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Cramér-Rao Lower Bounds for Hybrid Localization of Mobile Terminals

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***Abstract*—While in outdoor scenarios the Global Positioning System (GPS) provides accurate mobile station (MS) location estimates in the majority of cases, in dense urban and indoor scenarios GPS often cannot provide reliable MS location estimates, due to the attenuation or complete shadowing of the satellite signals. The existing cellular radio network (CRN)-based localization methods, however, provide MS location estimates in almost every scenario, but they do not reach the accuracy of the MS location estimates provided by GPS. Hybrid localization methods combine MS location information available from measured values of the CRN with MS location information provided by the measured values of GPS. In this paper, a hybrid localization method is proposed that combines received signal level and timing advance measured values from the Global System for Mobile Communication (GSM) and time of arrival measured values from GPS. The best achievable localization accuracy of the proposed hybrid localization method is evaluated in terms of the Cramér-Rao Lower Bound. It is shown that the hybrid localization method significantly improves the localization accuracy compared to existing CRN-based localization methods.**

I. Introduction

The demand for localization methods offering precise mobile station (MS) location estimates has increased in the last few years. On the one hand, this is due to the increasing interest in location based services (LBS) such as, e.g., fleet management, location sensitive billing, navigation and other promising applications that are based on accurate knowledge of the MS location and that will play a key role in future wireless systems [1]. On the other hand, the requirement of the United States Federal Communications Commission (FCC) that all wireless service providers have to report the location of all enhanced 911 (E-911) callers with specified accuracy, has pushed research and standardization activities in the field of MS localization [2].

Several localization methods have been proposed to solve the problem of locating a MS [3], [4]. Global navigation satellite system (GNSS)-based localization methods, e.g., such as the Global Positioning System (GPS), utilize the time of arrival (ToA) principle, i.e., the MS location is determined from the propagation time of the satellite (SAT) signals. If the MS receives SAT signals from at least four SATs, a three dimensional (3-D) MS location estimate can be found, where the fourth SAT signal is needed to resolve the unknown clock bias between the SAT and the MS clock [5]. GNSS-based localization methods have the disadvantage that especially in indoor and dense urban scenarios the number of SATs in view is often insufficient to determine a 3-D or even two dimensional (2-D) MS location estimate, due to the attenuation or complete shadowing of the SAT signals.

Cellular radio network (CRN)-based localization methods are based on, e.g., the received signal strength (RSS), round trip delay time (RTT), time difference of arrival (TDoA), angle of arrival (AoA) or ToA principle (an overview is given in [3], [4]). Although CRN-based localization methods have the advantage that the MS location estimates are almost everywhere available, they do not reach the accuracy of MS location estimates of the GNSS-based localization methods. In scenarios, where MS location estimates from the GNSS are not available and the MS location estimates from the CRN do not reach the desired localization accuracy, it is a promising approach to combine the information provided by the measured values of the CRN and the GNSS [6], [7]. The resulting hybrid localization methods are expected to enhance the accuracy and availability of MS location estimates.

The best achievable localization accuracy of hybrid localization methods can be evaluated in terms of the Cramér-Rao lower bound (CRLB). In [8], the CRLB for hybrid RSS/ToA and RSS/TDoA for wireless sensor networks is presented. The impact of quantizing RSS measured values on the CRLB for wireless sensor networks is presented in [9]. The localization accuracy of hybrid ToA/TDoA for cellular radio networks is investigated in [10].

In this paper, the CRLB of a hybrid localization method is evaluated that combines RSS and RTT measured values from the Global System for Mobile Communication (GSM) and ToA measured values from GPS. In existing GSM networks, only quantized RSS (RXLEV) and quantized RTT (TA) measured values are available. Thus, the corresponding CRLB that combines quantized measured values is additionally investigated and compared to CRLB of unquantized measured values. Simulation results show that the hybrid localization method will significantly improve the localization accuracy compared to existing CRN-based localization methods.

The remainder of this paper is organized as follows: In Section II, the well known general expressions for statistical models for the measured values, the Fisher information matrix and their relationship to the CRLB are introduced. In Section III, the statistical models for the RXLEV and TA are introduced and the well known statistical model for the ToA measured value is presented. In Section IV, the CRLB for the hybrid localization method is determined. In Section V, the achievable localization accuracy of hybrid localization method is evalu-

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ated by means of simulations. Section VI concludes the work.

II. Cramér-Rao Lower Bound

The Cramér-Rao Lower Bound (CRLB) gives a lower bound for the covariance matrix of any unbiased estimate of the unknown parameters [11]. Let $\hat{\boldsymbol{\theta}}$ be an estimate of the $n \times 1$ MS location vector $\boldsymbol{\theta}$, where $n = 2$ for MS localization in 2-D scenarios and $n = 3$ in 3-D scenarios. Let further $\boldsymbol{m} = [m_1, m_2, ..., m_N]^{\mathrm{T}}$ denote the vector of N measured values that are available from the different localization methods, where $[\cdot]^{\mathrm{T}}$ denotes the transpose of a vector. The covariance matrix of any unbiased estimator $\hat{\boldsymbol{\theta}}$ satisfies the following inequality

$$\mathrm{Cov}(\hat{\boldsymbol{\theta}}) \geq \mathbf{I}(\boldsymbol{\theta})^{-1} \quad (1)$$

[11], where $\mathrm{Cov}(\hat{\boldsymbol{\theta}}) \equiv E_{\boldsymbol{\theta}}\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T\}$, $E_{\boldsymbol{\theta}}\{\cdot\}$ denotes the expectation conditioned on $\boldsymbol{\theta}$, $\mathbf{I}(\boldsymbol{\theta})$ denotes the Fisher information matrix (FIM), and $[\cdot]^{-1}$ denotes the inverse of a matrix. The matrix inequality $\mathbf{A} \geq \mathbf{B}$ should be interpreted as the matrix $(\mathbf{A} - \mathbf{B})$ is positive semidefinite. The CRLB matrix is defined as the inverse of the FIM $\mathbf{I}(\boldsymbol{\theta})$. The (i, j)-th element of the FIM $\mathbf{I}(\boldsymbol{\theta})$ is given by

$$[\mathbf{I}(\boldsymbol{\theta})]_{i,j} = -E_{\boldsymbol{\theta}} \left\{ \frac{\partial^2 \log p(\boldsymbol{m}|\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j} \right\} \quad i, j = 1, 2, ..., n \quad (2)$$

