ON THE PERFORMANCE OF SDMA WITH SOFT DROPPING AND SINR BALANCING POWER CONTROL IN THE DOWNLINK OF MULTI-USER MIMO SYSTEMS

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ABSTRACT

The application of Space Division Multiple Access (SDMA) to the downlink of future Orthogonal Frequency Division Multiple Access (OFDMA) mobile radio systems is a promising solution to achieve high spectral efficiency. Besides enhancing system capacity, Quality of Service (QoS) requirements must also be respected to ensure reliable communication. In this work, a new SDMA strategy called Soft Dropping Algorithm (SDA) is proposed, which is composed of an SDMA grouping algorithm, namely the Greedy Correlation-Based Algorithm (GCBA), a Joint Precoding with Soft Dropping Power Allocation (JP-SD-PA) strategy based on soft dropping power control, and an SDMA group size tracking mechanism. It is flexible and takes into account both capacity and QoS aspects. The proposed SDMA strategy is shown to provide average capacity gains of 10% to 30%, as well as SINR gains of 2 dB to 3 dB to 90% of the UTs in the system, compared to an SINR Balancing Algorithm (SBA).

1. INTRODUCTION

The application of SDMA to the downlink of future OFDMA mobile radio systems is a promising solution to achieve high spectral efficiency. Using SDMA, frequency resources can be simultaneously reused by several User Terminals (UTs) separated in space by means of Multiple Input Multiple Output (MIMO) precoding techniques. Moreover, these precoding techniques benefit from the flat fading channel structure obtained due to Orthogonal Frequency Division Multiplexing (OFDM) transmission.

In the following, a group of UTs sharing a given subcarrier through SDMA is termed an SDMA group. How well UTs can be separated in space depends on the degree of spatial correlation among their channels. If UTs' spatial channels in an SDMA group are close to orthogonal, UTs can be efficiently multiplexed in space and capacity gains can be obtained. Oppositely, if their spatial channels are correlated, SDMA might bring no gains or even cause capacity losses.

Considering a Base Station (BS) with an n_T-element Antenna Array (AA) and K single-antenna UTs, there is a total number of K channels that can be selected for building an SDMA group. Therefore, a total number of $M = \sum_{g=1}^{G=n_T} \binom{K}{g}$ different SDMA groups are possible, where the group size G is usually upper limited by n_T. In general, finding the optimum SDMA group is a hard combinatorial problem which requires to compare all the M SDMA groups with each other, thus having exponential complexity [1, 2]. Therefore, sub-optimal SDMA algorithms are usually designed to build an efficient SDMA group with reduced complexity. In these algorithms, the efficiency of an SDMA group is usually measured by a grouping metric, which quantifies the degree of spatial compatiblity among the UTs in the group.

For example in [1], different heuristic SDMA algorithms are applied in a time division multiple access system having as grouping metric the minimum difference between target and estimated Signal-to-Interference plus Noise Ratios (SINRs) of the UTs in a group; in [3], SDMA groups are organized in a tree structure and group capacity or average Signal-to-Noise Ratio (SNR) of the UTs are used as grouping metrics; and in [4], the channel with the highest eigenvalue is first selected and additional channels are sequentially grouped with it in order of higher eigenvalues with and without successive precoding. These and other proposals achieve high capacity gains with non-exponential complexity, but rely on complex grouping metrics that require channel decompositions or computing precoding vectors [1, 3–5].

An efficient alternative to reduce complexity is employing grouping metrics based on the spatial correlation among UTs' channels. For example, in [6], a correlation metric is calculated for UTs pairwise and the best group becomes the one whose sum of the pairwise correlations among every pair of UTs in the group is minimum; in [7], a weighted norm of the spatial correlation metric of the UTs in a group is used

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as grouping metric and the best group is found by a Best Fit First (BFF) algorithm [1]. In [8], spatial correlation and channel gains are used as grouping metric in an algorithm which finds the best SDMA group by solving a convex optimization problem.

Also in order to limit complexity, a usual approach is to limit or predefine the group size to one (or some) fixed value(s), as e.g. in [2,4,5,9], thus avoiding to compare all M groups. A considerable capacity difference might be verified for different group sizes and, therefore, if the group size G does not match the ideal one, SDMA performance might be compromised [2–4]. For fixed group sizes, tracking the optimum group size, as e.g. in [3, 8], is beneficial since it might reduce the complexity of the SDMA algorithm and enhances its performance.

SDMA is not only influenced by the composition of the groups, but also by the applied precoding and power allocation strategies. Different optimization criteria can be followed, such as maximizing the sum capacity of the group under a total power constraint or minimizing the total transmit power under minimum rate constraints for UTs.

Since wireless communication services usually impose some constraints for reliable communication, not only the capacity enhancement should be pursued by SDMA strategies, but the QoS requirements of the UTs must also be taken into account. QoS requirements can often be expressed in terms of a target SINRs that must be attained, e.g., to ensure tolerable bit error rate or delay.

In [10] and references therein, the problem of maximizing the minimum SINR of the channels in the SDMA group with individual target SINRs and total power constraints is solved by a joint optimization of precoding and power allocation that exploits the duality between the Uplink (UL) and Downlink (DL) of Multi-User (MU) MIMO channels. Also in [10], a general feasibility condition for a set of target SINRs is given, which depends on the spectral radius of a weighted coupling matrix. In this context, whenever a set of target SINRs is not feasible, at least one of the target SINRs must be relaxed. Unfortunately, due to the problem structure, there is no simple rule to decide which target SINR to relax, since both precoding vectors and power allocation are jointly optimized.

