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Dynamic Resource Assignment (DRA) with Minimum Outage in Cellular Mobile Radio Networks

Philipp P. Hasselbach and Anja Klein
Communications Engineering Lab
Technische Universität Darmstadt, Darmstadt, Germany
Email: {p.hasselbach, a.klein}@nt.tu-darmstadt.de

Ingo Gaspard
T-Systems Enterprise Services GmbH
Darmstadt, Germany
Email: ingo.gaspard@t-systems.com

***Abstract*—The assignment of resources to cells of a cellular mobile radio network is an important task in design and operation of a network. In practice, the number of resource units that are available for assignment to the cells is limited. As a consequence, not all cells can generally receive as many resource units as demanded and outage, defined as the number of resource units requested but not assigned, occurs. In this paper, the dynamic assignment of resources to the cells of a cellular network is discussed under the assumption that the number of resource units available for the assignment is not in all cases sufficient to fulfil the resource demand of every cell and that the occurring outage has to be minimised. An efficient method of determining the number of resource units required for outage-free resource assignment is presented. This method is used to determine outage probability and a lower bound for the amount of outage. Finally, a policy based algorithm for the assignment of resources with very low complexity is presented and its performance in terms of amount of outage compared to the lower bound evaluated.**

I. Introduction

Future mobile radio networks are bound to support an increasing number of users with increasing capacity requirements. In order to be able to efficiently fulfil these rising demands, the overall spectrum efficiency of the networks has to be increased.

Cellular mobile radio networks are able to deliver high system capacity using limited resources due to the ability to use the same resources in several cells [1], [2]. This reuse of resources, however, leads to interference between cells using the same resources, the so called inter-cell interference [1], [2]. The assignment of resources to cells therefore has to be done wisely in order to assure an inter-cell interference below a certain maximum limit.

In real networks, the number of resource units available for the distribution among the cells of the network is limited. Fixed Channel Allocation (FCA) [3] enables a static assignment of resources to cells based on assumptions concerning the maximum capacity demands of the cells. It assures inter-cell interference below a maximum level but due to its static nature and the assumptions concerning the maximum demands of the cells, FCA leads to inefficient resources usage and therefore low spectral efficiency. In order to increase the spectral efficiency, Dynamic Channel Allocation (DCA) [3], which assigns the available resources to cells according to the actual demands of the cells, is applied. Since the number of resource units available for distribution among the cells is limited, outage, defined as the number of resource units requested but not assigned, occurs if the number of resource units is not sufficient.

FCA and DCA are both approaches to solving frequency assignment problems (FAPs) [4], [5]. In [4], a general FAP is discussed as a graph theory problem, [5] formulates different FAPs as combinatorial optimisation problems. The solution of FAPs, however, is very complex, since they belong to the class of NP-complete problems [4]. Several approaches have been made in order to reduce complexity using suboptimum heuristics, such as Neural Networks [6]–[8], Genetic and Evolutionary Algorithms [9]–[12], Local Search techniques [13], [14] or Particle Swarm Optimisation [15].

In this paper, a FAP that minimises network outage observing a minimum spatial separation between cells that use the same resource units is formulated. The number of resource units available for the distribution among the cells is fixed and in general not sufficient to fulfil the resource demand of every cell. An efficient approach for the determination of the number of resource units required by the network is proposed and investigated. This approach is then applied to find probability and amount of outage. Finally, a suboptimum algorithm solving the presented FAP with very low complexity is proposed and evaluated.

The paper is organised as follows: Section II presents the system model and formulates the FAP. Section III introduces and investigates the approach for determining the number of resource units required for outage-free resource assignment and for deriving the outage probability of the network. This approach is applied in Section IV to determine for a certain outage probability a lower bound for the amount of outage that occurs with the dynamic assignment of resources. In Section V, the suboptimum policy based algorithm to solve the presented FAP is proposed and compared to the lower bound derived in the preceding section. Section VI concludes the paper.

