# Transmit Antenna Selection with imperfect CQI feedback in Multi-user OFDMA systems

Alexander Kühne*, Anja Klein*, Xinning Wei**, and Tobias Weber**
*Communications Engineering Lab, Technische Universität Darmstadt, 64283 Darmstadt, Germany
**Institute of Communications Engineering, University of Rostock, 18119 Rostock, Germany
{a.kuehne, a.klein}@nt.tu-darmstadt.de, {xinning.wei, tobias.weber}@uni-rostock.de

*Abstract*— **In this paper, we analyse the impact of imperfect Channel Quality Information (CQI) feedback on a multi-user OFDMA system which employs transmit antenna selection. As CQI, the instantaneous signal-to-noise ratio (SNR) of the different subcarriers is employed which is used at the base station to allocate the subcarriers and to select the applied modulation scheme and transmit antenna. The imperfectness of the CQI feedback arises from channel estimation errors, SNR value quantisation, feedback errors and time delays. Two different antenna selection feedback schemes are introduced, which differ in the composition of the feedback signalled to the BS. Closed form expressions for the average data rate and bit error rate (BER) are analytically derived for both schemes. Finally, the maximum achievable data rate subject to a BER constraint is analysed and the robustness against imperfect CQI is compared for both antenna selection feedback schemes.**

## I. Introduction

Using multiple antennas at the transmitter and the receiver results in a significant improvement of the capacity and error probability of mobile radio communication links [1, 2]. However, the requirements in terms of hardware, which are associated with each antenna have be to considered. One way to benefit from the performance gain of multiple antennas with moderate hardware complexity is Antenna Selection (AS), which has been studied in previous works, for example in [3-5]. Orthogonal Frequency Division Multiple Access (OFDMA) is considered to be a suitable candidate for future radio networks where high data rates are required. It is applicable in multi-user systems and can be combined with multiple antenna systems [1, 2] using AS schemes. Using OFDMA, the overall channel can be divided in several subchannels in the time and frequency dimension. Having channel knowledge at the transmitter side, these subchannels, called subcarriers, can be allocated adaptively to the different users in order to exploit multi-user diversity [6]. Introducing AS, selection diversity can be exploited additionally. Having perfect channel knowledge at the transmitter, adaptive subcarrier allocation schemes achieve very good performances [7, 8]. However, in a realistic scenario only imperfect channel knowledge is available at the transmitter. This leads to a performance degradation when using adaptive techniques with imperfect channel knowledge compared to the case of having perfect channel knowledge. AS schemes employing space-time coding in the presence of imperfect channel knowledge due to channel estimation errors have been studied for example in [9, 10]. In [11], an adaptive Multi-user OFDMA FDD system is presented, where the available SNR on each subcarrier of each user is digitised with $N_Q$ bits and fed back as channel quality information (CQI) to the base station (BS). The BS selects a user and a modulation scheme for each subcarrier based on the CQI, where a certain target BER $BER_T$ has to be guaranteed. In this paper, we extend this OFDMA system to an OFDMA system employing Transmit AS with $n_T$ transmit antennas at the BS and Maximum Ratio Combining (MRC) at each mobile station (MS) having $n_R$ receive antennas. The imperfectness of the CQI arises from channel estimation errors, quantisation, feedback errors and time delays. We introduce two different AS feedback schemes and analytically derive expressions for the average BER and data rate of the selected user for both AS feedback schemes taking into account the impact of imperfect CQI feedback. We furthermore compare the different AS feedback schemes in terms of the achievable data rate and robustness to imperfect CQI.

The remainder of this paper is organised as follows. Section II presents the scenario assumptions and the considered adaptive transmission scheme. In Section III, the different sources of error of the CQI are briefly presented. In Section IV, the two AS feedback schemes are introduced. In Section V, the average data rate and BER for both feedback schemes are derived analytically for the case of imperfect CQI. In Section VI, the achievable data rates for both feedback schemes are illustrated and compared. Finally, conclusions are drawn in Section VII.

