) nodes are half-duplex and there are direct links between them, the required number of communication phases in order for each node to obtain the information from all other nodes is \( N \), as depicted in Figure 1.(a) for the case of 3 nodes, namely, S0, S1 and S2. Assuming there are no direct links between the nodes and they communicate through the assistance of an RS, if the RS applies one-way relaying protocol, the required number of phases is \( 2N \), as shown in Figure 1.(b) for 3 nodes example case. In the first phase, S0 sends \( x_0 \) to RS and in the second phase, the RS forwards \( x_0 \) to S1 and S2. In the third phase, S1 sends \( x_1 \) to RS and in the fourth phase, the RS forwards \( x_1 \) to S0 and S2. In the fifth phase, S2 sends \( x_2 \) to RS and in the sixth phase, the RS forwards \( x_2 \) to S0 and S1.

In this paper, we propose a non-regenerative multi-way relaying in which the required number of phases is only \( N \).
We choose non-regenerative due to the advantages of non-regenerative RS as mentioned above. The RS is equipped with multiple antennas to spatially separate the signals received from and transmitted to all nodes. Our work is a generalisation of non-regenerative two-way relaying, such that if \( N = 2 \), it becomes non-regenerative two-way relaying protocol. Figure 1(c) shows the proposed 3-phase multi-way relaying for 3 nodes. In the first phase, \( S_0 \) sends \( x_0 \), \( S_1 \) sends \( x_1 \) and \( S_2 \) sends \( x_2 \) simultaneously to the RS. In the second phase, the RS forwards \( x_0 \) to \( S_2 \), \( x_1 \) to \( S_0 \) and \( x_2 \) to \( S_1 \). In the third phase, the RS forwards \( x_0 \) to \( S_1 \), \( x_1 \) to \( S_2 \) and \( x_2 \) to \( S_0 \). After completing 3 communication phases, each node receives the data streams from all other nodes.

The contribution of this paper can be summarised as follows: 1. We propose a multi-way protocol which requires only \( N \) phases to support multi-way relaying for \( N \) nodes. 2. We derive the achievable sum rate for the multi-way relaying protocol for asymmetric and symmetric traffic cases. 3. We design low complexity linear transceive beamforming for \( N \)-phase multi-way relaying.

This paper is organised as follows. Section II describes the protocol and the system model of the \( N \)-phase non-regenerative multi-way relaying. Section III explains the achievable sum rate of the \( N \)-phase multi-way relaying. The design of the linear transceive beamforming is explained in Section IV. The sum rate performance analysis is given in Section V. Finally, Section VI provides the conclusion.

Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts \((\cdot)^T\) and \((\cdot)^H\) stand for matrix or vector transpose and complex conjugate transpose, respectively. The operators \(	ext{mod}_N(x)\), \(\mathbb{E}(X)\) and \(\text{tr}(X)\) denote the modulo \( N \) of \( x \), the expectation and the trace of \( X \), respectively, and \( \mathcal{CN}(0, \sigma^2) \) denotes the zero-mean complex normal distribution with variance \( \sigma^2 \).

II. PROTOCOL AND SYSTEM MODEL

In this section, the communication protocol and the system model of \( N \)-phase multi-way relaying are described. There are \( N \) single antenna nodes which want to communicate to each other through a multi-antenna RS with \( M \) antenna elements. Although in this paper we only consider single antenna nodes, our work can be readily extended to the case of multi-antenna nodes. It is assumed that \( M \geq N \) since the RS applies low complexity linear transceive beamforming.

In the first phase, all nodes transmit simultaneously to the RS in a so called multiple access (MAC) phase. The RS is equipped with multiple antennas to separate the signals from all nodes. In the following \((N-1)\) phases, the RS sends to an intended node the intended \((N-1)\) signals from \((N-1)\) other nodes in the so called broadcast (BC) phases. In each of these \((N-1)\) BC phases, the RS sends simultaneously \( N \) separated signals from the MAC phase to \( N \) different nodes. The separation of the signals in each of the \((N-1)\) BC phases is done spatially through the use of multiple antennas.

