Linear Transceive Filters for Relay Stations with Multiple Antennas in the Two-Way Relay Channel

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***Abstract*— This paper considers the two-hop relaying case where two nodes $S1$ and $S2$ in a wireless network can communicate with each other via an intermediate relay station (RS) which is equipped with multiple antennas and cannot transmit and receive simultaneously on the same channel resources. In one-way relaying, four orthogonal channel resources are required for the transmissions from $S1$ to $S2$ and from $S2$ to $S1$. MIMO two-way relaying has been introduced as an approach which requires only half the channel resources compared to one-way relaying due to the simultaneous transmission from $S1$ to $S2$ and vice versa. For MIMO two-way relaying, a spatial filter matrix is required at the RS which applies both, transmit and receive processing. The design of linear spatial filter matrices, termed transceive filter matrices, is given in this paper. In particular, linear transceive filters are derived which fulfill the zero forcing (ZF) and minimum mean square error (MMSE) criterion, respectively. It is shown that the linear MMSE transceive filter outperforms the linear ZF transceive filter in terms of overall bit error rate (BER). However, for different channel qualities on the two channels to the RS, the choice of the transceive filter influences which direction of communication has a better BER performance.**

I. Introduction

Recently, relaying gains much attention in the wireless communications research community [1]. For the two-hop relaying approach, two nodes $S1$ and $S2$ communicate with each other via an intermediate relay station (RS) assuming that a direct communication between the two nodes is not possible, e.g., due to shadowing or limited transmit power. Since the RS cannot transmit and receive simultaneously on the same channel resource it receives a signal on a first hop, applies signal processing and retransmits the signal on a second hop. It is assumed that the signal at the RS is neither decoded nor re-encoded. However, the RS can apply spatial filtering to its receive and transmit signal assuming multiple antennas at the RS. The design of linear spatial filters is discussed in this paper.

Two-way relaying which has been first introduced in [2] is a promising protocol in order to save channel resources in relay networks. The principle of this approach is based on the framework of network coding [3] in which data packets from different sources in a multi-node computer network are jointly encoded at intermediate network nodes, thus saving network resources. For two-way relaying, the nodes $S1$ and $S2$ transmit simultaneously on a first channel resource to an RS which receives a superposition of both signals. On the second channel resource, the RS retransmits this superposition. Due to the broadcast nature of the wireless channel, both nodes receive that superposition and may detect the desired signal from the other node by subtracting their own known signal. Obviously, two-way relaying requires only two orthogonal channel resources while one-way relaying would require four orthogonal channel resources for the communication in both directions, i.e., two resources for the transmission from $S1$ to $S2$ and two resources for the transmission from $S2$ to $S1$. In [4], it is shown that the spectral efficiency of two-way relaying with subtraction is significantly increased compared to one-way relaying.

In [5], it is proposed to extend two-way relaying to nodes and RSs with multiple antennas leading to multiple input multiple output (MIMO) two-way relaying. For MIMO two-way relaying, both nodes $S1$ and $S2$ have the same number of antennas and the number of antennas at the RS is at least twice as much. The discussion in [5] shows that the effort for signaling of channel state information (CSI) can be reduced if CSI is only required at the RS and not at $S1$ and $S2$. This CSI can be obtained by channel estimation at the RS in case of time division duplex systems, for example. If CSI is available at the RS, spatial transmit and receive processing [6] can applied at the RS. The spatial filter matrix at the RS which is termed transceive filter matrix makes receive processing and obtaining CSI at nodes $S1$ and $S2$ unnecessary.

In [5], the performance of MIMO two-way relaying is investigated by means of the sum rate which is the sum of the mutual information values in both directions of the communication. Compared to one-way relaying, the sum rate may be increased by a factor of two by MIMO two-way relaying with a linear transceive filter which fulfills the zero forcing (ZF) constraint. Compared to the two-way relaying protocol of [2], a higher sum rate is achieved by the ZF transceive filter in MIMO two-way relaying due to beamforming effects [5]. In this paper, the derivation of the linear ZF transceive filter which is not derived in [5] is given. Furthermore, the design of a linear transceive filter which fulfills the minimum mean square error (MMSE) criteria is given in this paper. For both types of transceive filters, it is shown that the spatial filtering at the RS may be divided into three steps. Firstly, the receive filter matrix separates the signals from $S1$ and $S2$. Secondly, the RS mapping matrix is introduced which ensures that each node is provided with its desired signal after retransmission from the

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RS. Thirdly, the transmit filter matrix is applied at the RS, which separates the signals designated to $S1$ and $S2$ before retransmission. The two transceive filters are compared to each other concerning their bit error rate (BER) performance.