II. System Model and Problem Description

A cellular network with N_C cells of radius R arranged in a hexagonal grid layout is considered. Each cell i, $i =$

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$1, \ldots, N_\mathrm{C}$, has a resource demand D_i which is counted in resource units and which can be different for different cells. The resource demands of the N_C cells are combined by the demand vector $\mathbf{D} = \left[\begin{array}{cccc} D_1 & D_2 & \ldots & D_{N_\mathrm{C}} \end{array} \right]$.

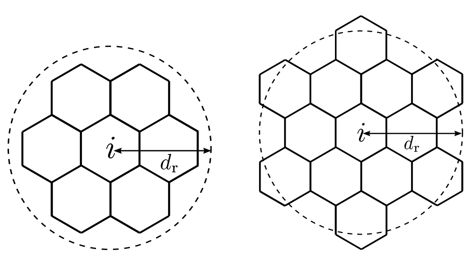
The cells are grouped to form clusters. Within a cluster, no resource unit can be reused. Clusters have certain shapes that are capable of covering a plane surface. Their size r is a rhombic number [1], [2].

Each resource unit can be used by several cells. The minimum spatial distance between the centres of two cells that may use the same resource unit is the reuse distance $d_\mathrm{r} = \sqrt{3r} \cdot R$ [1]. A resource unit can be of any type of resource, e. g. time slots, frequency bands, channels, subtones, codes, etc. Throughout this paper, the term Dynamic Resource Assignment (DRA) will therefore be used instead of DCA.

The neighbourhood $\mathbb{N}(i)$ of cell i are all cells whose centres are at a distance of less than the reuse distance d_r from the centre of cell i, including cell i. It is defined by

$$\mathbb{N}(i) = \{k | d(i,k) < d_\mathrm{r}\} \,, k \in \{1, \ldots, N_\mathrm{C}\} \,, \qquad (1)$$

with $d(i,k)$ the distance between the centres of cell i and cell k. Fig. 1 shows examples of the neighbourhoods of cell i for different values of r. Following the definition of neighbourhood, two cells i and k are neighbours if $k \in \mathbb{N}(i)$ and, vice versa, $i \in \mathbb{N}(k)$ holds.



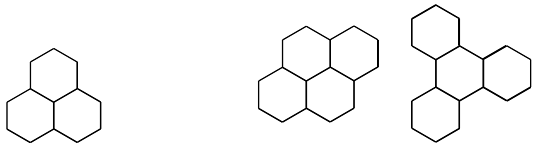
(a) $r = 3$. (b) $r = 4$.

Fig. 1. Neighbourhood of cell i for different values of r.

The neighbourhood group is defined as every group of r cells that are all neighbours. This holds true for all groups of r cells for that the intersection of their neighbourhoods is a set containing exactly these r cells:

$$\bigcap_{m=1}^{r} \mathbb{N}(k_m) = \{k_1, k_2, \ldots, k_r\} \,, k_m \in \{1, 2, \ldots, N_\mathrm{C}\} \,. \quad (2)$$

Fig. 2 shows examples of neighbourhood groups for different values of r. Note that for certain group sizes, neighbourhood groups may have several different shapes, as Fig. 2 demonstrates. In the following, r will be called neighbourhood group size.



(a) $r = 3$. (b) $r = 4$.

Fig. 2. Possible neighbourhood groups for different values of r.

Finally, the co-channel cells of cell i are all cells whose centres are at a distance of d_r from the centre of cell i,

$$\mathbb{C}(i) = \{k | d(i,k) = d_\mathrm{r}\} \,, k \in \{1, \ldots, N_\mathrm{C}\} \,, \qquad (3)$$

and all resources that are assigned to cell i are represented by the resource group $\mathbb{R}(i)$, defined by

$$\mathbb{R}(i) = \{l | a_{li} = 1\} \,, l \in \{1, \ldots, N_\mathrm{av}\} \,, \qquad (4)$$

with N_av the total number of resource units available and a_{li} the elements of a binary assignment matrix $\mathbf{A}$ whose elements are equal to one if resource unit l is assigned to cell i and zero if not.