In the MAC phase, all nodes transmit simultaneously to the RS. The received signal at the RS is given by

\[
r_{\text{RS}} = Hx + z_{\text{RS}},
\]

where \( H \in \mathbb{C}^{M \times N} = [h_0, \ldots, h_{(N-1)}] \) is the overall channel matrix, with \( h_i \in \mathbb{C}^{M \times 1} = [h_{i,1}, \ldots, h_{i,M}]^T \), \( i = 0, \ldots, (N - 1) \), the channel vector between node \( i \) and the RS. The channel coefficient \( h_{i,m}, m = 1, \ldots, M \), follows \( \mathcal{CN}(0, \sigma_{ii}^2) \). The vector \( x \in \mathbb{C}^{N \times 1} \) is a vector of \( (x_0, \ldots, x_{(N-1)})^T \), with \( x_i \) the signal of node \( i \) which follows \( \mathcal{CN}(0, \sigma_{ii}^2) \). The AWGN noise vector at the RS is denoted as \( z_{\text{RS}} \in \mathbb{C}^{M \times 1} = (z_{\text{RS}1}, \ldots, z_{\text{RS}M})^T \) with \( z_{\text{RS}m} \) follows \( \mathcal{CN}(0, \sigma_{ZRS}^2) \).

Since the RS is a non-regenerative RS, it will neither decode nor reencode the data streams. The RS simply performs transceive beamforming, which spatially separates the signals received from and transmitted to all nodes. In the \( n \)-th BC phase, \( n = 2, \ldots, N \), the RS applies transceive beamforming to the received signals in order to forward each of the intended signal to each of the intended node. If in the \( n \)-th phase a receive node \( k, k = 0, \ldots, (N - 1) \), receives from node \( i, i = 0, \ldots, (N - 1) \), their relationship is defined by

\[
i = \text{mod}_N(k + n - 1).
\]

Assuming reciprocal and stationary channels in \( N \) phases,
the downlink channel from the RS to the nodes is simply the transpose of the uplink channel $H$. The received signal vector of all nodes in the $n$-th BC phase can be written as

$$\mathbf{r}_{\text{nodes}}^n = H^T G^n (\mathbf{Hx} + z_{\text{RS}}) + z_{\text{nodes}},$$

(2)

and, accordingly, the received signal at node $k$ while receiving the data streams from node $i$ in the $n$-th BC phase is given by

$$r_{k,i}^n = h_k^T G^n h_i x_i + \sum_{j=0}^{N-1} h_k^T G^n h_j x_j + h_k^T G^n z_{\text{RS}} + z_k,$$

(3)

where $G^n$ is the $n$-th phase transceive beamforming matrix, $z_{\text{nodes}} = (z_0, \ldots, z_{N(N-1)})^T$ with $z_k$ follows $CN(0, \sigma_k^2)$. We assume, in each BC phase the RS transmits with power $P_{\text{RS}}$.

In multi-way relaying, each node wants all other nodes’ signals. However, BC’s interference signals may appear at all nodes. The BC’s interference signals are the received signals at node $k$ in the $n$-th BC phase which are not coming from node $i$. It consists of two parts, namely, self-interference which is node $k$’s own transmitted signal and other-stream interference signals which are not from node $k$ and node $i$. Since node $k$ knows its own signal, it may perform self-interference cancellation. Regarding other-stream interference signals, since node $k$ basically wants to receive all other nodes’ signals, it may perform successive other-stream interference cancellation. In the $n$-th BC phase, assuming node $k$ has been able to decode correctly all the data streams from the previous BC phases, it may subtract them from the received signal before it decodes the corresponding data streams. If the RS applies transceive beamforming which is able to nullify interference, such as ZF transceive beamforming, BC’s interference signals will not appear at each node.

### III. Achievable Sum Rate

In this section we define the signal to interference and noise ratio (SINR) of node $k$ when receiving from node $i$ in the $n$-th phase and derive the achievable sum rate for multi-way relaying for two cases, namely, asymmetric traffic and symmetric traffic. Asymmetric traffic case refers to the situation where we allow all nodes to transmit with different rate. Each node transmits with a rate which ensure that in the following $(N - 1)$ consecutive BC phases, all $(N - 1)$ intended receive nodes can decode the data streams correctly. Symmetric traffic case is the case where all nodes have to transmit simultaneously with the same rate which is defined by the lowest link between all the possible link combinations of receive and transmit node $(k, i)$.

#### A. Signal to Interference and Noise Ratio

Given the received signal as in (3), the SINR $\gamma_{k,i}$ for the link between node $k$ and $i$ is given by

$$\gamma_{k,i} = \frac{S}{I_s + I_{on} + Z_{\text{RS}} + Z_k},$$

(4)

with the useful signal

$$S = E\{ |h_k^T G^n h_i x_i|^2 \} = |h_k^T G^n h_i|^2 \sigma_i^2,$$

the self-interference

$$I_s = E\{ |h_k^T G^n h_k x_k|^2 \} = |h_k^T G^n h_k|^2 \sigma_k^2,$$

the other-stream interference

$$I_{on} = \sum_{j=0}^{N-1} E\{ |h_k^T G^n h_j x_j|^2 \} = \sum_{j=0}^{N-1} |h_k^T G^n h_j|^2 \sigma_j^2,$$

(5)