[11], where $p(\boldsymbol{m}|\boldsymbol{\theta})$ denotes the probability density function (pdf) of the measured values $\boldsymbol{m}$ conditioned on $\boldsymbol{\theta}$. In the following, the measured values m_k and m_l, $k = 1, ..., N$, $l = 1, ..., N$, $k \neq l$ are assumed to be statistically independent of each other. This assumption is considered to be a good approximation if the measured values result from different localization methods, each suffering from different errors. The conditional pdf $p(\boldsymbol{m}|\boldsymbol{\theta})$ can thus be determined from

$$p(\boldsymbol{m}|\boldsymbol{\theta}) = \prod_{k=1}^{N} p(m_k|\boldsymbol{\theta}), \quad (3)$$

where $p(m_k|\boldsymbol{\theta})$ denotes the likelihood function of each measured value m_k. In order to evaluate the CRLB, $p(m_k|\boldsymbol{\theta})$ has to be known. The likelihood function $p(m_k|\boldsymbol{\theta})$ can be determined from a statistical model of the measured value that is introduced in the following. Let us assume that each measured value m_k provides information about the MS location $\boldsymbol{\theta}$. Each measured value is additionally affected by errors that are assumed to be additive and that can be statistically described by a random variable n_k with known probability density function $p_{n_k}(n_k)$. The statistical model for the measured value is, thus, given by

$$m_k = f_k(\boldsymbol{\theta}) + n_k \quad (4)$$

[7], where $f_k(\boldsymbol{\theta})$ models the functional relationship between the MS's location $\boldsymbol{\theta}$ and the error-free measured value $\tilde{m}_k$, i.e., $\tilde{m}_k = f_k(\boldsymbol{\theta})$. The likelihood function $p(m_k|\boldsymbol{\theta})$ of the measured value m_k can be determined from (4), yielding

$$p(m_k|\boldsymbol{\theta}) = p_{n_i}(m_k - f_k(\boldsymbol{\theta})). \quad (5)$$

III. Examples for statistical models of measured values

A. Introduction

In the following, the statistical models of the measured values that can be obtained from the CRN, namely RTT and RSS, and from the GNSS, namely ToA, are presented. Due to the fact that in GSM the RTT and RSS measured values are quantized to finite precision, the corresponding relationships between quantized and unquantized measured values are additionally determined. The SAT location is given by $\mathbf{x}_{\mathrm{SAT}} = [x_{\mathrm{SAT}}, y_{\mathrm{SAT}}, z_{\mathrm{SAT}}]^{\mathrm{T}}$. The base station (BS) location $\mathbf{x}_{\mathrm{BS}} = [x_{BS}, y_{BS}]^{\mathrm{T}}$, as well as the MS's location $\boldsymbol{\theta} = [x, y]^{\mathrm{T}}$ to be estimated, are assumed to lie in the xy-plane. For the case of 3-D BS and MS locations and thus an estimation of the 3-D MS location vector, the statistical models can be obtained in a similar way.

B. Round Trip Time

In CRNs, the RTT is the time the radio signal requires to travel from the BS to the MS and back. It is a parameter that is used to synchronize the transmitted bursts of the MSs to the frame of the receiving BS [12]. Let $d_{\mathrm{BS}}(\boldsymbol{\theta})$ denote the BS to MS distance given by

$$d_{\mathrm{BS}}(\boldsymbol{\theta}) = \sqrt{(x_{\mathrm{BS}} - x)^2 + (y_{\mathrm{BS}} - y)^2}\,, \quad (6)$$

and $c_0 = 3 \cdot 10^8\ \mathrm{m/s}$ the speed of light. Then, the model for the functional relationship between the error-free measured value $\tilde{m}_{\mathrm{RTT}}$ and the MS location $\boldsymbol{\theta}$ is given by

$$\tilde{m}_{\mathrm{RTT}} = f_{\mathrm{RTT}}(\boldsymbol{\theta}) = \frac{2 \cdot d_{\mathrm{BS}}(\boldsymbol{\theta})}{c_0}. \quad (7)$$

The RTT measured values are affected by errors resulting form the propagation conditions - line-of-sight (LOS) or non-line-of-sight (NLOS) situation - and inaccuracies of the measurement equipment. It is assumed that the errors can be modelled as a Gaussian random variable n_{RTT} with mean μ_{RTT} accounting for the error due to NLOS situations and standard deviation σ_{RTT} [3]. Thus, the statistical model of the RTT measured value m_{RTT} is given by

$$m_{\mathrm{RTT}} = f_{\mathrm{RTT}}(\boldsymbol{\theta}) + n_{\mathrm{RTT}}. \quad (8)$$

In GSM, the RTT measured values m_{RTT} are rounded to the nearest integer bit period $T_b = 48/13\,\mu\mathrm{s}$, also known as Timing Advance (TA) measured values m_{TA} [12]. The statistical model for the TA measured value m_{TA} is given by

$$m_{\mathrm{TA}} = \begin{cases} \lfloor \frac{m_{\mathrm{RTT}}}{T_b} + \frac{1}{2} \rfloor, & 0 \leq \frac{m_{\mathrm{RTT}}}{T_b} < 62 \\ 63, & \frac{m_{\mathrm{RTT}}}{T_b} \geq 62, \end{cases} \quad (9)$$

where $\lfloor \cdot \rfloor$ denotes the nearest integer smaller than or equal to the argument.

