Duplex Schemes in Multiple-Antenna Two-Hop Relaying

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Abstract

In this paper, a novel scheme for two-hop relaying defined as space division duplex (SDD) relaying is proposed. In SDD relaying, multiple-antenna beamforming techniques are applied at the intermediate relay station (RS) in order to separate down- and uplink of a bi-directional two-hop communication between two nodes $S1$ and $S2$. For conventional amplify-and-forward two-hop relaying, there appears a loss in spectral efficiency due to the fact that the RS cannot receive and transmit simultaneously on the same channel resource. In SDD relaying, this loss in spectral efficiency is circumvented by giving up the strict separation of down- and uplink by either time division duplex or frequency division duplex. Two novel concepts for the linear beamforming filters at the RS are proposed, they can be designed either by a three-step or a one-step concept. SDD relaying introduces a new kind of interference which is defined as duplex-interference. An efficient method in order to combat duplex-interference is proposed. Furthermore, it is shown how the spectral efficiency of SDD relaying can be improved if the channels from $S1$ and $S2$ to the RS have different qualities.

I. I

There exists much ongoing work in the promising research field of two-hop relaying [1][2]. This paper is focused on bi-directional two-hop communication between two nodes $S1$ and $S2$ via an intermediate relay station (RS). It is assumed that the downlink traffic load from $S1$ to $S2$ via the RS is the same as the uplink traffic load from $S2$ to $S1$ via the RS. Due to the high dynamic range between the signal powers of up- and downlink signals, typical transceivers at $S1$, $S2$, and the RS cannot receive and transmit simultaneously on the same channel resource. In single-hop communication, where $S1$ and $S2$ can communicate directly with each other, this problem is typically solved by time division duplex (TDD) or frequency division duplex (FDD) [3]. In TDD, there exist two orthogonal time slots, one for the downlink and one for the uplink. In FDD, there exist two orthogonal frequency bands, one for the downlink and one for the uplink.

Most two-hop relaying schemes also assume a strict separation of down- and uplink by either TDD or FDD. These schemes are defined as one-way relaying schemes since down- and uplink can be regarded independently. In amplify-and-forward (AF) two-hop relaying [4], the RS either receives a signal from $S1$ or $S2$ on a first hop, amplifies this signal, and retransmits it to either $S2$ or $S1$ on a second hop. Due to the fact that a half-duplex RS cannot receive and transmit simultaneously on the same channel resource two orthogonal channel resources are required, one for the first hop and one for the second hop. If down- and uplink are separated by either TDD or FFD this means that the number of required channel resources is doubled compared to a single-hop communication. Regarding the spectral efficiency of two-hop relaying, this leads to a trade-off between the improved receive signal quality due to the reduced overall pathloss between $S1$ and $S2$, and the increase in required channel resources due to the two-hop approach. In literature, there exist a lot of one-way relaying schemes which try to overcome this conceptual drawback

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of two-hop relaying. However, there also exist schemes which relax the strict separation of down- and uplink by TDD or FDD which are also promising and in the focus of this work.

A. Related Work

One approach is to design two-hop relaying schemes which try to improve the spectral efficiency by allowing a smart reuse of channel resources among multiple one-way relaying connections [5][6][7].

In [5], multiple RSs are divided into two groups that alternately receive and transmit signals, i.e., while one group is receiving signals from the source node, the other group is transmitting signals to the destination node. Since the source always transmits, the number of required channel resources is the same as in the single-hop case. However, the performance can be significantly degraded by co-channel interference between the two groups of RSs. In [6], one source node communicates with K different destination nodes via K different RSs. Firstly, the source node transmits consecutively to the K RSs using K time slots. Secondly, all RSs transmit simultaneously to their assigned destination nodes in the relay time slot $K + 1$. Obviously, this protocol does not require double the resources compared to the single hop network, but only $(K + 1)/K$. However, the performance may be significantly degraded by co-channel interference from the RSs at the destination nodes. The problem of co-channel interference is also addressed in [7], where the co-channel interference is kept low by a smart selection of simultaneously transmitting RSs in the relay time slot.

Two other schemes which consider only one source and one destination node are proposed in [8]. For the first schemes, the communication between source and destination node is assisted by two RSs. While one RS receives from the source node the other RS transmits to the destination node in the same time slot. Since the source may transmit in every time slot, the number of required channel resources is the same as in the single-hop case. However, since the two RSs use the same channel resources there still exists co-channel interference in this scheme.

The second scheme which is termed two-way relaying is of particular interest for this work. It has been first introduced in [9] and it has attracted much similar work. In contrast to all previous schemes, two-way relaying is especially developed for bi-directional communication. For the first time, it relaxes the constraint that down- and uplink are transmitted on orthogonal time slots and/or orthogonal frequency bands. Hence, it uses neither TDD nor FDD. In two-way relaying, $S1$ and $S2$ transmit simultaneously on a first channel resource to a RS which receives a superposition of both signals. In general, there are two different approaches how to process the receive signal at the RS. For the decode-and-forward (DF) approach, the receive signal at the RS is decoded and the two separated signals from $S1$ and $S2$ are jointly re-encoded before retransmission. For the AF approach, the receive signal is only amplified at the RS before retransmission. For both approaches, a second channel resource is used for the retransmission and $S1$ and $S2$ may utilize their knowledge about the interference term which is coming from their own transmitted signal in order to detect the desired signal. In [10], the rate regions of DF two-way relaying are investigated. This work gives the optimal relative sizes of the first and second channel resource in order to maximize the achievable rate of DF two-way relaying. Two-way relaying is closely connected to network coding [11]. Actually, in network coding data packets from different sources in a multi-node computer network are jointly encoded at intermediate network nodes, thus saving network resources, i.e., DF two-way relaying can be interpreted as network coding in the original sense with the extension of allowing wireless links. In [12] the interconnection between AF two-way relaying and network coding is also established. Like in [13], it is assumed, that for DF two-way relaying three orthogonal channel resources are required. The first two resources are required for the transmission from $S1$ and $S2$ to the RS, respectively. This scheme guarantees that both signals can be decoded separately at the RS. The third resource is required for the retransmission of the jointly re-encoded signal from the RS. It is shown in [12] that AF two-way relaying provides a higher throughput for low noise levels at the RS than the considered DF two-way relaying which requires three instead of two orthogonal channel resources.

Another technique which promises to improve the spectral efficiency of two-hop relaying is the application of multiple antennas [2]. In [14], it is proposed that transmit and receive processing can be restricted to the RS, i.e., channel state information (CSI) is only required at the RS. In this approach, multiple RSs with multiple antennas apply beamforming in order to supply multiple destinations with their desired signals. However, [14] proposes only a one-way relaying scheme for multiple-antenna RSs. In [15], multiple antennas and CSI at the RS are applied in the context of DF two-way relaying. It is assumed that the signals from $S1$ and $S2$ are decoded at the RS and two different schemes are applied for the spatial pre-coding at the RS before the retransmission. For the first scheme, both decoded signals are re-encoded separately and linearly combined by applying a spatial pre-coding matrix coming from the singular value decomposition of the channel. For the second scheme, both decoded signals are combined by a bit-wise XOR operation and the spatial pre-coding is applied to the new single bit-stream. It is shown that the second approach outperforms the first approach in terms of achievable rate. Although the schemes in [15] apply multiple antennas in two-way relaying for the first time, decoding and re-encoding is still required at the RS.

B. Own Contribution

In this paper, an AF two-way relaying scheme with multiple antennas and linear signal processing at the RS is proposed leading to a new duplex scheme, defined as space division duplex (SDD). In SDD relaying, down- and uplink are transmitted on the same channel resources in time and frequency, but separated in space. This scheme circumvents the increase in required channel resources for two-hop relaying. Since the RS in the two-way relay channel is a receiver as well as a transmitter, linear receive and transmit beamforming can both be applied at the RS if CSI is available at the RS. The resulting spatial filter matrix at the RS is termed transceive filter matrix. Two novel concepts for the design of this transceive filter are proposed. It can be designed either in three independent steps or in one step. For both concepts, the linear transceive filters fulfilling the zero forcing (ZF) and the minimum mean square error (MMSE) criteria are derived and compared regarding their bit error rate (BER). SDD relaying introduces a new kind of interference between down- and uplink signals which is defined as duplex-interference. An efficient method in order to combat duplex-interference and to improve the BER performance is proposed in this paper. Furthermore, it is shown how the spectral efficiency of SDD relaying may be improved for the case of different channel qualities on the two channels from the RS to $S1$ and $S2$, respectively.

Regarding its spectral efficiency, SDD relaying is compared to other relaying schemes which require the same effort in terms of number of antennas, achieving CSI, and applied signal processing. Assuming multiple antennas, CSI availability at the RS, and linear signal processing, one could also exploit spatial diversity [16] at the RS instead of applying beamforming in SDD relaying. For that purpose, a one-way relaying scheme applying receive and transmit maximum ratio combining (MRC) [17][18] at the RS is proposed which is defined as MRC relaying. For MRC relaying, double the resources are required as for SDD relaying since down- and uplink have to be transmitted separately by either TDD or FDD. However, it provides diversity gain which can compensate the increase in required channel resources by allowing higher transmission rates. Furthermore, a relaying scheme applying a combination of receive MRC and transmit beamforming (BF) at the RS is proposed which is defined as MRC-BF relaying. In MRC-BF relaying, spatial diversity is exploited for the reception from $S1$ and $S2$ at the RS and the amount of required channel resources for the transmission from the RS to $S1$ and $S2$ is reduced.

The channel resource requirements and the applied signal processing at the RS for SDD relaying, MRC relaying and MRC-BF relaying are summarized in Fig. 1. In this paper, the spectral efficiencies of all proposed relaying schemes are investigated and compared to each other.

C. Notation

Throughout the paper, complex baseband transmission is assumed. Let $[\cdot]^T$, $[\cdot]^*$, $[\cdot]^H$, $\|\cdot\|_2^2$, $(\cdot)^{-1}$, $\det[\cdot]$, and $\mathrm{tr}\{\cdot\}$ denote the transpose, the conjugate,the conjugate transpose, the Euclidean norm, the inverse, the determinant of a matrix, and the sum of the main diagonal elements of a matrix, respectively. $\mathbf{I}_M$ and $\emptyset_M$ denote an identity matrix of size M and a null matrix of size $M \times M$, respectively. $\mathrm{E}\{\cdot\}$, $\mathrm{Re}\{\cdot\}$, and $\log_2(\cdot)$ denote the expectation, the real part, and the logarithm to the basis 2, respectively.

