Maximum Sum Rate of Non-regenerative Two-way Relaying in Systems with Different Complexities

(Invited Paper)

Timo Unger and Anja Klein
Technische Universität Darmstadt
Institute of Telecommunications, Communications Engineering Lab
Darmstadt, Germany
Email: {t.unger, a.klein}@nt.tu-darmstadt.de

***Abstract*—This paper considers the two-hop relaying case with bi-directional communication of two multiple-antenna nodes S1 and S2 via an intermediate multiple-antenna node, namely the relay station (RS). A general framework for the sum rate maximization in linear, non-regenerative two-way relaying with multiple-antenna nodes is proposed assuming all nodes have perfect channel state information (CSI) in order to perform linear beamforming (BF). There may be application cases in which the system cannot provide perfect CSI to all nodes or the nodes cannot be such complex. Hence, three different cases of system complexity are investigated which are shown to be special cases of the introduced framework. Firstly, all nodes can perform BF, secondly only S1 and S2 can perform BF, and thirdly only the RS can perform BF. The optimum performances of all three cases are compared to each other by means of numeric optimizations, and approaches to solve the resulting optimization problems are proposed. It is shown that applying BF exclusively at the RS provides the same performance as applying BF at all nodes.**

I. Introduction

Assuming that a direct wireless communication between two nodes, namely S1 and S2, is not possible, e.g., due to shadowing or limited transmit powers, two-hop relaying is a promising approach in order to enable a communication between S1 and S2. In non-regenerative two-hop relaying, a node, namely the relay station (RS), which is located between S1 and S2 and can communicate with both, S1 and S2, receives a data stream on a first hop, applies linear signal processing to this data stream and retransmits it on a second hop. Applying time division multiplexing (TDM) for half-duplex nodes, one time slot is required for the first hop transmission from S1 to the RS, and another time slot is required for the second hop transmission from the RS to S2. In conventional bi-directional communications, another two time slots are required for the transmission from S2 to S1 via the RS leading to a requirement of overall four time slots. Compared to a point-to-point bi-directional communication between S1 and S2, the number of required time slots is doubled in conventional two-hop relaying. There exist several efficient two-hop resource allocation schemes [1]–[3] which partially compensate this increase in required channel resources in two-hop relaying. In case of bi-directional communication between S1 and S2, two-way relaying [4] is a very promising approach in terms of resource efficiency since it requires only two time slots. In the first time slot, the source nodes S1 and S2 transmit simultaneously to the RS which retransmits the linearly processed superposition of S1's and S2's data streams in the second time slot. The destination nodes S1 and S2 can recover their desired data streams by subtracting their own transmitted but interfering data streams from the received data streams [4].

In two-way relaying, the sum rate is defined as the sum of the individual transmission rates from S1 to S2 and from S2 to S1, respectively. The maximum sum rate of non-regenerative two-way relaying with single antenna nodes has been investigated in [4]. To the best of our knowledge, there exists no closed-form solution for the maximum sum rate in case of multiple-antenna nodes. In this paper, a general framework for sum rate maximization in non-regenerative two-way relaying with multiple-antenna nodes is proposed. From conventional two-hop relaying with multiple-antenna nodes, it is known that the problem of rate maximization can be solved by applying beamforming (BF) exclusively at the RS [5]. For two-way relaying it has not been determined if exclusive BF at the RS is sufficient. For that reason, the following investigations start with a system that provides spatial channel state information (CSI) to all nodes and all nodes require computational capability in order to perform complex matrix operations. In particular, this means that S1, S2 and the RS require one-hop CSI which means that the transmitting nodes require transmit CSI about the channel which is used for their transmission. Furthermore, S1 and S2 also require two-hop CSI which is defined as the CSI about the channel between the RS and the respective other node. While providing one-hop CSI is relatively simple and known from conventional point-to-point communications, providing two-hop CSI is more critical [6].

There may be application cases in which the system cannot provide the required CSI to all nodes or the nodes cannot be such complex. Hence, depending on the general framework for sum rate maximization, a classification into three different cases of system complexity regarding CSI and node complexity is proposed:

Case1 : S1, S2, and the RS are provided with one-hop transmit CSI, and S1 and S2 also with two-hop transmit CSI. All nodes perform BF. This corresponds to a system without complexity constraints.

CASE2 : The RS has no CSI, and exclusively S1 and S2 perform BF which corresponds to a system with a low complexity RS.

CASE3 : S1 and S2 are only provided with one-hop CSI which allows the required interference subtraction in case of two-way relaying, and exclusively the RS performs BF. This corresponds to a system without two-hop CSI.

In this paper, the maximum sum rates of CASE1, CASE2, and CASE3 are compared to each other by means of numeric optimizations. For CASE1 and CASE3, a close-to-optimum approach is introduced for single-antenna source and destination nodes and an approach which reduces the number of optimization variables is proposed for multi-antenna source and destination nodes. For CASE2, a general sub-optimum problem solution is proposed.

The paper is organized as follows. Section II gives a general system model for non-regenerative two-way relaying with a decomposition into useful signal, interference and noise parts leading to the definition of the individual transmission rates. In Section III, the general problem of maximizing the sum rate is introduced and considered for the different cases. Section IV gives a comparison of the sum rate performances of CASE1, CASE2, and CASE3 and Section V concludes this work.

