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An Analytic Model for Outage Probability and Bandwidth Demand of the Downlink in Packet Switched Cellular Mobile Radio Networks

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Abstract—The ability to specify outage probability for cellular mobile radio networks is important in order to be able to assure a certain quality of service (QoS). In packet switched networks applying adaptive transmission, users have varying resource demands due to the adaptability. Furthermore, the medium, or a certain part of it, is not assigned exclusively to a user for the duration of a call or session. The determination of outage is therefore difficult. Time consuming simulations can be applied in order to determine a suitable network design capable of providing the required service. Alternatively, analytic or semi-analytic approaches can be applied. This paper presents an analytic model based on random variables (RVs) for the determination of the bandwidth demand of the downlink in packet switched cellular mobile radio networks applying adaptive transmission. The application of the model for determining outage probability is shown, an example of the application of the model in network design is given and the properties of the model are discussed. Comparisons with simulations are made in order to show the accuracy of the model.

I. INTRODUCTION

In circuit switched mobile radio networks, outage probability is usually defined depending on a minimum signal to noise and interference ratio (SINR) and the number of available channels, since a channel is exclusively assigned to a single user during a call or session [1]. In packet switched networks, however, the specification of outage is more difficult since the medium is not allocated exclusively to a user for the duration of a session. The problem gets even more challenging if the network applies adaptive transmission [2], [3], since the channel quality has effect on the number of resources required to achieve a certain service quality.

Simulations can be applied to find the outage probability of a system depending on the statistics of the user behaviour. Simulations, however, are very time consuming, especially for large networks accommodating many users. Alternatively, the stochastic information about the user behaviour can be applied directly to derive analytic models.

The topic of outage in Code Division Multiple Access (CDMA) has been discussed in several articles, as for example in [4]. Concerning Orthogonal Frequency Division Multiple Access (OFDMA), the topic has been treated in various aspects. Approaches for the analytic modelling of inter-cell interference in OFDMA have been presented [5], [6], the link outage probability has been treated analytically [7], the probability region, defining all achievable outage probability combinations of all active users, has been derived [8], and the average outage probability of a cell in a multi cell environment has been investigated using simulations [9].

In this paper, an analytic model for determining the bandwidth demand of the downlink in packet switched cellular mobile radio networks applying adaptive transmission is derived. The application of the model to determine the outage probability given a certain bandwidth supply or to determine the bandwidth demand given a maximum outage probability is shown, an example of the application of the model in network design is given and the properties of the model are discussed. Comparisons with simulations are made in order to show the accuracy of the model.

The paper is organised as follows. Section II introduces the system model and gives a description of the problem. Section III presents the determination of bandwidth demand and outage probability for a single user in a single cell scenario. Section IV uses the results from the previous section to derive bandwidth demand and outage probability in the multi-user case or for the case of a whole cell, respectively. In Section V, the multi cell case is investigated by considering inter-cell interference, an example of the application of the model is given and the properties of the model are discussed. Section VI concludes the paper.

II. SYSTEM MODEL AND PROBLEM DESCRIPTION

The downlink of a cellular mobile radio network with hexagonally shaped cells of radius $R$ is considered. The base stations (BSs) are located in the centre of the cells and are equipped with unidirectional antennas. Groups of $r$ cells form clusters, where the cluster size $r$ is a rhombic number [1], [10]. Two cells of different clusters may use the same resource which leads to inter-cell interference [1], [10]. The distance between two cells that use the same resources is called the reuse distance $D$ and is given by [1]

$$D = \sqrt{3r} \cdot R. \quad (1)$$

Exponential power loss propagation [1] is assumed and the SINR at the receiver is given by [1]

$$\gamma = \frac{P_S}{d^\alpha} \cdot \frac{1}{P_N + P_I} \quad (2)$$
with $P_S$ the transmit power of the BS, $P_N$ the noise power of the receiver, $P_I$ the interference power, $d$ the distance between BS and receiver and $\alpha$ the propagation coefficient. This simplistic propagation model has been chosen in order to ease the understanding of the presented approach. An extension to more realistic propagation models and the consideration of shadow fading can easily be carried out.