C. Received Signal Strength

In CRNs, the RSS value is an averaged value of the strength of a radio signal received by the MS from the BS. It is well known that the attenuation of the signal strength through a mobile radio channel is caused by three factors, namely path loss, fast fading and slow fading [13]. Let A denote the reference path loss at a BS to MS distance of $1\,\mathrm{km}$ and B the path loss exponent. Then, the path loss PL in dB is given by

$$PL = A + 10 \cdot B \cdot \log_{10}\left(\frac{d_{\mathrm{BS}}(\boldsymbol{\theta})}{\mathrm{km}}\right) \tag{10}$$

[3], where both parameters A and B strongly depend on the propagation conditions and the antenna settings of the transmitter. In a real system, the BS may be equipped with directional antennas in order to enhance the cell's capacity. For a fixed MS to BS distance this means that the MS located in the direction of maximum antenna gain receives a larger RSS value than a MS having the same propagation conditions but which is located in the direction of minimum antenna gain. In the following, it is assumed that normalized antenna gain models are a-priori available. As long as the 2-D MS location is determined, it is a reasonable assumption to assume 2-D normalized antenna gain models. Let $\varphi_{\mathrm{BS}}(\boldsymbol{\theta})$ denote the MS to BS angle given by

$$\varphi_{\mathrm{BS}}(\boldsymbol{\theta}) = \arctan\left(\frac{y_{\mathrm{BS}} - y}{x_{\mathrm{BS}} - x}\right). \tag{11}$$

Let further A_m denote the maximum attenuation, φ_0 the BS antenna's boresight direction and φ_{3dB} the half-power beamwidth of the BS antenna in degrees. Then, the model for the normalized antenna gain in dB scale is given by

$$g(\varphi_{\mathrm{BS}}(\boldsymbol{\theta})) = \begin{cases} -12\left(\frac{\varphi_{\mathrm{BS}}(\boldsymbol{\theta})-\varphi_0}{\varphi_{3dB}}\right)^2, & \left|\frac{\varphi_{\mathrm{BS}}(\boldsymbol{\theta})-\varphi_0}{\varphi_{3dB}}\right| \leq \sqrt{\frac{A_m}{12}} \\ -A_m, & \text{else} \end{cases} \tag{12}$$

[14]. The model for the functional relationship between the error-free measured value $\tilde{m}_{\mathrm{RSS}}$ and the MS location $\boldsymbol{\theta}$ is given by

$$\tilde{m}_{\mathrm{RSS}} = f_{\mathrm{RSS}}(\boldsymbol{\theta}) = P_t - \{PL - g(\varphi_{\mathrm{BS}}(\boldsymbol{\theta}))\}, \tag{13}$$

where P_t denotes the BS's equivalent isotropic radiated power. The RSS measured value m_{RSS} is further affected by errors due to fast fading and slow fading. As the RSS measured value is averaged over several time-consecutive measurements, the error due to fast fading can be removed and, thus, has not to be taken into account in the statistical model for the errors. It is well known that the error in dB due to slow fading can be modelled as an additive zero-mean Gaussian random variable n_{RSS} with standard deviation σ_{RSS} [3]. Thus, the statistical model of the mean RSS measured value m_{RSS} in dB scale is given by

$$m_{\mathrm{RSS}} = f_{\mathrm{RSS}}(\boldsymbol{\theta}) + n_{\mathrm{RSS}}. \tag{14}$$

In GSM, the RSS measured values m_{RSS}, measured in dBm, are quantized to finite precision, also known as received signal level (RXLEV) measured values m_{RXLEV} [12]. The statistical model for the RXLEV measured value m_{RXLEV} is given by

$$m_{\mathrm{RXLEV}} = \begin{cases} 0, & \frac{m_{\mathrm{RSS}}}{\mathrm{dBm}} + 110 \leq 0 \\ \lceil \frac{m_{\mathrm{RSS}}}{\mathrm{dBm}} + 110 \rceil, & 0 < \frac{m_{\mathrm{RSS}}}{\mathrm{dBm}} + 110 < 62 \\ 63, & \frac{m_{\mathrm{RSS}}}{\mathrm{dBm}} + 110 \geq 62, \end{cases} \tag{15}$$

where $\lceil \cdot \rceil$ denotes the nearest integer larger than or equal to the argument.

D. Time of Arrival

The GNSS-based localization methods utilize the concept of ToA, i.e., the MS is measuring the time the SAT signal requires to travel from the SAT to the MS [5]. In the following, the statistical model of the ToA measured value m_{ToA} is determined. Let $d_{\mathrm{SAT}}(\boldsymbol{\theta})$ denote the SAT to MS distance given by

$$d_{\mathrm{SAT}}(\boldsymbol{\theta}) = \sqrt{(x_{\mathrm{SAT}} - x)^2 + (y_{\mathrm{SAT}} - y)^2 + z_{\mathrm{SAT}}^2}\,. \tag{16}$$

Then, the model for the functional relationship $f_{\mathrm{ToA}}(\boldsymbol{\theta})$ between the error-free measured value $\tilde{m}_{\mathrm{ToA}}$ and the MS's location $\boldsymbol{\theta}$ is given by

$$\tilde{m}_{\mathrm{ToA}} = f_{\mathrm{ToA}}(\boldsymbol{\theta}) = \frac{d_{\mathrm{SAT}}(\boldsymbol{\theta})}{c_0}. \tag{17}$$

The measured value m_{ToA} is affected by errors resulting from delays as the signal propagates through the atmosphere, propagation conditions - LOS or NLOS situation - as well as resolution errors and receiver noise. These errors, given by n_{ToA}, are assumed to be Gaussian distributed with mean μ_{ToA} accounting for the error due to NLOS situations and standard deviation σ_{ToA} [3]. In general, the MS's clock is not synchronized to the clocks of the GPS SATs, resulting in an unknown receiver clock error [5]. The unknown receiver clock error can be compensated, e.g., by using timing information provided by the radio ressource protocol [15]. In the following, it is assumed that the MS's clock is synchronized to the clocks of the GPS SATs. Consequently, the statistical model of the measured value m_{ToA} is given by

$$m_{\mathrm{ToA}} = f_{\mathrm{ToA}}(\boldsymbol{\theta}) + n_{\mathrm{ToA}}. \tag{18}$$