II. System Model

Throughout the paper, complex baseband transmission is assumed. Let $[\cdot]^{\mathrm{T}}$, $[\cdot]^{*}$, $[\cdot]^{\mathrm{H}}$, $\|\cdot\|_2^2$, $(\cdot)^{-1}$, $|\cdot|$, $\det[\cdot]$, $\mathrm{diag}[\cdot]$, and $\mathrm{tr}\{\cdot\}$ denote the transpose, the conjugate, the conjugate transpose, the Euclidean norm, the inverse, the absolute value, the determinant, a diagonal matrix consisting of the main diagonal elements of the matrix argument, and the sum of the main diagonal elements of the matrix argument, respectively. An identity matrix and a null matrix of size M are denoted by $\mathbf{I}_M$ and $\mathbf{0}_M$, respectively. $\mathrm{E}\{\cdot\}$ and $\log_2(\cdot)$ denote the expectation and the logarithm to the basis two, respectively.

Nodes S1, S2 and the RS work in half-duplex mode [4] which means that the transmission and reception of each node are separated by orthogonal time slots. Without loss of generality, it is assumed that one data symbol per time slot and per transmit antenna element of the source nodes S1 and S2 is transmitted. In order to spatially separate the transmitted data symbols from the source node, the destination node requires at least as many receive antenna elements as transmitted data symbols. Due to the bi-directional communication between S1 and S2, the number of antenna elements at S1 and S2 has to be equal and is given by $M \geq 1$. The RS is equipped with $L \geq 1$ antenna elements.

Data vector $\mathbf{x}^{(i)} = \left[x_1^{(i)}, \ldots, x_M^{(1)}\right]^{\mathrm{T}}$ of data symbols $x_m^{(i)}$, $m = 1, \ldots, M$, is transmitted from Si to Sk for

$$i = \begin{cases} 1 & \text{if } k = 2 \\ 2 & \text{if } k = 1, \end{cases} \qquad k \neq i, \tag{1}$$

where indices i and k from Eq. (1) are valid throughout the whole paper unless otherwise stated. Throughout the paper, Gaussian signalling is considered which means that the data symbols are statistically independent and of zero-mean and unit variance. Data vector $\mathbf{x}^{(i)}$ is precoded by $\mathbf{Q}^{(i)} \in \mathbb{C}^{M\times M}$ at node Si. In the first time-slot, S1 and S2 transmit simultaneously to the RS. The radio channel between Si and the RS is described by the complex channel matrix $\mathbf{H}^{(i)} \in \mathbb{C}^{L\times M}$. Furthermore, channel reciprocity is assumed so that the channel from the RS to Si is given by $\mathbf{H}^{(i)^{\mathrm{T}}}$. The RS applies linear BF to the sum of the receive signals indicated by BF matrix $\mathbf{G} \in \mathbb{C}^{L\times L}$ and retransmits the filtered signal to S1 and S2 in the second time slot over channels $\mathbf{H}^{(k)^{\mathrm{T}}}$, $k = 1, 2$. At the destination node Sk, the receive signal is spatially filtered by $\mathbf{P}^{(k)} \in \mathbb{C}^{M\times M}$. Furthermore, the own transmitted data vector $\mathbf{x}^{(k)}$ is multiplied by $\mathbf{T}^{(k)}$ and added to the receive signal. The noise at the RS and the noise at the destination node Sk, are modeled by vectors $\mathbf{n}^{(\mathrm{RS})} \in \mathbb{C}^{L\times 1}$ and $\mathbf{n}_{\mathrm{R}}^{(k)} \in \mathbb{C}^{M\times 1}$, respectively, consisting of zero-mean, independent, circularly symmetric Gaussian random variables with variances $\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}$ and $\sigma^2_{\mathbf{n}_{\mathrm{R}}^{(k)}}$, respectively. The overall estimate $\hat{\mathbf{x}}^{(i)}$ for data vector $\mathbf{x}^{(i)}$ at destination node Sk is given by

$$\hat{\mathbf{x}}^{(i)} = \mathbf{A}_{\mathrm{uf}}^{(k)}\mathbf{x}^{(i)} + \mathbf{A}_{\mathrm{di}}^{(k)}\mathbf{x}^{(k)} + \mathbf{B}^{(k)}\mathbf{n}^{(k)}, \tag{2}$$

with the decomposition into complex matrices

$$\mathbf{A}_{\mathrm{uf}}^{(k)} = \mathbf{P}^{(k)}\mathbf{H}^{(k)^{\mathrm{T}}}\mathbf{G}\mathbf{H}^{(i)}\mathbf{Q}^{(i)}, \tag{3a}$$

$$\mathbf{A}_{\mathrm{di}}^{(k)} = \mathbf{P}^{(k)}\mathbf{H}^{(k)^{\mathrm{T}}}\mathbf{G}\mathbf{H}^{(k)}\mathbf{Q}^{(k)} + \mathbf{T}^{(k)}, \tag{3b}$$

$$\mathbf{B}^{(k)} = \left[\begin{array}{cc} \mathbf{P}^{(k)}\mathbf{H}^{(k)^{\mathrm{T}}}\mathbf{G}, & \mathbf{P}^{(k)} \end{array}\right], \tag{3c}$$

and the overall noise vector $\mathbf{n}^{(k)} = \left[\mathbf{n}^{(\mathrm{RS})^{\mathrm{T}}}, \quad \mathbf{n}_{\mathrm{R}}^{(k)^{\mathrm{T}}}\right]^{\mathrm{T}}$.

