

Power allocation in multi-carrier networks with unicast and multicast services

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Abstract—Since the provision of both unicast and multicast services is expected for the next generation of multi-carrier wireless systems, the allocation of resources for this combination of services is a relevant topic, which may have a significant impact on the system performance. The focus of this work lies on the allocation of power to the different downlink subchannels of a multi-carrier system containing both unicast and multicast users. The power allocation problem is analyzed considering on the one hand the maximization of the sum throughput and on the other hand the maximization of the minimum SNR. The solution of the former is presented, which depends on numerical optimization, and an algorithm similar to the waterfilling hypothesis testing is proposed for reducing the processing time, while for the latter a closed-form solution is demonstrated. A simplified power allocation algorithm based on multicast group quality criteria is also evaluated and shown to approximate the maximal achievable performance under certain circumstances.

I. INTRODUCTION

The provision of both unicast and multicast services is expected for the next generation of multi-carrier wireless systems. The allocation of resources for this combination of services is a relevant topic, which may have a significant impact on the system performance. Multicast services, which are specified for current 3GPP systems such as GSM/EDGE and WCDMA networks [1], have the characteristic that the same information has to be transmitted in the downlink to a certain group of users. In order to improve resource efficiency, the users of a multicast group which find themselves in the same cell can share the same radio resources (Point-to-Multipoint connection) [1]. In multi-carrier systems, such as OFDMA, this means that the subchannel allocation is done in a groupwise manner, i.e., the data transmitted on a subchannel allocated to a multicast group is received by all group members.

Some topics concerning the allocation of resources for multicast OFDM networks have been investigated in literature. In [2], an adaptive modulation method is proposed for multicast systems. In [3], a dynamic subchannel and bit allocation is evaluated, and in [4], proportional fairness scheduling aspects are introduced within the optimization problem.

The focus of this work lies specifically on the allocation of power to the different subchannels of a multi-carrier system containing both unicast and multicast users, while the subchannel allocation procedure is assumed to have already been performed, since algorithms for solving this problem have been previously proposed for both unicast [5, 6] and multicast users [3]. The power allocation problem does not

have a trivial solution, since the multicast users, which share the same resources, are subject to different radio link conditions. For this reason there are several possibilities for performing the power allocation, e.g., according to the best or worst user within each subchannel, or taking into account the requirements of each individual user.

The solution and analysis of the power allocation problem for this unicast/multicast context is the main contribution of this paper. The proposed algorithms consider the following two optimization criteria: the maximization of the sum throughput and the maximization of the minimum signal-to-noise ratio (SNR), both for a certain total transmit power constraint. It is shown that the former can be solved through an algorithm similar to the traditional waterfilling hypothesis testing [7]. A less complex approach, which takes a group measure for each subchannel, such as the best, worst, or average channel quality, is evaluated as well. The latter optimization, on the other hand, is fairness oriented, depending on the worst-user performance of each subchannel.

The paper is organized as follows. In section II, the power allocation problem is formulated and solved for the different considered optimization criteria. Section III presents the numerical results, which analyze the complexity and performance of the algorithms. Finally, in section IV, the main conclusions are drawn.

II. POWER ALLOCATION

A. System model

The system model corresponds to the downlink of a single cell in a cellular multi-carrier system. There are K available subchannels and N users within the cell. It is assumed that the subchannel allocation has already been performed, and therefore the information concerning which users are associated to which subcarrier is available to the power allocation algorithm. Additionally, error-tolerant hierarchical multicast data is considered [8, 9], which means that the perceived quality depends on the amount of correctly decoded information. This assumption is required by the throughput maximization algorithms, otherwise the low-quality multicast users within a channel would eventually stall the traffic flow due to subsequent retransmissions.