IV. Evaluation of the Cramér-Rao Lower Bound for Hybrid Localization

A. Introduction

In the following, the CRLB for the hybrid localization method is evaluated. Since the best achievable localization accuracy of the estimated MS location $\hat{\boldsymbol{\theta}} = [\hat{x}, \hat{y}]^{\mathrm{T}}$ is of main interest, it is reasonable to introduce the minimum mean square error (MMSE) of the MS location estimate $\hat{\boldsymbol{\theta}}$ [13]. Let $\min(\cdot)$ denote the minimum of the argument, $\|\mathbf{v}\|_2$ the L^2 norm of the vector $\mathbf{v}$ and $\mathrm{tr}\{\mathbf{A}\}$ the trace of a matrix $\mathbf{A}$. The MMSE of the MS location estimate is then given by

$$\begin{aligned} \mathrm{MMSE} &= \min(E_{\boldsymbol{\theta}}\{\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|_2\}) = \min(\mathrm{tr}\{\mathrm{Cov}(\hat{\boldsymbol{\theta}})\}) \\ &= \mathrm{tr}\{\mathbf{I}(\boldsymbol{\theta})^{-1}\} = \frac{[\mathbf{I}(\boldsymbol{\theta})]_{1,1} + [\mathbf{I}(\boldsymbol{\theta})]_{2,2}}{\mathrm{Det}\,\{\mathbf{I}(\boldsymbol{\theta})\}}, \end{aligned} \tag{19}$$

where $\mathrm{Det}\left\{\mathbf{I}(\boldsymbol{\theta})\right\}$ denotes the determinant of the symmetric 2×2 FIM $\mathbf{I}(\boldsymbol{\theta})$. Further, let $\mathbf{I}_{\mathrm{RTT}}(\boldsymbol{\theta})$ and $\mathbf{I}_{\mathrm{QRTT}}(\boldsymbol{\theta})$ denote the FIMs of the unquantized and quantized RTT measured value, $\mathbf{I}_{\mathrm{RSS}}(\boldsymbol{\theta})$ and $\mathbf{I}_{\mathrm{QRSS}}(\boldsymbol{\theta})$ the FIMs of the unquantized and quantized RSS measured value and $\mathbf{I}_{\mathrm{ToA}}(\boldsymbol{\theta})$ the FIM of the unquantized ToA measured value. As long as the measured values are assumed to be statistically independent, the corresponding FIMs can be added up [3]. Thus, the FIM of the hybrid localization method for unquantized RTT, RSS and ToA measured values can be determined from

$$\mathbf{I}(\boldsymbol{\theta}) = \mathbf{I}_{\mathrm{RTT}}(\boldsymbol{\theta}) + \mathbf{I}_{\mathrm{RSS}}(\boldsymbol{\theta}) + \mathbf{I}_{\mathrm{ToA}}(\boldsymbol{\theta}), \tag{20}$$

and for quantized RTT and RSS and unquantized ToA measured values from

$$\mathbf{I}(\boldsymbol{\theta}) = \mathbf{I}_{\mathrm{QRTT}}(\boldsymbol{\theta}) + \mathbf{I}_{\mathrm{QRSS}}(\boldsymbol{\theta}) + \mathbf{I}_{\mathrm{ToA}}(\boldsymbol{\theta}). \tag{21}$$

In the following, the FIM of unquantized RTT, RSS and ToA measured values and quantized RTT and RSS measured values are derived, from which the corresponding CRLBs for the hybrid localization method and the MMSE of the MS location estimate can be determined.

B. Cramér-Rao Lower Bound for unquantized measured values

As long as the measured values are unquantized and corrupted by additive errors that are zero-mean Gaussian distributed, the (i, j)-th element of $\mathbf{I}(\boldsymbol{\theta})$ can be found from the well known expression

$$\left[\mathbf{I}(\boldsymbol{\theta})\right]_{i,j} = \sum_{k=1}^{N} \frac{1}{\sigma_k^2} \cdot \frac{\partial f_k(\boldsymbol{\theta})}{\partial \theta_i} \cdot \frac{\partial f_k(\boldsymbol{\theta})}{\partial \theta_j} \tag{22}$$

[11]. The FIM of RTT measured values can be found from inserting (7) into (22). Let N_{RTT} denote the number of available RTT measured values and $\mathbf{u}_{\mathrm{BS},k}$ the unit vector originating at the MS location and directed towards the k-th BS given by

$$\mathbf{u}_{\mathrm{BS},k} = u_{\mathrm{BS}_x,k} \cdot \mathbf{u}_x + u_{\mathrm{BS}_y,k} \cdot \mathbf{u}_y \tag{23}$$

[10], where $\mathbf{u}_x$, $\mathbf{u}_y$ are the unit vectors in the x and y directions. Let further a_k be defined as

$$a_k = \left(\frac{2}{c_0 \cdot \sigma_{\mathrm{RTT},k}}\right)^2. \tag{24}$$

Then, the elements of the FIM of RTT measured values are given by:

$$\left[\mathbf{I}_{\mathrm{RTT}}(\boldsymbol{\theta})\right]_{1,1} = \sum_{k=1}^{N_{\mathrm{RTT}}} a_k \cdot u_{\mathrm{BS}_x,k}^2, \tag{25}$$

$$\left[\mathbf{I}_{\mathrm{RTT}}(\boldsymbol{\theta})\right]_{1,2} = \sum_{k=1}^{N_{\mathrm{RTT}}} a_k \cdot u_{\mathrm{BS}_x,k} \cdot u_{\mathrm{BS}_y,k}, \tag{26}$$

$$\left[\mathbf{I}_{\mathrm{RTT}}(\boldsymbol{\theta})\right]_{2,2} = \sum_{k=1}^{N_{\mathrm{RTT}}} a_k \cdot u_{\mathrm{BS}_y,k}^2. \tag{27}$$

The FIM of RSS measured values can be found from inserting (13) into (22). Let N_{RSS} denote the number of available RSS measured values. Let further b_k and $w_k(\boldsymbol{\theta})$ be defined as

$$b_k = \frac{10 \cdot B_k}{\sigma_{\mathrm{RSS},k} \cdot \log_e(10)} \tag{28}$$

and

$$w_k(\boldsymbol{\theta}) = \frac{1}{\sigma_{\mathrm{RSS},k}} \cdot \frac{\partial g_k(\varphi_{\mathrm{BS},k}(\boldsymbol{\theta}))}{\partial \varphi_{\mathrm{BS},k}(\boldsymbol{\theta})}. \tag{29}$$