In [11], Soft Dropping Power Control (SDPC) is proposed for distributed power control among a set of co-channel links in a multi-cell Single Input Single Output (SISO) system. In [12], SDPC is extended to the UL of a Single-User (SU) MIMO system. SDPC has a power-dependent target SINR that decreases with the allocated power. In this way, the links requiring more power target at lower SINR values, thus giving margin to a more efficient power usage. The SDPC incorporates a mechanism to automatically adjust the target SINR of the involved links and clearly points out which link to drop whenever necessary.

In this work, an SDMA strategy for the DL of MU MIMO systems is proposed. The proposed strategy is divided into:

- a greedy SDMA grouping algorithm, named GCBA, which builds an SDMA group with spatially compatible UTs' channels based on their spatial correlation;
- a Joint Precoding with Soft Dropping Power Allocation (JP-SD-PA) which adapts the SDPC algorithm to the considered scenario using the joint precoding and power allocation framework developed in [10];
- and a simple mechanism to track a suitable SDMA group size and to drop links if necessary.

The remainder of this paper is organized as follows. In section 2, the system model is described. In section 3, the proposed SDMA strategy is presented. In section 4, some results are analyzed. Finally, section 5 draws some conclusions.

2. SYSTEM MODEL

This section describes the scenario considered in this work. A single BS is considered in the modeling, with the interference from other BSs assumed as Gaussian and being incorporated directly as part of the Gaussian noise in the system. The BS has an n_T-element AA and there are K single-antenna UTs associated with the BS. A single frequency channel is considered, which is shared in space by the UTs in an SDMA group. The channel response is assumed to be flat and perfectly known at the transmitter. This scenario can be seen as a single sub-carrier, or a chunk of adjacent sub-carriers [13] for which a single sub-carrier is a good representative, in a OFDMA system using Time Division Duplexing (TDD) and having perfect channel estimation at the BS.

Each link between the BS and a UT k has an associated vector channel response $\mathbf{h}_k \in \mathbb{C}^{1\times n_T}$. Denoting transposition by $(\cdot)^T$, the channel matrix $\mathbf{H}_S \in \mathbb{C}^{K\times n_T}$ of all UTs together can be written by stacking the channel responses $\mathbf{h}_k$ as

$$\mathbf{H}_S = \begin{bmatrix}\mathbf{h}_1^T & \mathbf{h}_2^T & \dots & \mathbf{h}_K^T\end{bmatrix}^T. \tag{1}$$

The spatial correlation between two vector channels $\mathbf{h}_i$ and $\mathbf{h}_j$ is measured by the normalized scalar product ρ_{ij} [1, 2, 7, 8]. Let $|\cdot|$ denote the absolute value of a complex scalar and $\|\cdot\|_2$ denote the 2-norm. Then, ρ_{ij} is given by

$$\rho_{ij} = \frac{\left|\mathbf{h}_i\mathbf{h}_j^H\right|}{\|\mathbf{h}_i\|_2\,\|\mathbf{h}_j\|_2}, \tag{2}$$

which is used by GCBA in the next section as grouping metric.

Building an SDMA group $\mathcal{G}$ corresponds to adequately select a total of $G \le n_T$ vector channels $\mathbf{h}_i$ of $\mathbf{H}_S$, i.e., to optimally selecting G from the K rows of $\mathbf{H}_S$ according to the adopted grouping metric and problem constraints.

In the DL, the BS sends data symbols s_g, $g = 1,\dots,G$, to the UTs in the SDMA group $\mathcal{G}$. The data symbols s_g are assumed to be uncorrelated with average power $\sigma_s^2 = 1$ and

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are organized in the input data vector $\mathbf{s} \in \mathbb{C}^{G\times 1}$, which is precoded using the modulation matrix $\mathbf{M} \in \mathbb{C}^{n_T \times G}$, transmitted through the SDMA group channel $\mathbf{H} \in \mathbb{C}^{G\times n_T}$, and distorted by noise, which is represented by $\mathbf{z} \in \mathbb{C}^{G\times 1}$. $\mathbf{z}$ is considered to be spatially white with average power σ_z^2. The transmitted signals are demodulated by the demodulation matrix $\mathbf{D} \in \mathbb{C}^{G\times G}$ producing the estimated output data vector

$$\hat{\mathbf{s}} = \mathbf{DHMs} + \mathbf{z} \in \mathbb{C}^{G\times 1} \tag{3}$$

at the receivers.

Let $\operatorname{diag}\{\cdot\}$ denote a diagonal matrix whose diagonal elements are given in the vector argument. Then, the modulation matrix $\mathbf{M}$ can be written in terms of a precoding matrix $\mathbf{U}$ and a DL power vector $\mathbf{p}$ as

$$\mathbf{M} = \begin{bmatrix}\mathbf{m}_1 & \mathbf{m}_2 & \ldots & \mathbf{m}_G\end{bmatrix} = \mathbf{U}\operatorname{diag}\{\sqrt{\mathbf{p}}\}, \text{ with} \tag{4a}$$

$$\begin{aligned}\mathbf{U} &= \begin{bmatrix}\frac{\mathbf{m}_1}{\|\mathbf{m}_1\|_2} & \frac{\mathbf{m}_2}{\|\mathbf{m}_2\|_2} & \ldots & \frac{\mathbf{m}_G}{\|\mathbf{m}_G\|_2}\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \ldots & \mathbf{u}_G\end{bmatrix}, \text{ and}\end{aligned} \tag{4b}$$

$$\begin{aligned}\mathbf{p} &= \begin{bmatrix}\|\mathbf{m}_1\|_2^2 & \|\mathbf{m}_2\|_2^2 & \ldots & \|\mathbf{m}_G\|_2^2\end{bmatrix}^T\\ &= \begin{bmatrix}p_1 & p_2 & \ldots & p_G\end{bmatrix}^T.\end{aligned} \tag{4c}$$