The paper is organized as follows: In Section II, the system model of MIMO two-way relaying is introduced. Section III gives the linear ZF and MMSE transceive filters at the RS. The BER performance of the filters is analyzed by means of simulations in Section IV. Section V concludes this work.

II. System Model

In the following, the communication between two nodes $S1$ and $S2$ is considered which cannot exchange information directly, e.g., due to shadowing conditions, but only via an intermediate RS. The system model of MIMO two-way relaying given in [5] is summarized here as far as necessary for the following considerations. $S1$ and $S2$ are equipped with $M^{(1)}$ and $M^{(2)}$ antennas, respectively. For MIMO two-way relaying, it is assumed that $S1$ and $S2$ are equipped with

$$M^{(1)} = M^{(2)} = M \tag{1}$$

antennas and that the RS is equipped with

$$M_{\text{RS}} \geq M^{(1)} + M^{(2)} = 2M \tag{2}$$

antennas.
Data vector $\mathbf{x}^{(1)} = \left[x_1^{(1)}, \ldots, x_M^{(1)}\right]^T$ of data symbols $x_n^{(1)}$, $n = 1, \ldots, M$, shall be transmitted from $S1$ to $S2$, and data vector $\mathbf{x}^{(2)} = \left[x_1^{(2)}, \ldots, x_M^{(2)}\right]^T$ of data symbols $x_n^{(2)}$, $n = 1, \ldots, M$, shall be transmitted from $S2$ to $S1$, where $[\cdot]^T$ denotes the transpose. The overall data vector $\mathbf{x} = \left[\mathbf{x}^{(1)^T}, \mathbf{x}^{(2)^T}\right]^T$ is defined with covariance matrix $\mathbf{R}_{\mathbf{x}} = \mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\}$ where $\mathrm{E}\{\cdot\}$ and $[\cdot]^H$ denote the expectation and the conjugate transpose, respectively. For simplicity, but without loss of generality, the wireless channel is assumed to be flat fading. Hence, the channel between Sk, $k = 1, 2$, and the RS may be described by the channel matrix

$$\mathbf{H}^{(k)} = \begin{bmatrix} h_{1,1}^{(k)} & \cdots & h_{1,M}^{(k)} \\ \vdots & \ddots & \vdots \\ h_{M_{\text{RS}},1}^{(k)} & \cdots & h_{M_{\text{RS}},M}^{(k)} \end{bmatrix}, \tag{3}$$

where $h_{m,n}^{(k)}$, $m = 1, \ldots, M_{\text{RS}}$ and $n = 1, \ldots, M$, are complex fading coefficients. In Fig. 1, the described relay network is depicted for the case of $M = 1$ and $M_{\text{RS}} = 2$. In MIMO two-way relaying, the data vectors $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are exchanged between $S1$ and $S2$ during two orthogonal time slots. During the first time slot, $S1$ and $S2$ transmit simultaneously to the RS. Since spatial filtering shall only be applied at the RS, only scalar transmit filters $\mathbf{Q}^{(1)} = q^{(1)}\mathbf{I}_M$ and $\mathbf{Q}^{(2)} = q^{(2)}\mathbf{I}_M$ are applied at $S1$ and $S2$, where $\mathbf{I}_M$ is an identity matrix of size M. These transmit filters are required in order to fulfill the transmit energy constraints. Assuming that $E^{(1)}$ and $E^{(2)}$