Then, the elements of the FIM of RSS measured values are given by

$$\left[\mathbf{I}_{\mathrm{RSS}}(\boldsymbol{\theta})\right]_{1,1} = \sum_{k=1}^{N_{\mathrm{RSS}}} \left[\frac{b_k \cdot u_{\mathrm{BS}_x,k} + w_k(\boldsymbol{\theta}) \cdot u_{\mathrm{BS}_y,k}}{d_{\mathrm{BS},k}(\boldsymbol{\theta})}\right]^2, \tag{30}$$

$$\left[\mathbf{I}_{\mathrm{RSS}}(\boldsymbol{\theta})\right]_{1,2} = \sum_{k=1}^{N_{\mathrm{RSS}}} \left[\left(\frac{b_k \cdot u_{\mathrm{BS}_x,k} + w_k(\boldsymbol{\theta}) \cdot u_{\mathrm{BS}_y,k}}{d_{\mathrm{BS},k}(\boldsymbol{\theta})}\right) \cdot \left(\frac{b_k \cdot u_{\mathrm{BS}_y,k} - w_k(\boldsymbol{\theta}) \cdot u_{\mathrm{BS}_x,k}}{d_{\mathrm{BS},k}(\boldsymbol{\theta})}\right)\right], \tag{31}$$

$$\left[\mathbf{I}_{\mathrm{RSS}}(\boldsymbol{\theta})\right]_{2,2} = \sum_{k=1}^{N_{\mathrm{RSS}}} \left[\frac{b_k \cdot u_{\mathrm{BS}_y,k} - w_k(\boldsymbol{\theta}) \cdot u_{\mathrm{BS}_x,k}}{d_{\mathrm{BS},k}(\boldsymbol{\theta})}\right]^2. \tag{32}$$

The FIM of ToA measured values can be found from inserting (17) into (22). Let N_{ToA} denote the number of available ToA measured values and $\mathbf{u}_{\mathrm{SAT},k}$ the unit vector originating at the MS location and directed towards the k-th SAT, given by

$$\mathbf{u}_{\mathrm{SAT},k} = u_{\mathrm{SAT}_x,k} \cdot \mathbf{u}_x + u_{\mathrm{SAT}_y,k} \cdot \mathbf{u}_y + u_{\mathrm{SAT}_z,k} \cdot \mathbf{u}_z, \tag{33}$$

where $\mathbf{u}_z$ is the unit vector in the z direction. The projection of the unit vector $\mathbf{u}_{\mathrm{SAT},k}$ into the xy-plane is given by

$$\mathbf{p}_{\mathrm{SAT},k} = u_{\mathrm{SAT}_x,k} \cdot \mathbf{u}_x + u_{\mathrm{SAT}_y,k} \cdot \mathbf{u}_y. \tag{34}$$

Let further c_k be defined as

$$c_k = \left(\frac{1}{c_0 \cdot \sigma_{\mathrm{ToA},k}}\right)^2. \tag{35}$$

The elements of the FIM of ToA measured values are given by

$$\left[\mathbf{I}_{\mathrm{ToA}}(\boldsymbol{\theta})\right]_{1,1} = \sum_{k=1}^{N_{\mathrm{ToA}}} c_k \cdot u_{\mathrm{SAT}_x,k}^2, \tag{36}$$

$$\left[\mathbf{I}_{\mathrm{ToA}}(\boldsymbol{\theta})\right]_{1,2} = \sum_{k=1}^{N_{\mathrm{ToA}}} c_k \cdot u_{\mathrm{SAT}_x,k} \cdot u_{\mathrm{SAT}_y,k}, \tag{37}$$

$$\left[\mathbf{I}_{\mathrm{ToA}}(\boldsymbol{\theta})\right]_{2,2} = \sum_{k=1}^{N_{\mathrm{ToA}}} c_k \cdot u_{\mathrm{SAT}_y,k}^2. \tag{38}$$

[8]. The CRLB for the hybrid localization method of unquantized measured values can then be found from (20) and (1). In the following, a closed form expression of the MMSE of the MS location estimate is determined. For the sake of simplicity, it is assumed that the BS antennas have a uniform normalized antenna gain $g_k(\varphi_{\mathrm{BS},k}(\boldsymbol{\theta}))$. From this it follows

that $w_k(\boldsymbol{\theta}) = 0$. Let $\mathcal{A}_{k,i}$ denote the area of the parallelogram determined by $\mathbf{u}_{\mathrm{BS},k}$ and $\mathbf{u}_{\mathrm{BS},i}$ [10], given by

$$\mathcal{A}_{k,i} = \|\mathbf{u}_{\mathrm{BS},k} \times \mathbf{u}_{\mathrm{BS},i}\|_2, \tag{39}$$

$\mathcal{B}_{k,i}$ denote the area of the parallelogram determined by $\mathbf{u}_{\mathrm{BS},k}$ and $\mathbf{p}_{\mathrm{SAT},i}$, given by

$$\mathcal{B}_{k,i} = \|\mathbf{u}_{\mathrm{BS},k} \times \mathbf{p}_{\mathrm{SAT},i}\|_2 \tag{40}$$

and $\mathcal{C}_{k,i}$ denote the area of the parallelogram determined by $\mathbf{p}_{\mathrm{SAT},k}$ and $\mathbf{p}_{\mathrm{SAT},i}$, given by

$$\mathcal{C}_{k,i} = \|\mathbf{p}_{\mathrm{SAT},k} \times \mathbf{p}_{\mathrm{SAT},i}\|_2, \tag{41}$$

where $\mathbf{a} \times \mathbf{b}$ denotes the cross product between two vectors $\mathbf{a}$ and $\mathbf{b}$. Then, the MMSE of the MS location estimate for unquantized measured values is given by

$$\mathrm{MMSE} = \frac{\sum_{k=1}^{N_{\mathrm{RTT}}} a_k + \sum_{k=1}^{N_{\mathrm{RSS}}} \frac{b_k^2}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta})} + \sum_{k=1}^{N_{\mathrm{ToA}}} c_k \cdot \|\mathbf{p}_{\mathrm{SAT},k}\|_2}{\mathrm{Det}\left\{\mathbf{I}\left(\boldsymbol{\theta}\right)\right\}}, \tag{42}$$