Matrix $\mathbf{A}_{\mathrm{uf}}^{(k)} \in \mathbb{C}^{M\times M}$ is linked with the useful receive signal vector containing $\mathbf{x}^{(i)}$ at Sk. Matrix $\mathbf{A}_{\mathrm{di}}^{(k)} \in \mathbb{C}^{M\times M}$ is linked with the duplex interference vector which contains the interference for the data symbols of $\mathbf{x}^{(i)}$ at Sk by simultaneously received data symbols of $\mathbf{x}^{(k)}$. For $M \geq 1$, this means that each received data symbol may be interfered by M other data symbols intended for the other destination node. Matrix $\mathbf{B}^{(k)} \in \mathbb{C}^{M\times(L+M)}$ is the overall noise filter matrix and describes the filtering of the noise at the RS and at the destination node Sk.

The destination node Sk can subtract the duplex interference coming from its own transmitted data vector $\mathbf{x}^{(k)}$. For that purpose, matrix $\mathbf{T}^{(k)}$ has to be chosen as follows:

$$\mathbf{T}^{(k)} = -\mathbf{P}^{(k)}\mathbf{H}^{(k)^{\mathrm{T}}}\mathbf{G}\mathbf{H}^{(k)}\mathbf{Q}^{(k)}. \tag{4}$$

According to Eq. (4), Sk only requires one-hop CSI about its own channel $\mathbf{H}^{(k)}$ to the RS in order to calculate $\mathbf{T}^{(k)}$. Assuming channel reciprocity and a sufficiently long channel coherence time, this one-hop CSI can be estimated at Sk with relatively low effort [7]. In the following, it is assumed that the subtraction of duplex interference can always be performed.

The covariance matrices of the desired data vector $\mathbf{x}^{(i)}$ and the overall noise vector $\mathbf{n}^{(k)}$ are denoted by $\mathbf{R}_{\mathbf{x}^{(i)}}$ and $\mathbf{R}_{\mathbf{n}^{(k)}}$. Under these assumptions, the one-directional rate for the transmission from node Si to node Sk is defined as

$$C^{(k)} = \frac{1}{2}\log_2\left(\det\left[\mathbf{I}_M + \left(\mathbf{A}_{\text{uf}}^{(k)}\mathbf{R}_{\mathbf{x}^{(i)}}\mathbf{A}_{\text{uf}}^{(k)^{\text{H}}}\right)\mathbf{F}^{(k)^{-1}}\right]\right), \tag{5}$$

with $\mathbf{F}^{(k)} = \mathbf{B}^{(k)}\mathbf{R}_{\mathbf{n}^{(k)}}\mathbf{B}^{(k)^{\text{H}}}$. The pre-log factor $1/2$ in Eq. (5) is introduced in order to indicate the increase in the number of required time-slots by the factor of 2 for the one-directional transmission from Si to Sk compared to a one-directional single-hop transmission from Si to Sk. The second summand of the determinant in Eq. (5) gives the ratio between the power of the useful signal and the power of the distortions which consist of the sum of the intersymbol interference and the overall noise.

III. Sum rate maximization

The spectral efficiency of the two-way relaying approach can be analyzed by means of the sum rate of the individual transmission rates $C^{(1)}$ and $C^{(2)}$ since both transmissions are processed within the same time slots. In order to maximize the spectral efficiency, the linear BF matrices introduced in Section II have to be chosen such that the sum rate is maximized. The most general notation of this optimization problem is given by

$$\max_{\left\{\mathbf{P}^{(1)},\mathbf{P}^{(2)},\mathbf{G},\mathbf{Q}^{(1)},\mathbf{Q}^{(2)}\right\}} C^{(1)} + C^{(2)}, \tag{6a}$$

$$\text{subject to:}\quad \mathrm{E}\left\{\left\|\mathbf{Q}^{(i)}\mathbf{x}^{(i)}\right\|_2^2\right\} \leq E^{(i)}, \quad i = 1, 2, \tag{6b}$$

$$\mathrm{E}\left\{\left\|\mathbf{G}\left(\sum_{i=1}^{2}\mathbf{H}^{(i)}\mathbf{Q}^{(i)}\mathbf{x}^{(i)} + \mathbf{n}^{(\text{RS})}\right)\right\|_2^2\right\} \leq E^{(\text{RS})}, \tag{6c}$$

where the sum rate is maximized under the transmit energy constraints (6b) at the source nodes and the transmit energy constraint (6c) at the RS, where $E^{(i)}$ and $E^{(\text{RS})}$ are the maximum transmit energies of Si and the RS, respectively. In the most general case, all elements of the linear BF matrices $\mathbf{P}^{(1)}, \mathbf{P}^{(2)}, \mathbf{G}, \mathbf{Q}^{(1)}$, and $\mathbf{Q}^{(2)}$ are arbitrary complex numbers. Assuming joint decoding over the co-located antennas of the destination nodes S1 and S2, the optimization problem gets independent of $\mathbf{P}^{(1)}$ and $\mathbf{P}^{(2)}$. Nevertheless for some of the optimization cases, $\mathbf{P}^{(k)}$ can be used in order to diagonalize the overall transmission from Si to Sk leading to a simplified derivation of $\mathbf{G}$ and $\mathbf{Q}^{(i)}$.

In the following, the optimization problem (6) is adapted to Case1, Case2, and Case3 introduced in Sec. I.