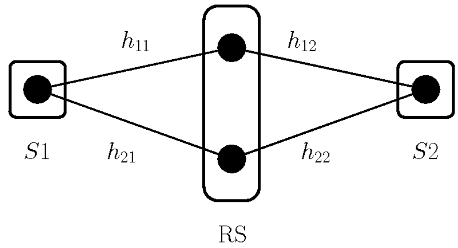

Fig. 1. Relay network for $M^{(1)} = 1$ antenna at $S1$, $M^{(2)} = 1$ antenna at $S2$, and $M_{\text{RS}} = 2$ antennas at the RS

are the maximum transmit energies of nodes $S1$ and $S2$, the transmit energy constraints are given by

$$\mathrm{E}\left\{\| \, q^{(k)}\mathbf{x}^{(k)} \, \|_2^2\right\} \leq E^{(\text{k})}, \quad k = 1, 2, \tag{4}$$

where $\| \cdot \|_2^2$ is the Euclidian norm of a vector. The overall transmit filter is given by the block diagonal matrix

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}^{(1)} & \mathbf{I}_M \\ \mathbf{I}_M & \mathbf{Q}^{(2)} \end{bmatrix}. \tag{5}$$

Defining the overall channel matrix $\mathbf{H} = \left[\mathbf{H}^{(1)}, \mathbf{H}^{(2)}\right]$, the receive vector $\mathbf{y}_{\text{RS}}$ at the RS is given by

$$\mathbf{y}_{\text{RS}} = \mathbf{H}\mathbf{Q}\mathbf{x} + \mathbf{n}_{\text{RS}}, \tag{6}$$

where $\mathbf{n}_{\text{RS}}$ is an additive white Gaussian noise vector with covariance matrix $\mathbf{R}_{\mathbf{n}_{\text{RS}}} = \mathrm{E}\left\{\mathbf{n}_{\text{RS}}\mathbf{n}_{\text{RS}}^H\right\}$.
At the RS, the receive vector $\mathbf{y}_{\text{RS}}$ is spatially filtered by a linear transceive filter $\mathbf{G}$ leading to the RS transmit vector

$$\mathbf{x}_{\text{RS}} = \mathbf{G}\mathbf{y}_{\text{RS}} = \mathbf{G}\mathbf{H}\mathbf{Q}\mathbf{x} + \mathbf{G}\mathbf{n}_{\text{RS}}. \tag{7}$$

The RS transmit vector has to fulfill the transmit energy constraint at the RS

$$\mathrm{E}\left\{\| \, \mathbf{x}_{\text{RS}} \, \|_2^2\right\} \leq E_{\text{RS}}, \tag{8}$$

where E_{RS} is the maximum transmit energy at the RS. During the second time slot, the RS transmit vector is simultaneously transmitted to $S1$ and $S2$. Since spatial filtering shall only be applied at the RS, a scalar receive filter $\mathbf{P} = p\mathbf{I}_{2M}$ at $S1$ and $S2$ is assumed. Note that the channel matrix from the RS to nodes $S1$ and $S2$ is the transpose $\mathbf{H}^T$ of channel matrix $\mathbf{H}$ assuming that the channel is constant during two consecutive time slots. In the following, the estimate for data vector $\mathbf{x}_2$ at $S1$ is termed $\hat{\mathbf{x}}_1$ and the estimate for data vector $\mathbf{x}_1$ at $S2$ is termed $\hat{\mathbf{x}}_2$. With these assumptions, the overall estimated data vector $\hat{\mathbf{x}} = \left[\hat{\mathbf{x}}_1^T, \hat{\mathbf{x}}_2^T\right]^T$ after the scalar receive filter is given by

$$\hat{\mathbf{x}} = p\left(\mathbf{H}^T\mathbf{G}\mathbf{H}\mathbf{Q}\mathbf{x} + \mathbf{H}^T\mathbf{G}\mathbf{n}_{\text{RS}} + \mathbf{n}_{\text{R}}\right), \tag{9}$$

where it is assumed that $\mathbf{n}_{\text{R}}$ is an additive white Gaussian noise vector with covariance matrix $\mathbf{R}_{\mathbf{n}_{\text{R}}} = \mathrm{E}\left\{\mathbf{n}_{\text{R}}\mathbf{n}_{\text{R}}^H\right\}$.