where the determinant is given by

$$\begin{aligned}
\mathrm{Det}\left\{\mathbf{I}(\boldsymbol{\theta})\right\} = & \sum_{k=1}^{N_{\mathrm{RTT}}} \sum_{\substack{i=1\\ i>k}}^{N_{\mathrm{RTT}}} a_k \cdot a_i \cdot \mathcal{A}_{k,i} \\
& + \sum_{k=1}^{N_{\mathrm{RTT}}} \sum_{i=1}^{N_{\mathrm{RSS}}} \frac{a_k \cdot b_i^2}{d_{\mathrm{BS},i}^2(\boldsymbol{\theta})} \cdot \mathcal{A}_{k,i} \\
& + \sum_{k=1}^{N_{\mathrm{RTT}}} \sum_{i=1}^{N_{\mathrm{ToA}}} a_k \cdot c_i \cdot \mathcal{B}_{k,i} \\
& + \sum_{k=1}^{N_{\mathrm{RSS}}} \sum_{i=1}^{N_{\mathrm{ToA}}} \frac{b_k^2 \cdot c_i}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta})} \cdot \mathcal{B}_{k,i} \\
& + \sum_{k=1}^{N_{\mathrm{RSS}}} \sum_{\substack{i=1\\ i>k}}^{N_{\mathrm{RSS}}} \frac{b_k^2 \cdot b_i^2}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta}) \cdot d_{\mathrm{BS},i}^2(\boldsymbol{\theta})} \cdot \mathcal{A}_{k,i} \\
& + \sum_{k=1}^{N_{\mathrm{ToA}}} \sum_{\substack{i=1\\ i>k}}^{N_{\mathrm{ToA}}} c_k \cdot c_i \cdot \mathcal{C}_{k,i}.
\end{aligned} \tag{43}$$

C. Cramér-Rao Bound for quantized measured values

In GSM, the RTT measured values are quantized to $K = 64$ levels, $i = 0, ..., 63$, $i \in \mathbb{Z}$. Each level i is composed of a lower bound $Q_{\mathrm{RTT,lb}}(i)$, denoting the minimum RTT measured in level i and an upper bound $Q_{\mathrm{RTT,ub}}(i)$, representing the maximum RTT measured in level i. According to (9), the lower and upper bounds of the corresponding TA measured values $m_{\mathrm{TA}} = i$ are given by

$$Q_{\mathrm{RTT,lb}}(i) = \begin{cases} 0, & i = 0 \\ (2i-1) \cdot \frac{T_b}{2}, & i = 1, ..., 63 \end{cases} \tag{44}$$

and

$$Q_{\mathrm{RTT,ub}}(i) = \begin{cases} (2i+1) \cdot \frac{T_b}{2}, & i = 0, ..., 62 \\ \infty, & i = 63. \end{cases} \tag{45}$$

The FIM of quantized RTT measured values can be found, following the derivation given in [9]. Let N_{QRTT} denote the number of available TA measured values and $\Phi(\cdot)$ the standard normal cumulative distribution function of the argument. The conditional pdf $p_k(0|\boldsymbol{\theta})$ is given by

$$p_k(0|\boldsymbol{\theta}) = \frac{1}{\sigma_{\mathrm{RTT},k}\sqrt{2\pi}} \cdot \exp\left\{-\frac{[f_{\mathrm{RTT},k}(\boldsymbol{\theta})]^2}{2 \cdot \sigma_{\mathrm{RTT},k}^2}\right\}, \tag{46}$$

and $h_{k,i}$ is defined as

$$h_{k,i} = \frac{1}{2\pi} \sum_{i=0}^{63} \frac{\left[\exp\left\{-\frac{1}{2}(r_1)^2\right\} - \exp\left\{-\frac{1}{2}(r_2)^2\right\}\right]^2}{\Phi\left(r_2\right) - \Phi\left(r_1\right)}, \tag{47}$$

with the following functions

$$r_1 = \frac{Q_{\mathrm{RTT,lb}}(i) - f_{\mathrm{RTT},k}(\boldsymbol{\theta})}{\sigma_{\mathrm{RTT},k}} \tag{48}$$

and

$$r_2 = \frac{Q_{\mathrm{RTT,ub}}(i) - f_{\mathrm{RTT},k}(\boldsymbol{\theta})}{\sigma_{\mathrm{RTT},k}}. \tag{49}$$

Then, the elements of the FIM of quantized RTT measured values are given by

$$\begin{aligned}
\left[\mathbf{I}_{\mathrm{QRTT}}(\boldsymbol{\theta})\right]_{1,1} = & \sum_{k=1}^{N_{\mathrm{QRTT}}} f_{\mathrm{RTT},k}(\boldsymbol{\theta}) \cdot p_k(0|\boldsymbol{\theta}) \\
& \cdot \left[\left(a_k + \frac{1}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta})}\right) u_{\mathrm{BS}_x,k}^2 - \frac{1}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta})}\right] \\
& + \sum_{k=1}^{N_{\mathrm{QRTT}}} a_k \cdot u_{\mathrm{BS}_x,k}^2 \cdot h_{k,i},
\end{aligned} \tag{50}$$

$$\begin{aligned}
\left[\mathbf{I}_{\mathrm{QRTT}}(\boldsymbol{\theta})\right]_{1,2} = & \sum_{k=1}^{N_{\mathrm{QRTT}}} f_{\mathrm{RTT},k}(\boldsymbol{\theta}) \cdot p_k(0|\boldsymbol{\theta}) \cdot \Bigg[\Bigg(a_k \\
& + \frac{1}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta})}\Bigg) u_{\mathrm{BS}_x,k} u_{\mathrm{BS}_y,k} - \frac{1}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta})}\Bigg] \\
& + \sum_{k=1}^{N_{\mathrm{QRTT}}} a_k \cdot u_{\mathrm{BS}_x,k} \cdot u_{\mathrm{BS}_y,k} \cdot h_{k,i},
\end{aligned} \tag{51}$$

$$\begin{aligned}
\left[\mathbf{I}_{\mathrm{QRTT}}(\boldsymbol{\theta})\right]_{2,2} = & \sum_{k=1}^{N_{\mathrm{QRTT}}} f_{\mathrm{RTT},k}(\boldsymbol{\theta}) \cdot p_k(0|\boldsymbol{\theta}) \\
& \cdot \left[\left(a_k + \frac{1}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta})}\right) u_{\mathrm{BS}_y,k}^2 - \frac{1}{d_{\mathrm{BS},k}^2(\boldsymbol{\theta})}\right] \\
& + \sum_{k=1}^{N_{\mathrm{QRTT}}} a_k \cdot u_{\mathrm{BS}_y,k}^2 \cdot h_{k,i},
\end{aligned} \tag{52}$$