The active users are assumed to be uniformly distributed within each of the cells. The link data rate $DR$ of a user assuming perfect link adaptation (gaussian signalling) is expressed by [11]

$$DR = B \cdot \log_2(1 + \gamma)$$

(3)

with $B$ the link bandwidth. Consequently, the bandwidth $B$ required to provide a certain link data rate depends on the SINR of the link. In the multi user case, either bandwidth or transmission time or both has to be shared between the simultaneously active users.

The service quality is in this paper expressed in terms of link data rate. A user is in outage, if the demanded link data rate is not achieved. Outage probability concerning a user is the probability that a user does not get the demanded link data rate. Concerning a cell, outage probability is defined as the probability that the cell can not give to all its users the demanded link data rate.

The problem addressed in this paper is the modelling of a network accommodating several simultaneously active users competing for a shared medium and having different link qualities. Due to the adaptive transmission, the resource demand of the users to achieve a fixed data rate varies. The more resources a user requires, the less resources are available for the remaining users, such that each user has effect on the data rate, and thus service quality, of the remaining users. With the above definition of outage probability, the resource demand can be used to evaluate the outage probability or, given a maximum outage probability, to determine the required resources.

III. SINGLE USER CASE

This section derives bandwidth demand and outage probability of a single user in a single cell scenario where no inter-cell interference occurs. The cell will be assumed to be of circular shape in order to ease derivations, and users are assumed to be uniformly distributed over the cell, as stated in Section II. In the case of a single user, this means that the user is located at any position of the cell with equal probability.

Assuming a given data rate, the bandwidth required for a transmission with that data rate depends on the SINR, according to Equation (3). Neglecting interference and with given noise power, the SINR depends on the receive power of the signal. The receive power depends on the distance between BS and user and finally, the distance between BS and user can be calculated from the position of the user in the cell.

Following this argumentation, a random variable (RV) that represents the bandwidth demand can be calculated by deriving a RV that represents the distance between BS and user using the assumption of equal position probability. From this RV, a RV representing the signal to noise ratio (SNR) at the receiver can be obtained via RV transformation [12]. From the distribution of the SNR, finally, the distribution of the required bandwidth can be gained and the bandwidth requirement can be used to determine the outage probability.

Fig. 1 shows the single cell scenario. According to [12], the joint density for equal position probability is given by

$$p_{r,\varphi} = \begin{cases} \frac{1}{\pi R^2} & \text{for } 0 < r < R, \ 0 < \varphi < 2\pi \\ 0 & \text{else} \end{cases} \quad (4)$$

Denoting the distance between user and BS as RV $r$, probability density function (PDF) and cumulative distribution function (CDF) of $r$ are given by [12]

$$f_r(r) = \frac{2}{R^2} \cdot \int_0^{2\pi} p_{r,\varphi} d\varphi = \frac{2r}{R^2} \quad (5)$$

$$F(r) = \int_0^r f_r(r) dr = \frac{r^2}{R^2} \quad (6)$$

Fig. 1. Single Cell Scenario.

With the assumption of exponential power loss propagation of (2), the SNR at the receiver is given by

$$s(r) = \frac{P_S}{(r^2 + h^2)\pi} \cdot \frac{1}{P_N} \quad (7)$$

with $h$ the difference in the height of the BS and the user terminal. The PDF of a RV $s$ representing the SNR at the receiver can be derived from RV $r$ using RV transformation [12]:

$$f_s(s) = f_r(\sqrt{s}) \cdot \left| \frac{dr}{ds} \right|$$

(8)

With

$$r(s) = \sqrt{\frac{P_s}{P_N}} \cdot \frac{1}{s^{-\alpha/2} - h^2} \quad (9)$$

the PDF of RV $s$ is defined by

$$f_s(s) = \frac{2}{\alpha \cdot R^2} \left( \frac{P_S}{P_N} \right)^{\frac{\alpha}{2}} \cdot s^{-\left(\frac{\alpha}{2} + 1\right)} \quad (10)$$

and the CDF of $s$ can be derived according to [12] by integration over the PDF and considering boundary conditions to yield to

$$F(s) = 1 - \frac{1}{R^2} \left[ \left( \frac{P_S}{P_N} \right)^{\frac{\alpha}{2}} s^{-\frac{\alpha}{2}} - h^2 \right] \quad (11)$$