A. Case1*: System without complexity constraints*

In Case1, all nodes in the system are provided with CSI. In order to perform BF at the RS, it only requires one-hop CSI about its channels from or to the node Si. This CSI can be provided to the RS with relatively low effort. However, in order to perform BF at Sk, it also requires two-hop CSI about the other channel $\mathbf{H}^{(i)}$ between Si and the RS. This CSI can be either attained by periodic insertion of pilot symbols into the data streams of node Si and estimating the overall channel $\mathbf{H}^{(k)^{\text{T}}}\mathbf{G}\mathbf{H}^{(i)}$ at node Sk [8], or by signalling the CSI of channel $\mathbf{H}^{(i)}$ by the RS which can estimate it directly. Both approaches require significantly high effort, increase the complexity of the system and degrade the effective throughput of the system [6].

In general, it is difficult to determine the optimum set $\left\{\mathbf{G}_{\text{opt}}, \mathbf{Q}_{\text{opt}}^{(1)}, \mathbf{Q}_{\text{opt}}^{(2)}\right\}$ since problem (6) is continuous non-linear constrained. There exist sequential quadratic programming (SQP) algorithms [9] which apply Newton's method in order to determine an optimum.

Since SQP algorithms are iterative algorithms with high complexity, a sub-optimum approach for $M = 1$ and $L \geq 2$ with significantly lower complexity is proposed in the following. In this case, the transmit and receive filter matrices at Si and Sk, respectively, are reduced to scalar variables. The transmit filter is chosen to fulfill the transmit energy constraint at Si leading to $\mathbf{Q}^{(i)} = q^{(i)} = \sqrt{E^{(i)}}$. Channel matrix $\mathbf{H}^{(i)}$ is reduced to a channel vector $\mathbf{H}^{(i)} = \mathbf{h}^{(i)}$, and the duplex interference can be always subtracted. Denoting the signal-to-noise ratio (SNR) at Sk by $\text{SNR}^{(k)}$, the transmission rate $C^{(k)}$ from Eq. (5) is of the form $C^{(k)} = \log_2\left(1 + \text{SNR}^{(k)}\right)$. Instead of maximizing the logarithm, one could maximize $\text{SNR}^{(k)}$ as an approximation. Since the SNR is typically maximized by a matched filter (MF) approach, the following sub-optimum solution is defined as MF two-way relaying. In MF two-way relaying, the BF matrix at the RS is decomposed into

$$\mathbf{G} = \mathbf{G}_{\text{T}}\mathbf{W}\mathbf{G}_{\text{R}}, \tag{7}$$

with the receive filter $\mathbf{G}_{\text{R}}$, the weighting matrix $\mathbf{W} = \text{diag}\left[w_1, w_2\right]$ with real-valued weighting coefficients w_1 and w_2 and the transmit filter $\mathbf{G}_{\text{T}}$. The receive SNR at the RS for data symbol $x_1^{(i)}$ is maximized for the receive MF $\mathbf{h}^{(i)^{\text{H}}}$ matched to channel $\mathbf{h}^{(i)}$ [10]. Hence, the overall receive MF applied to the received data streams of the RS is a concatenation of the MFs to channels $\mathbf{h}^{(1)}$ and $\mathbf{h}^{(2)}$ given by

$$\mathbf{G}_{\text{R}} = \begin{bmatrix} \mathbf{h}^{(1)^{\text{H}}} \\ \mathbf{h}^{(2)^{\text{H}}} \end{bmatrix}. \tag{8}$$

The receive SNR at node Sk for data symbol $x_1^{(i)}$ is maximized for the transmit MF $\mathbf{h}^{(k)^*}$ matched to channel $\mathbf{h}^{(k)^{\text{T}}}$ [11]. Hence, the overall transmit matched filter applied to the transmitted data streams of the RS is a concatenation of the MFs to channels $\mathbf{h}^{(2)^{\text{T}}}$ and $\mathbf{h}^{(1)^{\text{T}}}$ given by

$$\mathbf{G}_{\text{T}} = \left[\mathbf{h}^{(2)^*}, \mathbf{h}^{(1)^*}\right]. \tag{9}$$

A similar approach of combining receive and transmit MFs is proposed in [12], namely maximal-ratio-reception maximal-ratio-transmission (MRR-MRT). In MRR-MRT two-way relaying, the weighting matrix is given by $\mathbf{W} = \mathbf{I}_2$. However, MRR-MRT two-way relaying only performs well if the SNRs on the channel $\mathbf{h}^{(1)}$ and $\mathbf{h}^{(2)}$ are in a similar range. In this case, the sum rate can be maximized by maximizing each single rate $C^{(k)}$ individually. In case of different SNRs on the channels $\mathbf{h}^{(1)}$ and $\mathbf{h}^{(2)}$, the sum rate is maximized by giving different weights to the data streams dedicated to S1 and S2 at the RS. The optimum values for w_1 and w_2 can also be determined by SQP methods and the resulting approach is named optimum MF two-way relaying. The number of optimization variables is reduced to two real-valued weights by applying the receive and transmit MFs. In the following, a sub-optimum approach for determining the weights is proposed which is named sub-optimum MF two-way relaying. With the receive and transmit MFs, Eq. (5) results in

$$C^{(k)} = \log_2\left(1 + \frac{E^{(i)}\left|\mathbf{h}^{(i)}\right|^4\left|\mathbf{h}^{(k)}\right|^4 w_i^2}{\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}\mathbf{h}^{(k)\mathrm{T}}\mathbf{G}\mathbf{G}^{\mathrm{H}}\mathbf{h}^{(k)*} + \sigma^2_{\mathbf{n}_{\mathrm{R}}^{(k)}}}\right). \quad (10)$$