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III. Linear Transceive Filters

One major drawback of two-way relaying with subtraction [2] is that both nodes $S1$ and $S2$ require CSI about their own link to the RS as well as CSI about the link from the other node to the RS in order to design an adequate receive filter. Exchanging this CSI requires signaling effort. Compared to this signaling effort, it is relatively easy to obtain CSI at the RS, e.g., by estimating the channel at the RS in a time division duplex (TDD) system. For the case of one-way relaying with multiple antenna RSs, it is proposed in [7] that transmit and receive processing can be restricted to the RSs, i.e., CSI is only required at the RSs. A similar approach is applied in MIMO two-way relaying. However, there is a difference between the effort for obtaining CSI in [7] and the proposed MIMO two-way relaying protocol. In [7], CSI of the channels from nodes $S1$ and $S2$ to the RS can be obtained by pilot signaling. But CSI of the channels from the RS to the nodes can only be obtained by feedback from the nodes to the RS since up- and downlink are separated on orthogonal channel resources. In MIMO two-way relaying, CSI of the channels from the nodes to the RS as well as from the RS to nodes $S1$ and $S2$ is obtained by only one pilot signal since up- and downlink are processed simultaneously. Applying the obtained CSI, the transceive filter for MIMO two-way relaying can be designed which consists of three independent filters.

Firstly, the receive vector at the RS $\mathbf{y}_{\text{RS}}$ is multiplied with the linear receive filter matrix $\mathbf{G}_{\text{R}}$ resulting in the RS estimation vector

$$\hat{\mathbf{x}}_{\text{RS}} = \left[\hat{\mathbf{x}}_{\text{RS}}^{(1)^T}, \hat{\mathbf{x}}_{\text{RS}}^{(2)^T}\right]^T \quad (10)$$

with the estimate $\hat{\mathbf{x}}_{\text{RS}}^{(1)}$ for $\mathbf{x}^{(1)}$ and the estimate $\hat{\mathbf{x}}_{\text{RS}}^{(2)}$ for $\mathbf{x}^{(2)}$, respectively. The receive filter separates the signals from $S1$ and $S2$. Note that the covariance matrix of $\hat{\mathbf{x}}_{\text{RS}}$ which is required for the following filter design is given by

$$\mathbf{R}_{\hat{\mathbf{x}}_{\text{RS}}} = \text{E}\left\{\hat{\mathbf{x}}_{\text{RS}}\hat{\mathbf{x}}_{\text{RS}}^H\right\} = \mathbf{G}_{\text{R}}\left(\mathbf{H}\mathbf{Q}\mathbf{R}_{\mathbf{x}}\mathbf{Q}^H\mathbf{H}^H + \mathbf{R}_{\mathbf{n}_{\text{RS}}}\right)\mathbf{G}_{\text{R}}^H. \quad (11)$$

Secondly, the RS estimation vector $\hat{\mathbf{x}}_{\text{RS}}$ is multiplied with the RS mapping matrix

$$\mathbf{G}_{\Pi} = \begin{bmatrix} \emptyset_M & \mathbf{I}_M \\ \mathbf{I}_M & \emptyset_M \end{bmatrix} \quad (12)$$

where $\emptyset_M$ is a null matrix with M rows and M columns. The RS mapping matrix is introduced in order to ensure that, $S1$ is provided with an estimate of data vector $\mathbf{x}^{(2)}$ and $S2$ is provided with an estimate of data vector $\mathbf{x}^{(1)}$.

Thirdly, the mapped RS estimation vector is multiplied with transmit filter matrix $\mathbf{G}_{\text{T}}$ leading to the RS transmit vector

$$\mathbf{x}_{\text{RS}} = \mathbf{G}_{\Pi}\mathbf{G}_{\text{T}}\hat{\mathbf{x}}_{\text{RS}} \quad (13)$$

from Eq. (7). The transmit filter separates the signals to $S1$ and $S2$ before retransmission and substitutes receive processing and in particular subtracting the own signal at $S1$ and $S2$. Thus, the effort at the two communicating nodes $S1$ and $S2$ is decreased and the CSI signaling effort in the network can be reduced while maintaining the increased spectral efficiency of the two-way relay channel. The overall transceive filter matrix is given by

$$\mathbf{G} = \mathbf{G}_{\text{T}}\mathbf{G}_{\Pi}\mathbf{G}_{\text{R}}. \quad (14)$$

In the following, two different linear MIMO transceive filters $\mathbf{G}$ at the RS are derived which are based on the ZF and MMSE criterion, respectively. The filters are based on the results for linear transmit and receive filters in [8].