## II. System model

### A. Scenario assumptions

In this work, we consider a one cell Orthogonal Frequency Division Multiple Access (OFDMA) downlink scenario in a Frequency Division Duplex (FDD) system with $N$ subcarriers with index $n = 1, \cdots, N$, where one BS and $U$ MSs with user index $u = 1, \cdots, U$ are located in the cell. The BS is equipped with $n_T$ transmit antennas and each MS with $n_R$ receive antennas. Furthermore, it is assumed that all users have the same requirements in terms of data rate. The entries of the $n_T \times n_R$ MIMO channel of each subcarrier are assumed to be uncorrelated.

The transfer factor $H_u^{(i,j)}(n,k)$ of the channel from transmit antenna $i$ with $i = 1, \cdots, n_T$ to receive antenna $j$ with $j = 1, \cdots, n_R$ of each user $u$ on subcarrier with index $n$ in each time slot $k \in \mathbf{N}$ is modeled as a complex Gaussian distributed random process with variance one. In the following, only fast fading is considered, i.e. it is assumed that all users experience the same average SNR $\bar{\gamma}$. From this, it follows that the instantaneous SNR $\gamma_u^{(i,j)}(n,k)$ of user $u$ on subcarrier $n$ in time slot $k$ from transmit antenna $i$ to receive antenna $j$ is given by

$$\gamma_u^{(i,j)}(n,k) = \bar{\gamma} \cdot \left| H_u^{(i,j)}(n,k) \right|^2 . \tag{1}$$

### B. Adaptive transmission

Based on the CQI available at the BS, transmit AS, adaptive subcarrier allocation and modulation scheme selection are performed. First, the best transmit antenna for each user for each subcarrier is identified. Next, the subcarriers are allocated to the different users. In this paper, a Max-SNR Scheduler is employed to allocate the subcarriers to the users with the best SNR conditions based on the result of the AS, where one subcarrier is allocated to only one user exclusively. If several users have the best channel condition, the subcarrier is allocated randomly to one of these users. After performing the subcarrier allocation, the modulation scheme is selected for each allocated subcarrier based on the CQI value, where it is assumed that the transmit power for each subcarrier is the same. In this work, M-PSK and M-QAM modulation are considered. Finally, the data of the scheduled user is transmitted over the allocated subcarrier using the selected transmit antenna. At the receiver, Maximum Ratio Combining (MRC) is applied leading to a summation of the SNR values of all receive antennas at the output of the combiner.

## III. Modelling imperfect CQI

The CQI is assumed to be imperfect, where four sources of error are considered, which are briefly described in this section. For further details, we refer to [12].

First, the CQI is only an estimate that contains a certain estimation error which is modeled by an additional Gaussian distributed error with variance $\sigma_E^2$. The error variance $\sigma_E^2 \in [0, 1]$ depends on the conditions of the channel and the applied estimation scheme and, according to [13], is given by $\sigma_E^2 = \frac{1}{1+T_\tau P_\tau}$ where $T_\tau$ is the number of training symbols per coherence time and $P_\tau$ the SNR during the training phase. Second, the CQI is quantised in $W = 2^{N_Q}$ quantisation levels with $W + 1$ quantisation bounds $\gamma_l$ with $l = 0, \cdots, W$, where $\gamma_0 = 0, \gamma_W = \infty$ and $N_Q$ denotes the number of quantisation bits per subcarrier. The quantised CQI values are then digitised according to a certain bit coding scheme which is characterised by a $W \times W$ matrix $\mathbf{B}$. The $(i, j)$-th element $b_{i,j}$ of matrix $\mathbf{B}$ with $i, j = 1, \cdots, W$ contains the number of bits which differ comparing the bit coding of the $i$-th quantisation level $[\gamma_{i-1}, \gamma_i]$ with the bit coding of the $j$-th $[\gamma_{j-1}, \gamma_j]$ quantisation level. Third, when detecting the feedback bits at the BS, bit errors may occur with an average feedback BER $p_b$. Finally, the CQI is outdated due to the time delay $T$ between the time instance when measuring the SNR and the actual time of data transmissions. The outdated CQI can be modeled by correlation, where the realisation of the actual channel and the outdated channel are being correlated with a correlation coefficient of $\rho = J_0(2\pi f_D T)$, with $f_D$ the Doppler frequency and $J_0(x)$ denoting the 0th-order Bessel function of the first kind. In the following, we assume that the correlation coefficient $\rho$, the average feedback BER $p_b$ and error variance $\sigma_E^2$ are the same for all users.