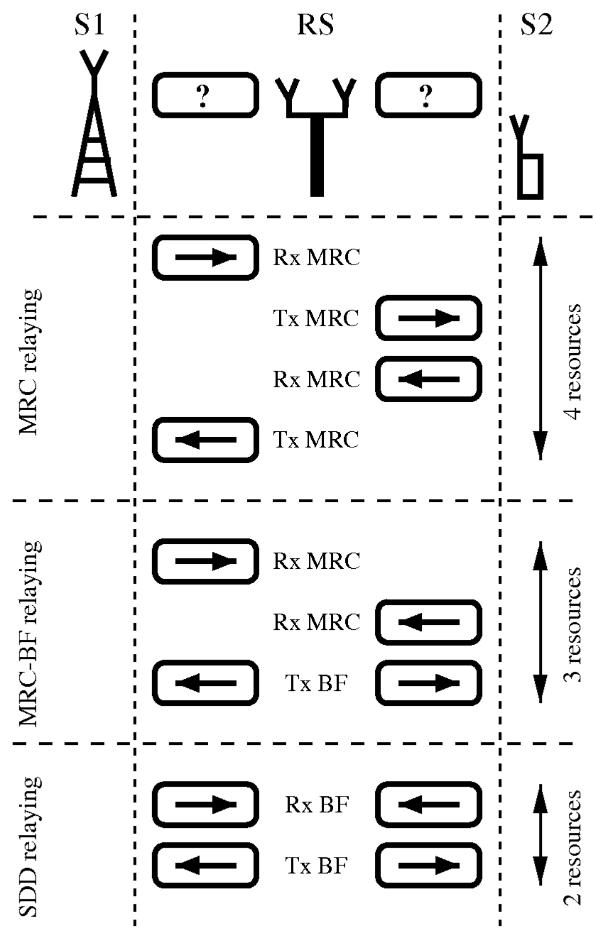

Fig. 1. Channel resource requirements of different relaying schemes with applied signal processing at the RS: receive MRC (Rx MRC), transmit MRC (Tx MRC), receive beamforming (Rx BF), transmit beamforming (Tx BF)

D. Outline

The system model of SDD relaying is given in Section II. Section III introduces the ZF and MMSE transceive filters which are firstly given for a three-step design concept and secondly they are derived by a one-step design concept. In Section IV, the duplex-interference in SDD relaying is considered. Section V, shortly introduces MRC and MRC-BF relaying. The required amount of CSI for the different relaying schemes and extensions of these schemes are discussed in Section VI. In Section VII, the sum rate for SDD relaying is given. Simulation results regarding the BER performance and the spectral efficiency of SDD relaying are presented in Section VIII. Section IX concludes this work.

II. S M SDD R

In the following, the communication between two nodes $S1$ and $S2$ is considered which exchange information via an intermediate RS since they cannot exchange information directly, e.g., due to shadowing conditions. Due to the half-duplex constraint, all stations cannot transmit and receive simultaneously on the same channel resource. $S1$ and $S2$ are equipped with $M^{(1)}$ and $M^{(2)}$ antennas, respectively. For the proposed SDD relaying scheme

$$M^{(1)} = M^{(2)} = M \tag{1}$$

is required while it is assumed that the RS is equipped with

$$M^{(\mathrm{RS})} \geq M^{(1)} + M^{(2)} = 2M \tag{2}$$

antennas.

The data vector $\mathbf{x}^{(1)} = \left[x_1^{(1)}, \ldots, x_M^{(1)}\right]^T$ of data symbols $x_n^{(1)}$, $n = 1, \ldots, M$, shall be transmitted from $S1$ to $S2$, and the data vector $\mathbf{x}^{(2)} = \left[x_1^{(2)}, \ldots, x_M^{(2)}\right]^T$ of data symbols $x_n^{(2)}$, $n = 1, \ldots, M$, shall be transmitted from $S2$ to $S1$. The corresponding transmit covariance matrices are given by $\mathbf{R}_{\mathbf{x}^{(\mathbf{k})}} = \mathrm{E}\left\{\mathbf{x}^{(k)}\mathbf{x}^{(k)^H}\right\}$, $k = 1, 2$. The overall data vector is defined as $\mathbf{x} = \left[\mathbf{x}^{(1)^T}, \mathbf{x}^{(2)^T}\right]^T$ with covariance matrix $\mathbf{R}_{\mathbf{x}} = \mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\}$. For simplicity, the wireless channel is assumed to be flat fading, so that all following considerations are applicable e.g. to multi-carrier systems. Hence, the channel between Sk, $k = 1, 2$, and the RS may be described by the channel matrix

$$\mathbf{H}_{\mathrm{R}}^{(k)} = \begin{bmatrix} h_{1,1}^{(k)} & \cdots & h_{1,M}^{(k)} \\ \vdots & \ddots & \vdots \\ h_{M^{(\mathrm{RS})},1}^{(k)} & \cdots & h_{M^{(\mathrm{RS})},M}^{(k)} \end{bmatrix}, \tag{3}$$

where $h_{m,n}^{(k)}$, $m = 1, \ldots, M^{(\mathrm{RS})}$ and $n = 1, \ldots, M$, are complex fading coefficients. The overall channel matrix for the transmission from $S1$ and $S2$ to the RS is defined as

$$\mathbf{H}_{\mathrm{R}} = \left[\mathbf{H}_{\mathrm{R}}^{(1)} \quad \mathbf{H}_{\mathrm{R}}^{(2)}\right]. \tag{4}$$

The channel between the RS and Sk, $k = 1, 2$, is described by the channel matrix

$$\mathbf{H}_{\mathrm{T}}^{(k)} = \begin{bmatrix} \tilde{h}_{1,1}^{(k)} & \cdots & \tilde{h}_{1,M^{(\mathrm{RS})}}^{(k)} \\ \vdots & \ddots & \vdots \\ \tilde{h}_{M,1}^{(k)} & \cdots & \tilde{h}_{M,M^{(\mathrm{RS})}}^{(k)} \end{bmatrix}, \tag{5}$$

where $\tilde{h}_{n,m}^{(k)}$, $n = 1, \ldots, M$ and $m = 1, \ldots, M^{(\mathrm{RS})}$, are complex fading coefficients. Assuming channel reciprocity, channel matrix $\mathbf{H}_{\mathrm{T}}^{(k)}$ is the transpose of $\mathbf{H}_{\mathrm{R}}^{(k)}$, i.e., $\mathbf{H}_{\mathrm{T}}^{(k)} = \mathbf{H}_{\mathrm{R}}^{(k)^T}$, if the channel is constant during one transmission cycle which includes the transmission from $S1$ to $S2$ and the transmission from $S2$ to $S1$. For the following considerations, the more general case of $\mathbf{H}_{\mathrm{T}}^{(k)} \neq \mathbf{H}_{\mathrm{R}}^{(k)^T}$ is regarded. The overall channel matrix for the transmission from the RS to $S2$ and $S1$ is defined as

$$\mathbf{H}_{\mathrm{T}} = \begin{bmatrix} \mathbf{H}_{\mathrm{T}}^{(2)} \\ \mathbf{H}_{\mathrm{T}}^{(1)} \end{bmatrix}. \tag{6}$$

In SDD relaying, the data vectors $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are exchanged between $S1$ and $S2$ during two orthogonal time-slots. During the first time-slot, $S1$ and $S2$ transmit simultaneously to the RS. Since spatial filtering shall only be applied at the RS, only scalar transmit filters $\mathbf{Q}^{(1)} = q^{(1)}\mathbf{I}_M$ and $\mathbf{Q}^{(2)} = q^{(2)}\mathbf{I}_M$ are applied at $S1$ and $S2$. These transmit filters are required in order to fulfill the transmit energy constraints at $S1$ and $S2$. Assuming that $E^{(1)}$ and $E^{(2)}$ are the transmit energies of nodes $S1$ and $S2$, the transmit energy constraints are given by

$$\mathrm{E}\left\{\| \, q^{(k)}\mathbf{x}^{(k)} \, \|_2^2\right\} = E^{(\mathrm{k})}, \quad k = 1, 2. \tag{7}$$

Assuming positive and real scalar transmit filters, the transmit energy constraints from (7) lead to

$$q^{(k)} = \sqrt{\frac{E^{(k)}}{\mathrm{tr}\left\{\mathbf{R}_{\mathbf{x}^{(k)}}\right\}}} \quad k = 1, 2, \tag{8}$$

i.e., the transmit energy of each node is equally shared among all transmit antennas of the node. The overall transmit filter is given by the block diagonal matrix

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}^{(1)} & \emptyset_M \\ \emptyset_M & \mathbf{Q}^{(2)} \end{bmatrix}. \tag{9}$$

The receive vector $\mathbf{y}_{\mathrm{RS}}$ at the RS is given by

$$\mathbf{y}_{\mathrm{RS}} = \mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{x} + \mathbf{n}_{\mathrm{RS}}, \tag{10}$$

where $\mathbf{n}_{\mathrm{RS}}$ is an additive white Gaussian noise vector with covariance matrix $\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}} = \mathrm{E}\left\{\mathbf{n}_{\mathrm{RS}}\mathbf{n}_{\mathrm{RS}}^{H}\right\}$. The covariance matrix of the RS receive vector $\mathbf{y}_{\mathrm{RS}}$ results in

$$\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}} = \mathrm{E}\left\{\mathbf{y}_{\mathrm{RS}}\mathbf{y}_{\mathrm{RS}}^{H}\right\} = \mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{R}_{\mathbf{x}}\mathbf{Q}^{H}\mathbf{H}_{\mathrm{R}}^{H} + \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}. \tag{11}$$

At the RS a linear transceive filter $\mathbf{G}$ is designed which shall ensure that $S1$ receives an estimate of data vector $\mathbf{x}^{(2)}$ and $S2$ receives an estimate of data vector $\mathbf{x}^{(1)}$. There are several possibilities how $\mathbf{G}$ can be designed which will be discussed in Section III. After applying transceive filter $\mathbf{G}$, the RS transmit vector is given by

$$\mathbf{x}_{\mathrm{RS}} = \mathbf{G}\mathbf{y}_{\mathrm{RS}} = \mathbf{G}\left(\mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{x} + \mathbf{n}_{\mathrm{RS}}\right) \tag{12}$$