In GSM, the RSS measured values are quantized to $K = 64$ levels, $i = 0, ..., 63$, $i \in \mathbb{Z}$. Again, each level i is composed of a lower bound $Q_{\mathrm{RSS,lb}}(i)$, denoting the minimum RSS measured in level i and an upper bound $Q_{\mathrm{RSS,ub}}(i)$, representing the maximum RSS measured in level i. According to (15), the lower and upper bounds of the corresponding RXLEV measured values $m_{\mathrm{RXLEV}} = i$ are given by

$$Q_{\mathrm{RSS,lb}}(i) = \begin{cases} -\infty, & i = 0 \\ (i-1) - 110, & i = 1, ..., 63 \end{cases} \tag{53}$$

and

$$Q_{\mathrm{RSS,ub}}(i) = \begin{cases} i - 110, & i = 0, ..., 62 \\ \infty, & i = 63. \end{cases} \tag{54}$$

The elements of the FIM of quantized RSS measured values are given by

$$[\mathbf{I}_{\mathrm{QRSS}}(\boldsymbol{\theta})]_{1,1} = \sum_{k=1}^{N_{\mathrm{QRSS}}} \left[\frac{b_k \cdot u_{\mathrm{BS}_x,k} + w_k(\boldsymbol{\theta}) \cdot u_{\mathrm{BS}_y,k}}{d_{\mathrm{BS},k}(\boldsymbol{\theta})} \right]^2 \cdot h_{k,i}, \tag{55}$$

$$[\mathbf{I}_{\mathrm{QRSS}}(\boldsymbol{\theta})]_{1,2} = \sum_{k=1}^{N_{\mathrm{QRSS}}} \left[\left(\frac{b_k \cdot u_{\mathrm{BS}_x,k} + w_k(\boldsymbol{\theta}) \cdot u_{\mathrm{BS}_y,k}}{d_{\mathrm{BS},k}(\boldsymbol{\theta})} \right) \cdot \left(\frac{b_k \cdot u_{\mathrm{BS}_y,k} - w_k(\boldsymbol{\theta}) \cdot u_{\mathrm{BS}_x,k}}{d_{\mathrm{BS},k}(\boldsymbol{\theta})} \right) \right] \cdot h_{k,i}, \tag{56}$$

$$[\mathbf{I}_{\mathrm{QRSS}}(\boldsymbol{\theta})]_{2,2} = \sum_{k=1}^{N_{\mathrm{QRSS}}} \left[\frac{b_k \cdot u_{\mathrm{BS}_y,k} - w_k(\boldsymbol{\theta}) \cdot u_{\mathrm{BS}_x,k}}{d_{\mathrm{BS},k}(\boldsymbol{\theta})} \right]^2 \cdot h_{k,i}, \tag{57}$$

where N_{QRSS} is the number of available RXLEV measured values and $h_{k,i}$ is defined in (47) with the exception that r_1 and r_2 have to be replaced by

$$r_1 = \frac{Q_{\mathrm{RSS,lb}}(i) - f_{\mathrm{RSS},k}(\boldsymbol{\theta})}{\sigma_{\mathrm{RSS},k}} \tag{58}$$

and

$$r_2 = \frac{Q_{\mathrm{RSS,ub}}(i) - f_{\mathrm{RSS},k}(\boldsymbol{\theta})}{\sigma_{\mathrm{RSS},k}}. \tag{59}$$

The CRLB for the hybrid localization method of quantized RTT and RSS measured values and unquantized ToA measured values can be then found from (21) and (1). The MMSE of the MS location estimate can be found from (19) and (21). For the sake of clarity, the closed form expression of the MMSE of the MS location estimate is omitted.

V. Simulation Scenario and Results

In the following, the simulation scenario is explained and simulation results are presented. It is assumed that the BSs are organized in an ideal 2-D CRN with hexagonal cell structure and cell radius R_{cell}. The BS locations are assumed to be fixed and known, and each BS is equipped with an omnidirectional BS antenna. In order to evaluate the CRLB of the hybrid localization method, only the N_{BS} nearest BSs are considered for MS localization as shown in Fig. 1, where the nearest BS is assumed to be the serving BS. The GPS SAT locations are assumed to be known since they can be calculated from the navigation message that is transmitted by each SAT. The simulation parameters are given in Table I and are assumed to be equal for all BSs and all SATs for the sake of simplification. The following combinations of measured values are investigated:

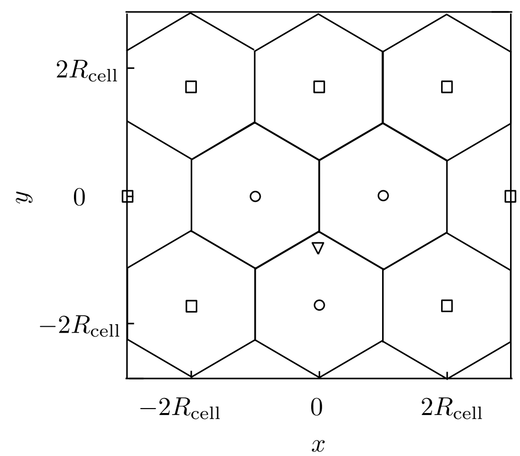

Fig. 1. CRN with the MS (▽) and $N_{\mathrm{BS}} = 3$ BSs (○) that are involved in the MS localization process. The BSs that are not involved in the MS localization process are marked as squares (□).

- One TA and RXLEV measured value from the serving BS and one RXLEV measured value from $N_{\mathrm{BS}} - 1$ neighbouring BSs (GSM method)
- Measured values of GSM method and, in addition, one ToA measured value from one GPS SAT (Hybrid 1 method)
- Measured values of GSM method and, in addition, one ToA measured value from each of a total of two GPS SATs (Hybrid 2 method)
- Two ToA measured values provided by two different GPS SATs (2 Satellite method)

The hybrid localization method is evaluated in terms of the average root mean square error ($\overline{\mathrm{RMSE}}$), which gives the localization accuracy for all possible MS locations in the CRN and is given by

$$\overline{\mathrm{RMSE}} = E_{\boldsymbol{\theta}} \left\{ \sqrt{\mathrm{tr}\left\{ \mathbf{I}(\boldsymbol{\theta})^{-1} \right\}} \right\}. \tag{60}$$

In Fig. 2, the simulation results for the $\overline{\mathrm{RMSE}}$ in dependence of the cell radius for the GSM, Hybrid 1 and Hybrid 2 method for (dense) urban scenarios are shown. For comparison, the results for the $\overline{\mathrm{RMSE}}$ assuming unquantized RTT and RSS are given. The GSM method provides the worst results in terms of localization accuracy, which can be explained by the fact that the investigated GSM measured values cannot provide the same level of accuracy as the GPS measured values. The