## IV. Antenna selection feedback schemes

Applying AS at the transmitter side, information about the channel quality of different transmit antennas has to be fed back to the BS in order to identity the best transmit antenna. In this work, we consider two AS feedback schemes. Using the first feedback scheme, referred to as AS Feedback scheme ASF 1, for each subcarrier all $n_T$ digitised CQI values are fed back to the BS, resulting in a total amount of

$$F_L = n_T N_Q \tag{2}$$

feedback bits per user and per subcarrier referred to as feedback load $F_L$. In this case, the transmit AS is performed at the BS, i.e. using ASF 1, the multi-user system having $U$ users can be interpreted as a system employing only one transmit antenna but having $n_T \cdot U$ users.

Applying the second AS Feedback scheme ASF 2, only the CQI of the best antenna together with the label of the best antenna are fed back, resulting in

$$F_L = N_Q + \log_2(n_T) \tag{3}$$

feedback bits per user and per subcarrier. In this case, the transmit AS is performed at the MS before the quantisation of the SNR values.

## V. Data rate and bit error rate using transmit antenna selection

In this section, we analytically derive expressions for the average data rate and the average uncoded BER performance using the two AS feedback schemes taking into account all types of imperfect CQI introduced in Section III. Furthermore, it is shown how to maximise the data rate subject to a BER constraint.

### A. Average data rate using imperfect CQI

The average data rate $\overline{R}$ using transmit AS and MRC at the receiver and applying adaptive modulation is defined as the sum rate of the different modulation constellations weighted by their probability. Therefore, we introduce $p_{\hat{\gamma}}^{(m)}(\hat{\gamma})$ as the probability density function (PDF) of an

SNR value of a selected user, which is assumed at the BS to be in the $m$-th quantisation level $[\gamma_{m-1}, \gamma_m]$. The PDF can be formulated as

$$p_{\hat{\gamma}}^{(m)}(\hat{\gamma}) = \sum_{k=1}^{card(\mathcal{M})} d_{m,k} \cdot p_{\hat{\gamma}}^{(m,k)}(\hat{\gamma}), \quad (4)$$

with $d_{m,k} = (1-p_b)^{N_Q - b_{m,k}} \cdot p_b^{b_{m,k}}$ determining the probability that an SNR value of a selected user, which is measured at the MS to be in the $k$-th quantisation level $[\gamma_{m-1}, \gamma_m]$ is assumed at the BS to be in the $m$-th quantisation level $[\gamma_{k-1}, \gamma_k]$. Furthermore, $p_{\hat{\gamma}}^{(m,k)}(\hat{\gamma})$ determines the PDF of an SNR value of a selected user which is measured at the MS to be in the $k$-th quantisation level is assumed at the BS to be in the $m$-th quantisation level. The average date rate is then given by

$$\overline{R} = \sum_{m=1}^{card(\mathcal{M})} c_m \cdot \int_{\gamma_{m-1}}^{\gamma_m} p_{\hat{\gamma}}^{(m)}(\hat{\gamma})\, d\hat{\gamma}, \quad (5)$$

with $\mathcal{M}$ denoting a certain selection of modulation schemes, where $card(\mathcal{M})$ denotes the cardinality of $\mathcal{M}$ and $c_m$ denotes the number of bits per symbol corresponding to the modulation scheme. The interval in which a particular modulation scheme is applied is determined by the bounds $\gamma_{m-1}$ and $\gamma_m$, with $m = 1, ..., card(\mathcal{M})$. Note that these bounds are identical to the quantisation bounds introduced in Section III.