In this scenario, both unicast and multicast users are present in the system and different multicast groups are supported. The subchannel allocation matrix $\mathbf{A}_{N \times K}$, with elements $A_{i,j} \in$

$\{0, 1\}$, determines which users are active within each subchannel, where 0 and 1 correspond to the inactive and active states, respectively. Since no intracell interference is assumed, only users of the same multicast group may share one subchannel, i.e., combinations of unicast/multicast or multicast users of different groups are not allowed in the same subchannel. Denoting by \mathcal{K}_{uc} and \mathcal{K}_{mc} the set of subchannel indices assigned to unicast and multicast users, respectively, it follows that:

$$\begin{cases} \sum_{n=1}^N A_{n,k} = 1 & \forall k \in \mathcal{K}_{uc}, \\ \sum_{n=1}^N A_{n,k} \geq 1 & \forall k \in \mathcal{K}_{mc}. \end{cases} \quad (1)$$

The power allocation problem consists of determining the power vector $\mathbf{p}_{K \times 1} = [p_1 \dots p_K]^T$, which indicates the amount of power p_k allocated to each subchannel k . The allocation can be done according to different optimization criteria, such as the maximization of the throughput or the maximization of the minimum SNR. The algorithms proposed in the following subsections, which have different characteristics with regard to their complexity, capacity, and fairness, are namely: Sum Throughput Maximization (STM), Group Criterion for Throughput Maximization (GCTM), and Fair Power Allocation (FPA).

B. Sum throughput maximization (STM)

In this section, the STM algorithm is described and illustrated. This algorithm has the purpose of maximizing the total throughput of the system, which is here defined as the sum of the bitrate perceived by the individual users. The throughput of user n associated to subcarrier k is denoted by $R_{n,k}$, and if Gaussian signalling is assumed it can be written as:

$$R_{n,k} = \log_2(1 + p_k G_{n,k}), \quad (2)$$

where p_k is the power allocated to subcarrier k and $G_{n,k}$ is an element of matrix $\mathbf{G}_{N \times K}$, which corresponds to the path gain $H_{n,k}$ (including path-loss, fading and noise) conditioned to the subchannel allocation, i.e., $G_{n,k} = H_{n,k} \cdot A_{n,k}$. In order to compose the matrix \mathbf{G} , information on the channel gains of the allocated subchannels is required, which is assumed to be available at the transmitter.

The optimization problem can be expressed as:

$$\begin{aligned} \mathbf{p}_{opt} = \underset{\mathbf{p}}{\operatorname{argmax}} & \sum_{k=1}^K \sum_{n=1}^N \log_2(1 + p_k G_{n,k}), \\ \text{subject to:} & \begin{cases} p_k \geq 0, & \forall k \in \mathcal{K}, \\ \sum_{k=1}^K p_k = P_T. \end{cases} \end{aligned} \quad (3)$$

where the first constraint avoids negative power levels, P_T is the total available power, and \mathcal{K} denotes the set of all subchannel indices $k = 1, \dots, K$.

The application of Lagrange optimization and the Karush-Kuhn-Tucker (KKT) necessary conditions for optimality [10]

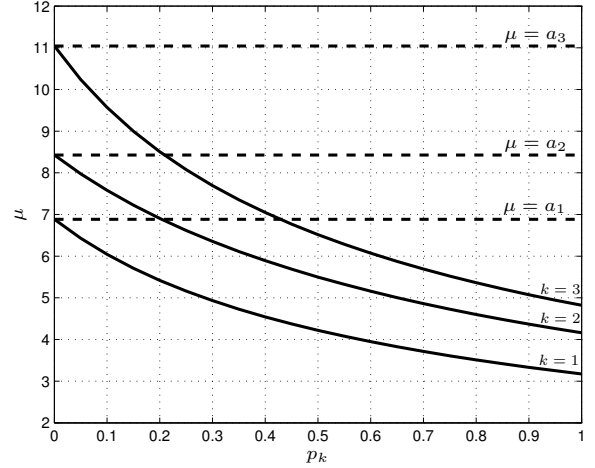


Fig. 1. Sum throughput maximization for 3 subchannels and $P_T = 1$.

results in the following system of equations:

$$\begin{cases} p_k \geq 0, & \forall k \in \mathcal{K}, \\ \sum_{k=1}^K p_k = P_T, \\ \mu \geq \sum_{n=1}^N \frac{G_{n,k}}{1 + p_k G_{n,k}}, & \forall k \in \mathcal{K}, \\ p_k \left(\mu - \sum_{n=1}^N \frac{G_{n,k}}{1 + p_k G_{n,k}} \right) = 0, & \forall k \in \mathcal{K}. \end{cases} \quad (4)$$

where μ is a Lagrange multiplier. From (4), it follows that μ is related to the power of each subchannel k according to:

$$\begin{cases} p_k = 0 & \text{for } \mu \geq \sum_{n=1}^N G_{n,k}, \quad (5a) \\ \mu = \sum_{n=1}^N \frac{G_{n,k}}{1 + p_k G_{n,k}} & \text{for } \mu < \sum_{n=1}^N G_{n,k}. \quad (5b) \end{cases}$$