(6)

(7)

(8)

the RS’s propagated noise

$$Z_{\text{RS}} = E\{ |h_k^T G^n z_{\text{RS}}|^2 \} = |h_k^T G^n|^2 \sigma_{z_{\text{RS}}}^2$$

(9)

In every $n$-th BC phase, node $k$ may perform interference cancellation. It subtracts the a priory self-interference as well as the a priory other-stream interference from the previous BC phases. With interference cancellation, the SINR can be rewritten as

$$\gamma_{k,i} = \frac{S}{I_{can} + Z_{\text{RS}} + Z_k},$$

(10)

where

$$I_{can} = \sum_{j=0}^{N-1} |h_k^T G^n h_j|^2 \sigma_j^2,$$

(11)

is the BC’s interference signals without self-interference and other-streams interference that have been decoded in the previous BC phases, with $L = \{ \mod_N(k + q - 1), \forall q, q = 2, \ldots, (n - 1) \}$.

#### B. Asymmetric Traffic

The achievable sum rate of non-regenerative multi-way relaying is given by

$$SR = \frac{1}{N} (N - 1) \sum_{i=0}^{N-1} R_i,$$

(12)

where

$$R_i = \min (R_{k,i}), \forall k, k = 0, \cdots, (N - 1), k \neq i$$

(13)

is the minimum rate among all nodes $k$ when they receive the data streams from node $i$ with

$$R_{k,i} = \log_2(1 + \gamma_{k,i})$$

(14)

and $\gamma_{k,i}$ given in subsection III-A.

The achievable sum rate of multi-way relaying is defined by (13) since the nodes transmit only once. The factor $(N - 1)$ is due to the fact that in $(N - 1)$ consecutive BC phases there are $(N - 1)$ nodes which receive the same data streams from a certain transmit node. The scaling factor $\frac{1}{N}$ is due to $N$ channel uses for $N$-phase multi-way relaying.
C. Symmetric Traffic

In certain scenarios, there might be a requirement to have a symmetric traffic between all nodes. All nodes communicate with the same traffic rate defined by the minimum of $R_i, \forall i, i = 0, \ldots, (N - 1)$. The achievable sum rate becomes

$$SR_{symm} = \frac{1}{N} (N - 1) N (\min(R_i)).$$

(15)

IV. TRANSCIEVE BEAMFORMING

In this section, the design of three low complexity transceive beamformers for $N$-phase non-regenerative multi-way relaying is explained. Three different criteria are considered, namely, ZF, MMSE and MSNR. The receive and transmit beamforming based on these criteria for two-way relaying is given in [8] and, here, we extend them to the case of multi-way relaying. For $N$-phase non-regenerative multi-way relaying, as there is only one MAC phase, the receive beamforming is needed to be computed only once.

A. Zero Forcing

For multi-way relaying, the ZF receive beamforming is given by

$$G_{Re} = (H^H R_{zRe}^{-1} H)^{-1} H^H R_{zRe}^{-1}$$

and the $n$-th phase ZF transmit beamforming is given by

$$G^n_{Tx} = \frac{1}{p^n_{ZF}} H^H_{Tx^n} (H_{Tx^n}^H H^H_{Tx^n})^{-1},$$

(17)

with

$$p^n_{ZF} = \sqrt{\text{tr}\{ (H^H H_{Re}^{-1} H)^{-1} (H_{Tx^n}^H H^H_{Tx^n})^{-1} \}}$$

(18)

and

$$\Upsilon_{Re} = H R_x^H + R_{zRe},$$

(19)

where $H_{Tx^n}$ is the $n$-th BC phase downlink channel matrix, $R_{zRS} = \mathbb{E}\{z_{zRS} z_{zRS}^H\}$ is the covariance matrix of RS' noise, $R_x = \mathbb{E}\{x x^H\}$ is the covariance matrix of the transmitted signal and $E_{RS}$ is the transmit power of the RS.

B. Minimum Mean Square Error

For multi-way relaying, the MMSE receive beamforming is given by

$$G_{Re} = R_x H^H \Upsilon_{Re}^{-1}$$

(20)

and the $n$-th phase MMSE transmit beamforming is given by

$$G^n_{Tx} = \frac{1}{p^n_{MMSE}} \Upsilon_{Tx^n}^{-1} H^H_{Tx^n},$$

(21)

with

$$p^n_{MMSE} = \sqrt{\text{tr}\{ H R_x H_{Tx^n}^H \Upsilon_{Tx^n}^{-2} H_{Tx^n}^H R_x H^H_{Tx^n} \Upsilon_{Re}^{-1} \}}$$

(22)

and

$$\Upsilon_{Tx^n} = H_{Tx^n}^H H_{Tx^n} + \frac{\text{tr}\{ R_{znodes} \}}{E_{RS}} I_M,$$

(23)

where $R_{znodes} = \mathbb{E}\{z_{nodes} z_{nodes}^H\}$ is the covariance matrix of the noise vector of all nodes.