The problem discussed in this paper is the assignment of a limited number N_av of available resource units to the N_C cells of a network such that outage is minimised and the reuse distance observed. Outage occurs if the number of resource units that are assigned to a cell is lower than the resource demand of the cell. The number of resource units that are demanded by a cell but not assigned is given by

$$O_i = \begin{cases} D_i - |\mathbb{R}(i)| & \text{if } |\mathbb{R}(i)| < D_i \\ 0 & \text{else} \end{cases} \qquad (5)$$

with $|\mathbb{R}(i)|$ the number of elements in $\mathbb{R}(i)$. The outage probability of the network is given by

$$p_\mathrm{out} = p\left(\sum_{i=1}^{N_\mathrm{C}} O_i > 0\right) \qquad (6)$$

and the amount of outage of the network is defined by

$$O_\mathrm{NW} = \frac{\sum\limits_{i=1}^{N_\mathrm{C}} O_i}{\sum\limits_{i=1}^{N_\mathrm{C}} D_i} \,. \qquad (7)$$

The problem can now be expressed as a constrained combinatorial optimisation problem:

$$\min_{\mathbf{A}} \; O_\mathrm{NW} \qquad (8)$$

$$\text{s.t.} \quad a_{li} \cdot a_{lk} = 0 \quad \forall \quad l, i \wedge k \in \mathbb{N}(i) \qquad (9)$$

$$\mathbf{A} \in \{0,1\}^{(n_\mathrm{av} \times N_\mathrm{C})} \,, \; n_\mathrm{av} \leq N_\mathrm{av} \qquad (10)$$

Equation (9) is the interference constraint, it assures that resource units are reused such that the maximum inter-cell interference is yielded to (i.e., the reuse distance is observed). Equation (10) assures that the number of resource units used in the assignment does not exceed the number of available resource units N_av.

III. Resource Demand and Outage Probability of a Network

This section introduces an efficient method of determining the number of resource units required to fulfil the demand of every cell of a network, expressed by the demand vector

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D. This number is called the network resource demand N_{R}. From this resource demand, the outage probability p_{out} of the network can be determined if the number N_{av} of available resource units is given.

Lemma 1: The resource demand of a cellular network is given by the sum demand of the neighbourhood group with the largest sum demand.

Proof: The network described in Section II can also be modeled by a graph, the so called interference graph [4], [16]

$$G = (V, E) \tag{11}$$

with V the set of the vertices representing the positions of all cells in the plane and E the collection of all pairs of vertices or cells, respectively, whose centres are at a distance of less than the reuse distance [4], [5]. The graph of (11) has different multiplicities at its nodes since the cells can have varying demands of several resource units. The multiplicities can be treated by splitting each node with a multiplicity of larger than one into a number of copies that is equal to the respective multiplicity. The resulting graph is called a split interference graph [5].

It is noted that the FAP corresponds to the graph colouring problem [17]. For the proof of Lemma 1, at first some terms from graph theory will be defined and analogies to terms introduced in Section II will be pointed out. In a second step, a graph theoretic proof will be made applying terms and relations from graph colouring.

Some definitions that are important in the following will be made [4], [17]. The definitions hold for a split interference graph with multiplicities of one. A graph is complete if $uv \in E$ holds for all $u, v \in V$ and $u \neq v$. A graph F is a subgraph of graph G if the set of vertices of F is a subset of V and if the collection of all pairs of vertices of graph F is a subcollection of E. A clique H of G is a complete subgraph of G that is not properly contained in another subgraph of G. The clique number $\omega(G)$ is given by the number of vertices of the largest clique of G. The chromatic number $\chi(G)$ is the number of colours that are required to colour the graph.

With these definitions, it can be seen that the definition of the neighbourhood group, as it is given in Section II, corresponds to the definition of the clique, since each neighbourhood group is represented in the interference graph G of the network by a complete subgraph of G that is not properly contained in another subgraph of G. The clique number therefore corresponds to the sum demand of the neighbourhood group with the largest sum demand.