A single level μ therefore determines the power of all subchannels. It should be noted that it is not possible to explicitly express p_k as a function of μ in (5). However, (5b) can be rewritten as the following polynomial in p_k :

$$\sum_{j=1}^N (p_k + G_{j,k}^{-1} - N\mu^{-1}) \prod_{i=1, i \neq j}^N (p_k + G_{i,k}^{-1}) = 0, \quad (6)$$

which has degree N and only one positive real root.

The problem now consists of finding an adequate μ such that the resulting power vector satisfies the total power constraint. The optimal solution can be numerically calculated by performing a one-dimensional search over μ [10].

In order to better illustrate the problem, Fig. 1 depicts μ as a function of p_k according to (5) for a system containing three subchannels and $P_T = 1$. This example represents a particular system snapshot, which is characterized by the instantaneous values of the path gain matrix \mathbf{G} . Each curve corresponds to a subchannel k and monotonically decreases with increasing p_k . For the considered power range, the dashed lines indicate the maximum μ of each curve, which is achieved for $p_k = 0$ and is denoted by a_k . From (5), it follows that $a_k = \sum_{n=1}^N G_{n,k}$.

By analyzing the problem, it can be seen that a hypothesis testing similar to that of the traditional waterfilling algorithm [7] can also be done for this more general unicast/multicast case, with the purpose of reducing the processing time of the one-dimensional search for μ . The algorithm, which is described below, assumes that for a given μ , each p_k is obtained by finding the real positive root of (6).

- 1) Assign the subchannel indices according to the increasing order of a_k and set $\tilde{k} = 1$.
- 2) Set $\mu = a_{\tilde{k}}$ and compute $p_{\tilde{k}+1}, \dots, p_K$.
If $\sum_{k=\tilde{k}+1}^K p_k \leq P_T$, then proceed to step 3,
otherwise set $\tilde{k} = \tilde{k} + 1$ and repeat step 2.
- 3) Find $\mu \in]a_{\tilde{k}-1}, a_{\tilde{k}}] \mid \sum_{k=\tilde{k}}^K p_k = P_T$.
Assume that $a_0 = 0$ for the case in which $\tilde{k} = 1$.
Set $p_1, \dots, p_{\tilde{k}-1}$ to zero and compute $p_{\tilde{k}}, \dots, p_K$.

The algorithm does not eliminate the need for a numerical method in order to calculate μ , but as it can be seen from step 3, it may benefit from a narrower search space and reduced dimension (vector \mathbf{p} with some zero elements), which may result in significant gains in terms of processing time.

C. Group criterion for throughput maximization (GCTM)

In this section, the GCTM algorithm is presented, which also aims at the maximization of the sum throughput, but corresponds to a simplification of the STM algorithm. It assumes that the users of a multicast group do not have their quality indicators (channel gains) taken into account individually. Instead, for each subchannel, a single indicator is considered for the whole group.

Let g_k represent the group quality indicator for subchannel k , then the optimization problem becomes:

$$\begin{aligned} \mathbf{p}_{opt} = \underset{\mathbf{p}}{\operatorname{argmax}} \sum_{k=1}^K \log_2(1 + p_k g_k), \\ \text{subject to: } \begin{cases} p_k \geq 0, & \forall k \in \mathcal{K}, \\ \sum_{k=1}^K p_k = P_T, \end{cases} \end{aligned} \quad (7)$$

which can be solved directly by the waterfilling algorithm.

The group indicator for each subchannel can be expressed as a function of the previously defined gain \mathbf{G} , i.e., $g_k = f(\mathbf{G}_k)$, where \mathbf{G}_k is the k^{th} column of matrix \mathbf{G} . The functions considered in this work are the following: maximum (GCTM-Max), minimum (GCTM-Min), and the arithmetic mean (GCTM-Mean) of the elements of \mathbf{G}_k . More details on which of them are more adequate to better approximate the solution of the STM algorithm are presented in section III-C.