A. Zero Forcing Transceive Filter

1) Zero Forcing Receive Filter: For the ZF criterion, the receive filter $\mathbf{G}_{\text{R,ZF}}$ at the RS has to be designed such that the mean squared error of the estimate vector $\hat{\mathbf{x}}_{\text{RS}}$ for data vector $\mathbf{x}$ is minimized. With the ZF constraint and the transmit power constraint of (4), the ZF optimization may be formulated as

$$\{\mathbf{G}_{\text{R,ZF}}, q_{\text{ZF}}^{(1)}, q_{\text{ZF}}^{(2)}\} = \underset{\{\mathbf{G}_{\text{R}}, q^{(1)}, q^{(2)}\}}{\arg\min} \ \text{E}\left\{\| \hat{\mathbf{x}}_{\text{RS}} - \mathbf{x} \|_2^2\right\} \quad (15a)$$

$$\text{subject to:} \quad \hat{\mathbf{x}}_{\text{RS}} = \mathbf{x} \quad \text{for} \quad \mathbf{n}_{\text{RS}} = \emptyset_{M_{\text{RS}}\times 1} \quad (15b)$$

$$\text{E}\left\{\| q^{(k)}\mathbf{x}^{(k)} \|_2^2\right\} \leq E^{(k)} \quad k = 1, 2. \quad (15c)$$

As shown in [8], the ZF optimization problem in (15a) is not convex. However, the Karush-Kuhn-Tucker (KKT) conditions [9] can be used to determine the global minimum of (15a) under the constraints (15b) and (15c) which leads to the ZF receive filter

$$\mathbf{G}_{\text{R,ZF}} = \left(\mathbf{Q}^H\mathbf{H}^H\mathbf{R}_{\mathbf{n}_{\text{RS}}}^{-1}\mathbf{H}\mathbf{Q}\right)^{-1}\mathbf{Q}^H\mathbf{H}^H\mathbf{R}_{\mathbf{n}_{\text{RS}}}^{-1} \quad (16)$$

with the scalar transmit filter coefficients

$$q_{\text{ZF}}^{(k)} = \sqrt{\frac{E^{(k)}}{\text{tr}\left\{\mathbf{R}_{\mathbf{x}^{(k)}}\right\}}} \quad k = 1, 2 \quad (17)$$

where $\mathbf{R}_{\mathbf{x}^{(k)}}$ is the covariance matrix of $\mathbf{x}^{(k)}$ and $\text{tr}\{\cdot\}$ denotes the sum of the main diagonal elements of a matrix.

2) Zero Forcing Transmit Filter: The ZF transmit filter $\mathbf{G}_{\text{T,ZF}}$ at the RS has to be designed such that the mean squared error of the estimate vector $\hat{\mathbf{x}}$ for transmit vector $\mathbf{G}_{\Pi}\hat{\mathbf{x}}_{\text{RS}}$ is minimized. With the ZF constraint and the RS transmit power constraint of (8), the ZF optimization may be formulated as

$$\{\mathbf{G}_{\text{T,ZF}}, p_{\text{ZF}}\} = \underset{\{\mathbf{G}_{\text{T}}, p\}}{\arg\min} \text{E}\left\{\| \hat{\mathbf{x}} - \mathbf{G}_{\Pi}\hat{\mathbf{x}}_{\text{RS}} \|_2^2\right\} \quad (18a)$$

$$\text{subject to:} \quad \hat{\mathbf{x}} = \hat{\mathbf{x}}_{\text{RS}} \quad \text{for} \quad \mathbf{n}_{\text{R}} = \emptyset_{M_{\text{RS}}\times 1} \quad (18b)$$

$$\text{E}\left\{\| \mathbf{x}_{\text{RS}} \|_2^2\right\} \leq E_{\text{RS}}. \quad (18c)$$