Setting the SINR in Equation (3) equal to the SNR as defined in (6), resolving (3) for the bandwidth $B$ and again performing RV transformation derives PDF and CDF of RV $B$, which represents the bandwidth $B$ required to transmit at a given data rate $DR$:

$$f_{B}^{(SC)}(B) = \frac{2}{\alpha A \sqrt{\pi}} \left( \frac{P_{B}}{\gamma_{N}} \right)^{\frac{\alpha}{2}} \left( 2^{\frac{\alpha B}{\gamma_{N}} - 1} \right)^{-\left(\frac{\alpha}{2} + 1\right)} \cdot A,$$  \hspace{1cm} (12)

$$F^{(SC)}(B) = \frac{1}{R^2} \left[ \left( 1 - \frac{P_{B}}{\gamma_{N}} \right)^{\frac{\alpha}{2}} \left( 2^{\frac{\alpha B}{\gamma_{N}} - 1} \right)^{-\frac{\alpha}{2}} - h^2 \right].$$  \hspace{1cm} (13)

Fig. 2 compares for a user demanding a data rate of $DR=10^{4}\text{ bit}$ the PDF of (12) to the normalised histogram of the bandwidth requirement calculated for the whole cell area in a grid of one meter. The figure shows that the PDF properly describes the bandwidth required for a single user in a single cell scenario.

Noting that the CDF $F^{(SC)}(B)$ of RV $B$ gives the probability that the required bandwidth is smaller than $B$ and remembering the definition of outage probability of Section II, the outage probability of a user can directly be taken from the CDF $F^{(SC)}(B)$ as

$$p_{\text{user}} = 1 - F^{(SC)}(B_{av}),$$  \hspace{1cm} (14)

which is the percentage for that the available link bandwidth $B_{av}$ is smaller than the required bandwidth.

### IV. Multi User Single Cell Case

Now, several users accommodated by the same cell are considered. As mentioned in Section II, multiple access can be achieved by separation of the users in time or frequency domain or both. From the point of view of theoretic derivation, users can be separated in time domain as well as in frequency domain. For reasons of simplicity, the derivations of this section are done assuming user separation in frequency domain. The derivations also apply to OFDMA since the exclusive assignment of a single subcarrier is equivalent to the assignment of several subcarriers for a corresponding fraction of the time.

$K$ independent users are assumed to be active in the considered cell. Every user $i$ has a data rate demand $DR_{i}$ and experiences a certain SINR $\gamma_{i}$. Data rate demand and SINR of all users are independent. From the demanded data rate $DR_{i}$ and the SINR $\gamma_{i}$, the required bandwidth $B_{i}$ can be calculated for each user using Equation (3). Section III derives a RV $B$ which represents the bandwidth required by an individual user. In the following, RV $B_{i}$ represents the bandwidth required by user $i$.

Since user separation in frequency domain is assumed, the bandwidth required by a cell is expressed by the sum of the bandwidths required by the individual users and RV $B_{\text{cell}}$ representing the bandwidth required by the cell is given by

$$B_{\text{cell}} = \sum_{i=1}^{K} B_{i}.\,$$

Assuming independent users, RV $B_{\text{cell}}$ is, according to the central limit theorem [12], for sufficiently large values of $K$ normal distributed: $B_{\text{cell}} \sim \mathcal{N}(\mu_{\text{cell}}, \sigma^{2}_{\text{cell}})$. According to [12], mean $\mu_{\text{cell}}$ and variance $\sigma^{2}_{\text{cell}}$ of RV $B_{\text{cell}}$ are given by the sum of mean and variance, respectively, of the RVs representing the required bandwidths of the individual users:

$$\mu_{\text{cell}} = \sum_{i=1}^{K} \mu_{i}, \quad \sigma^{2}_{\text{cell}} = \sum_{i=1}^{K} \sigma^{2}_{i}. \hspace{1cm} (15)$$

In the special case of identical distributed bandwidth requirements of the individual users, which occurs according to (12) and (13) if all users have the same data rate demand and assuming equal noise power for all users, the calculation of mean and variance of the bandwidth required by a cell can be simplified, following the arguments in [12], to

$$\mu_{\text{cell}} = K \cdot \mu_{0}, \quad \sigma^{2}_{\text{cell}} = K \cdot \sigma^{2}_{0} \hspace{1cm} (16)$$

with $\mu_{0}$ and $\sigma^{2}_{0}$ the mean and variance of the required bandwidth of a single user. The case of equal data rate demand is relevant if only a single service, e.g. telephony, is considered.