Assuming Gaussian signaling and ideal matched filtering at the receivers, the DL SINR of the g^{th} UT in $\mathcal{G}$ is given by

$$\gamma_g = \frac{p_g \mathbf{u}_g^H \mathbf{h}_g^H \mathbf{h}_g \mathbf{u}_g}{\sigma_z^2 + \sum\limits_{j=1, j\neq g}^{G} p_j \mathbf{u}_j^H \mathbf{h}_g^H \mathbf{h}_g \mathbf{u}_j} \tag{5}$$

with $\mathbf{h}_g$ the g^{th} row of $\mathbf{H}$, $\mathbf{u}_g$ the g^{th} column $\mathbf{U}$, and p_g the g^{th} element of $\mathbf{p}$. The group capacity $C(\mathcal{G})$ of the SDMA group $\mathcal{G}$ is thus given by

$$C(\mathcal{G}) = \sum_{g=1}^{G} \log_2(1+\gamma_g). \tag{6}$$

In the next sections, (5) is used in the joint optimization precoding and power allocation, and (6) is used to compare the performance of the algorithms in terms of capacity.

3. SDMA STRATEGY

In this section, the proposed SDMA strategy is described. An optimum SDMA strategy requires a joint optimization of UTs' channel selection, precoding, and power allocation, thus being a hard non-convex non-linear optimization problem with prohibitive complexity.

For this reason, the strategy proposed here divides the problem into an SDMA grouping problem, and a joint precoding and power allocation problem. Moreover, dropping one (or some) UTs if target SINRs cannot be met and tracking an adequate size for SDMA groups are also considered.

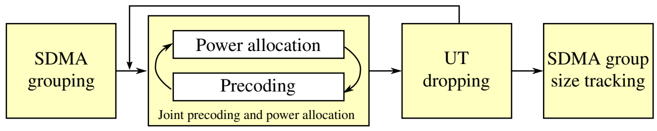

Fig. 1. Block diagram of the proposed SDMA strategy.

The block diagram of the proposed SDMA strategy shown in Fig. 1.

Section 3.1 and section 3.2 describe the proposed SDMA grouping algorithm and the joint precoding and power allocation strategy, respectively. Section 3.3 presents the adopted dropping criteria and the SDMA group size tracking mechanism.

3.1. SDMA grouping algorithm

In this section, the GCBA is described. If joint precoding and power allocation optimization is done, it is interesting to have an SDMA algorithm that finds a group of spatially compatible UTs based on a grouping metric that does not rely on precoding vectors or allocated powers, since they will be adjusted afterwards.

For a fixed group size G, an SDMA group $\mathcal{G}$ with G minimally spatially correlated channels is a good sub-optimal solution. Denoting the 1-norm of a vector by $\|\cdot\|_1$, such a group of low correlated channels can be obtained by solving the integer optimization problem

$$\mathbf{x}^\star = \arg\min_{\mathbf{x}} \left\{\frac{1}{2}\mathbf{x}^T\mathbf{R}\mathbf{x}\right\}, \tag{7a}$$

$$\text{subject to: } \|\mathbf{x}\|_1 = G, \tag{7b}$$

$$x_c = 1,\ c \in 1,\ldots,n_R \tag{7c}$$

$$x_i \in \{0,1\},\ i = 1,\ldots,n_R, \tag{7d}$$

where $\mathbf{x} = \begin{bmatrix}x_1 & x_2 & \ldots & x_{n_R}\end{bmatrix}^T$ is a binary selection vector. The constraint (7c) is introduced to ensure that a given UT s be in $\mathcal{G}$. For example, this UT can be the highest-priority UT selected for transmission by the resource scheduler, or be randomly selected. If $\mathbf{x}^\star$ is the optimum solution of (7), the optimum SDMA group $\mathcal{G}^\star$ corresponds to select the rows $\mathbf{h}_i$ of $\mathbf{H}_S$ for which $x_i^\star = 1$, $i = 1,\ldots,K$.

Solving (7) using integer or convex optimization [14, 15] might require considerably high computational effort. Instead of this, the Greedy Correlation-Based Algorithm (GCBA) proposed here is used to build a sub-optimal SDMA group composed of highly spatially uncorrelated UTs. It is a greedy algorithm in which an initial UT, indexed by c, is firstly selected, e.g., by the scheduler. Then, an SDMA group $\mathcal{G}$ is built containing only this UT. After that, at each iteration of GCBA, the UT least correlated with all the UTs already admitted to $\mathcal{G}$ is added to the group until the desired group size G is reached. GCBA is described in Table 1.

GCBA builds the group for which joint optimization of power allocation and precoding vectors is done in section 3.2.

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Table 1. Greedy Correlation-Based Algorithm.

1. Set the SDMA group $\mathcal{G} = \{c\}$.
2. For $g = 1$ to $G-1$
   a. Set $\mathcal{G} = \mathcal{G} \cup \underset{j\in\{1,\ldots,K\}\setminus\mathcal{G}}{\arg\min} \left\{ \sum_{i\in\mathcal{G}} \rho_{ij} \right\}$.