The RS transmit vector $\mathbf{x}_{\mathrm{RS}}$ has to fulfill the transmit energy constraint at the RS

$$\mathrm{E}\left\{\| \mathbf{x}_{\mathrm{RS}} \|_2^2\right\} \leq E^{(\mathrm{RS})}, \tag{13}$$

where $E^{(\mathrm{RS})}$ is the maximum transmit energy at the RS. In the following, the estimate for data vector $\mathbf{x}^{(1)}$ at $S2$ is termed $\hat{\mathbf{x}}^{(1)}$ and the estimate for data vector $\mathbf{x}^{(2)}$ at $S1$ is termed $\hat{\mathbf{x}}^{(2)}$. For each receiving node, the scalar receive filters $\mathbf{P}^{(1)} = p^{(1)}\mathbf{I}_M$ at $S2$ and $\mathbf{P}^{(2)} = p^{(2)}\mathbf{I}_M$ at $S1$ with filter coefficients $p^{(1)}$ and $p^{(2)}$ are assumed. The overall receive filter matrix results in

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}^{(1)} & \emptyset_M \\ \emptyset_M & \mathbf{P}^{(2)} \end{bmatrix}. \tag{14}$$

The overall estimated data vector $\hat{\mathbf{x}} = \left[\hat{\mathbf{x}}^{(1)^T}, \hat{\mathbf{x}}^{(2)^T}\right]^T$ is given by

$$\hat{\mathbf{x}} = \mathbf{P}\left(\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{x} + \mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{n}_{\mathrm{RS}} + \mathbf{n}_{\mathrm{R}}\right) \tag{15}$$

where $\mathbf{n}_{\mathrm{R}} = \left[\mathbf{n}_{\mathrm{R}}^{(2)^T}\mathbf{n}_{\mathrm{R}}^{(1)^T}\right]^T$ is the combined additive white Gaussian noise vector of $S2$ and $S1$ with $\mathbf{n}_{\mathrm{R}}^{(2)}$ and $\mathbf{n}_{\mathrm{R}}^{(1)}$ being the noise vectors at $S2$ and $S1$, respectively. The covariance matrix of $\mathbf{n}_{\mathrm{R}}$ is defined by $\mathbf{R}_{\mathbf{n}_{\mathrm{R}}} = \mathrm{E}\left\{\mathbf{n}_{\mathrm{R}}\mathbf{n}_{\mathrm{R}}^{H}\right\}$.

III. L T F SDD R

In the following, it is assumed that instantaneous CSI about $\mathbf{H}_{\mathrm{R}}$ and $\mathbf{H}_{\mathrm{T}}$ is available at the RS. In this case, there are two concepts how the transceive filter $\mathbf{G}$ at the RS can be designed. For the first concept, $\mathbf{G}$ is assumed as a combination of a linear receive filter $\mathbf{G}_{\mathrm{R}}$, a weight matrix $\mathbf{G}_{\Pi}$, and a linear transmit filter $\mathbf{G}_{\mathrm{T}}$ where all filters can be determined independently, i.e., the transceive filter is designed in three steps. For the second concept, $\mathbf{G}$ is designed in one step without separating it into a receive and a transmit filter part.

A. Three-Step Design for the Linear Transceive Filter

In the first step, the RS receive vector $\mathbf{y}_{\mathrm{RS}}$ is multiplied with the linear receive filter matrix $\mathbf{G}_{\mathrm{R}}$ resulting in the RS estimation vector

$$\hat{\mathbf{x}}_{\mathrm{RS}} = \left[\hat{\mathbf{x}}_{\mathrm{RS}}^{(1)^T}, \hat{\mathbf{x}}_{\mathrm{RS}}^{(2)^T}\right]^T \tag{16}$$

with the estimate $\hat{\mathbf{x}}_{\mathrm{RS}}^{(1)}$ for $\mathbf{x}^{(1)}$ and the estimate $\hat{\mathbf{x}}_{\mathrm{RS}}^{(2)}$ for $\mathbf{x}^{(2)}$, respectively.

In the second step, $\hat{\mathbf{x}}_{\mathrm{RS}}$ is multiplied with the RS weight matrix

$$\mathbf{G}_{\Pi} = \begin{bmatrix} \sqrt{\beta}\mathbf{I}_M & \emptyset_M \\ \emptyset_M & \sqrt{(1-\beta)}\mathbf{I}_M \end{bmatrix} \tag{17}$$

where the parameter β with $0 \leq \beta \leq 1$ is a weight factor which is applied to the RS estimation vectors before retransmission. For $\beta = 0.5$, the RS estimation vectors are equally weighted, for $\beta = 1$ only $\hat{\mathbf{x}}_{\mathrm{RS}}^{(1)}$ is transmitted and for $\beta = 0$ only $\hat{\mathbf{x}}_{\mathrm{RS}}^{(2)}$ is transmitted.

In the third step, the weighted RS estimation vector is multiplied with transmit filter matrix $\mathbf{G}_{\mathrm{T}}$ leading to the RS transmit vector

$$\mathbf{x}_{\mathrm{RS}} = \mathbf{G}_{\mathrm{T}}\mathbf{G}_{\Pi}\hat{\mathbf{x}}_{\mathrm{RS}} \tag{18}$$

from Eq. (12). The transmit filter $\mathbf{G}_{\mathrm{T}}$ separates the vectors designated to $S1$ and $S2$ before retransmission and substitutes receive processing at $S1$ and $S2$. The overall transceive filter matrix is given by

$$\mathbf{G} = \mathbf{G}_{\mathrm{T}}\mathbf{G}_{\Pi}\mathbf{G}_{\mathrm{R}}. \tag{19}$$

In the following, two different linear transceive filters $\mathbf{G}$ are considered which are based on the ZF and MMSE criterion, respectively. The derivation of the filters is exactly like in a single-hop MIMO system and can be verified in [19]. Hence, only the resulting filters are summarized here.

1) ZF Receive Filter:

$$\mathbf{G}_{\mathrm{R,ZF}} = \left(\mathbf{Q}_{\mathrm{R}}^{H}\mathbf{H}_{\mathrm{R}}^{H}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}^{-1}\mathbf{H}_{\mathrm{R}}\mathbf{Q}\right)^{-1}\mathbf{Q}^{H}\mathbf{H}_{\mathrm{R}}^{H}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}^{-1} \tag{20}$$

2) ZF Transmit Filter:

$$\mathbf{G}_{\mathrm{T,ZF}} = \frac{1}{p_{\mathrm{ZF}}}\mathbf{H}_{\mathrm{T}}^{H}\left(\mathbf{H}_{\mathrm{T}}\mathbf{H}_{\mathrm{T}}^{H}\right)^{-1} \tag{21}$$

with the scalar receive filters

$$p_{\mathrm{ZF}}^{(1)} = p_{\mathrm{ZF}}^{(2)} = p_{\mathrm{ZF}} = \sqrt{\frac{\mathrm{tr}\left\{\left(\mathbf{H}_{\mathrm{T}}\mathbf{H}_{\mathrm{T}}^{H}\right)^{-1}\mathbf{G}_{\Pi}\mathbf{G}_{\mathrm{R}}\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}\mathbf{G}_{\mathrm{R}}^{H}\mathbf{G}_{\Pi}^{H}\right\}}{E^{(\mathrm{RS})}}}. \tag{22}$$

3) MMSE Receive Filter:

$$\mathbf{G}_{\mathrm{R,MMSE}} = \mathbf{R}_{\mathbf{x}}\mathbf{Q}^{H}\mathbf{H}_{\mathrm{R}}^{H}\left(\mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{R}_{\mathbf{x}}\mathbf{Q}^{H}\mathbf{H}_{\mathrm{R}}^{H} + \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\right)^{-1} \tag{23}$$

4) MMSE Transmit Filter:

$$\mathbf{G}_{\mathrm{T,MMSE}} = \frac{1}{p_{\mathrm{MMSE}}}\left(\mathbf{H}_{\mathrm{T}}^{H}\mathbf{H}_{\mathrm{T}} + \frac{\mathrm{tr}\left\{\mathbf{R}_{\mathbf{n}_{\mathrm{R}}}\right\}}{E^{(\mathrm{RS})}}\mathbf{I}\right)^{-1}\mathbf{H}_{\mathrm{T}}^{H} \tag{24}$$

with the scalar receive filters

$$p_{\mathrm{MMSE}}^{(1)} = p_{\mathrm{MMSE}}^{(2)} = p_{\mathrm{MMSE}} = \sqrt{\frac{\mathrm{tr}\left\{\left(\mathbf{H}_{\mathrm{T}}^{H}\mathbf{H}_{\mathrm{T}} + \frac{\mathrm{tr}\left\{\mathbf{R}_{\mathbf{n}_{\mathrm{R}}}\right\}}{E^{(\mathrm{RS})}}\mathbf{I}\right)^{-2}\mathbf{H}_{\mathrm{T}}^{H}\mathbf{G}_{\Pi}\mathbf{G}_{\mathrm{R}}\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}\mathbf{G}_{\mathrm{R}}^{H}\mathbf{G}_{\Pi}^{H}\mathbf{H}_{\mathrm{T}}\right\}}{E^{(\mathrm{RS})}}}. \tag{25}$$

Since the derived receive and transmit filters $\mathbf{G}_{\mathrm{R}}$ and $\mathbf{G}_{\mathrm{T}}$ require the same channel coefficients if channel reciprocity can be assumed there also exists a high potential for saving processing effort at the RS. For example, the calculation of the inverse of $\mathbf{H}_{\mathrm{T}}\mathbf{H}_{\mathrm{T}}^{H}$ in Eq. (21) may be reused for the calculation of the inverse of $\mathbf{H}_{\mathrm{R}}^{H}\mathbf{H}_{\mathrm{R}}$ in Eq. (20) if $\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}$ and $\mathbf{Q}$ are diagonal matrices with equal entries on their main diagonal.

B. One-Step Design for the Linear Transceive Filter

In the following, the ZF and MMSE criterion are applied directly to the estimate of Eq. (15), i.e., the transceive filter design is not separated into an independent receive and transmit filter design like introduced in the previous section. For the one-step concept, there exist no RS estimation vectors. Hence, it is not possible to give different weights to each direction of communication before the retransmission like introduced in (17). Since the one-step concept is not based on former results for receive and transmit beamforming the optimization problems are formulated and solved in the following.