As shown in [8], this optimization problem is not convex. However, the KKT conditions can be used to determine the ZF transmit filter

$$\mathbf{G}_{\text{T,ZF}} = \frac{1}{p_{\text{ZF}}}\mathbf{H}^*\left(\mathbf{H}^T\mathbf{H}^*\right)^{-1} \quad (19)$$

with the scalar receive filter

$$p_{\text{ZF}} = \sqrt{\frac{\text{tr}\left\{\left(\mathbf{H}^{\mathbf{T}}\mathbf{H}^*\right)^{-1}\mathbf{G}_{\Pi}\mathbf{R}_{\hat{\mathbf{x}}_{RS}}\mathbf{G}_{\Pi}^H\right\}}{E_{\text{RS}}}}. \quad (20)$$

Timo Unger and Anja Klein, 'Linear Transceive Filters for Relay Stations with Multiple Antennas in the Two-Way Relay Channel', in Proc. of 16th IST Mobile and Wireless Communications Summit, Budapest, Hungary, July 2007.

Since the derived receive and transmit filters $\mathbf{G}_{\text{R,ZF}}$ and $\mathbf{G}_{\text{T,ZF}}$ require the same channel coefficients in TDD systems there also exists a high potential for saving processing effort at the RS. For example, the calculation of the inverse of $\mathbf{H}^T\mathbf{H}^*$ in Eq. (19) may be reused for the calculation of the inverse of $\mathbf{H}^H\mathbf{H}$ in Eq. (16) if $\mathbf{R}_{\mathbf{n}_{\text{RS}}}$ and $\mathbf{Q}$, respectively, are diagonal matrices with equal entries on their main diagonal.

B. Mimimum Mean Square Error Transceive Filter

1) Mimimum Mean Square Error Receive Filter: For the MMSE criterion, the receive filter $\mathbf{G}_{\text{r,MMSE}}$ at the RS has to be designed such that the mean squared error of the estimate vector $\hat{\mathbf{x}}_{\text{RS}}$ for data vector $\mathbf{x}$ is minimized. Additionally, the transmit power constraint of (4) at $S1$ and $S2$ has to be met. The constrained MMSE optimization may be formulated as

$$\{\mathbf{G}_{\text{R,MMSE}},, q^{(1)}_{\text{MMSE}}, q^{(2)}_{\text{MMSE}}\} = \underset{\{\mathbf{G}_{\text{R}}, q^{(1)}, q^{(2)}\}}{\arg\min} \; \text{E}\left\{\| \hat{\mathbf{x}}_{\text{RS}} - \mathbf{x} \|_2^2\right\} \tag{21a}$$

$$\text{subject to:} \quad \text{E}\left\{\| q^{(k)}\mathbf{x}^{(k)} \|_2^2\right\} \leq E^{(k)} \quad k = 1, 2. \tag{21b}$$

As shown in [8], the MMSE optimization problem in (21a) is not convex. However, the KKT conditions can be used to determine the global minimum of (21a) under the constraint (21b) leading to

$$\mathbf{G}_{\text{R,MMSE}} = \mathbf{R}_{\mathbf{x}}\mathbf{Q}^H\mathbf{H}^H \left(\mathbf{H}\mathbf{Q}\mathbf{R}_{\mathbf{x}}\mathbf{Q}^H\mathbf{H}^H + \mathbf{R}_{\mathbf{n}_{\text{RS}}}\right)^{-1} \tag{22}$$

with the scalar transmit filter coefficients

$$q^{(k)}_{\text{MMSE}} = \sqrt{\frac{E^{(k)}}{\text{tr}\left\{\mathbf{R}_{\mathbf{x}^{(k)}}\right\}}} \quad k = 1, 2. \tag{23}$$