For $\mathbf{G}$ from Eq. (7), the filtered noise of the RS which is the first summand in the denominator of Eq. (10) depends on w_i and w_k. However, it can be shown that this first summand is dominated by weight w_i, and hence Eq. (10) can be approximated by

$$C^{(k)} \approx \log_2\left(1 + \frac{E^{(i)}\left|\mathbf{h}^{(i)}\right|^4\left|\mathbf{h}^{(k)}\right|^4 w_i^2}{\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}\left|\mathbf{h}^{(k)}\right|^4 w_i^2 + \sigma^2_{\mathbf{n}_{\mathrm{R}}^{(k)}}}\right). \quad (11)$$

Now, the optimum w_1 and w_2 for maximizing the sum rate under the RS energy constraint can be determined by Lagrangian optimization methods [9]. The problem is similar to the problem of finding optimum weights for different data streams at the RS in a one-way relaying approach introduced in [5]. The optimum weights are derived as

$$w_i^2 = \frac{\tilde{w}_i^2}{E^{(i)}\left|\mathbf{h}^{(i)}\right|^4 + \sigma^2_{\mathbf{n}^{(\mathrm{RS})}}}, \quad (12)$$

with

$$\tilde{w}_i^2 = \left[\sqrt{\left(\frac{\rho^{(k)}\nu^{(i)}}{2}\right)^2 + \tilde{\mu}\rho^{(k)}\nu^{(i)}} - \frac{\rho^{(k)}\nu^{(i)}}{2} - \frac{\sigma^2_{\mathbf{n}_{\mathrm{R}}^{(k)}}}{\left|\mathbf{h}^{(k)}\right|^4}\right]^+,$$

where $[\cdot]^+$ is the argument itself if the argument is positive and zero if it is negative, $\nu^{(i)} = E^{(i)}\frac{\left|\mathbf{h}^{(i)}\right|^4}{\left|\mathbf{h}^{(k)}\right|^4}$, and $\rho^{(k)} = \frac{\sigma^2_{\mathbf{n}_{\mathrm{R}}^{(k)}}}{\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}}$. The Lagrangian multiplier $\tilde{\mu}$ has to be chosen such that the RS energy constraint is fulfilled.

B. CASE2: *System with low complexity RS*

In CASE2, the RS has no CSI. Hence, the BF matrix at the RS is reduced to a simple scalar real-valued amplification factor g equal at all transmit antennas of the RS, which means $\mathbf{G} = g\mathbf{I}_L$. The RS only measures the received signal power and adapts the amplification factor g in order to fulfill its transmit energy constraint. In this well-known amplify-and forward relaying approach, the CSI complexity is still high since S1 and S2 require one-hop and two-hop CSI in order to perform BF.
Applying Eq. (6c), the amplification factor g can be derived as

$$g = \sqrt{\frac{E^{(\mathrm{RS})}}{\mathrm{tr}\left\{\mathrm{E}\left\{\sum_{i=1}^{2}\mathbf{H}^{(i)}\mathbf{Q}^{(i)}\mathbf{R}_{\mathbf{x}^{(i)}}\mathbf{Q}^{(i)\mathrm{H}}\mathbf{H}^{(i)\mathrm{H}}\right\}\right\} + L\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}}}, \quad (13)$$

The amplification factor g from Eq. (13) still depends on the BF matrices $\mathbf{Q}^{(i)}$ at the source nodes. An optimum set $\left\{g_{\mathrm{opt}}, \mathbf{Q}^{(1)}_{\mathrm{opt}}, \mathbf{Q}^{(2)}_{\mathrm{opt}}\right\}$ can be found by using SQP algorithms, too.
In the following, a sub-optimum solution for this problem is proposed. For that purpose, it assumed that the BF matrix at the source nodes is given by

$$\mathbf{Q}^{(i)} = \sqrt{\frac{E^{(i)}}{M}}\tilde{\mathbf{Q}}^{(i)}, \quad (14)$$

where $\tilde{\mathbf{Q}}^{(i)}$ is a unitary precoding matrix. Under this assumption, the amplification factor at the RS simplifies to

$$g = \sqrt{\frac{E^{(\mathrm{RS})}}{\mathrm{tr}\left\{\sum_{i=1}^{2}\frac{E^{(i)}}{M}\mathrm{E}\left\{\mathbf{H}^{(i)}\mathbf{H}^{(i)\mathrm{H}}\right\}\right\} + L\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}}}. \quad (15)$$

From Eq. (15), it can be seen that g gets independent from the BF at the source nodes and can be determined as a constant in the considered optimization problem. For constant g and without duplex interference, the rates $C^{(1)}$ and $C^{(2)}$ get independent from each other and can be maximized separately. In the following, the singular value decomposition (SVD) of the overall channel from source Si to destination Sk is defined by

$$g\mathbf{H}^{(k)\mathrm{T}}\mathbf{H}^{(i)} = \mathbf{U}^{(k,i)}\mathbf{\Lambda}^{(k,i)^{1/2}}\mathbf{V}^{(k,i)\mathrm{H}}, \quad (16)$$

where $\mathbf{U}^{(k,i)} \in \mathbb{C}^{M\times M}$ and $\mathbf{V}^{(k,i)} \in \mathbb{C}^{M\times M}$ are unitary, and $\mathbf{\Lambda}^{(k,i)^{1/2}} \in \mathbb{C}^{M\times M}$ is non-negative and diagonal. The columns of $\mathbf{U}^{(k,i)}$ are the left eigenvectors and the columns of $\mathbf{V}^{(k)}$ are the right eigenvectors, and the diagonal entries of $\mathbf{\Lambda}^{(k)^{1/2}} = \mathrm{diag}\left[\sqrt{\lambda_1^{(k,i)}}, \ldots, \sqrt{\lambda_R^{(k,i)}}\right]$, $R \leq M$, are the non-negative square roots of the eigenvalues of the overall channel which are sorted by their magnitude in descending order. In order to diagonalize the overall transmission from Si to Sk, the following BF matrices are used:

$$\mathbf{Q}^{(i)} = \sqrt{\frac{E^{(i)}}{M}}\mathbf{V}^{(k,i)}\mathbf{W}^{(k,i)}, \quad (17)$$

$$\mathbf{P}^{(k)} = \mathbf{U}^{(k,i)\mathrm{H}}, \quad (18)$$

where $\mathbf{W}^{(k,i)} = \mathrm{diag}\left[w_1^{(k,i)}, \ldots, w_R^{(k,i)}\right]$ with $w_r^{(k,i)} \in \mathbb{R}^+$, $r = 1, \ldots, R$, is a weighting matrix for the different data symbols of the transmit vector $\mathbf{x}^{(i)}$. This approach leads to the independent transmission rate

$$C^{(k)} = \log_2\det\left[\mathbf{I} + \mathbf{\Lambda}^{(k,i)^{1/2}}\mathbf{W}^{(k,i)}\mathbf{\Lambda}^{(k,i)^{1/2}}\mathbf{D}^{(k)^{-1}}\right], \quad (19)$$

with

$$\mathbf{D}^{(k)} = g^2\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}\mathbf{U}^{(k,i)^{\mathrm{H}}}\mathbf{H}^{(k)^{\mathrm{T}}}\mathbf{H}^{(k)^*}\mathbf{U}^{(k,i)} + \sigma^2_{\mathbf{n}^{(k)}_{\mathrm{R}}}\mathbf{I}_M. \quad (20)$$

The left part of the sum in Eq. (20) indicates that the filtered noise of the RS is no longer spatially white. For that reason, the problem of determining weighting matrix $\mathbf{W}^{(k,i)}$ is no typical water filling problem [13]. However, the equivalent noise power per receive antenna at the output of the receive filter $\mathbf{P}^{(k)}$ can be described by

$$\tilde{\sigma}^2_{\mathbf{n}^{(k)}} = \frac{g^2\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}}{M}\mathrm{tr}\left\{\mathbf{H}^{(k)^{\mathrm{T}}}\mathbf{H}^{(k)^*}\right\} + \sigma^2_{\mathbf{n}^{(k)}_{\mathrm{R}}}. \quad (21)$$

By replacing $\mathbf{D}^{(k)}$ in Eq. (19) by spatially white noise with the equivalent noise power $\tilde{\sigma}^2_{\mathbf{n}^{(k)}}$, i.e., $\mathbf{D}^{(k)} = \tilde{\sigma}^2_{\mathbf{n}^{(k)}}\mathbf{I}_M$, the maximization problem becomes a water filling problem where the weights $w_r^{(k,i)}$ can be found by the water pouring algorithm [14].

C. CASE3: *System without two-hop CSI*

In CASE3, BF is not performed at S1 and S2 since two-hop CSI is not available. But subtraction of the duplex interference is still possible due to the availability of one-hop CSI. Hence, only scalar filter coefficients are employed at the transmit antennas of S1 and S2, which means, $\mathbf{Q}^{(i)} = q^{(i)}\mathbf{I}_M$. In this case, the transmit node can only adapt its transmit energy in order to fulfill the transmit energy constraint (6b). The transmit energy is distributed equally among M transmit antennas leading to $q^{(i)} = \sqrt{\frac{E^{(i)}}{M}}$. In order to perform BF at the RS, it only requires one-hop CSI about its channels from or to the node Si. This CSI can be provided to the RS with relatively low effort.

For $M = 1$, the optimization problem is exactly the same as introduced for CASE1 in Sec. III-A which means that the MF two-way relaying approach from Eq. (7) gives a feasible sub-optimum solution. For $M > 1$ and $L \geq 2M$ the following approach in order to reduce the number of L^2 complex optimization variables in $\mathbf{G}$ is proposed. For that purpose, the composed channel matrix

$$\mathbf{H}_{\mathrm{R}} = \left[\mathbf{H}^{(1)}, \mathbf{H}^{(2)}\right] \quad (22)$$

is defined which describes the overall receive channel from S1 and S2 at the RS. Since the overall transmit channel matrix $\mathbf{H}_{\mathrm{T}}$ is a simple transformation of the overall receive channel matrix given by $\mathbf{H}_{\mathrm{T}} = \mathbf{\Omega}\mathbf{H}_{\mathrm{R}}^{\mathrm{T}}$, with

$$\mathbf{\Omega} = \begin{bmatrix} \mathbf{0}_M & \mathbf{I}_M \\ \mathbf{I}_M & \mathbf{0}_M \end{bmatrix}, \quad (23)$$

it is sufficient to consider only the SVD of the overall receive channel in the following. The SVD of $\mathbf{H}_{\mathrm{R}}$ may be described by

$$\mathbf{H}_{\mathrm{R}} = \left[\tilde{\mathbf{U}}_{\mathrm{R}}, \bar{\mathbf{U}}_{\mathrm{R}}\right]\mathbf{\Lambda}_{\mathrm{R}}^{1/2}\mathbf{V}_{\mathrm{R}}^{\mathrm{H}}, \quad (24)$$