3.2. Joint precoding and power power allocation optimization

3.2.1. Joint Precoding with SINR Balancing Power Allocation (JP-SB-PA)

This section reviews the joint precoding and power allocation optimization framework proposed in [10], on which the new strategy proposed in this work is based. The strategy from [10] is termed here as Joint Precoding with SINR Balancing Power Allocation (JP-SB-PA).

In [10], an UL/DL duality theory is proposed for the joint optimization of precoding vectors and power allocation. Therein, the maximization of the minimum ratio $\frac{\gamma_g}{\nu_g}$ between the actual SINR γ_g and the target SINRs ν_g of spatially multiplexed UTs under a total power constraint P is solved by means of an iterative alternating optimization of precoding vectors and power allocations. The problem is formulated in DL as

$$\{\mathbf{U}^\star, \mathbf{p}^\star\} = \arg\max_{\{\mathbf{U},\mathbf{p}\}} \min \left\{ \frac{\gamma_g}{\nu_g} \right\},\ g = 1,\ldots,G, \tag{8a}$$

$$\text{subject to: } \|\mathbf{u}_g\|_2 = 1, \tag{8b}$$

$$\|\mathbf{p}\|_1 = P, \tag{8c}$$

with the associated dual UL problem

$$\{\mathbf{U}^\star, \mathbf{q}^\star\} = \arg\max_{\{\mathbf{U},\mathbf{q}\}} \min \left\{ \frac{\phi_g}{\nu_g} \right\},\ g = 1,\ldots,G, \tag{9a}$$

$$\text{subject to: } \|\mathbf{u}_g\|_2 = 1, \tag{9b}$$

$$\|\mathbf{q}\|_1 = P, \tag{9c}$$

where the UL SINR ϕ_g is given by

$$\phi_g = \frac{q_g \mathbf{u}_g^H \mathbf{h}_g^H \mathbf{h}_g \mathbf{u}_g}{\mathbf{u}_g^H \left(\sum_{j=1, j\neq g}^{G} q_j \mathbf{h}_j^H \mathbf{h}_j + \sigma_z^2 \mathbf{I} \right) \mathbf{u}_g} \tag{10}$$

and $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & \ldots & q_G \end{bmatrix}^T$ is the uplink power vector.

In the following, let t denote the iteration index of the described procedure. For a fixed UL power vector $\mathbf{q}$, which starts with an initial arbitrary value $\mathbf{q} = \mathbf{q}^{(0)}$, the optimum precoding vectors in $\mathbf{U}^{(t)}(\mathbf{q}^{(t-1)})$ correspond to the Minimum Variance Distortionless Response (MVDR) precoding vectors [16] that maximize individually each $\phi_g(\mathbf{q}^{(t-1)})$, $g = 1,\ldots,G$, in (10) [10]. Then, at iteration t the optimum $\mathbf{q}^{(t)}$ is obtained by solving

$$\begin{bmatrix} \mathbf{\Gamma}\mathbf{\Psi}^T & \mathbf{\Gamma}\sigma \\ \frac{1}{P}\mathbf{1}^T\mathbf{\Gamma}\mathbf{\Psi}^T & \frac{1}{P}\mathbf{1}^T\mathbf{\Gamma}\sigma \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} = \lambda_{max} \begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} \tag{11}$$

where λ_{max} is the dominant eigenvalue of the eigenproblem (11), and

$$[\mathbf{\Psi}]_{ij} = \begin{cases} \mathbf{u}_j^H \mathbf{h}_i^H \mathbf{h}_i \mathbf{u}_j, & i \neq j \\ 0, & i = j \end{cases}, \tag{12a}$$

$$\mathbf{\Gamma} = \operatorname{diag}\left\{ \begin{bmatrix} \frac{\nu_1}{\mathbf{u}_1^H \mathbf{h}_1^H \mathbf{h}_1 \mathbf{u}_1} & \cdots & \frac{\nu_G}{\mathbf{u}_G^H \mathbf{h}_G^H \mathbf{h}_G \mathbf{u}_G} \end{bmatrix} \right\}, \tag{12b}$$

$$\sigma = \begin{bmatrix} \sigma_z^2 & \ldots & \sigma_z^2 \end{bmatrix}^T, \tag{12c}$$

with the index t dropped for notation simplicity.

This alternating optimization is repeated until $\lambda_{max}^{(t)} - \lambda_{max}^{(t-1)} \leq \epsilon$ and $\mathbf{U}^\star$ is obtained. Then, using $\mathbf{U}^\star$, the optimum DL power vector is obtained by solving the eigenproblem

$$\begin{bmatrix} \mathbf{\Gamma}\mathbf{\Psi} & \mathbf{\Gamma}\sigma \\ \frac{1}{P}\mathbf{1}^T\mathbf{\Gamma}\mathbf{\Psi} & \frac{1}{P}\mathbf{1}^T\mathbf{\Gamma}\sigma \end{bmatrix} \begin{bmatrix} \mathbf{p}^\star \\ 1 \end{bmatrix} = \lambda_{max} \begin{bmatrix} \mathbf{p}^\star \\ 1 \end{bmatrix} \tag{13}$$

[10]. In the case in which the target SINR is the same for all the UTs in the SDMA group, i.e, $\nu_g = \nu, g = 1,\ldots,G$, the algorithm proposed in [10] balances the SINR γ_g of all UTs on the maximum balanced SINR value $\gamma^\star$. This situation occurs, for example, when all UTs in the SDMA group use the same data service.

3.2.2. Joint Precoding with Soft Dropping Power Allocation (JP-SD-PA)

This section presents the proposed JP-SD-PA strategy, which is based on the SDPC algorithms proposed in [11, 12, 17].