D. Fair power allocation (FPA)

The algorithms considered so far have aimed at the maximization of the sum throughput, which is not a fair criterion in terms of user performance, since the users may achieve bitrates

which largely differ from one another. In this section, the FPA algorithm is described, which has the purpose of introducing fairness within the power allocation procedure.

The optimization objective of the FPA algorithm is to maximize the lowest Signal-to-Noise Ratio (SNR) within the cell. Let the SNR perceived by user n on subchannel k be defined as $p_k G_{n,k}$, then the optimization problem can be written as:

$$\begin{aligned} \mathbf{p}_{opt} = \underset{\mathbf{p}}{\operatorname{argmax}} \min_{\{n,k\}} (p_k G_{n,k}), \\ \text{for } n = 1, \dots, N \quad \text{and } k = 1, \dots, K, \\ \text{subject to: } \begin{cases} p_k \geq 0, & \forall k \in \mathcal{K}, \\ \sum_{k=1}^K p_k = P_T, \end{cases} \end{aligned} \quad (8)$$

where the \min_+ operator is here assumed to return the minimum non-zero element.

Since the allocated power does not depend on n , the problem can be rewritten as follows:

$$\mathbf{p}_{opt} = \underset{\mathbf{p}}{\operatorname{argmax}} \min_k (p_k g'_k), \quad (9)$$

where $g'_k = \min_n G_{n,k}$ and the same range of n and k , as well as the same constraints of (8), are assumed.

This means that only the worst user within each subchannel needs to be considered. The objective is that these worst users in the different subchannels achieve the same SNR γ for the optimal power vector \mathbf{p}_{opt} , which implies that $p_k g'_k = \gamma$ for all subchannels. Assuming that $\mathbf{d}_{K \times 1}$ represents a vector with elements $d_k = g'^{-1}_k$, $\|\cdot\|_1$ denotes the 1-norm of a vector, and $P_T = \|\mathbf{p}_{opt}\|_1$ is the total power constraint in vector form, the following system of equations can be established:

$$\begin{cases} \mathbf{p}_{opt} = \gamma \mathbf{d}, \\ P_T = \gamma \|\mathbf{d}\|_1, \end{cases} \quad (10)$$

whose solution is given by:

$$\mathbf{p}_{opt} = P_T \frac{\mathbf{d}}{\|\mathbf{d}\|_1}. \quad (11)$$

III. COMPLEXITY AND PERFORMANCE RESULTS

A. Network scenario

The system consists of a single cell serving a certain number N_g of user groups. Among these groups there are N_{ug} unicast groups, each containing one user, and N_{mg} multicast groups, such that $N_g = N_{ug} + N_{mg}$. For simplicity, it is assumed that all multicast groups have the same size, which is denoted by L_{mg} , only one subchannel is allocated to each group, and the number of available subchannels is equal to the number of user groups, i.e., $K = N_g$.

The users are uniformly distributed over one hexagonal sector of a tri-sectorized cell and a single-antenna base station is located at the sector corner. The considered propagation effects include the distance-based path-loss attenuation (with exponent $\alpha = 3.5$), as well as uncorrelated Rayleigh fading, which is modelled as circularly symmetric complex Gaussian

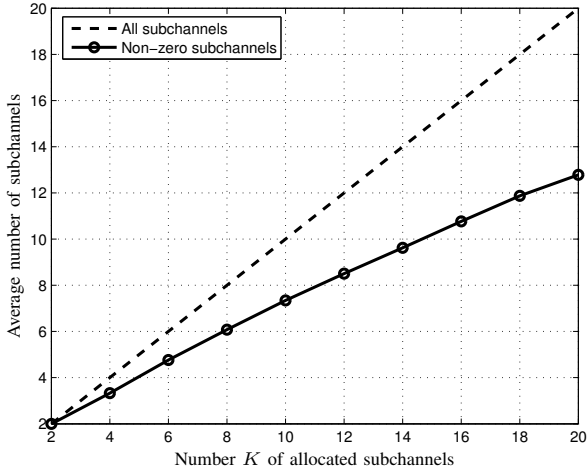


Fig. 2. Effective length of the power allocation vector for scenario S1, STM algorithm, $N_{ug} = N_{mg} = K/2$, and $L_{mg} = 3$.

random variables with variance σ^2 . The path-loss is modelled by assuming that the cell border is at a distance $d_b = 1$ from the base station and that the fading variance of a user with distance $d \leq d_b$ is given by $\sigma^2 = 1/d^\alpha$ [11]. Additive white Gaussian noise is also assumed and the transmit power is adjusted to provide an average SNR of 10dB at the cell border. The simulation results are obtained considering 10,000 independent channel realizations.