TABLE I
Simulation parameters

Parameter	Value	Parameter	Value
A in dB	132.8	P_t in dBm	33
B in dB	3.8	σ_{RTT} in μs	1
σ_{RSS} in dB	8	σ_{ToA} in ns	$33.\bar{3}$

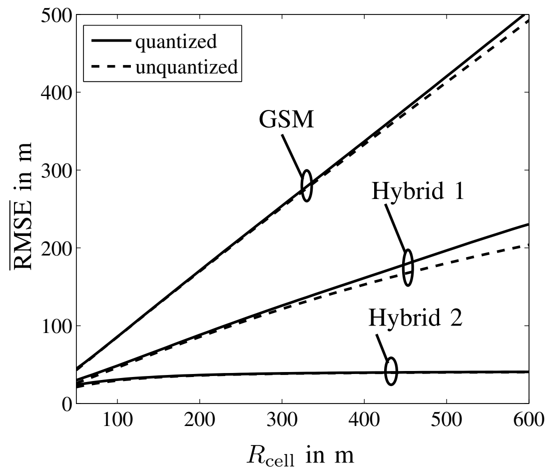

Fig. 2. $\overline{\text{RMSE}}$ vs. R_{cell} for the GSM, Hybrid 1 and Hybrid 2 method for quantized and unquantized measured values for $N_{\text{BS}} = 3$.

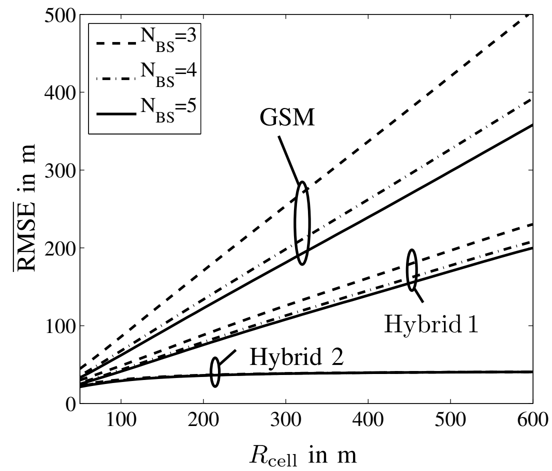

Fig. 3. $\overline{\text{RMSE}}$ vs. R_{cell} for the GSM, Hybrid 1 and Hybrid 2 method for quantized measured values for different $N_{\text{BS}} = \{3, 4, 5\}$.

localization accuracy can be significantly improved with the Hybrid 1 method, by additionally taking into account one ToA measured value from one GPS SAT. In contrast to the Hybrid 1 method, the Hybrid 2 method takes into account two ToA measured values from two different GPS SATs, leading to a further improvement of the localization accuracy. The impact of the quantization on the achievable localization accuracy can also be seen from Fig. 2. For the GSM and Hybrid 1 method the $\overline{\text{RMSE}}$ degrades only for large cell radii, whereas for the Hybrid 2 method, the quantization does not significantly affect the $\overline{\text{RMSE}}$ for almost the entire range of investigated cell radii. In Fig. 3, the $\overline{\text{RMSE}}$ in dependence of the cell radius for the GSM, Hybrid 1 and Hybrid 2 method for different values of N_{BS} is shown. If N_{BS} and, thus, the number of available RXLEV measured values is increased, the $\overline{\text{RMSE}}$ can be improved. For the GSM method the improvement is significant, for the Hybrid 1 method moderate and for the Hybrid 2 method marginal. The marginal accuracy improvement of the Hybrid 2 method can be explained by the fact that the two ToA measured values from the GPS SATs have a higher level of accuracy than the RXLEV measured values from the CRN, so that the improvement of the localization accuracy due to additionally taking into account RXLEV measured values from the CRN is marginal. Moreover, the achievable localization accuracy of the Hybrid 2 method strongly depends on the geometric constellation of the SATs and BSs relative to the MS. For the 2 Satellite method the geometric constellation of the SATs relative to the MS can be expressed by the geometric dilution of precision (GDOP) value for 2-D scenarios [16]. If the GPS satellites are close together in the sky, the geometry between the SATs and the MS is said to be weak and the corresponding GDOP value is large. If the GPS SATs are far away from each other, the geometry between the SATs and the MS is said to be strong and the corresponding GDOP value is small. In Fig. 4, the localization accuracy for the Hybrid 2 method and the 2 Satellite method for different average GDOP values and $N_{\text{BS}} = 3$ is presented. It can be clearly seen that the larger the GDOP value, the better is the performance of the Hybrid 2 method compared to the 2 Satellite method. Thus, in scenarios where the GDOP value tends to be large as, e.g., in urban street canyons, the localization accuracy of the Hybrid 2 method is expected to be significantly better than the localization accuracy of the 2 Satellite method.

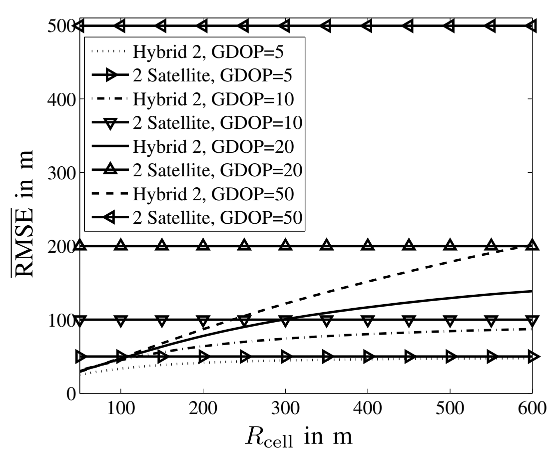

Fig. 4. $\overline{\text{RMSE}}$ vs. R_{cell} for the Hybrid 2 and 2 Satellite method for different average GDOP values for $N_{\text{BS}} = 3$.

VI. CONCLUSION

In this paper, the CRLB for a hybrid localization method is determined that combines RXLEV and TA measured values from GSM and ToA measured values from GPS. It has been shown by simulations that compared to combining only RXLEV and TA measured values from GSM the localization accuracy can be significantly improved by additionally taking into account one or two ToA measured values from GPS. In scenarios with low GDOP values it is sufficient to combine two ToA measured values from GPS, whereas in scenarios with high GDOP values the corresponding hybrid localization method should be used.

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