1) ZF Transceive Filter: For the ZF criterion, the transceive filter $\mathbf{G}$ at the RS has to be designed such that the mean squared error of the estimate vector $\hat{\mathbf{x}}$ for data vector $\mathbf{x}$ is minimized. With the ZF constraint and the RS transmit power constraint of (13), the ZF optimization may be formulated as

$$\{\mathbf{G}_{\mathrm{ZF}}, p_{\mathrm{ZF}}^{(1)}, p_{\mathrm{ZF}}^{(2)}\} = \underset{\{\mathbf{G}, p^{(1)}, p^{(2)}\}}{\arg\min}\, \mathrm{E}\left\{\| \hat{\mathbf{x}} - \mathbf{x} \|_2^2\right\} \tag{26a}$$

$$\text{subject to:} \quad \hat{\mathbf{x}} = \mathbf{x} \quad \text{for} \quad \mathbf{n}_{\mathrm{RS}} = \emptyset_{M^{(\mathrm{RS})}\times 1} \text{ and } \mathbf{n}_{\mathrm{R}} = \emptyset_{M\times 1} \tag{26b}$$

$$\mathrm{E}\left\{\| \mathbf{x}_{\mathrm{RS}} \|_2^2\right\} \leq E^{(\mathrm{RS})} \quad k = 1, 2. \tag{26c}$$

From the derivation in Appendix A it can be seen that the ZF transceive filter is given by

$$\mathbf{G}_{\mathrm{ZF}} = \frac{1}{p_{\mathrm{ZF}}} \left(\mathbf{H}_T^H \mathbf{H}_{\mathrm{T}}\right)^{-1} \mathbf{H}_{\mathrm{T}}^H \mathbf{Q}^H \mathbf{H}_{\mathrm{R}}^H \left(\mathbf{H}_{\mathrm{R}} \mathbf{Q} \mathbf{Q}^H \mathbf{H}_{\mathrm{R}}^H\right)^{-1} \tag{27}$$

with the scalar receive filters

$$p_{\mathrm{ZF}}^{(1)} = p_{\mathrm{ZF}}^{(2)} = p_{\mathrm{ZF}} = \sqrt{\frac{\mathrm{tr}\left\{\left(\mathbf{H}_T^H \mathbf{H}_{\mathrm{T}}\right)^{-1} \mathbf{H}_{\mathrm{T}}^H \mathbf{Q}^H \mathbf{H}_{\mathrm{R}}^H \left(\mathbf{H}_{\mathrm{R}} \mathbf{Q} \mathbf{Q}^H \mathbf{H}_{\mathrm{R}}^H\right)^{-1} \mathbf{R}_{\mathbf{y}_{RS}} \left(\mathbf{H}_{\mathrm{R}} \mathbf{Q} \mathbf{Q}^H \mathbf{H}_{\mathrm{R}}^H\right)^{-1} \mathbf{H}_{\mathrm{R}} \mathbf{Q} \mathbf{H}_{\mathrm{T}} \left(\mathbf{H}_T^H \mathbf{H}_{\mathrm{T}}\right)^{-1}\right\}}{E^{(\mathrm{RS})}}}. \tag{28}$$

Comparing Eq. (27) with the single filters in (20) and (21) shows that both solutions are very similar since both concepts simply reverse the two channels $\mathbf{H}_{\mathrm{R}}$ and $\mathbf{H}_{\mathrm{T}}$.

2) MMSE Transceive Filter: The MMSE transceive filter $\mathbf{G}_{\mathrm{MMSE}}$ at the RS has to be designed such that the mean squared error of the estimate vector $\hat{\mathbf{x}}$ for transmit vector $\mathbf{x}$ is minimized. With the RS transmit power constraint of (13), the MMSE optimization may be formulated as

$$\{\mathbf{G}_{\mathrm{MMSE}}, p_{\mathrm{MMSE}}^{(1)}, p_{\mathrm{MMSE}}^{(2)}\} = \underset{\{\mathbf{G}, p^{(1)}, p^{(2)}\}}{\arg\min}\, \mathrm{E}\left\{\| \hat{\mathbf{x}} - \mathbf{x} \|_2^2\right\} \tag{29a}$$

$$\text{subject to:} \quad \mathrm{E}\left\{\| \mathbf{x}_{\mathrm{RS}} \|_2^2\right\} \leq E^{(\mathrm{RS})}. \tag{29b}$$

From the derivation in Appendix B it can be seen that the MMSE transceive filter is given by

$$\mathbf{G}_{\mathrm{MMSE}} = \frac{1}{p_{\mathrm{MMSE}}} \left(\mathbf{H}_T^H \mathbf{H}_{\mathrm{T}}\right)^{-1} \mathbf{H}_{\mathrm{T}}^H \mathbf{R}_{\mathbf{x}}^H \mathbf{Q}^H \mathbf{H}_{\mathrm{R}}^H \left(\mathbf{H}_{\mathrm{R}} \mathbf{Q} \mathbf{R}_{\mathbf{x}} \mathbf{Q}^H \mathbf{H}_{\mathrm{R}}^H + \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\right)^{-1} \tag{30}$$

with the scalar receive filters

$$p_{\mathrm{MMSE}}^{(1)} = p_{\mathrm{MMSE}}^{(2)} = p_{\mathrm{MMSE}} = \sqrt{\frac{\mathrm{tr}\left\{\left(\mathbf{H}_T^H \mathbf{H}_{\mathrm{T}}\right)^{-1} \mathbf{H}_{\mathrm{T}}^H \mathbf{R}_{\mathbf{x}}^H \mathbf{Q}^H \mathbf{H}_{\mathrm{R}}^H \left(\mathbf{R}_{\mathbf{y}_{RS}}^H\right)^{-1} \mathbf{H}_{\mathrm{R}} \mathbf{Q} \mathbf{R}_{\mathbf{x}} \mathbf{H}_{\mathrm{T}} \left(\mathbf{H}_T^H \mathbf{H}_{\mathrm{T}}\right)^{-1}\right\}}{E^{(\mathrm{RS})}}}. \tag{31}$$

The solution in (30) is somehow different from the solutions in (23) and (24). This comes from the fact that the RS transmit energy constraint has to be relaxed in order to get an analytical solution for the MMSE one-step concept. For a detailed description on this circumstance please see Appendix B. Due to this difference in both solutions, different BER performances of the one-step and the three-step concept are expected. The three-step concept should outperform the one-step concept since it does not require a relaxation of its constraints.

IV. S D -I SDD R

In the following, knowledge about the own transmitted vectors $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ shall be exploited at $S1$ and $S2$, respectively, in order to improve the performance of SDD relaying like proposed in [20]. For that purpose, $\hat{\mathbf{x}}$ from Eq. (15) is decomposed into an overall useful receive signal vector $\mathbf{x}_{\mathrm{uf}} = \left[\mathbf{x}_{\mathrm{uf}}^{(1)^T}, \mathbf{x}_{\mathrm{uf}}^{(2)^T}\right]^T$

and an overall duplex-interference vector $\mathbf{x}_{\mathrm{if}} = \left[\mathbf{x}_{\mathrm{if}}^{(1)^T}, \mathbf{x}_{\mathrm{if}}^{(2)^T}\right]^T$ each consisting of the corresponding vectors at S1 and S2. Furthermore, matrix $\mathbf{A} = \mathbf{P}\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{R}}\mathbf{Q}$ is defined which may be written as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{\mathrm{uf}}^{(1)} & \mathbf{A}_{\mathrm{if}}^{(2)} \\ \mathbf{A}_{\mathrm{if}}^{(1)} & \mathbf{A}_{\mathrm{uf}}^{(2)} \end{bmatrix} \tag{32}$$

with matrices $\mathbf{A}_{\mathrm{uf}}^{(1)}$, $\mathbf{A}_{\mathrm{if}}^{(1)}$, $\mathbf{A}_{\mathrm{uf}}^{(2)}$, and $\mathbf{A}_{\mathrm{if}}^{(2)}$ each of size $M \times M$. Matrices $\mathbf{A}_{\mathrm{uf}}^{(1)}$ and $\mathbf{A}_{\mathrm{uf}}^{(2)}$ correspond to the useful receive signal vectors containing $\mathbf{x}^{(1)}$ at $S2$ and containing $\mathbf{x}^{(2)}$ at $S1$, respectively. Matrices $\mathbf{A}_{\mathrm{if}}^{(2)}$ and $\mathbf{A}_{\mathrm{if}}^{(1)}$ correspond to the duplex-interference from $\mathbf{x}^{(2)}$ at $S2$ and from $\mathbf{x}^{(1)}$ at $S1$, respectively. Applying this notation, Eq. (15) can be rewritten as

$$\hat{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{A}_{\mathrm{uf}}^{(1)} & \emptyset_M \\ \emptyset_M & \mathbf{A}_{\mathrm{uf}}^{(2)} \end{bmatrix} \mathbf{x}}_{\mathbf{x}_{\mathrm{uf}}} + \underbrace{\begin{bmatrix} \emptyset_M & \mathbf{A}_{\mathrm{if}}^{(2)} \\ \mathbf{A}_{\mathrm{if}}^{(1)} & \emptyset_M \end{bmatrix} \mathbf{x}}_{\mathbf{x}_{\mathrm{if}}} + \mathbf{P}\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{n}_{\mathrm{RS}} + \mathbf{P}\mathbf{n}_{\mathrm{R}}. \tag{33}$$

Subtracting the overall duplex-interference vector $\mathbf{x}_{\mathrm{if}}$ from the estimation vector $\hat{\mathbf{x}}$ at $S1$ and $S2$, the improved overall estimation vector in SDD relaying is given by

$$\hat{\mathbf{x}}_{\mathrm{imp}} = \hat{\mathbf{x}} - \mathbf{x}_{\mathrm{if}}. \tag{34}$$

Since the duplex-interfence is eliminated, the overall signal-to-noise-and-interference ratio (SINR) at $S1$ and $S2$ is increased for the estimate in (34) compared to the estimate in (15). This corresponds to a signal-to-noise ratio (SNR) gain in the BER performance which is analyzed in the simulations. Note that this improvement can only be verified for linear transceive filters which introduce interference among simultaneously received and transmitted data symbols like the MMSE transceive filter for example. A linear filter which fulfills the ZF constraint does not introduce duplex-interference at $S1$ and $S2$, i.e., for the linear ZF transceive filter no SNR gain can be achieved due to subtraction of duplex-interference (SDI).

V. R R S

In SDD relaying, the receive and transmit signals at the RS are neither decoded nor encoded. Therefore, SDD relaying can still be interpreted as an AF relaying scheme which applies linear signal processing at the RS. The down- and uplink signals are separated by multiple-antenna beamforming techniques. With the proposed linear transceive filters from Section III, no further signal processing is required at $S1$ and $S2$. In this section, two other relaying schemes are proposed, namely MRC relaying and MRC-BF relaying which are already known from Fig. 1. Compared to SDD relaying, the same effort in terms of number of antennas, achieving CSI, and applied signal processing is required in MRC and MRC-BF relaying. Since both schemes apply state-of-the-art signal processing at the RS, they are only shortly summarized here.