According to [4], [5], [17], the clique number $\omega(G)$ is a lower bound for the chromatic number $\chi(G)$. In case of a perfect graph, $\chi(G) = \omega(G)$ holds [17], which means that the clique number is a reachable lower bound.

In order to proof Lemma 1, it has to be shown that the interference graph of a network is perfect. This is done by noting that chordal, or triangulated, graphs are perfect [17]. Chordal graphs do not contain cycles of length larger than three [17] and remembering that any pair of neighboured cells corresponds to an edge of graph G, it can be seen that the interference graph of a hexagonal cellular network that covers a convex area without holes is chordal and thus perfect.

The number of colours required to colour the interference graph of a network is therefore equal to the chromatic number of the graph, which means that the number of resource units required to fulfil the resource demand of every cell is equal to the largest sum demand of any possible neighbourhood group.

Denoting the set of the sum demands of all possible neighbourhood groups by $\mathbb{S}$, the network resource demand is therefore given by

$$N_{\mathrm{R}} = \max \{\mathbb{S}\}. \tag{12}$$

Probability density function (PDF) and cumulative distribution function (CDF) of the network resource demand can be calculated numerically using Lemma 1. Fig. 3 shows the CDF $F(N_{\mathrm{R}})$ of the network resource demand for different neighbourhood group sizes r and for a number N_{C} of cells of 16 and 25 times, respectively, the neighbourhood group size. The resource demands of the cells are assumed to be independent and uniformly distributed in the interval $[50, 150]$.

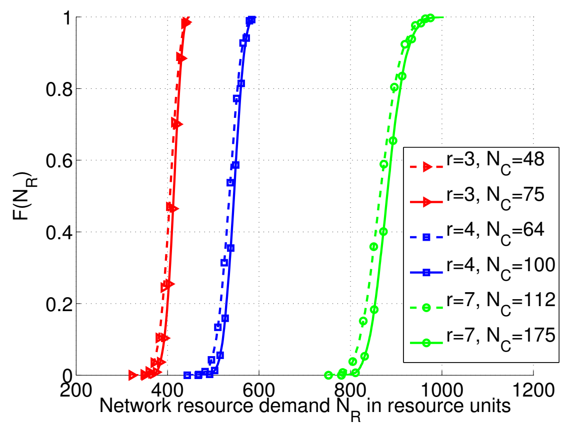

Fig. 3. CDF of the network resource demand N_{R} for different neighbourhood group sizes r and number of cells N_{C}. The cell demands are assumed to be independent and uniformly distributed in the interval $[50, 150]$.

The outage probability of the network is given by the percentage for which the number N_{av} of available resource units is smaller than the network resource demand N_{R}:

$$p_{\mathrm{out}} = 1 - F(N_{\mathrm{av}}). \tag{13}$$

Fig. 3 thus shows that for larger networks and larger neighbourhood group sizes, more resource units are required in order to guarantee a certain outage probability.

Given the network size and knowing the statistical properties of the demand of the cells, the outage probability related to a certain number N_{av} of available resource units can therefore be calculated from the CDF of the network resource demand N_{R}. Also, if a maximum allowed outage probability is given and if the statistical properties of the cell demands are known, the network resource demand N_{R} to comply with

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the maximum allowed outage probability can be read from the CDF.

IV. Amount of Outage

Following Lemma 1, the sum demand of a group can be fulfilled as long as it is less than or equal to the number N_{av} of available resource units. Otherwise, outage occurs in the respective neighbourhood group.

The neighbourhood groups in which outage occur can thus be easily determined and a modified demand vector $\tilde{\mathbf{D}}$, which never causes the sum demand of a neighbourhood group to exceed the number N_{av} of available resource units and which at the same time causes only minimum amount of outage compared to the original demand vector $\mathbf{D}$, can be derived. In the following, a method for obtaining such a modified demand vector is presented.