A simple subchannel allocation (SSA) algorithm is implemented, which approximates the maximization of the sum throughput given an equal power distribution. The considered algorithm iteratively allocates a subchannel to each user group according to the highest average group channel gain. After an allocation, the corresponding user group and channel are no longer taken into account by the further steps. The procedure is repeated until all user groups are allocated one channel.

The evaluation of the results considers two distinct scenarios. The first one, denoted as scenario S1, represents a worst-case situation in which the users have path gains of the same order, with $\sigma^2 = 1$, and no specific subchannel allocation algorithm is employed (random allocation). This scenario can be interpreted as all users being at the same distance from the base station or, equivalently, it can be assumed that path gain compensation has been performed. Scenario S2, on the other hand, takes into account the different path-loss of the users, with $\sigma^2 = 1/d^\alpha$, as well as the previously described SSA algorithm.

B. Complexity analysis of the STM algorithm

In this section, the complexity of the STM algorithm is analyzed. The other algorithms are not considered, because they either have a closed-form solution, in the case of FPA, or their complexity is the same as that of traditional waterfilling, in the case of GCTM.

It has been shown in section II-B that the allocation of power based on sum throughput maximization can have its processing effort reduced by employing an algorithm similar to the traditional waterfilling, which consists of iteratively testing the hypothesis that a certain subchannel be allocated zero

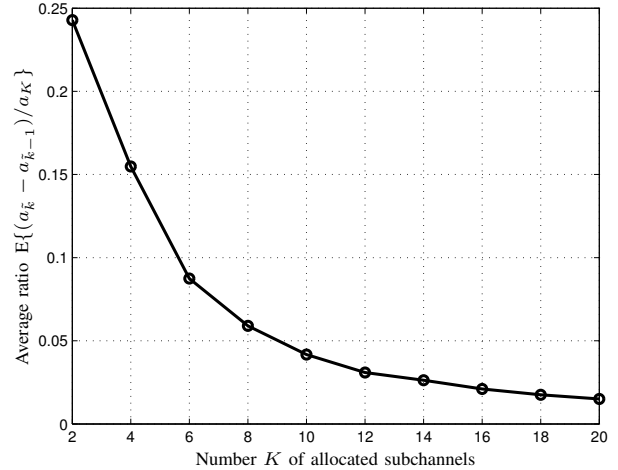


Fig. 3. Ratio between the search space range with and without hypothesis testing for scenario S1, STM algorithm, $N_{ug} = N_{mg} = K/2$, and $L_{mg} = 3$.

power. The advantage of this approach is the reduction of both the power vector dimension and the range of the search space, which results in decreased computational effort when searching for μ , conform section II-B.

In the following, it is analyzed to which extent it is expected that the effective power vector length, i.e., the number of non-zero power elements within \mathbf{p} , and the search space be reduced when applying the hypothesis testing of section II-B. The simulation scenario S1 is considered and among K allocated subchannels the same number of unicast and multicast groups is assumed, i.e., $N_{ug} = N_{mg} = K/2$, with each multicast group being composed of three users ($L_{mg} = 3$).

In Fig. 2, it can be seen that the absolute difference between the total number K of subchannels and the number K_{nz} of non-zero subchannels increases for larger values of K . For a small number of subchannels the difference is negligible, but for an intermediate/large amount, the reduction of the effective power vector length may lead to significant gains in terms of processing effort.

The average ratio between the search space range for the cases with and without hypothesis testing, which can be defined as $E\{(a_k - a_{k-1})/a_K\}$, where $E\{\cdot\}$ is the expectation operator, is shown in Fig. 3. The ratio rapidly decreases as a few subchannels are added. For more than 10 subchannels it can be seen that the hypothesis testing is capable of reducing the search space to less than 5% of the total range.