A. MRC Relaying

MRC relaying is a one-way relaying protocol, i.e., the bi-directional communication between $S1$ and $S2$ requires four orthogonal channel resources. MRC is a well-known approach for combating fading of the wireless channel [17]. Originally, signals which are received via multiple diversity branches are combined that way that the SNR at the receiver is maximized. MRC can also be applied to the transmit signal [18]. In two-hop relaying, one may apply both, receive and transmit MRC, since each antenna at the RS represents a diversity branch for reception as well as for transmission. In MRC relaying, on the first channel resource $S1$ transmits $\mathbf{x}^{(1)}$ to the RS. Firstly, receive MRC is applied to the receive vector at the RS, i.e., the MRC receive filter at the RS is matched to channel $\mathbf{H}_{\mathrm{R}}^{(1)}$ from $S1$ to the RS. Secondly, transmit MRC is applied at the RS, i.e., the MRC transmit filter at the RS is matched to channel $\mathbf{H}_{\mathrm{T}}^{(2)}$ from the RS to $S2$. On the second channel resource, the RS retransmits the filtered vector to $S2$ leading to the estimate $\hat{\mathbf{x}}^{(1)}$. Using the third and fourth channel resource, the same scheme is applied for the transmission of $\mathbf{x}^{(2)}$ from $S2$ to $S1$ via the RS.

In contrast to SDD relaying, down- and uplink can be separated conventionally by either TDD or FDD in MRC relaying.

B. MRC-BF Relaying

For MRC-BF relaying, three orthogonal channel resources are required for the bi-directional communication between $S1$ and $S2$. On the first channel resource, $S1$ transmits $\mathbf{x}^{(1)}$ to the RS. Receive MRC is applied to the receive vector at the RS, i.e., the MRC receive filter at the RS is matched to channel $\mathbf{H}_{\mathrm{R}}^{(1)}$ from $S1$ to the RS. The estimate $\hat{\mathbf{x}}_{\mathrm{RS}}^{(1)}$ is stored at the RS for further signal processing. On the second channel resource, $S2$ transmits $\mathbf{x}^{(2)}$ to the RS. Receive MRC is applied to the receive vector at the RS, i.e., the MRC receive filter at the RS is matched to channel $\mathbf{H}_{\mathrm{R}}^{(2)}$ from $S2$ to the RS. The two estimates $\hat{\mathbf{x}}_{\mathrm{RS}}^{(1)}$ and $\hat{\mathbf{x}}_{\mathrm{RS}}^{(2)}$ after the MRC receive filters are spatially separated by a linear transmit beamforming filter which can be taken from the set of transmit filters in Section III-A. On the third channel resource, the filtered estimates at the RS are simultaneously retransmitted to $S1$ and $S2$.

In MRC-BF relaying on the first two channel resources, down- and uplink can be separated by either TDD or FDD, but on the third channel resource, down- and uplink are separated by SDD. This means that MRC-BF relaying is a mixture of different duplex schemes.

Note that the order of MRC and beamforming could also be reversed which would lead to another relaying scheme. In this scheme, firstly receive beamforming and secondly transmit MRC would be applied at the RS. Since this scheme is very similar to MRC-BF relaying it is not considered in the following.

VI. CSI R

Throughout the paper, it is assumed that the considered CSI is instantaneously and perfectly known. However, there exists much space for future work which investigates the impact of non-instantaneous and imperfect CSI to the proposed relaying schemes. In this section, SDD relaying is analyzed concerning the location where CSI is required and how it can be achieved at this location.

SDD relaying without SDI requires CSI only at the RS. CSI of the channels $\mathbf{H}_{\mathrm{R}}^{(1)}$ and $\mathbf{H}_{\mathrm{R}}^{(2)}$ from $S1$ and $S2$ to the RS, respectively, can be obtained by inserting a pilot signal into the transmit signal of each node and estimating each channel at the RS independently. For a sufficiently long channel coherence time which allows to assume channel reciprocity, the same channel coefficients can be used for the retransmission from the RS to $S1$ and $S2$. This means that no CSI feedback channels are required for SDD relaying without SDI.

The performance of SDD relaying may be improved if CSI is also available at $S1$ and $S2$. In this case, SDD relaying with SDI like introduced in Section IV can be applied. For SDD relaying with SDI, it is assumed that the RS still estimates both channels $\mathbf{H}_{\mathrm{R}}$ and $\mathbf{H}_{\mathrm{T}}$ in order to design the transceive filter. Furthermore, the matrices $\mathbf{A}_{\mathrm{if}}^{(1)}$ and $\mathbf{A}_{\mathrm{if}}^{(2)}$ from (32) are determined at the RS and signaled to $S1$ and $S2$, respectively, via a feedback channel. Knowing these matrices and the own transmitted vectors $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ at $S1$ and $S2$, respectively, it is possible to subtract the duplex-interference $\mathbf{x}_{\mathrm{if}}^{(2)}$ at $S1$ and $\mathbf{x}_{\mathrm{if}}^{(1)}$ at $S2$.

In MRC relaying, CSI about the same channels like in SDD relaying is required at the RS. Therefore, CSI can be achieved in the same way. Due to the separation of down- and uplink by either TDD or FDD there exists no duplex-interference in MRC relaying, i.e., CSI signaling from the RS to $S1$ and $S2$ cannot improve the performance.

In MRC-BF relaying, CSI about the same channels like in SDD relaying is required at the RS. Therefore, CSI can be achieved in the same way. Like in SDD relaying, duplex-interference is generated at $S1$ and $S2$ due to the transmit beamforming filter in MRC-BF relaying. The required CSI for SDI can be achieved via a feedback channel like in SDD relaying.

Table I gives an overview which schemes require CSI estimation at the RS and which schemes additionally require CSI signaling from the RS to $S1$ and $S2$.

	CSI estimation at RS	CSI signaling: RS → $S1/S2$
MRC relaying	X	
MRC-BF relaying	X	
SDD relaying	X	
MRC-BF relaying with SDI	X	X
SDD relaying with SDI	X	X

TABLE I

CSI

A final remark shall be given on SDD relaying combined with cooperative relaying [1]. Since $S1$ and $S2$ always receive and transmit simultaneously in SDD relaying, it is not possible to exploit the direct channel between $S1$ and $S2$ for a cooperative relaying approach. Hence, SDD relaying is a relaying scheme which is especially developed for scenarios where the direct channel between $S1$ and $S2$ is not available, e.g., due to shadowing or limited transmit power. Since $S1$ and $S2$ receive and transmit on different channel resource, cooperation is possible for MRC and MRC-BF relaying in general. However, additional effort would be required in this case and cooperation goes beyond the scope of this paper.

VII. S R SDD R

In the following, the sum rate of a system is defined as the sum of the mutual information values for all transmissions using the same channel resources. It is a measure for the spectral efficiency of the considered relaying schemes. In [21], it is shown that for a MIMO system with

$$\tilde{\mathbf{y}} = \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{B}}\tilde{\mathbf{n}} \tag{35}$$

the mutual information is given by

$$C_{\text{MIMO}} = \log_2\left(\det\left[\mathbf{I} + \frac{\tilde{\mathbf{A}}\mathbf{R}_{\tilde{\mathbf{x}}}\tilde{\mathbf{A}}^H}{\tilde{\mathbf{B}}\mathbf{R}_{\tilde{\mathbf{n}}}\tilde{\mathbf{B}}^H}\right]\right) \tag{36}$$

where $\tilde{A}$ and $\tilde{B}$ depend on the underlying MIMO system and $\mathbf{R}_{\tilde{\mathbf{x}}}$ and $\mathbf{R}_{\tilde{\mathbf{n}}}$ are the transmit vector and receive noise vector covariance matrices, respectively.

In the following, the duplex-interference in SDD relaying from Eq. (33) is regarded as additional noise, leading to the overall interference and noise vector

$$\mathbf{n}^{(k)} = \left[\mathbf{x}^{(i)^T} \quad \mathbf{n}_{RS}^T \quad \mathbf{n}_R^{(k)^T}\right]^T, \qquad k = \begin{cases} 1 & \text{for } i = 2 \\ 2 & \text{for } i = 1 \end{cases} \tag{37}$$

at node i, and covariance matrix $\mathbf{R}_{\mathbf{n}^{(k)}} = \mathrm{E}\left\{\mathbf{n}^{(k)}\mathbf{n}^{(k)^H}\right\}$. Furthermore, the overall interference and noise matrix $\mathbf{B}_{\text{TW}}^{(k)}$ is given by

$$\mathbf{B}_{\text{TW}}^{(k)} = \begin{bmatrix} \mathbf{A}_{\text{if}}^{(i)} & \emptyset_{M\times M^{(RS)}} & \emptyset_{M\times M} \\ \emptyset_{M\times M} & \mathbf{P}^{(k)}\mathbf{H}_T^{(k)}\mathbf{G} & \mathbf{P}^{(k)} \end{bmatrix}, \qquad k = \begin{cases} 1 & \text{for } i = 2 \\ 2 & \text{for } i = 1 \end{cases} \tag{38}$$

at node i. Under these assumptions, the mutual information in SDD relaying at each node is given by

$$C_{\text{TW}}^{(k)} = \frac{1}{2}\log_2\left(\det\left[\mathbf{I}_M + \frac{\mathbf{A}_{\text{uf}}^{(k)}\mathbf{R}_{\mathbf{x}^{(k)}}\mathbf{A}_{\text{uf}}^{(k)^H}}{\mathbf{B}_{\text{TW}}^{(k)}\mathbf{R}_{\mathbf{n}^{(k)}}\mathbf{B}_{\text{TW}}^{(k)^H}}\right]\right) \text{ for } k = 1, 2 \tag{39}$$

where $C_{\text{TW}}^{(1)}$ is the mutual information at node $S2$ and $C_{\text{TW}}^{(2)}$ is the mutual information at node $S1$. The pre-log factor 1/2 is introduced in order to indicate the increase in required channel resources for each direction of communication due to the two-hop relaying approach. Because of the simultaneous transmission of down- and uplink, the sum rate of SDD relaying results in

$$C_{\text{TW}} = C_{\text{TW}}^{(1)} + C_{\text{TW}}^{(2)}. \tag{40}$$

Note that in case of SDI at $S1$ and $S2$ like introduced in Section IV, matrices $\mathbf{A}_{\text{if}}^{(i)}$, $i = 1, 2$, are set to zero, i.e., $\mathbf{A}_{\text{if}}^{(i)} = \emptyset_M$, since there exists no duplex-interference for this scheme.