Herein, it is proposed to employ the alternating optimization framework of [10] to implement the JP-SD-PA strategy, in which precoding vectors are determined in the same way as with the JP-SB-PA strategy, i.e., by computing the UL MVDR precoding vectors $\mathbf{u}_g$ maximizing (10) for each individual UT g. However, the UL powers q_g allocated to each UT g are obtained using SDPC [11]. The power allocation in the JP-SD-PA strategy employs a power-dependent target SINR ν_g given by

$$\nu_g^{(t)} = \min\left\{ \max\left\{ \nu_m \left(\frac{p_g^{(t-1)}}{p_M} \right)^\alpha, \nu_m \right\}, \nu_M \right\} \tag{14}$$

$$\text{with } \alpha = \frac{\log_{10}(\nu_M/\nu_m)}{\log_{10}(p_m/p_M)}$$

where ν_m, ν_M and p_m, p_M are respectively the minimum and maximum target SINRs and allocable powers of UT g. The power allocation in UL used in the the JP-SB-PA strategy is given by

$$q_g^{(t)} = q_g^{(t-1)} \left(\frac{\nu_g^{(t)}}{\phi_g^{(t-1)}} \right)^\beta \tag{15}$$

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where β is a control parameter.

Different from JP-SB-PA, JP-SB-PA does not balance the SINRs of the UTs, but allows the UTs in better channel conditions to aim at higher SINR values, thus increasing the power efficiency of the system. As with the JP-SB-PA strategy, UL powers and precoding vectors are optimized alternately. After $\mathbf{q}$ converges with a given precision ϵ to $\mathbf{q}^\star$, the DL powers are calculated.

Let $\mathbf{\Upsilon}$ and η be defined respectively as

$$[\mathbf{\Upsilon}]_{ij} = \begin{cases} \frac{1}{\nu_i^\star}, & i = j \\ \frac{\mathbf{u}_j^H \mathbf{h}_i^H \mathbf{h}_i \mathbf{u}_j}{\mathbf{u}_i^H \mathbf{h}_i^H \mathbf{h}_i \mathbf{u}_i}, & i \neq j \end{cases} \tag{16a}$$

$$\eta = \begin{bmatrix} \frac{\sigma_z^2}{\mathbf{u}_1^H \mathbf{h}_1^H \mathbf{h}_1 \mathbf{u}_1} & \cdots & \frac{\sigma_z^2}{\mathbf{u}_G^H \mathbf{h}_G^H \mathbf{h}_G \mathbf{u}_G} \end{bmatrix}^T. \tag{16b}$$

Then, the optimum DL power vector $\mathbf{p}^\star$ is given as

$$\mathbf{p}^\star = \left(\begin{bmatrix} \mathbf{\Upsilon} \\ \mathbf{1}_G^T \end{bmatrix}^T \begin{bmatrix} \mathbf{\Upsilon} \\ \mathbf{1}_G^T \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{\Upsilon} \\ \mathbf{1}_G^T \end{bmatrix}^T \begin{bmatrix} \eta \\ P \end{bmatrix}. \tag{17}$$

In this way, it is not necessary to solve an eigenvalue problem to obtain the DL power allocation, as in (13) in the JP-SB-PA strategy. The proposed JP-SD-PA strategy is described in Table 2.

Table 2. JP-SD-PA strategy.

1. Set $t = 0$ and define with $\mathbf{q}^{(t)} = \mathbf{q}^{(0)} = (P/G)\mathbf{1}_G$.
2. Set $t = t + 1$.
3. Compute the UL precoding vectors $\mathbf{u}_g^{(t)}$, $g = 1, \ldots, G$, by solving the G MVDR problems.
4. Compute $\phi_g^{(t-1)}$ using (10), $\nu_g^{(t)}$ using (14), and then update UL power $q_g^{(t)}$ using (15).
5. Normalize $\mathbf{q}^{(t)} = P\mathbf{q}^{(t)} / \left\| \mathbf{q}^{(t)} \right\|_1$.
6. If $\min \left\{ \left| q_g^{(t)} - q_g^{(t-1)} \right| \right\} > \epsilon$, $g = 1, \ldots, G$, go to step 2.
7. Set $\mathbf{u}_g^\star = \mathbf{u}_g^{(t)}$ and $\nu_g^\star = \phi_g^{(t)}$, $g = 1, \ldots, G$.
8. Calculate $\mathbf{p}^\star$ using (17).

In order to ensure convergence of JP-SD-PA, $q_g^{(t)}$ in (15) must be an standard interference function $I(q_g^{(t-1)})$ and satisfy, for $q \geq 0$ [17]:

- Positivity: $I(q) \geq 0$.
- Monotonicity: $I(q) \geq I(q')$, $p \geq p'$.
- Scalability: $aI(q) \geq I(aq)$, $a \geq 1$.

In [12], SDPC is extended to the UL of a SU MIMO system and $I(\mathbf{q})$ is shown to be standard whenever $\beta < (1-\alpha)^{-1}$. Since no structural differences exist between the SU MIMO and the MU Single Input Multiple Output (SIMO) channels, a proof very similar to that provided in [12] for the UL of a SU MIMO channel is provided here for the UL the MU MIMO considered in this work.

Positivity for $I(\mathbf{q})$ follows directly, since all involved terms are positive.