In order to determine $p_{\hat{\gamma},(1)}^{(m,k)}(\hat{\gamma})$ using ASF 1 we introduce the function $F_{n_R}(\gamma)$ which is given by

$$F_{n_R}(\gamma) = 1 - \exp\left(-\frac{\gamma}{E\{\hat{\gamma}\}}\right) \sum_{w=0}^{n_R-1} \frac{1}{w!} \left(\frac{\gamma}{E\{\hat{\gamma}\}}\right)^w . \quad (6)$$

Introducing

$$p_j^{(1)} = \sum_{i=1}^{card(\mathcal{M})} d_{j,i} \cdot \left(F_{n_R}(\gamma_i) - F_{n_R}(\gamma_{i-1})\right), \quad (7)$$

we can calculate the scaling factor

$$a_m^{(1)} = \frac{\left(\sum_{j=1}^{m} p_j^{(1)}\right)^{n_T U} - \left(\sum_{j=1}^{m-1} p_j^{(1)}\right)^{n_T U}}{p_m^{(1)}}. \quad (8)$$

Using the step function $\sigma(x)$ and $E\{\hat{\gamma}\} = \bar{\gamma}(1-\sigma_E^2)$ it can be shown that

$$p_{\hat{\gamma},(1)}^{(m,k)}(\hat{\gamma}) = \frac{a_m^{(1)}}{(n_R-1)!} \cdot \frac{\hat{\gamma}^{n_R-1}}{E\{\hat{\gamma}\}^{n_R}} \cdot \exp\left(-\frac{\hat{\gamma}}{E\{\hat{\gamma}\}}\right) \cdot \left[\sigma(\hat{\gamma}-\gamma_{k-1}) - \sigma(\hat{\gamma}-\gamma_k)\right]. \quad (9)$$

Inserting (9) in (4) and (5), the average data rate using ASF 1 is given by

$$\overline{R}^{(1)} = \sum_{m=1}^{card(\mathcal{M})} c_m \left[\left(\sum_{j=1}^{m} p_j^{(1)}\right)^{n_T U} - \left(\sum_{j=1}^{m-1} p_j^{(1)}\right)^{n_T U}\right] \quad (10)$$

In case of using ASF 2, two aspects have to be taken into account. First, the distribution of the SNR of the selected user is different compared to ASF 1, since the AS is done at the MS before digitising the SNR values. Second, feedback bit errors also have to be considered for the digitised antenna label, i.e. a possibly wrong antenna selection at the BS has to be taken into account. In the following, three cases have to be considered. First, the antenna label of the best antenna is received correctly at the BS. In this case, the PDF of the SNR values are given by

$$p^{(a)}(\gamma) = \frac{n_T}{(n_R-1)!} \cdot \exp\left(-\frac{\gamma}{E\{\hat{\gamma}\}}\right) \cdot \frac{\gamma^{n_R-1}}{E\{\hat{\gamma}\}^{n_R}} \left[F_{n_R}(\gamma)\right]^{n_T-1}. \quad (11)$$

Second, the antenna label of the best antenna is not received correctly at the BS, but the SNR value of the wrongly selected antenna is in the same quantisation level $[\gamma_{k-1}, \gamma_k]$ as the SNR value of the correct antenna. The PDF of the SNR values for this second case is given by

$$p^{(b)}(\gamma, \gamma_k) = \frac{n_T}{n_T-1} \cdot \frac{1}{(n_R-1)!} \cdot \exp\left(-\frac{\gamma}{E\{\hat{\gamma}\}}\right) \cdot \frac{\gamma^{n_R-1}}{E\{\hat{\gamma}\}^{n_R}} \left(\left[F_{n_R}(\gamma_k)\right]^{n_T-1} - \left[F_{n_R}(\gamma)\right]^{n_T-1}\right). \quad (12)$$

Third, the antenna label of the best antenna is not received correctly at the BS and the SNR value of the wrongly selected antenna is in a quantisation level below the level $[\gamma_{k-1}, \gamma_k]$ of the SNR value of the correct antenna. The PDF is then given by

$$p^{(c)}(\gamma, \gamma_k, \gamma_{k-1}) = \frac{n_T}{n_T-1} \cdot \frac{1}{(n_R-1)!} \cdot \exp\left(-\frac{\gamma}{E\{\hat{\gamma}\}}\right) \cdot \frac{\gamma^{n_R-1}}{E\{\hat{\gamma}\}^{n_R}} \cdot \left(\left[F_{n_R}(\gamma_k)\right]^{n_T-1} - \left[F_{n_R}(\gamma_{k-1})\right]^{n_T-1}\right). \quad (13)$$