C. Performance in terms of throughput and fairness

This section presents the performance analysis of the proposed algorithms in terms of the achievable throughput as well as the fairness among the users. First, the relative performance among the sum throughput maximization algorithms, namely STM and GCTM, is compared for different scenarios, then the FPA algorithm is included and the absolute throughput achieved by all algorithms is analyzed, and finally the algorithms are compared in terms of the worst-user SNR.

In section II-C, the GCTM algorithm has been presented as an alternative for performing the sum throughput maximization,

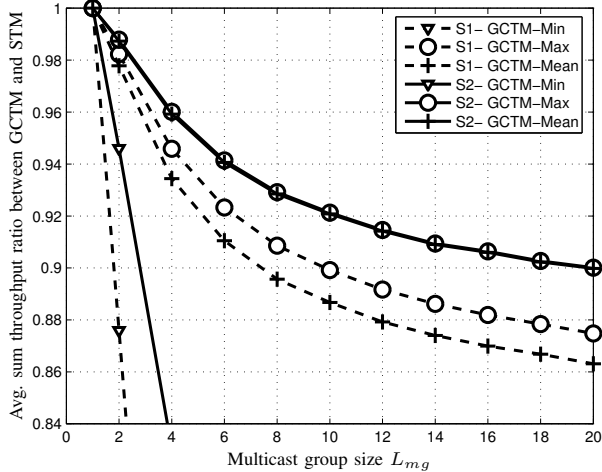


Fig. 4. Sum throughput ratio between GCTM and STM for different group criteria, for scenarios S1 and S2, $N_{ug} = N_{mg} = 2$, and $K = 4$.

which consists of assuming a single quality indicator for each subchannel and applying the waterfilling algorithm.

The performance of GCTM is shown in Fig. 4 for the simulation scenarios S1 and S2, with $N_{ug} = N_{mg} = 2$ and $K = 4$, and for some different $f(\mathbf{G}_k)$ functions, which are namely: maximum (GCTM-Max), minimum (GCTM-Min), and arithmetic mean (GCTM-Mean). The figure depicts the average sum throughput ratio between the GCTM and STM algorithms, i.e., $E\{R_{GCTM}/R_{STM}\}$, as a function of the multicast group size L_{mg} .

For scenario S1, it can be seen that GCTM-Max is the algorithm which best approximates the performance of STM. The performance gets worse for an increasing group size, but is still close to 88% for $L_{mg} = 20$. The GCTM-Min presents the worst result, while GCTM-Mean has an intermediate performance. For scenario S2, better results are achieved and both GCTM-Mean and GCTM-Max present similar performance. This performance gain is explained by the fact that scenario S2 implements the subchannel allocation algorithm SSA, instead of random allocation, as well as the different path-loss perceived by the users.

The cumulative distribution function (CDF) of the sum throughput is shown in Fig. 5 for scenario S2 and a group size of 10 users. Note that the high sum throughput values are a result of the large amount of multicast users, which have resource sharing capabilities. The relative behavior among the GCTM and STM curves with regard to Fig. 4 is maintained, being GCTM-Max and GCTM-Mean the ones which better approximate the STM algorithm. The GCTM-Min is a rather inadequate criterion for GCTM, which is explained due to the fact that the waterfilling algorithm may happen to allocate low power to a multicast subchannel, since the power is adjusted according to the worst user, even if there are other users with very good channel gains which would significantly contribute to increase the sum throughput.

In order to compare the degree of fairness of the different algorithms, the measure of the worst-user SNR is employed,

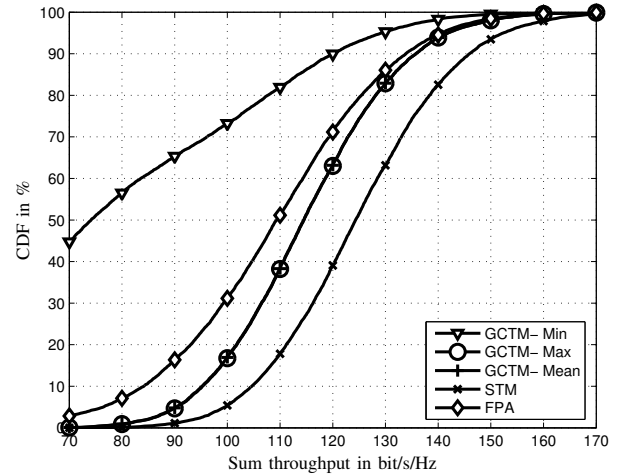


Fig. 5. CDF of the sum throughput of the different algorithms for scenario S2, $N_{ug} = N_{mg} = 2$, $K = 4$, and $L_{mg} = 10$.