Dropping the iteration index t, considering a fixed precoding matrix $\mathbf{U}$ according to the framework of [10], and considering $q_g \geq q_g'$, monotonicity is obtained if

$$\begin{aligned} & I(q_g) \geq I(q_g') \Rightarrow \\ & q_g^{(1+\alpha\beta-\beta)} \mathcal{I} \geq {q'}_g^{(1+\alpha\beta-\beta)} \mathcal{I} \Leftrightarrow \\ & 1 + \alpha\beta - \beta \geq 0 \Leftrightarrow \beta \leq (1-\alpha)^{-1}, \end{aligned} \tag{18}$$

where the term $\mathcal{I}$ is given by

$$\mathcal{I} = \left[\frac{\nu_m}{p_M^\alpha} \cdot \frac{\mathbf{u}_g^H \left(\sum\limits_{j=1, j\neq g}^{G} q_j \mathbf{h}_j^H \mathbf{h}_j + \sigma_z^2 \mathbf{I} \right) \mathbf{u}_g}{\mathbf{u}_g^H \mathbf{h}_g^H \mathbf{h}_g \mathbf{u}_g} \right]^\beta. \tag{19}$$

For scalability,

$$\begin{aligned} & aI(q_g) \geq I(aq_g) \Rightarrow \\ & aq_g^{(1+\alpha\beta-\beta)} \mathcal{I} \geq (aq_g)^{(1+\alpha\beta-\beta)} \mathcal{I} \Leftrightarrow \\ & 1 \geq 1 + \alpha\beta - \beta \Leftrightarrow \beta \geq \alpha\beta \Leftrightarrow \alpha \leq 1. \end{aligned} \tag{20}$$

Indeed, considering the DL of a MU MIMO, obtaining the same proofs is straightforward.

3.3. Dropping criteria and SDMA group size tracking

This section describes the dropping criteria and the scheme used to track the adequate SDMA group size.

Depending on the radio channel conditions, not all the links of the UTs in an SDMA group might be supported with the required target SINR. In this case, target SINRs must be adjusted or one or more UTs must be dropped from the SDMA group.

In [10], the feasibility condition for a set of target SINRs ν_g, $g = 1, \ldots, G$, requires that $\lambda_{max}^{-1} \leq 1$ in (13). Because (13) depends on $\mathbf{U}^\star$, finding a set of feasible target SINRs that enhances system capacity might become considerably complex, requiring to solve (8) several times. Moreover, because both power and precoding vectors are jointly adjustable, there is no fixed rule to determine from which UT to reduce the target SINR.

After solving (8) using JP-SB-PA, which balances the SINR of all UTs, either all or none of the UTs are supported. Unfortunately, adjusting the target SINRs within the JP-SB-PA strategy proposed in [10] while keeping its fast convergence properties renders a hard optimization problem. It might also be mentioned that using (14) to adjust targets with JP-SB-PA lead to non-convergence of the strategy. Alternatively, in order to avoid solving (8) several times for different sets of target SINRs, it is proposed here to drop the UT g consuming the most power p_g whenever the target SINRs are not jointly feasible.

For JP-SD-PA, a suitable dropping criterion is to remove the UT g with worst SINR γ_g. Since JP-SB-PA does not balance SINRs of the UTs and internally adjusts target SINRs, that UT is also often the one consuming most power. Note that dropping does not mean here insatisfaction for UTs, but only relates to power-efficiency required for an UT to aim at a given target SINR.

Whenever dropping occurs, the SDMA group size is reduced. On the one hand, a large group size might lead to higher SDMA gains. On the other hand, a too large group size leads to unnecessary computations due to successive droppings. Therefore, it is suggested here to set the group size to be used for building the next SDMA group with GCBA, e.g, in the next time-slot, as

$$G^{(l+1)} = \min \left\{ n_T, G^{(l)} + 1 \right\} \tag{21}$$

where l indicates the l^{th} run or time-slot in which the proposed strategy is applied. The complete strategy proposed here is summarized in Table 3 and is termed SDA. Whenever JP-SB-PA is considered instead of JP-SD-PA, the strategy is termed SBA.

Table 3. Summary for SBA/SDA strategies.

1. Start $G^{(l)} = G^{(0)} = n_T$.
2. Set $l = l + 1$.
3. Build an SDMA group using GCBA from Table 1.
4. Optimize power and precoding vectors using JP-SB-PA from section 3.2.1/JP-SD-PA in Table 2 from section 3.2.2.
5. If $\gamma_g \leq \nu$ for some g, drop a UT and return to step 4.
6. Adjust $G^{(l)}$ according to (21) for the next group.

4. ANALYSIS AND RESULTS

In this section, the performance of SDA is studied through simulations and compared with the performance of SBA. A BS with a Uniform Linear Array (ULA) of $n_T = 8$ elements separated by half wavelength is assumed. A total number of $K = 32$ single-antenna users are randomly placed in the cell area. UTs have an average speed of 10 km/h. For both SDA and SBA, at each time-slot (run) one UT, indexed by c, is randomly selected as initial UT for GCBA and is not allowed to be dropped from the SDMA group. All UTs are assumed to always have data to transmit and to have the same target SINR $\nu = 5$ dB. A total power P of 30 dBm is assumed. p_M and p_m are set to 30 dBm and -5 dBm, respectively. ν_m is set to 5 dB. Different values are assigned to ν_M in order to obtain different values for α and get more insight on the performance of SDA. Slow fading and path loss are assumed to be ideally compensated by power control and only the fast fading is considered. Channel matrices are obtained using the WINNER Phase I Model (WIM) [18] and one sample is considered at each 0.25ms. The most relevant simulation parameters are summarized in Table 4.

Table 4. Simulation parameters.