The bandwidth required by a cell can therefore easily be modeled if the statistical properties, i.e. mean and variance, of the bandwidths required by the individual users are known. Section III gives in (12) and (13) PDF and CDF of the bandwidth required by an individual user but unfortunately, mean and variance can not be determined from these equations in a closed form, since the respective integrals can not be solved analytically.

A semi-analytic approach, however, is possible since the bandwidth depends linearly on the data rate, see (3). Exploiting the considerations of [12] concerning linearity enables the calculation of mean and variance of the bandwidth required by a cell using the formulas

$$\mu_{\text{cell}} = \mu_{\text{unit}} \cdot \sum_{i=1}^{K} DR_{i}, \quad \sigma^{2}_{\text{cell}} = \sigma^{2}_{\text{unit}} \sum_{i=1}^{K} DR_{i}^{2} \hspace{1cm} (17)$$

with $\mu_{\text{unit}}$ and $\sigma^{2}_{\text{unit}}$ the mean and the variance of a data rate demand of a certain unit of data rate of which the $DR_{i}$ are
multiples.

The values of $\mu_{\text{unit}}$ and $\sigma^2_{\text{unit}}$ have to be determined numerically. This can easily be done assuming a single user with a data rate demand equal to the data rate unit. With these assumptions, the PDF of Equation (12) is calculated at discrete points, from which $\mu_{\text{unit}}$ and $\sigma^2_{\text{unit}}$ can be calculated. Note that the numerical calculation has to be done only once for a unit of data rate.

Fig. 3 shows a comparison of the normalised histogram of an empirical calculation of the bandwidth required by a cell and a PDF gained by the analytic approach presented in this paper. The results are for a single cell scenario with $K = 5$ active users, each one demanding a data rate of $D R = 10^{\text{kbit}}$. The histogram is made from 100000 realisations of the bandwidth required for $K = 5$ independent active users with uniform position probability over the cell area. The figure shows that even for a low number of active users, the presented method enables an accurate approximation of the distribution of the bandwidth required by the cell.

An exception from the central limit theorem generally applies if there are single or few RVs that have dominant probability densities [12], which in this case occurs if single or few users have substantially higher data rate demands than others. The distribution of the required bandwidth is then given by the convolution of the probability densities of the dominant users [12]. If there are several dominant users, however, the central limit theorem applies to the dominant users, and the presented method is applicable, again.

Following the argumentation of Section III concerning the outage probability of a user, the outage probability of the cell can directly be taken from the CDF $F_{\text{cell}}(B_{\text{cell}})$ of RV $B_{\text{cell}}$ as

$$p_{\text{cell}} = 1 - F_{\text{cell}}(B_{\text{cell,av}}), \quad (18)$$

which is the percentage for which the bandwidth $B_{\text{cell,av}}$ available to the cell is smaller than the required bandwidth.

V. MULTI CELL CASE

A scenario as described in Section II is considered. In a first approach, inter-cell interference is assumed to be constant over the whole area of the considered cell. Assuming exponential power loss propagation, as given by Equation (2), inter-cell interference is approximated by the inter-cell interference at a certain reference position within the considered cell. The approximation is given by

$$P_I = \sum_{i=1}^{N_t} \frac{P_{I,i}}{(d_i^2 + h_{ii}^2)^2} \quad (19)$$

with $N_t$ the number of interferers, $P_{I,i}$ the transmit power of interferer $i$, $d_i$ the distance between interferer $i$ and the reference position and $h_{ii}$ the difference in the height of interferer $i$ and the user terminal. The assumption of constant interference power level is acceptable "unless very small reuse distances are used" [1].