where $\tilde{\mathbf{U}}_{\mathrm{R}} \in \mathbb{C}^{M^{(\mathrm{RS})}\times 2M}$ and $\bar{\mathbf{U}}_{\mathrm{R}} \in \mathbb{C}^{M^{(\mathrm{RS})}\times(L-2M)}$ contain the left singular vectors of $\mathbf{H}_{\mathrm{R}}$. Matrix $\tilde{\mathbf{U}}_{\mathrm{R}}$ forms an orthogonal basis for the range of $\mathbf{H}_{\mathrm{R}}$. It may be used as an adaptation to the non-zero eigenmodes of the overall channel at the RS. By setting

$$\mathbf{G} = \tilde{\mathbf{U}}_{\mathrm{R}}^*\tilde{\mathbf{G}}\tilde{\mathbf{U}}_{\mathrm{R}}^{\mathrm{H}} \quad (25)$$

the number of L^2 complex optimization variables in $\mathbf{G}$ is reduced to $(2M)^2$ in matrix $\tilde{\mathbf{G}}$. Of course, the same approach may also be applied in order to simplify the optimization in CASE1.

IV. SIMULATION RESULTS

In this section, the average sum rates of CASE1, CASE2, and CASE3 are compared to each other. The presented results are achieved from Monte Carlo simulations assuming statistically independent channel fading realizations with spatially white and Rayleigh distributed channel coefficients of zero mean and variance one. It is assumed that the average SNR for the transmission from Si to the RS is the same as the average SNR for the transmission from the RS to Si which leads to the following definition: $\mathrm{SNR}^{(i)} = E^{(\mathrm{RS})}/\sigma^2_{\mathbf{n}^{(i)}} = E^{(i)}/\sigma^2_{\mathbf{n}^{(\mathrm{RS})}}$. Fig. 1 gives the average sum rate depending on $\mathrm{SNR}^{(2)}$ with $\mathrm{SNR}^{(1)} = 10$dB and $\mathrm{SNR}^{(1)} = 20$dB, respectively, for the different approaches proposed in Sec. III-A for $M = 1$. Note

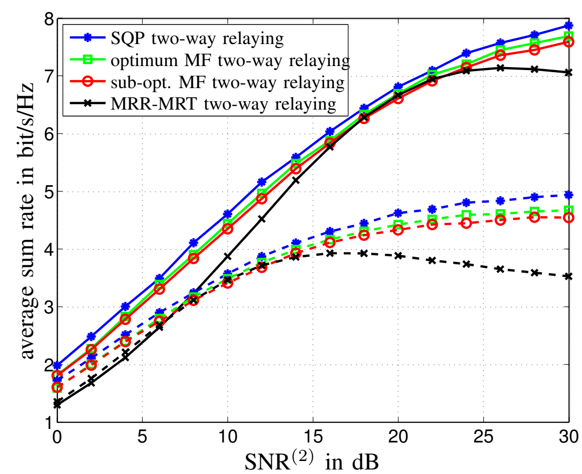

Figure 1. Average sum rate for different approaches in CASE1 and CASE3, $M = 1$, $L = 4$ (dashed lines: $\mathrm{SNR}^{(1)} = 10$dB, solid lines: $\mathrm{SNR}^{(1)} = 20$dB)

that CASE1 and CASE3 are the same optimization problems for $M = 1$. The RS is equipped with $L = 4$ antennas. The following relative observations are valid for $\mathrm{SNR}^{(1)} = 10$dB as well as for $\mathrm{SNR}^{(1)} = 20$dB. The maximum sum rate is achieved for SQP two-way relaying where $\mathbf{G}_{\mathrm{opt}}$ is determined by SQP methods provided by MATLAB®.

Although the optimum MF two-way relaying approach requires less computational complexity than the SQP two-way relaying approach, the average sum rate of the optimum MF two-way relaying approach comes very close to the maximum average sum rate achieved by SQP two-way relaying. The sub-optimum MF two-way relaying approach performs almost as well as the optimum MF two-way relaying approach. Finally, it is shown in Fig. 1 that the MRR-MRT two-way relaying approach like proposed in [12] has a significantly worse performance especially in case of different SNRs on the two channels to the RS. For $\mathrm{SNR}^{(1)} = 10\mathrm{dB}$, the average sum rate has a maximum for $\mathrm{SNR}^{(2)} \approx 17\mathrm{dB}$ and even decreases for $\mathrm{SNR}^{(2)} > 17\mathrm{dB}$. This comes from the fact that maximizing the receive power for the transmission over channel $\mathbf{h}^{(2)^{\mathrm{T}}}$ by the transmit MF also increases the noise for the transmission over channel $\mathbf{h}^{(1)^{\mathrm{T}}}$. This means that $C^{(2)}$ is increased while $C^{(1)}$ is decreased. For $\mathrm{SNR}^{(2)} > 17\mathrm{dB}$ the increase in $C^{(2)}$ is smaller than the decrease in $C^{(1)}$ which means that the sum rate gets smaller for equal weights. Hence, weighting is absolutely necessary in case of applying the MF approach for $M = 1$.