Parameter	**Value**
System bandwidth	1.25 MHz
Center frequency	5 GHz
# of subcarriers	128
WIM scenario	C2
Sampling rate	1 sample each 0.25 ms
# K of single-antenna UTs	32
UTs' speed	10 km/h
# n_T of elements of the BS ULA	8
ULA element separation	half wavelength
Element radiation pattern	omni
SDMA grouping algorithm	GCBA (see Table 1)
Initial SDMA group size $G^{(0)}$	8
Joint precoding and power allocation	JP-SB-PA, JP-SD-PA
β parameter	$(1-\alpha)^{-1}$
SINR target ν	5 dB
Minimum SINR target ν_m	5 dB
Maximum SINR target ν_M	15 dB, 35 dB, and 75 dB
Initial power allocation	Equal Power Allocation (EPA)
Total transmit power P	1 W = 30 dBm
Minimum transmit power p_m	-5 dBm
Maximum transmit power p_M	30 dBm

First, it is important to compare the performance of SBA and SDA in terms of the achieved system capacity. In Fig. 2, the average capacity of the system is shown considering SBA and SDA for different average SNR values.

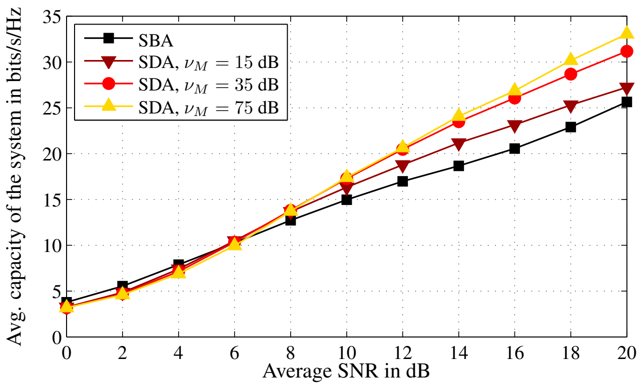

Fig. 2. Average capacity of the system for the proposed strategy with SBA and SDA for different average SNR values.

In Fig. 2, it can be seen that for high average SNR values SDA can provide average capacity gains of 10% to 30% with respect to SBA depending on the parameter setting. For low SNR values, it can be observed that SDA is only slightly worse than SBA. The gains verified for high average SNR

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values happen for two reasons. First, JP-SD-PA does not balance the SINR of the UTs in the SDMA group built by GCBA. Thus, differently from JP-SB-PA, a power margin is created, which allows for enhancing the throughput of the UTs in good channel conditions, while UTs in bad channel conditions aim at the minimum target SINR ν_m. Second, because SDA looks for an efficiency trade-off between target SINR and power, it might lead to droppings more often than SBA. Indeed, because SBA balances the SINR of the UTs, it minimizes dropping of UTs. Ideally, SBA should be combined with a suitable mechanism to dynamically adjust the target SINRs of the UTs. However, this problem is left open here and might deserve future investigation.

The increased dropping rate of SDA compared to SBA can can also be observed by comparing the average SDMA group sizes obtained for both strategies after dropping, which is shown in Fig. 3.

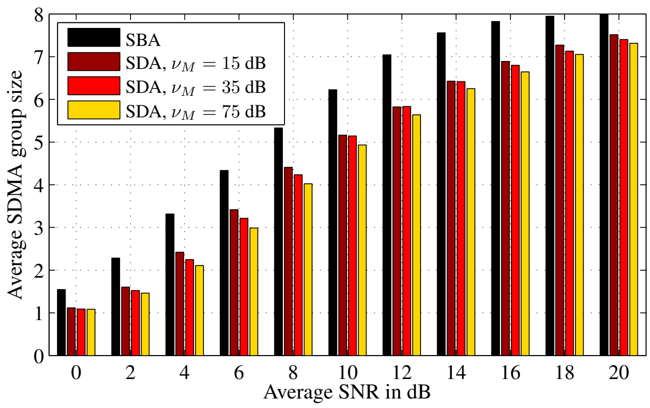


Fig. 3. Average SDMA group sizes after dropping obtained for SBA and SDA for different average SNR values.

In Fig. 3, it can be seen that the SDMA groups in the SDA case contain approximately one UTs less than in the SBA case. Because the total transmit power is shared among less UTs, the power margin exploited by SDA is further increased leading to capacity gains. Indeed, other group size adjustment mechanisms than the one proposed in section 3.3 could be used to enhance the performance of both SDA and SBA strategies, e.g., as in [2, 7, 8].

Because the proposed SDA strategy is supposed to ensure minimum QoS levels, it is interesting to see the distribution of the SINR perceived by the UTs in the system. In Fig. 4, the 10^{th} percentile of the SINR distribution of the UTs, i.e., the SINR perceived by 90% of the UTs, is shown.

From Fig. 4, it can be observed that the target SINR requirement $\nu = 5$ dB is respected for 90% of the UTs in the whole range of SNR values investigated. Thus, in the proposed SDMA strategy, both SDA and SBA meet very well the objective of ensuring the minimum QoS levels. For the reasons previously mentioned, SDA also provides SINR levels for the UTs which are 2-3 dB higher than those of SBA.