Since the interference power is assumed to be constant over the whole cell area, its approximation $P_I$ can be treated the same way as the receiver noise $P_N$ in the preceding sections. PDF and CDF of the bandwidth required by a single user in a single cell scenario from (12) and (13) can thus be extended to yield the case of a multi cell scenario considering constant inter-cell interference to

$$f_{\text{B}}^{(MC)}(B) = \frac{2}{\alpha R^2} \left( \frac{P_S}{P_N + P_I} \right)^{\frac{1}{\alpha}} \left( 2^\frac{DR}{B R} - 1 \right)^{-1} \cdot A, \quad (20)$$

$$F_{\text{B}}^{(MC)}(B) = \frac{1}{R^2} \left[ \left( \frac{P_S}{P_N + P_I} \right)^{\frac{1}{\alpha}} \left( 2^\frac{DR}{B R} - 1 \right)^{-1} - h^2 \right]. \quad (21)$$

Concerning the bandwidth required by the cell, the same approach as in Section IV can be followed, using the statistical properties of the distributions of the bandwidths required by users in a multi-cell scenario from (20) and (21): First, a data rate unit of which the user demands are multiples has to be chosen, for example $1^{\text{kbit}}$. Then, $\mu_{\text{unit}}$ and $\sigma^2_{\text{unit}}$ have to be determined numerically as described in Section IV but using (20) and (21) instead of (12) and (13). Finally, the bandwidth required by the cell can be modeled from the data rate demands of the users using (17).

An example of the application of the presented model is given now. It is assumed that all cells transmit at the same transmit power, $P_S = P_{I,i} = P_{tx}$, $\forall i$, and have the same height differences $h_i = h$, $\forall i$. Only the first tier of co-channel cells is regarded, see Fig. 4 for an example for a reuse factor of $r = 3$. The centre of the considered cell is chosen as reference point for the approximation $P_I$ of the interference power, as given by (19). $P_I$ can then be derived to yield $P_I = \frac{P_{tx} \cdot 2^\alpha}{(D^2 + h^2)^2}$ with $D$ the reuse distance. Due to the approximation of the interference power, an error is introduced. The relative error of the model is given by

$$e = \frac{B_{\text{cell}}^{(\text{an})} - B_{\text{cell}}^{(\text{num})}}{B_{\text{cell}}^{(\text{num})}} \quad (22)$$
with $B^{(\text{an})}_{\text{cell}}$ and $B^{(\text{num})}_{\text{cell}}$ the analytically and numerically calculated results of the bandwidth required by a cell. Fig. 5 shows the relative error $e$ for the bandwidth required by a cell for a cell outage probability $P^{(\text{cell})}_{\text{out}}$ of 5% and for $K = 5$ active users, each demanding a data rate of $DR = 10^{\text{kbit}}$. The results are shown for different cluster sizes $r$ and reuse distances. The analytical results are calculated using the method presented in this paper, the numerical results are obtained from calculating the bandwidth requirements for 100000 realizations of $K = 5$ users.

It can be seen that the analytic model assuming constant interference power over the whole area of a cell is a good approximation of the real case. The accuracy of the model is higher for larger cluster sizes and larger reuse distances. The reason is that with increasing reuse distance, the inter-cell interference is more equal over the considered cell, as it was assumed by the approximation of the inter-cell interference power in (19). Consequently, the model can also be expected to be more accurate with increasing propagation coefficient $\alpha$. The higher accuracy for larger cluster sizes is due to the fact that for constant reuse distance $D$, the cell radius $R$ is smaller for larger $r$, cf. Equation (1), again leading to more equal interference power over the cell area. From a certain reuse distance on, the interference power is significantly lower than the receiver noise $P_N$, such that interference can be neglected and the analytic model is very accurate, cf. Fig. 5.

Note that due to the choice of the reference point for the approximation $P_1$ of the interference power, the actual interference power is always larger or equal to the approximation $P_1$. The analytically calculated required bandwidth $B^{(\text{an})}_{\text{cell}}$ is therefore never larger than the actually required bandwidth and the relative error $e$ of the model of (22) is thus always smaller than zero.

VI. CONCLUSION

This paper presents an analytic model for the bandwidth required in packed switched cellular mobile radio networks applying adaptive transmission. The model is suited to determine outage probability, jointly considering all users accommodated by a cell. It provides an alternative to time consuming simulations since it enables an efficient way of determining outage probability independent of user positions. It thus allows fast investigation of the performance of cellular mobile radio networks and is useful in the design of cellular mobile radio networks. Given a minimum service quality, i.e. a minimum data rate for each user, and a maximum outage probability, the model can also be used for determination of the bandwidth demand of cellular mobile radio networks and in the development of resource assignment strategies and algorithms.

REFERENCES