C. Maximisation of Signal to Noise Ratio

For multi-way relaying, the MSNR receive beamforming is given by

$$G_{Re} = R_x H^H \Upsilon_{Re}^{-1}$$

(24)

and the $n$-th phase MSNR transmit beamforming is given by

$$G^n_{Tx} = \frac{1}{p^n_{MSNR}} H^H_{Tx^n},$$

(25)

with

$$p^n_{MSNR} = \sqrt{\frac{\text{tr}\{ H_{Tx^n} H^H_{Tx^n} R_x H^H_{Tx^n} \Upsilon_{Re}^{-1} R_x H_{Tx^n} \}}{E_{RS}}}.$$

(26)

D. $n$-th BC Phase Transceive Beamforming

Assuming reciprocal and stationary channel within $N$ phases, we can compute the transmit beamforming only once by defining the transmit channel matrix $H^H_{Tx} = H^H_{Tx} = H^\dagger$. We use the permutation matrix $\Pi^n$ to define the relationship between receiving node $k$, transmitting node $i$ and the corresponding phase $n$. The $\Pi^n$ is given by the operation $\Pi^n = \text{colperm}(I_3, (n - 1))$ with $I_3$ an identity matrix of size $N$. The $\Pi^n$ permutes circularly to the right the columns of the identity matrix $(n - 1)$ times. For example, for 3 nodes multi-way relaying, the permutation matrix $\Pi^n = \left( \begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{array} \right)$. Finally, the $n$-th phase transceive beamforming for the multi-way relaying is defined as

$$G^n = G_{Tx^n} \Pi^n G_{Re}.$$

(27)

If the channel coefficients are not reciprocal and stationary, we need to compute the transmit beamforming for each BC phase. We define the $n$-th BC phase downlink channel matrix as $H_{Tx^n} = H^H_{Tx^n} \Pi^n$ and, consequently, we need to change (2) and (3) accordingly. The $n$-th phase transceive beamforming for multi-way relaying is given by

$$G^n = G_{Tx^n} G_{Re}.$$

(28)

V. PERFORMANCE ANALYSIS

In this section, the sum rate performance is analysed in a scenario where $N = 3$ single antenna nodes communicate to each other with the help of an RS with $M = 3$ antenna elements. We set $\sigma^2_{zRS} = 1, \forall k, \sigma^2_{z_i} = 1, \forall i, E_{RS} = 1$ and SNR = $\sigma^2_{z_i}$.

Figure 2 shows the sum rate performance of multi-way relaying using three linear transceive beamformers for the asymmetric case, with and without interference cancellation. The MMSE performs the best at the expense of using feedback channel since the RS needs the information of the noise variance of the nodes. As it can be seen, MMSE converges to ZF in the high SNR region and, even though it is not shown in the figure, in very low SNR MMSE converges to MSNR. In mid to high SNR, the MSNR reaches saturation as a consequence of not trying to manage the interference among the streams.

Interesting to be noticed is the impact of interference cancellation. ZF, which forces the interference to be zero,
does not have any performance improvement since there is no appearance of the BC’s interfering signals at all nodes. Using ZF at the RS helps reducing the complexity at all nodes as they do not need to apply interference cancellation. MMSE is able to obtain a performance gain if interference cancellation is applied. Since the MSNR maximises the SNR without managing the interference, it obtains the highest performance gain if interference cancellation is applied at all nodes.

Figure 3 shows the sum rate performance of multi-way relaying for symmetric case. The sum rate performance in the symmetric case is lower than in the asymmetric case and the performance improvement due to the interference cancellation is not as high as in the case of asymmetric case. This is the consequence of taking the rate of the weakest link as the transmission rate of all nodes.

VI. CONCLUSION

In this paper, we introduce non-regenerative multi-way relaying where N single antenna nodes want to communicate to each other through the assistance of a multi-antenna RS. We propose an N-phase multi-way relaying protocol which consists of one MAC phase and (N − 1) BC phases. We derive the achievable sum rate for non-regenerative N-phase multi-way relaying for both asymmetric and symmetric traffic, with and without interference cancellation. Three low computational complexity transceive beamforming, namely ZF, MMSE and MSNR, are designed for the N-phase multi-way relaying. From the sum rate analysis, MMSE outperforms the other beamformers at the expense of using feedback channel to obtain the noise variance of the nodes. MSNR achieves the highest performance gain if interference cancellation is performed at all nodes.

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