Lemma 2: The minimum outage can be achieved if those cells, that are most often contained in the neighbourhood groups whose sum demands exceed the number N_{av} of available resource units, are assigned less resource units than demanded.

Proof: Let $\mathbb{O}$ be the set of neighbourhood groups whose sum demand exceeds the number N_{av} of available resource units. A_i denotes the number of neighbourhood groups that belong to $\mathbb{O}$ and that contain cell i. Each resource unit that is demanded by cell i but not assigned to the cell then reduces the sum demand of A_i of the neighbourhood groups from set $\mathbb{O}$ by one resource unit. With the definition of (7), it is therefore most efficient in terms of minimum amount of outage to deny resource units to those cells that appear most often in the neighbourhood groups of set $\mathbb{O}$, i.e. the cells with largest A_i.

Fig. 4 shows the CDF of the amount of network outage of DRA for different neighbourhood group sizes r and a number of cells of 25 times the group size. The resource demands of the cells are assumed to be independent and uniformly distributed in the interval $[50, 150]$ and the number N_{av} of available resource units has been chosen from Fig. 3 such that the network outage probability p_{out} is 95%. The CDF was determined using Lemma 2.

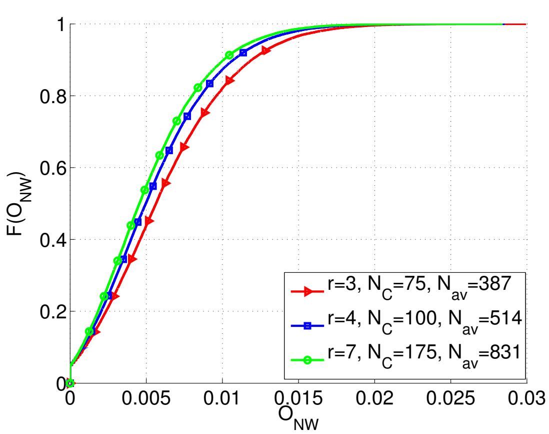

Fig. 4. CDF of the amount of outage of the network in DRA for different neighbourhood group sizes r and $25 \cdot r$ cells. The cell demands are independent and uniformly distributed in the interval $[50, 150]$ and the number N_{av} of available resource units has been chosen for 95% network outage probability.

With the chosen parameters, DRA therefore achieves in five percent of all cases an outage of zero, cf. Fig. 4. Although the network outage probability is high, the amount of outage that occurs in the network is below three percent. The results shown correspond to the optimum solution of the FAP formulated in Section II.

Fig. 4 also shows that the amount of outage decreases with increasing neighbourhood group size r. This is due to the fact that for larger neighbourhood group sizes, the sum demand of the group is calculated over more independent cells, making it less likely to exceed the number N_{av} of available resource units.

V. Resource Assignment Algorithm

This section presents a policy based resource assignment algorithm for solving the FAP formulated in Section II with very low complexity. The algorithm tries to assign as many resource units as possible with the reuse distance d_{r}, meaning that two cells whose centres are at a distance of d_{r} shall use as much as possible the same resource units.

The N_{C} cells are divided into r cell groups $\mathbb{G}_m^{\mathrm{C}}$, $m = 1 \ldots r$. The groups are constructed such that the centres of all cells within each group are at a distance of multiples of the reuse distance d_{r}. The N_{av} available resource units are divided into r resource groups $\mathbb{G}_m^{\mathrm{R}}$, $m = 1 \ldots r$.

A modified demand vector $\tilde{\mathbf{D}}$ is derived from demand vector $\mathbf{D}$ using Lemma 2. According to Section IV, this modified demand vector is such that N_{av} resource units are sufficient to fulfil the modified demand of every cell and that the difference to the original demand vector $\mathbf{D}$ corresponds to the minimum amount of outage.

The proposed algorithm uses the modified demand vector $\tilde{\mathbf{D}}$ to determine the cell demand in the assignment of resources. The algorithm is described below.