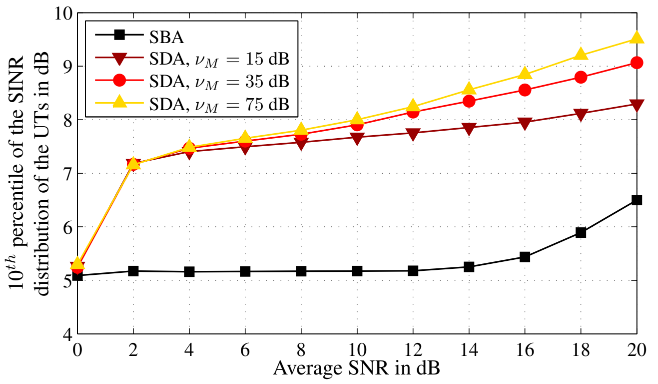
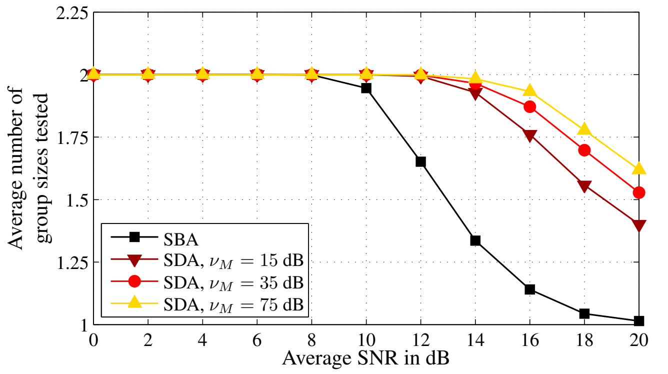
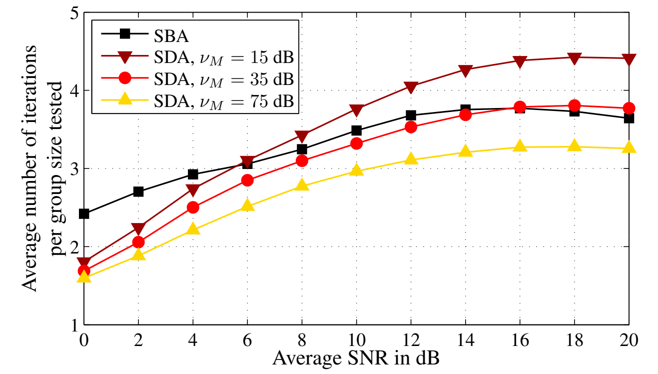


Fig. 4. 10% outage capacity of the system for the proposed strategy with SBA and SDA for different average SNR values.

As stated before, an interesting property of JP-SB-PA is its fast convergence in SBA, which should be compared with that of JP-SD-PA in SDA. In Fig. 5, the average number of group sizes tested and the average number of iterations required for each tested group size is shown.



(a) Average number of group sizes tested.



(b) Average number of iterations per group size tested.

Fig. 5. Average number of groups tested by SBA and SDA and average number of iterations required by SBA and SDA per group tested.

In Fig. 5(a), it can be noted that the number of SDMA groups tested is kept equal to or below 2 confirming the efficiency of the dropping and group size tracking mechanisms.

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For low to moderate SNR values, the number of groups tested by SBA and SDA is the same. For high SNR values, SDA tests more SDMA groups since it drops UTs more often than SBA, as previously explained. In Fig. 5(b), it can be observed that depending on the parameter setting, the number of iterations required by SDA might be smaller or larger than the one required by SBA.

With SDA, for a constant ν_m, when ν_M increases α decreases, and consequently, β goes towards 0. This leads to a smaller number of iterations required by JP-SD-PA to converge in SDA, since the term $\nu_g^{(t)}/\phi_g^{(t-1)}$ in (15) approaches 1. This can be noted for example in Fig. 5(b) for $\nu_M = 75$ dB. However, as it can be noted in Fig. 5(a), more SDMA groups are tested. In the limit case, i.e., when $\beta = 0$, no power adjustments are done and $q^\star = q^{(0)}$. In this case, the proposed strategy reduces to SDMA grouping followed by equal power allocation, by MVDR precoding, and by link droppings. Anyway, it can be seen in Fig. 2 and Fig. 4 when $\nu_M = 75$ dB that such strategy could lead to considerable capacity gains and since the randomly selected initial UT, indexed by c, in the GCBA is not allowed to be dropped from the SDMA group, long-term fairness is also ensured.

On the other hand, when ν_M approaches ν_m, it yields $\alpha \to 0$ and $\beta \to 1$. A larger average number of iterations might be required by JP-SD-PA to converge in SDA, as illustrated in Fig. 5(b) when $\nu_M = 15$ dB, as well as a reduced number of SDMA groups might be tested, as it can be seen in Fig. 5(a). In the limit case, i.e., when $\beta = 1$, SDA performance converges to the the performance of SBA and balances the SINRs of the UTs. This trend can be seen in Fig. 2 when $\nu_M = 15$ dB.

Therefore, by adjusting SDA parameters, increased flexibility can be provided to the proposed SDMA strategy than considering SBA, while both SDA and SBA present similar complexity. An exact comparison of the complexity of both SDA and SBA, as well as a comparison of their performance for different target SINR ν_g for each UT g, deserve further study and might be theme of future investigations.

5. CONCLUSIONS

In this paper, an SDMA strategy termed SDA has been proposed. The SDA is composed of a simple SDMA grouping algorithm, namely GCBA, a joint precoding power allocation strategy called JP-SD-PA, and simple dropping and SDMA group size tracking mechanisms. The performance of SDA has been studied and compared to the performance of SBA. SDA has been shown to provide average capacity gains of 10% to 30%, as well as SINR gains of 2 dB to 3 dB to 90% of the UTs in the system. Moreover, by suitable parameter setting, the proposed strategy might provide different trade-offs between capacity gains and computational effort.

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