Again, we determine the scaling factor

$$a_m^{(2)} = \frac{\left(\sum_{j=1}^{m} p_j^{(2)}\right)^{U} - \left(\sum_{j=1}^{m-1} p_j^{(2)}\right)^{U}}{p_m^{(2)}}, \quad (14)$$

with

$$p_j^{(2)} = \sum_{i=1}^{card(\mathcal{M})} d_{j,i} \cdot \left(\left[F_{n_R}(\gamma_i)\right]^{n_T} - \left[F_{n_R}(\gamma_{i-1})\right]^{n_T}\right). \quad (15)$$

Now, it can be shown that the PDF $p_{\hat{\gamma},(2)}^{(m,k)}(\hat{\gamma})$ using ASF 2 is given by

$$p_{\hat{\gamma},(2)}^{(m,k)}(\hat{\gamma}) = a_m^{(2)} \cdot \left(1-(1-p_b)^Q\right) \cdot p^{(c)}(\hat{\gamma}, \gamma_k, \gamma_{k-1}) \cdot \left[\sigma(\hat{\gamma}) - \sigma(\hat{\gamma}-\gamma_{k-1})\right] + a_m^{(2)} \Big((1-p_b)^Q p^{(a)}(\hat{\gamma}) + \left(1-(1-p_b)^Q\right) \cdot p^{(b)}(\hat{\gamma}, \gamma_k)\Big) \cdot \left[\sigma(\hat{\gamma}-\gamma_{k-1}) - \sigma(\hat{\gamma}-\gamma_k)\right] \quad (16)$$

with $Q = \log_2(n_T)$ denoting the number of bits for the digitised antenna label, where we assume $n_T = 2^n$,

$n \in \mathcal{N}$. Inserting (16) in (4) and (5), the average data rate using ASF 2 is given by

$$\overline{R}^{(2)} = \sum_{m=1}^{card(\mathcal{M})} c_m \left[ \left( \sum_{j=1}^{m} p_j^{(2)} \right)^U - \left( \sum_{j=1}^{m-1} p_j^{(2)} \right)^U \right]. \tag{17}$$

### B. Average bit error rate using imperfect CQI

The average BER $\overline{BER}$ using imperfect CQI is formulated as the sum of the number of errors of the different modulation constellations divided by the average bit rate. To determine $\overline{BER}$ we introduce the BER $BER_m$ of the applied modulation scheme with index $m$ which is a function of the instantaneous SNR $\gamma$ and which can be approximated by [14]

$$BER_m(\gamma) = 0.2 \cdot \exp(-\beta_m \gamma) \tag{18}$$

with $\beta_m = \frac{1.6}{2^{c_m}-1}$ using M-QAM modulation and $\beta_m = \frac{7}{2^{1.9c_m}+1}$ using M-PSK modulation respectively. Furthermore, we introduce $p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ as the conditional PDF of the actual SNR $\gamma$ and the outdated SNR $\hat{\gamma}$ with estimation errors. Assuming one transmit antenna and MRC at the receiver, it can be shown that the conditional PDF is given by

$$\begin{aligned} p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) &= \frac{1}{\bar{\gamma}\sigma_r^2} \cdot \exp\left( -\frac{\rho^2 \cdot \hat{\gamma} + \gamma}{\bar{\gamma}\sigma_r^2} \right) \\ &\quad \cdot I_{n_R-1}\left( \frac{2\rho\sqrt{\gamma \cdot \hat{\gamma}}}{\bar{\gamma}\sigma_r^2} \right), \end{aligned} \tag{19}$$

with $\sigma_r^2 = 1 - \rho^2(1 - \sigma_E^2)$ and $I_n(x)$ denoting the $n$th-order modified Bessel function of the first kind. Now, the average BER is given by

$$\begin{aligned} \overline{BER} &= \frac{1}{\overline{R}} \sum_{m=1}^{card(\mathcal{M})} c_m \int_{\gamma_{m-1}}^{\gamma_m} p_{\hat{\gamma}}^{(m)}(\hat{\gamma}) \\ &\quad \cdot \left[ \int_0^\infty BER_m(\gamma) p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})\, d\gamma \right] d\hat{\gamma}. \end{aligned} \tag{20}$$

Inserting (9), (4), (19) and (18) in (20), the average BER using ASF 1 can be determined by Eq. (21) shown on the top of the next page.
In the case of using ASF 2, the average BER is determined by inserting (16), (4), (19) and (18) in (20) resulting in Eq. (22) as shown on the next page with $G = \sum_{w=0}^{n_R-1} w \cdot i_w$ and $i_w$ the integer numbers from the last sum of Eq. (22).