1) Search for the neighbourhood group with maximum sum demand and assign resource units as demanded by the group members. The interference constraint of (9) has to be observed in the assignment, meaning that only resource units, which are not yet assigned to any neighbours can be assigned to a cell. This initial assignment determines size and elements of the r resource groups $\mathbb{G}_m^{\mathrm{R}}$.
2) Search for cells that are neighbours to $r-1$ of any of the cells which already have been resources assigned. Assign resource units to these cells from the resource group with the same index m as the cell group to which the cells belong. Assign preferably the same resource units as assigned to the co-channel cells of a cell as defined in (3). The interference constraint of (9) has to be observed.
3) If the resource demand of a cell is not fulfilled by step 2), assign as many resource units as possible observing the interference constraint (9) from the resource group

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$\mod(m,r)+1$, with $\mod$ the modulo operation, to the cell. Repeat this step with resource units from resource group $\mod(m+1,r)+1$, $\mod(m+2,r)+1$, ... until the resource demand of the cell is fulfilled or all r resource groups have been considered.

4) Go back to step 2) until all N_{C} cells have been treated.

According to the above description, the algorithm runs in $O(N_{\mathrm{C}})$ time. Note that the complexity of the algorithm does not depend on the number N_{av} of available resource units, allowing thus very fine granularity and the use of very small resource unit sizes.

The presented algorithm is suboptimum in terms of the amount of outage and therefore in general not able to find an optimum solution to the FAP of Section II, although the number N_{av} of available resource units is sufficient to fulfil the modified demand vector $\tilde{\mathbf{D}}$. The algorithm thus increases the amount of network outage, compared to an optimum algorithm, and the results of Section IV are a lower bound for the performance of the algorithm.

Fig. 5 shows the CDF of the amount of network outage of the algorithm in comparison to the lower bounds and for the same parameter values as in Section IV. It can be seen

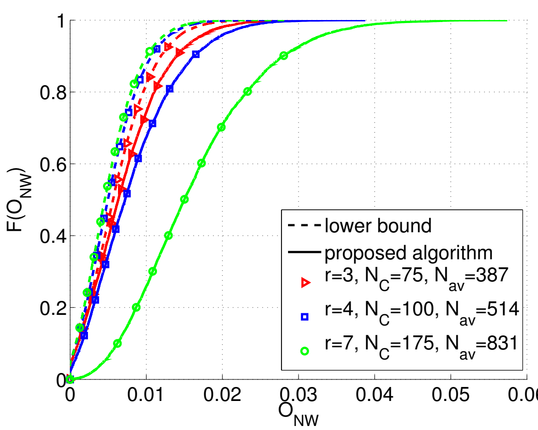

Fig. 5. CDF of the amount of outage of the network for the proposed policy based resource assignment algorithm for different neighbourhood group sizes r in comparison to the lower bounds of Fig. 4.

that in contrast to Fig. 4, the amount of outage increases with increasing neighbourhood group size r. The reason is that in case of a cell demand exceeding the number of resource units available from the respective resource group $\mathbb{G}_m^{\mathrm{R}}$, resource units are assigned from the next resource group, resource group $\mathbb{G}^{\mathrm{R}}{}_{\mod(m,r)+1}$. This choice is more probable to be a bad choice in terms of amount of outage in case of larger neighbourhood group sizes.

VI. Conclusion

In this paper, the dynamic assignment of a limited number of resource units to the cells of a cellular mobile radio network is discussed under the assumption that the number of resource units available for the assignment is not in all cases sufficient to fulfil the resource demand of every cell. A new method of determining the number of resource units that are required to fulfil a certain resource demand is presented. This method allows to efficiently determine probability and amount of outage of a cellular network using DRA. The method is also interesting in connection with resource assignment algorithms since it can be used to derive a modified demand vector that can be fulfilled with the available number of resource units and causes minimum outage, compared to the actual cell demand. A suboptimum policy based resource assignment algorithm using such a modified demand vector is presented. The algorithm has very low complexity and is therefore attractive for the application in self-organising techniques for adapting a cellular network to changing operating conditions and environments.

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