2) Mimimum Mean Square Error Transmit Filter: The MMSE transmit filter $\mathbf{G}_{\text{T,MMSE}}$ at the RS has to be designed such that the mean squared error of the estimate vector $\hat{\mathbf{x}}$ for transmit vector $\mathbf{G}_{\Pi}\hat{\mathbf{x}}_{\text{RS}}$ is minimized. With the RS transmit power constraint of (8), the MMSE optimization may be formulated as

$$\{\mathbf{G}_{\text{T,MMSE}}, p_{\text{MMSE}}\} = \underset{\{\mathbf{G}_{\text{T}}, p\}}{\arg\min} \text{E}\left\{\| \hat{\mathbf{x}} - \mathbf{G}_{\Pi}\hat{\mathbf{x}}_{\text{RS}} \|_2^2\right\} \tag{24a}$$

$$\text{subject to:} \quad \text{E}\left\{\| \mathbf{x}_{\text{RS}} \|_2^2\right\} \leq E_{\text{RS}}. \tag{24b}$$

As shown in [8], this optimization problem is not convex. However, the KKT conditions can be used to determine the MMSE transmit filter

$$\mathbf{G}_{\text{T,MMSE}} = \frac{1}{p_{\text{MMSE}}}\left(\mathbf{H}^*\mathbf{H}^T + \frac{\text{tr}\left\{\mathbf{R}_{\mathbf{n}_{\text{R}}}\right\}}{E_{\text{RS}}}\mathbf{I}\right)^{-1}\mathbf{H}^* \tag{25}$$

with the scalar receive filter

$$p_{\text{MMSE}} = \sqrt{\frac{\text{tr}\left\{\left(\mathbf{H}^*\mathbf{H}^T + \frac{\text{tr}\left\{\mathbf{R}_{\mathbf{n}_{\text{R}}}\right\}}{E_{\text{RS}}}\mathbf{I}\right)^{-2}\mathbf{H}^*\mathbf{R}_{\hat{\mathbf{x}}_{RS}}\mathbf{H}^T\right\}}{E_{\text{RS}}}}. \tag{26}$$

Similarly to the ZF transceive filter, there exists a high potential for saving processing effort at the RS since receive and transmit filters require the same channel coefficients.

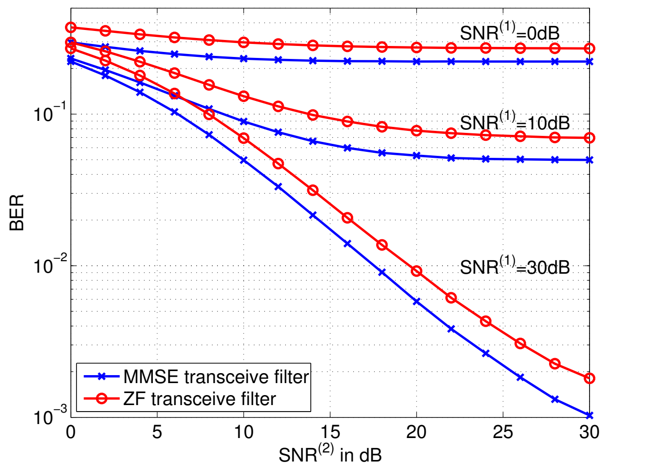

Fig. 2. Comparison of overall BER performance at $S2$ for the linear ZF and MMSE transceive filter for different fixed values of SNR$^{(1)}$

IV. Simulation Results

In this section, the BER performance of the linear ZF and MMSE transceive filters is evaluated by means of simulations. It is assumed that $S1$ and $S2$ are equipped with $M = 1$ antenna each and the RS is equipped with $M_{\text{RS}} = 2$ antennas according to the requirement in Eq. (2). The data symbols of $S1$ and $S2$ are QPSK modulated. The channel coefficients are spatially white and their amplitude is Rayleigh distributed. For the following investigations, the average signal-to-noise ratio SNR$^{(1)}$ of the first channel from $S1$ to the RS is fixed at a certain value and the BER is depicted depending on the average SNR$^{(2)}$ of the second channel from $S2$ to the RS.