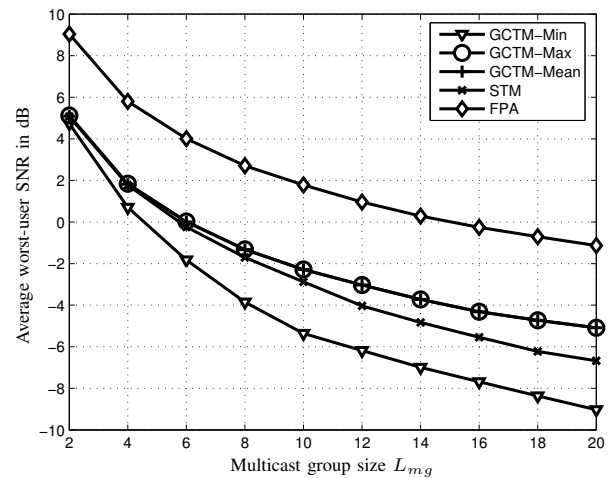


Fig. 6. Comparison of the different algorithms in terms of the average worst-user SNR for scenario S2, $N_{ug} = N_{mg} = 2$, and $K = 4$.

which corresponds to the lowest SNR perceived among all users in all subchannels.

In Fig. 6, the average worst-user SNR is depicted as a function of the multicast group size for the different power allocation methods. The FPA algorithm presents the best performance in terms of fairness, as already expected, and it presents a gain of roughly 5dB with regard to the GCTM-Max algorithm, which is maintained throughout the whole group size range. The GCTM-Mean curve practically overlaps the GCTM-Max, and the GCTM-Min is again the one with the worst performance. Additionally, it is seen that the GCTM-Mean/Max algorithms have performance similar to STM, but for $L_{mg} \geq 12$, STM becomes worse in terms of fairness. It is important to note that the fairness advantage of FPA comes at the cost of reduced sum throughput when compared to GCTM and STM, which can be seen from Fig. 5.

Fairness is an important aspect to be taken into account, especially for users of multicast services. In the case of the considered error-tolerant hierarchical multicast, it is probably more advantageous to prefer the sum throughput maximization,

since the capacity can be maximized at the cost of a few users with low-quality audio/video transmission. However, for services which do not tolerate errors, such as file download, low quality users may compromise the throughput of everyone within the multicast group, due to retransmission mechanisms [12], and therefore a fair algorithm may be more adequate.

IV. CONCLUSIONS

The provision of both unicast and multicast services in multi-carrier networks requires an efficient management of the available radio resources. Since the users belonging to a certain multicast group are able to share the same subchannels, the resource allocation has to be done in a groupwise manner. This characteristic increases the complexity of the resource allocation algorithms, since the different radio link qualities perceived by the users need to be taken into account.

In this work, the power allocation problem has been analyzed within a unicast/multicast context, and the following algorithms have been proposed and investigated: sum throughput maximization (STM), group criterion for throughput maximization (GCTM), and fair power allocation (FPA). The first two aim at maximizing the sum capacity, while the last one maximizes the minimum perceived SNR.

The solution of the STM problem has been presented, which depends on numerical optimization, and an algorithm similar to the waterfilling hypothesis testing has been proposed for reducing the processing effort. It was shown that by employing the hypothesis testing, both the effective power vector dimension and the search space range can be significantly reduced, especially for a large number of allocated subchannels.

The GCTM algorithm, which consists of a simplification of STM that employs a single quality indicator per subchannel, has been shown to provide a reasonable approximation of STM. The best group function was verified to be the maximum channel gain (GCTM-Max), whose performance is degraded for increased multicast group sizes, but up to an intermediate size it still achieves roughly 90% of the STM performance.

Finally, the fairness of the algorithms with regard to the worst-user SNR has been compared. It was shown that the fair

power allocation is able to provide an SNR at least 5dB higher than the other algorithms, while the STM and GCTM-Max had similar performances, but with the latter being slightly better for large group sizes.

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