### C. Maximising data rate

From the last subsections, we have seen that the average data rate and BER depend on the quantisation levels $[\gamma_{m-1}, \gamma_m]$, the applied modulation schemes $\mathcal{M}$, and the bit coding scheme $\mathbf{B}$. Furthermore, the parameters describing the imperfectness of the CQI feedback, which are the correlation coefficient $\rho$ between the actual and the outdated channel, the estimation error variance $\sigma_E^2$, and the average BER $p_b$ of the feedback channel, also have an impact on the average data rate and BER. Assuming that these parameters are known to the BS, we can maximise the data rate subject to an average BER constraint $\overline{BER} \leq BER_T$ with the target BER $BER_T$ by finding the optimal quantisation levels $[\gamma_{m-1}, \gamma_m]$ together with the optimal set of applied modulation schemes $\mathcal{M}$ and bit coding schemes $\mathbf{B}$ leading to

$$\begin{aligned} \bar{R}_{opt} &= \max_{[\gamma_{m-1},\gamma_m],\mathbf{B},\mathcal{M}} \left( \overline{R} \right) \\ &\quad \text{subject to} \\ &\quad \overline{BER} \leq BER_T. \end{aligned} \tag{23}$$

In this work, this optimisation is done numerically.

## VI. Numerical Results

In the following, we compare the two different AS feedback schemes in terms of maximum achievable data rate. The number of transmit antennas is $n_T = 2$, the number of receive antenna is $n_R = 1$, the number of users is $U = 25$ and the average SNR is $\bar{\gamma} = 10$ dB, resulting in an estimation error variance $\sigma_E^2 = 0.09$. The average feedback BER is fixed to $p_b = 10^{-3}$. For the first case, the target BER is $BER_T = 10^{-2}$ and $BER_T = 10^{-3}$ for the second case. The applied modulation schemes are QPSK, 8-PSK, 16-QAM and 32-QAM. Using $N_Q = 1$ quantisation bits, both feedback schemes require a feedback load of $F_L = 2$. In Fig. 1 the achievable average data rate is presented as a function of the normalised time delay $f_D T$ for the two feedback schemes. As one can see, the maximum achievable data

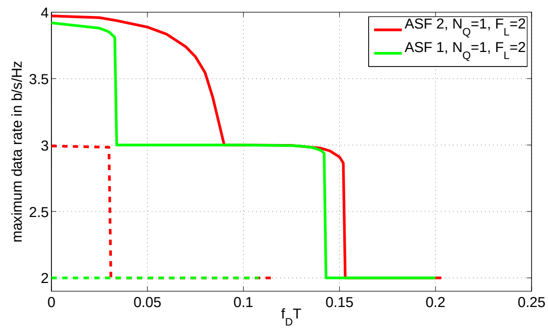


Fig. 1. Maximum data rate versus normalised time delay $f_D T$ for $U = 25$, $n_T = 2$, $n_R = 1$, $\bar{\gamma} = 10$ dB, $p_b = 10^{-3}$ and $\sigma_E^2 = 0.09$; solid lines: $BER_T = 10^{-2}$, dashed lines: $BER_T = 10^{-3}$

rate still providing the given target BER decreases for an increasing normalised time delay $f_D T$, since the selected modulation schemes have to be more robust to cope with the outdated channel information. At a certain value of $f_D T$, the target BER can no longer be fullfilled, e.g. for ASF 2, the target BER $BER_T = 10^{-2}$ can only be guaranteed for $f_D T < 0.21$ using the given modulation schemes. It appears that for the less demanding BER requirement $BER_T = 10^{-2}$, the achievable data rate is higher, since modulation schemes with a higher number of bits per symbol can be employed. Furthermore, it appears that ASF 2 clearly outperforms ASF 1 in terms of the achievable data rate and robustness against outdated CQI