A. Maximizing the Sum Rate

Both mutual information values $C_{\mathrm{TW}}^{(1)}$ and $C_{\mathrm{TW}}^{(2)}$ depend on the quality of both channels, $\mathbf{H}_{\mathrm{R/T}}^{(1)}$ between $S1$ and the RS and $\mathbf{H}_{\mathrm{R/T}}^{(2)}$ between $S2$ and the RS, i.e., even if one channel is much better than the other channel, both, down- and uplink, are degraded by the worse channel.

For the three-step concept of the transceive filter design from Section III-A, it is possible to give different weights β to the two RS estimation vectors $\hat{\mathbf{x}}_{\mathrm{RS}}^{(1)}$ and $\hat{\mathbf{x}}_{\mathrm{RS}}^{(2)}$ after the receive filter $\mathbf{G}_R$. Assigning equal weights to both RS estimation vectors at the RS before retransmission may lead to a sub-optimum sum rate if one RS estimation vector is received over a good channel while the other RS estimation vector is received over a bad channel. The sum rate of Eq. (40) can be maximized by optimizing β from Eq. (17). The underlying optimization problem is formulated as

$$\beta_{opt} = \arg\max_{\beta} \left\{ C_{\mathrm{TW}}^{(1)} + C_{\mathrm{TW}}^{(2)} \right\} \tag{41a}$$

$$\text{subject to:} \quad 0 \le \beta \le 1. \tag{41b}$$

There exists no closed form solution to this optimization problem. However, it can be transferred into a convex problem and may be solved by numeric optimization.

For the one-step design from Section III-B, this optimization is not possible since there exist no estimation vectors at the RS which could be weighted. The filter design for the one-step concept is adapted to the overall channel which is a combination of $\mathbf{H}_{\mathrm{R}}$ and $\mathbf{H}_{\mathrm{T}}$, but it cannot be adapted to each channel separately which is the case for the three-step design.

B. Approximation for Maximizing the Sum Rate

In the following, the optimization problem in (41) is simplified leading to a closed form approximation for β_{opt} in the three-step transceive filter design. Let us assume a fading channel with an average SNR on the channel from $S1$ to the RS given by $\rho^{(1)}$ and an average SNR on the channel from $S2$ to the RS given by $\rho^{(2)}$. In this case, the overall average SNR for AF relaying at receiving node $S2$ results in [4]

$$\rho_{\mathrm{ov}}^{(1)} = \frac{\beta\rho^{(1)}\rho^{(2)}}{\rho^{(1)} + \beta\rho^{(2)} + 1} \tag{42}$$

and the overall SNR at receiving node $S1$ results in

$$\rho_{\mathrm{ov}}^{(2)} = \frac{(1-\beta)\rho^{(1)}\rho^{(2)}}{(1-\beta)\rho^{(1)} + \rho^{(2)} + 1}. \tag{43}$$

Approximating the mutual information values of Eq. (39) by the single input single output (SISO) mutual information

$$\tilde{C}_{\mathrm{TW}}^{(k)} = \frac{1}{2}\log_2\left(1 + \rho_{\mathrm{ov}}^{(k)}\right) \quad \text{for } k = 1, 2 \tag{44}$$

the sum rate may be approximated in the high SNR region by

$$\tilde{C}_{\mathrm{TW}} = \frac{1}{2}\log_2\left(\rho_{\mathrm{ov}}^{(1)}\right) + \frac{1}{2}\log_2\left(\rho_{\mathrm{ov}}^{(2)}\right). \tag{45}$$

Substituting (42) and (43) into (45) and setting the deviation of (45) equal to zero the approximation leads to

$$\beta_{app} = \begin{cases} 0.5 & \text{for } \rho^{(1)} = \rho^{(2)} \\ \frac{\rho^{(1)}+1-\sqrt{(\rho^{(1)}+1)(\rho^{(2)}+1)}}{\rho^{(1)}-\rho^{(2)}} & \text{for } \rho^{(1)} \neq \rho^{(2)} \end{cases}. \tag{46}$$

Note that the sum rate which is calculated by Eq. (45) is different from the exact sum rate in Eq. (40). However, in order to determine the optimum parameter β this approximation provides reasonable results with low effort, which is also confirmed by the simulation results.

VIII. S R

In this section, the BER performance and the average sum rate of SDD relaying is analyzed by means of simulations. For all simulations, it is assumed that nodes $S1$ and $S2$ are each equipped with $M = 1$ antenna and the RS is equipped with $M^{(\mathrm{RS})} = 2$ antennas. For the BER performance analyses, the data symbols of $S1$ and $S2$ are QPSK modulated. For the sum rate analyses, Gaussian data signals are assumed. The channel coefficients are spatially white and Rayleigh distributed with zero mean and variance 1. The noise vectors are complex zero mean Gaussian with variance σ^2_{RS} at the RS, variance σ^2_1 at $S1$, and variance σ^2_2 at $S2$, respectively. The presented results are achieved from Monte Carlo simulations with statistically independent channel fading realisations where $\rho^{(1)} = E^{(\mathrm{RS})}/\sigma^2_1 = E^{(1)}/\sigma^2_{\mathrm{RS}}$ denotes the average SNR between $S1$ and the RS and $\rho^{(2)} = E^{(\mathrm{RS})}/\sigma^2_2 = E^{(2)}/\sigma^2_{\mathrm{RS}}$ denotes the average SNR between $S2$ and the RS.

A. Comparison of One-Step and Three-Step Designs

For the following investigations, the average SNR $\rho^{(1)}$ of the first channel from $S1$ to the RS is fixed at a certain value and the BER is depicted depending on the average SNR $\rho^{(2)}$ of the second channel from $S2$ to the RS. Fig. 2 gives BER performance at node $S2$ for the linear ZF and MMSE transceive filters from Section III which are either designed in one step or in three steps. For the one-step design, $\beta = 0.5$

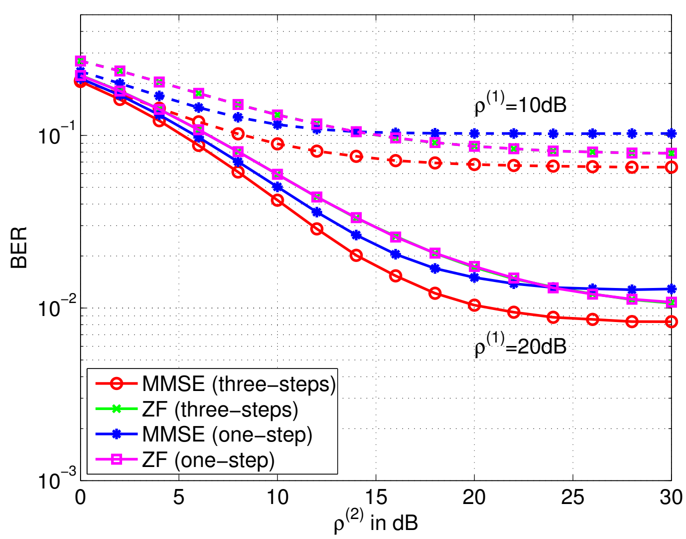

Fig. 2. Comparison of BER performance at $S2$ for the ZF and MMSE transceive filters with one-step and three-step designs (dashed lines: $\rho^{(1)} = 10$dB, solid lines: $\rho^{(1)} = 20$dB)

is chosen since the optimization of the sum rate is not of interest for the following investigations. For all transceive filters, the BER performance has an error floor which increases for decreasing $\rho^{(1)}$, i.e., all curves show a saturation region where an increase of $\rho^{(2)}$ does no longer improve the BER performance due to the fixed value of $\rho^{(1)}$. From receive and transmit oriented spatial filters, it is known that the linear MMSE receive and transmit filters outperform the linear ZF receive and transmit filters [19]. This result is also found for the transceive filters in SDD relaying for the one-step design which applies the same receive and transmit filters like in [19]. The one-step and the three-step designs for the linear ZF transceive filter lead exactly to the same BER performance. This has already been expected from the derivation of the filters since both solutions simply reverse the two overall channels $\mathbf{H}_{\mathrm{R}}$ and $\mathbf{H}_{\mathrm{T}}$. For the MMSE transceive filter, the three-step design performs better than the one-step design. This could also be expected from the design of the filters since the one-step design does not consider the RS energy constraint in its optimization which leads to a sub-optimum solution.

Fig. 3 gives the BER performance at node $S1$ for the linear ZF and MMSE transceive filters which are

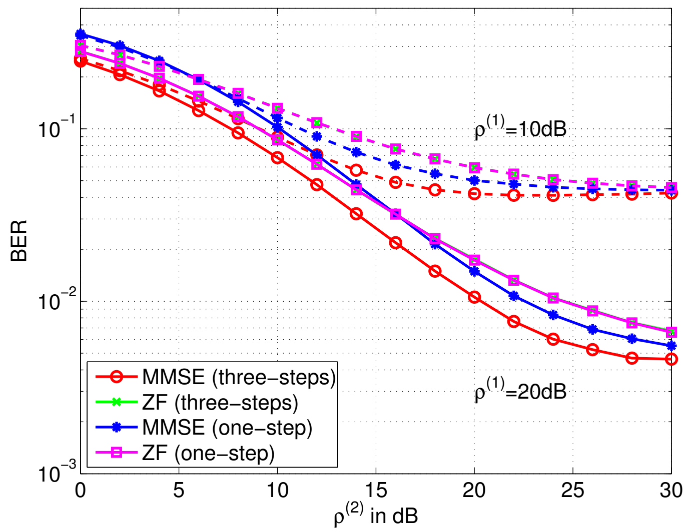

Fig. 3. Comparison of BER performance at $S1$ for the ZF and MMSE transceive filters with one-step and three-step designs (dashed lines: $\rho^{(1)} = 10$dB, solid lines: $\rho^{(1)} = 20$dB)

either designed in one step or in three steps. The BER performance at $S1$ is different from the one at $S2$, since down- and uplink are individually orthogonalized by the transceive filters depending on the SNR of each single channel. For the linear ZF transceive filter, the one-step and the three-step designs still perform in the same way. The one-step MMSE transceive filter performs better than the ZF transceive filters for high $\rho^{(2)}$ and worse for low $\rho^{(2)}$. At $S2$ the behavior is the other way round. For the BER performances at $S1$ as well as at $S2$, this means that the one-step MMSE transceive filter outperforms the ZF transceive filters only if the SNR on the first hop which is either $\rho^{(1)}$ for the BER at $S2$ or $\rho^{(2)}$ for the BER at $S1$, is higher than the SNR on the second hop. Like at $S2$, the MMSE three-step transceive filter outperforms all other filters regarding the BER performance at $S1$. In the following, only the three-step transceive filters are considered as they provide the same or even better performance than the one-step filters.