In Fig. 2, the maximum average sum rate is depicted depending on $\mathrm{SNR}^{(1)} = 10\mathrm{dB}$ and $\mathrm{SNR}^{(1)} = 20\mathrm{dB}$, respectively, for the three different cases. S1 and S2 are equipped with $M = 2$ antennas and the RS is equipped with $L = 8$ antennas. The optimum BF matrices are determined by SQP

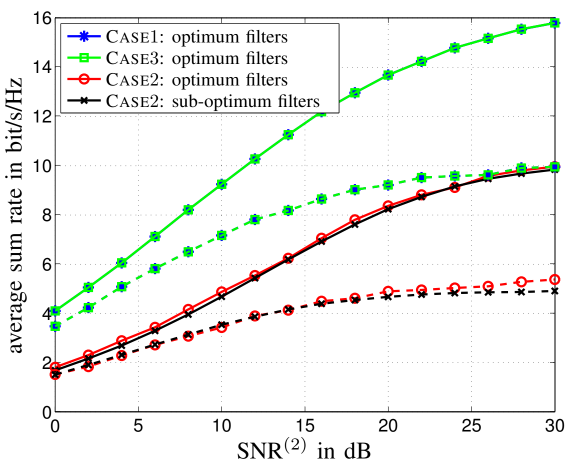

Figure 2. Average sum rate for all optimum cases; average sum rate for sub-optimum CASE2, $M = 2$, $L = 8$ (dashed lines: $\mathrm{SNR}^{(1)} = 10\mathrm{dB}$, solid lines: $\mathrm{SNR}^{(1)} = 20\mathrm{dB}$)

methods. The figure gives an overview of how the different system complexities influence the maximum achievable sum rate by numeric optimizations. The highest sum rate is achieved by CASE1 and CASE3. Obviously, precoding at the source nodes is not required in the two-way relaying approach and spatial filtering at the RS is sufficient in order to achieve the optimum performance. The same result has already be obtained for conventional two-hop relaying [5]. CASE2 provides a significantly lower sum rate than the other two cases since the high number of antennas at the RS is not exploited. In CASE2, the number of antennas at the RS has no influence on the sum rate since the overall channel seen by S1 and S2 is always an $M \times M$ channel where M is the sum rate limiting variable. This means that for CASE2, only $L = 1$ is of practical interest since other configurations at the RS only increase the hardware complexity at the RS without any performance improvement. For CASE2, the sum rate of the proposed sub-optimum filtering at S1 and S2 is also depicted in Fig. 2. The performance of the sub-optimum approach comes very close to the optimum and is only slightly degraded by the assumption of spatially white noise at the receiver outputs.

V. CONCLUSIONS

In this paper, a general framework for the sum rate maximization in non-regenerative two-way relaying with multiple-antenna nodes is given. The sum rate maximization problem is adapted to three different cases regarding CSI and node complexity. In general, the results show that exclusive BF at a multiple-antenna RS is very promising. Although this approach has a reduced system complexity, it outperforms the case where BF is exclusively performed at the source and destination nodes which has a higher system complexity in the considered scenarios. For single-antenna source and destination nodes there exist quite simple and efficient BF approaches at the RS which come close to the optimum performance.

ACKNOWLEDGMENT

The authors would like to thank the TPC for the invitation of this paper.

REFERENCES

[1] H. Shi, T. Asai, and H. Yoshino, "A relay node division duplex relaying approach for MIMO relay networks," in *Proc. IEEE Symposium on Personal, Indoor and Mobile Radio Communications*, Helsinki, Finland, Sep. 2006.

[2] O. Munoz, J. Vidal, and A. Agustin, "A game theoretic approach for cooperative MIMO schemes with cellular reuse of the relay slot," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, vol. 4, Montreal, Canada, May 2004, pp. iv–581 – iv–584.

[3] H. Hu, H. Yanikomeroglu, D. Falconer, and S. Periyalwar, "Range extension without capacity penalty in cellular networks with digital fixed relays," in *Proc. IEEE Global Telecommunications Conference*, Texas, Dallas, USA, Nov. 2004.

[4] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 379–389, Feb. 2007.

[5] O. Munoz, J. Vidal, and A. Agustin, "Linear transceiver design in non-regenerative relays with channel state information," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2593–2604, Jun. 2007.

[6] T. Cui, F. Gao, and A. Nallanathan, "Optimal training design for channel estimation in amplify and forward relay networks," in *Proc. IEEE Global Communications Conference*, Washington DC, USA, Nov. 2007.

[7] L. Tong, B. M. Sadler, and M. Dong, "Pilot-assisted wireless transmissions," *IEEE Signal Processing Magazine*, vol. 21, no. 6, pp. 12–25, Nov. 2004.

[8] C. S. Patel and G. L. Stüber, "Channel estimation for amplify and forward relay based cooperation diversity systems," *IEEE Transactions on Wireless Communications*, vol. 6, no. 6, pp. 2348–2355, Jun. 2007.

[9] S. Boyd and L. Vandenberghe, *Convex Optimization*, 1st ed. Cambridge, UK: Cambridge University Press, 2004.

[10] J. Proakis, *Digital Communications*, 4th ed. NY, USA: McGraw-Hill, New York, 2001.

[11] J. K. Cavers, "Single-user and multiuser adaptive maximal ratio transmission for rayleigh channels," *IEEE Transactions on Vehicular Technology*, vol. 49, pp. 2043–2050, Nov. 2000.

[12] Y.-C. Liang and R. Zhang, "Optimal analogue relaying with multi-antennas for physical layer network coding," in *Proc. IEEE International Conference on Communications*, Beijing, China, May 2008.

[13] D. P. Palomar and J. R. Fonollosa, "Practical algorithms for a family of waterfilling solutions," *IEEE Transactions on Signal Processing*, vol. 53, no. 2, pp. 686–695, Feb. 2005.

[14] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, 1st ed. Cambridge, UK: Cambridge University Press, 2003.