$$\overline{BER}^{(1)} = \frac{0.2}{\overline{R}^{(1)}} \sum_{m=1}^{card(\mathcal{M})} a_m^{(1)} \cdot c_m \sum_{k=1}^{card(\mathcal{M})} d_{m,k} \cdot \frac{1}{1+\beta_m\bar{\gamma}} \left( F_{n_R}\left( \frac{\gamma_k \cdot (1+\beta_m\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m\bar{\gamma}\sigma_r^2)} \right) - F_{n_R}\left( \frac{\gamma_{k-1} \cdot (1+\beta_m\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m\bar{\gamma}\sigma_r^2)} \right) \right) \tag{21}$$

$$\begin{aligned}\overline{BER}^{(2)} = {} & \frac{0.2}{\overline{R}^{(2)}} \cdot \frac{n_T}{n_T-1} \sum_{m=1}^{card(\mathcal{M})} a_m^{(2)} \cdot c_m \sum_{k=1}^{card(\mathcal{M})} d_{m,k} \cdot \left[ \frac{1-(1-p_b)^Q}{(1+\beta_m\bar{\gamma})^{n_R}} \left( [F_{n_R}(\gamma_k)]^{n_T-1} \cdot \left( F_{n_R}\left( \frac{\gamma_k \cdot (1+\beta_m\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m\bar{\gamma}\sigma_r^2)} \right) \right.\right.\right. \\ & \left.\left. - F_{n_R}\left( \frac{\gamma_{k-2} \cdot (1+\beta_m\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m\bar{\gamma}\sigma_r^2)} \right)\right) - [F_{n_R}(\gamma_{k-1})]^{n_T-1} \cdot \left( F_{n_R}\left( \frac{\gamma_{k-1} \cdot (1+\beta_m\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m\bar{\gamma}\sigma_r^2)} \right) - F_{n_R}\left( \frac{\gamma_{k-2} \cdot (1+\beta_m\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m\bar{\gamma}\sigma_r^2)} \right)\right)\right) \\ & + (n_T(1-p_b)^Q - 1) \sum_{l=0}^{n_T-1} (-1)^l \binom{n_T-1}{l} \sum_{i_0+\ldots+i_{n_R-1}=l} \binom{l}{i_0, \ldots, i_{n_R-1}} \cdot \frac{(1+\beta_m\bar{\gamma}\sigma_r^2)^G}{(l+1+\beta_m(\bar{\gamma}(l+1)-p^2 l E\{\hat{\gamma}\}))^{n_R+G}} \\ & \cdot \frac{(n_R-1+G)!}{(n_R-1)!} \cdot \frac{1}{\prod_{w=0}^{n_R-1}(w!)^{i_w}} \cdot \left( F_{n_R+G}\left( \frac{\gamma_k(l+1+\beta_m(\bar{\gamma}(l+1)-p^2 l E\{\hat{\gamma}\}))}{E\{\hat{\gamma}\}(1+\beta_m\bar{\gamma}\sigma_r^2)} \right) \right. \\ & \left.\left. - F_{n_R+G}\left( \frac{\gamma_{k-1}(l+1+\beta_m(\bar{\gamma}(l+1)-p^2 l E\{\hat{\gamma}\}))}{E\{\hat{\gamma}\}(1+\beta_m\bar{\gamma}\sigma_r^2)} \right)\right)\right]\end{aligned} \tag{22}$$

feedback while requiring the same amount of feedback. The reason for this effect is the AS which is performed at the MS employing ASF 2. In this case, the SNR values resulting from the use of different transmit antennas are compared before the quantisation which leads to less errors selecting the antenna.

In Fig. 2, all parameters remain the same beside the number of transmit antenna which is now $n_T = 4$. In this case, the feedback load for ASF 1 is $F_L = 4$ using $N_Q = 1$ quantisation bit. Feedback scheme ASF 2 only requires $F_L = 3$ feedback bits using $N_Q = 1$ quantisation bit. From Fig. 2 it can be seen that ASF 2 again outperforms ASF 1 in spite of requiring less feedback load. If we also employ ASF 2 with $F_L = 4$, we can now spend one more bit for quantisation ($N_Q = 2$). In this case, the difference between ASF 1 and ASF 2 in terms of achievable data rate is even bigger due to a better resolution of the SNR values using $N_Q = 2$ quantisation bits.