B. Subtraction of Duplex-Interference

Fig. 4 gives the BER performance at $S2$ for the one-step MMSE transceive filter with and without SDI like introduced in Section IV. Furthermore, the BER performance at $S1$ for the MMSE transceive filter with SDI is depicted in the figure. Like in the previous cases, the BER performance has an error floor which increases for decreasing $\rho^{(1)}$. There exists a significant improvement for the BER performance at $S2$ for the linear MMSE transceive filter if SDI is applied. For a target BER of 10^{-2}, the SNR gain due to SDI is approximately 4dB for $\rho^{(1)} = 20$dB. Another effect of the SDI approach is that the BER at $S1$ and $S2$ is now the same. Since the duplex-interference is cancelled at both nodes, the performance only depends on the effective SNR at $S1$ and $S2$ which is determined by the overall noise behind all filters. The effective SNR is the same at both nodes due to the symmetry of the network. Symmetry in this case means that the receive signal at $S1$ is firstly transmitted over the channel with $\rho^{(2)}$ and secondly over the channel with $\rho^{(1)}$, and the receive signal at $S2$ is firstly transmitted over the channel with $\rho^{(1)}$ and secondly over the channel with $\rho^{(2)}$ which leads to the same overall noise for down- and uplink. This is an interesting result since the same BER performance at the receiving nodes can be achieved easily by applying SDI in SDD relaying. For beamforming schemes which cannot apply SDI due to unknown transmit signals at the receiving nodes, a much more complicated algorithm has to be applied in order to achieve the same BER performance at the receiving nodes [22].

In Fig. 5, the average sum rate of the linear ZF transceive filter, the MMSE transceive filter without SDI, and the MMSE transceive filter with SDI is given. Note that SDI cannot improve the performance of the linear ZF transceive filter since the two channels are perfectly orthogonalized by this filter which

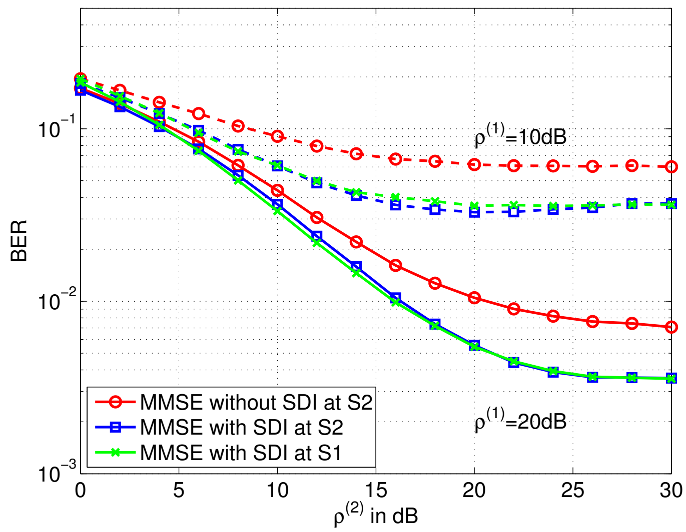

Fig. 4. Comparison of BER performance at $S1$ and $S2$ for the MMSE transceive filter with and without SDI (dashed lines: $\rho^{(1)}$ = 10dB, solid lines: $\rho^{(1)}$ = 20dB)

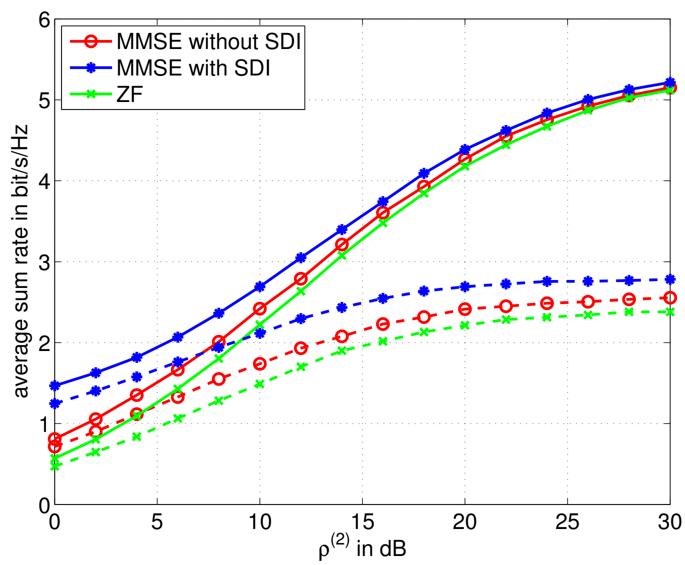

Fig. 5. Comparison of average sum rate for the ZF and MMSE transceive filters with and without SDI (dashed lines: $\rho^{(1)}$ = 10dB, solid lines: $\rho^{(1)}$ = 20dB)

implicitly suppresses duplex-interference at $S1$ and $S2$ at the cost of noise enhancement at the receivers. For increasing $\rho^{(2)}$, the average sum rate converges to a constant maximum which depends on $\rho^{(1)}$. For small $\rho^{(1)}$, the sum rate converges faster with increasing $\rho^{(2)}$ and the maximum sum rate is lower than for high $\rho^{(1)}$. For low $\rho^{(2)}$, the linear ZF transceive filter has a worse performance than the linear MMSE transceive filter. If $\rho^{(1)}$ and $\rho^{(2)}$ are sufficiently high, the overall noise at $S1$ and $S2$ which consists of the noise at the RS and the noise at $S1$ and $S2$ itself can be neglected. In this case, the sum rate of the linear ZF transceive filter converges to the sum rate of the MMSE transceive filter. This effect can already be seen for $\rho^{(1)}$ = 20dB and high values of $\rho^{(2)}$. It can be seen from Fig. 5 that SDI significantly increases the sum rate for the MMSE transceive filter if the SNR on both channels is low. For high values of $\rho^{(1)}$ and $\rho^{(2)}$, there exists almost no duplex-interference and the sum rates of the MMSE transceive filter with and without SDI converge.

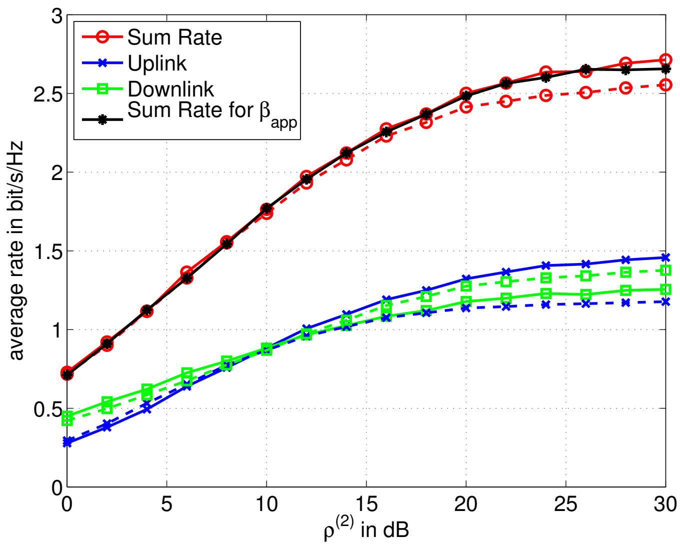

Fig. 6. Average sum rate and average rate of down- and uplink of MMSE transceive filter depending on $\rho^{(2)}$ for fixed $\rho^{(1)} = 10$dB (solid lines: β from the optimization in (41); dashed lines: $\beta = 0.5$)

C. Maximizing Sum Rate

In Section VII-A, maximizing the sum rate by giving different weights to the RS estimation vectors in case of different channel qualities on the two channels is discussed. In Fig. 6 for fixed $\rho^{(1)} = 10$dB, the average sum rate depending on $\rho^{(2)}$ of a three-step MMSE transceive filter achieved for the numeric optimization of β_{opt} from (41) is compared to the value achieved for fixed $\beta = 0.5$ and the value achieved by the approximation β_{app} from (46). Additionally, the rates of down- and uplink for β_{opt} and $\beta = 0.5$ are depicted separately. For equal channel qualities on both channels ($\rho^{(1)} = \rho^{(2)}$), all approaches provide the same average sum rate. However, for increasing difference of the channel qualities on both channels the sum rates diverge. The optimization of β provides a higher sum rate than the fixed approach with $\beta = 0.5$. Let us consider the uplink for $\rho^{(2)} > 10$dB, which means that the first hop of the uplink has a higher SNR than the first hop of the downlink. Assigning equal weights ($\beta = 0.5$) to both RS estimation vectors results in a lower rate in the uplink than in the downlink since the second hop of the uplink has a lower SNR. But, if the RS estimation vector of the uplink gets a higher weight due to the optimization, then the situation changes and the uplink achieves a higher rate than the downlink. In general, this means that the sum rate may be increased by introducing a higher weight to the RS estimation vector which is received over the better channel on the first hop. This can be explained as follows. The noise at the RS is filtered by the MMSE receive filter which leads to different effective SNR values for the two receive vectors from $S1$ and $S2$ after the filter. The receive vector which comes over the better channel has a higher SNR, provides a higher mutual information, and consequently it should get a higher weight before retransmission. The approximation for β_{app} comes very close to the optimum sum rate for β_{opt}, although the approximation requires a significantly lower effort like the optimum solution.