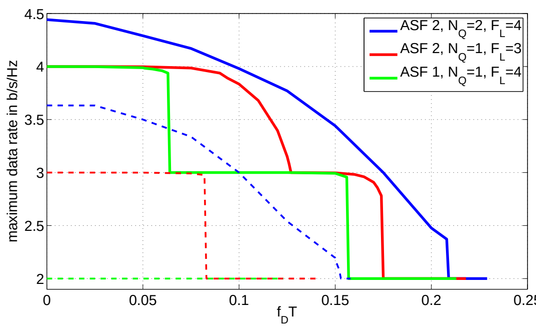


Fig. 2. Maximum data rate versus normalised time delay $f_D T$ for $U = 25$, $n_T = 4$, $n_R = 1$, $\bar{\gamma} = 10$ dB, $p_b = 10^{-3}$ and $\sigma_E^2 = 0.09$; solid lines: $BER_T = 10^{-2}$, dashed lines: $BER_T = 10^{-3}$

## VII. Conclusions

In this paper, we analytically derived closed form expressions for the average data rate and BER of an OFDMA system employing transmit AS and MRC at the receiver with imperfect CQI feedback. We introduced two different AS feedback systems, where the first scheme ASF 1 feeds back all CQI values of each transmit antenna while the second scheme ASF 2 only feeds back the CQI of the best antenna together with the digitised label of the best antenna. It appears that using the same amount of feedback load, scheme ASF 2 clearly outperforms scheme ASF 1 in terms of the achievable data rate and robustness against outdated CQI feedback.

## References


[1] I.E. Telatar, "Capacity of multi-antenna gaussian channels," Lucent Technologies/Bell Labs Innovations, Techn. Memo., Oct. 1995.

[2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Commun., pp. 311-335, Mar. 1998.

[3] A. F. Molisch, M. Z. Win, and J. H. Winters, "Capacity of MIMO systems with antenna selection," in Proc. *ICC*, May 2001.

[4] A. Ghrayeb and T. M. Duman, "Performance analysis of MIMO systems with antenna selection over quasi-static fading channels," *IEEE Trans. Veh. Techn.*, pp. 281-288, Mar. 2003.

[5] I. Bahceci, T. M Duman, and Y. Altunbasak, "Antenna selection for multiple antenna transmission systems: performance analysis and code construction," *IEEE Trans. Inf. Techn.*, Oct. 2003.

[6] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in Proc. *ICC*, June 1995.

[7] M. Schubert, H. Boche, "Solution of multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Techn.*, pp. 18-28, Jan. 2004.

[8] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. Signal Proc.*, pp. 461-471, Feb. 2004.

[9] Q. Ma and C. Tepedelenlioglu, "Antenna Selection for space-time coded systems with imperfect channel estimation," *IEEE Trans. Wireless Commun.*, 710-719, Feb. 2007.

[10] T. Gucluoglu and E. Panayirci, "Performance of Tranmsit and Receive Antenna Selection in the Presence of Channel Estimation Errors," *IEEE Commun. Letters*, pp. 371-373, May 2008.

[11] A. Kühne and A. Klein, "Adaptive MIMO-OFDM using OSTBC with imperfect CQI feedback," *Workshop Smart Ant.*, Feb. 2008.

[12] A. Kühne, A. Klein, "Throughput analysis of Multi-user OFDMA systems using imperfect CQI feedback and diversity techniques," accepted for *IEEE Journal on Selected Areas in Commun.*

[13] B. Hassibi and B. Hochwald, "How much training is needed in multiple antenna wireless links ?," *IEEE Trans. Inf. Theory*, pp. 951-963, April 2003.

[14] S. T. Chunk and A. Goldsmith, "Degrees of freedom in adaptive modulation: A unified view," *IEEE Trans. Commun.*, pp. 1561-1571, Sep. 2001.