D. Average Sum Rate of Different Relaying Schemes

In the following, the average sum rate of MRC relaying and MRC-BF relaying with a linear MMSE transmit filter is compared to the sum rate of SDD relaying with a linear MMSE transceive filter at the RS. In Fig. 7, the average sum rate of the three schemes is depicted depending on $\rho^{(2)}$ for $\rho^{(1)} = 10$dB and $\rho^{(1)} = 20$dB. Obviously, the linear MMSE transceive filter in the SDD relaying approach outperforms the MRC and MRC-BF relaying approaches in terms of average sum rate. This comes from the fact that in SDD relaying, down- and uplink are transmitted simultaneously. In MRC relaying, down- and uplink require separated channel resources and in MRC-BF relaying indeed the number of required channel

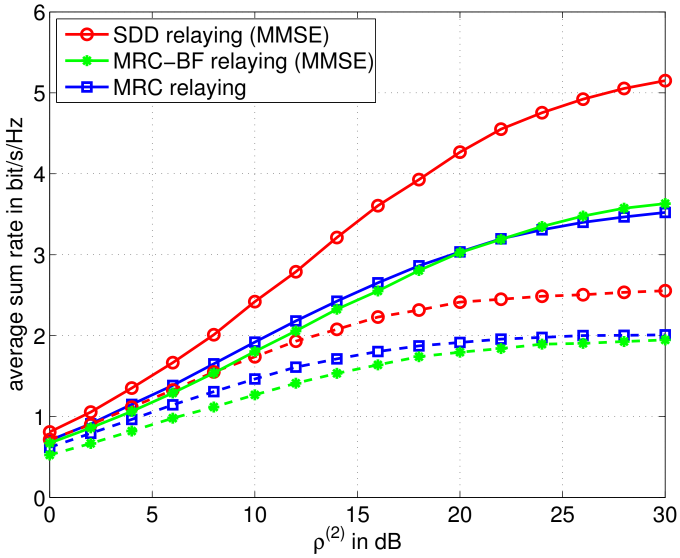

Fig. 7. Comparison of average sum rate for SDD relaying, MRC relaying, and MRC-BF relaying (dashed lines: $\rho^{(1)} = 10$dB, solid lines: $\rho^{(1)} = 20$dB)

resources is decreased compared to MRC relaying, but still higher than in SDD relaying. There exists a trade-off between the number of required channel resources and the effective receive SNR at $S1$ and $S2$. SDD relaying provides the worst effective receive SNR since both antennas at the RS are used for spatial separation of down- and uplink. However, the reduced amount of required channel resources more than compensates the worst effective receive SNR compared to the other schemes. MRC provide the best effective receive SNR since both antennas at the RS are used to improve the effective receive SNR. But the improved effective receive SNR cannot compensate the increased amount of required resources compared to SDD relaying. The effective receive SNR and the number of required channel resources of MRC-BF relaying lies inbetween the values of MRC relaying and SDD relaying. However, this trade-off in MRC-BF relaying seems to provide the worst performance since it suffers from duplex-interference in contrast to MRC relaying.

IX. C

In this paper, SDD relaying is proposed as a novel relaying scheme saving channel resources for equal traffic load in down- and uplink. In SDD relaying, down- and uplink are separated by applying transceive filters at the RS which can be designed by a one-step or a three-step concept. It turns out that the three-step transceive filters provide a better BER performance than the one-step filters and that the three-step transceive filters are more flexible, e.g., it is only possible for the three-step transceive filters to give different weights to down- and uplink which leads to an increased sum rate. For a linear MMSE transceive filter, duplex-interference can be subtracted at $S1$ and $S2$ which results in an improved BER performance and balances the BER performances of down- and uplink. The linear MMSE transceive filter always outperforms the linear ZF transceive filter. For sake of comparison, MRC and MRC-BF relaying are proposed as other relaying schemes which apply multiple antennas and linear signal processing at the RS. It is shown that SDD relaying provides a higher spectral efficiency than MRC and MRC-BF relaying due to the simultaneous transmission of down- and uplink.

A A
D ZF T F

As shown in [23], the ZF optimization problem in (26a) is not convex. However, the Karush-Kuhn-Tucker (KKT) conditions [24] can be used to solve (26a) under the constraints (26b) and (26c). For simplicity but without loss of generality, it is assumed that the scalar receive filters at $S1$ and $S2$ are the same, i.e., $p^{(1)} = p^{(2)} = p$. By applying the Lagrangian function

$$L(\mathbf{G}, p, \mu, \mathbf{\Lambda}) = \mathrm{tr}\left\{|p|^2 \left(\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^H\mathbf{H}_{\mathrm{T}}^H + \mathbf{R}_{\mathbf{n}_{\mathrm{R}}}\right)\right\} + \mu\left(\mathrm{tr}\left\{\mathbf{G}\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}\mathbf{G}^H\right\} - E^{(\mathrm{RS})}\right) - 2\mathrm{Re}\left\{\mathrm{tr}\left\{\mathbf{\Lambda}\left(p\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{R}}\mathbf{Q} - \mathbf{I}\right)\right\}\right\} \quad (47)$$

the KKT conditions are given by

$$\frac{\partial L(\mathbf{G}, p, \mu, \mathbf{\Lambda})}{\partial \mathbf{G}} = |p|^2\, \mathbf{H}_{\mathrm{T}}^*\mathbf{G}^*\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}^*\mathbf{H}_{\mathrm{T}}^T + \mu\mathbf{G}^*\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}^* - p\mathbf{Q}\mathbf{H}_{\mathrm{R}}^T\mathbf{H}_{\mathrm{T}}^T\mathbf{\Lambda}^T = \emptyset \quad (48a)$$

$$\frac{\partial L(\mathbf{G}, p, \mu, \mathbf{\Lambda})}{\partial p} = \mathrm{tr}\left\{p^*\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^H\mathbf{H}_{\mathrm{T}}^H - \mathbf{\Lambda}\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{R}}\right\} = 0 \quad (48b)$$

$$\mu\left(\mathrm{tr}\left\{\mathbf{G}\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}\mathbf{G}^H\right\} - E^{(\mathrm{RS})}\right) = 0 \quad (48c)$$

$$\mathbf{\Lambda}\left(p\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{R}}\mathbf{Q} - \mathbf{I}\right) = \emptyset \quad (48d)$$

From the fourth KKT (48d), for $\det[\mathbf{\Lambda}] \neq 0$ one gets

$$\mathbf{G} = \frac{1}{p}\left(\mathbf{H}_T^H\mathbf{H}_{\mathrm{T}}\right)^{-1}\mathbf{H}_{\mathrm{T}}^H\mathbf{Q}^H\mathbf{H}_{\mathrm{R}}^H\left(\mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{Q}^H\mathbf{H}_{\mathrm{R}}^H\right)^{-1} \quad (49)$$

Applying $\mathbf{G}$ from (49), p may be determined from the third KKT (48d) for $\mu \neq 0$ leading to the solution given in (28).

A B

D MMSE T F

As shown in [23], the MMSE optimization expression in (29a) is not convex. In this case, the KKT conditions are only required to solve (29a) under the transmit power constraint (29b). For simplicity but without loss of generality, it is assumed that the scalar receive filters at $S1$ and $S2$ are the same, i.e., $p^{(1)} = p^{(2)} = p$. By applying the Lagrangian function

$$\begin{aligned} L(\mathbf{G}, p, \mu) = {} & \mathrm{tr}\left\{\mathbf{R}_{\mathbf{x}} + |p|^2\,\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{R}_{\mathbf{x}}\mathbf{Q}^H\mathbf{H}_{\mathrm{R}}^H\mathbf{G}^H\mathbf{H}_{\mathrm{T}}^H - p\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{R}_{\mathbf{x}} - p^*\mathbf{R}_{\mathbf{x}}\mathbf{Q}^H\mathbf{H}_{\mathrm{R}}^H\mathbf{G}^H\mathbf{H}_{\mathrm{T}}^H\right\} \\ & + \mathrm{tr}\left\{|p|^2\left(\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^H\mathbf{H}_{\mathrm{T}}^H + \mathbf{R}_{\mathbf{n}_{\mathrm{R}}}\right)\right\} + \mu\left(\mathrm{tr}\left\{\mathbf{G}\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}\mathbf{G}^H\right\} - E^{(\mathrm{RS})}\right) \end{aligned} \quad (50)$$

the KKT conditions are given by

$$\frac{\partial L(\mathbf{G}, p, \mu)}{\partial \mathbf{G}} = |p|^2\,\mathbf{H}_{\mathrm{T}}^*\mathbf{G}^*\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}^*\mathbf{H}_{\mathrm{T}}^T - p\mathbf{R}_{\mathbf{x}}^T\mathbf{Q}^T\mathbf{H}_{\mathrm{R}}^T\mathbf{H}_{\mathrm{T}}^T + \mu\mathbf{G}^*\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}^* = \emptyset \quad (51a)$$

$$\frac{\partial L(\mathbf{G}, p, \mu)}{\partial p} = \mathrm{tr}\left\{-\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{H}_{\mathrm{R}}\mathbf{Q}\mathbf{R}_{\mathbf{x}} + p^*\mathbf{H}_{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}\mathbf{G}^H\mathbf{H}_{\mathrm{T}}^H + p^*\mathbf{R}_{\mathbf{n}_{\mathrm{R}}}\right\} = 0 \quad (51b)$$

$$\mu\left(\mathrm{tr}\left\{\mathbf{G}\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}\mathbf{G}^H\right\} - E^{(\mathrm{RS})}\right) = 0. \quad (51c)$$

From the first KKT condition (51a) it can bee seen that there exists no analytical solution for $\mathbf{G}$ due to the existence of the power constraint (29b). Neglecting the power constraint in (29b), the first KKT can be rewritten as

$$\frac{\partial L(\tilde{\mathbf{G}})}{\partial \tilde{\mathbf{G}}} = \mathbf{H}_{\mathrm{T}}^*\tilde{\mathbf{G}}^*\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}^*\mathbf{H}_{\mathrm{T}}^T - \mathbf{R}_{\mathbf{x}}^T\mathbf{Q}^T\mathbf{H}_{\mathrm{R}}^T\mathbf{H}_{\mathrm{T}}^T = \emptyset. \quad (52)$$

Solving Eq. (52) for $\tilde{\mathbf{G}}$, finally leads to

$$\tilde{\mathbf{G}} = \left(\mathbf{H}_T^H\mathbf{H}_{\mathrm{T}}\right)^{-1}\mathbf{H}_{\mathrm{T}}^H\mathbf{R}_{\mathbf{x}}^H\mathbf{Q}^H\mathbf{H}_{\mathrm{R}}^H\mathbf{R}_{\mathbf{y}_{\mathrm{RS}}}^{-1} \quad (53)$$

Introducing the normalization factor $1/p$ to (53) in order to fulfill the power constraint (29b) at the RS subsequently one gets

$$\mathbf{G} = \frac{1}{p}\tilde{\mathbf{G}} \quad (54)$$

with p from (31). Note that $1/p$ does not come from the optimization process itself, but is a somehow artificial weighting factor similar to the approach in [25].

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