In [5], the sum rate of the transmission from $S1$ to $S2$ and the transmission from $S2$ to $S1$ is considered as a performance measure since the performance gains of MIMO two-way relaying come from the simultaneous transmission in both directions. Since this paper considers the BER performance of transceive filters in MIMO two-way relaying, the overall BER is defined which is the average over both BER values at $S1$ and $S2$, respectively. In Fig. 2, the overall BER is given for the linear ZF and MMSE transceive filter, respectively. For all curves, the performance significantly depends on SNR$^{(1)}$ which determines a minimum achievable BER. All curves show a saturation region where an increase of SNR$^{(2)}$ does no longer improve the overall BER performance. From transmit and receive oriented spatial filters it is known that the linear MMSE transceive filter outperforms the linear ZF transceive filter by a constant SNR gain [8]. The same result is found for the transceive filters in MIMO two-way relaying where the SNR gain of the MMSE transceive filter is about 2dB outside the saturation region.

In the following, the BER performances at $S1$ and $S2$ are considered independently. In Fig. 3, the BER performance at $S1$ and $S2$, respectively, is depicted for the linear ZF transceive filter. Considering the BER for equal SNR$^{(1)}$, there exists always one intersection point between the BER at $S1$ and $S2$ which is located at SNR$^{(1)}$ = SNR$^{(2)}$. As long as SNR$^{(2)}$ > SNR$^{(1)}$, the BER at $S1$ is lower than the BER at

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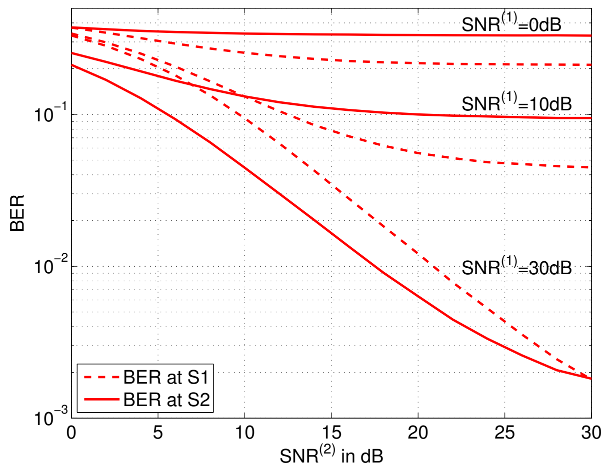

Fig. 3. Comparison of BER performance at $S1$ and $S2$ for the linear ZF transceive filter for different fixed values of $\mathrm{SNR}^{(1)}$

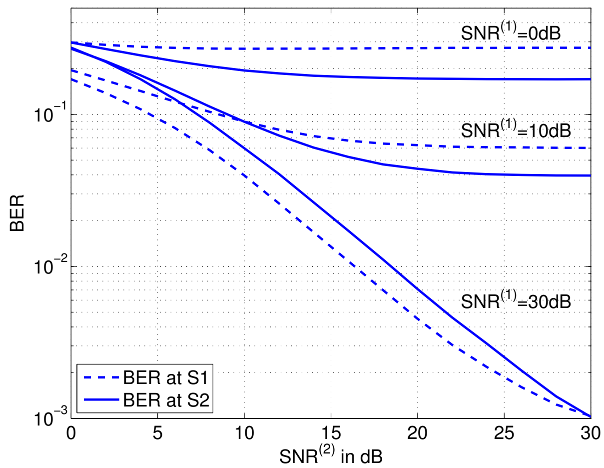

Fig. 4. Comparison of BER performance at $S1$ and $S2$ for the linear MMSE transceive filter for different fixed values of $\mathrm{SNR}^{(1)}$

$S2$. For $\mathrm{SNR}^{(2)} < \mathrm{SNR}^{(1)}$, the BER at $S2$ is lower than the BER at $S1$. The ZF receive filter as well as the ZF transmit filter lead to unbiased estimates at $S1$ and $S2$. For the ZF receive filter, the noise at the RS is filtered which leads to different SNR values for the two receive vectors from $S1$ and $S2$ after the filter. In particular, the receive vector which comes over the better channel has a higher SNR. For the ZF transmit filter, the noise at $S1$ and $S2$ is not filtered and both receive vectors at $S1$ and $S2$ have the same SNR after the filter, i.e., the data vector which is transmitted over the better channel does not benefit from the better channel. Hence, the linear ZF transceive filter is more sensitive to the channel quality on the first hop and the data stream which is coming over the better channel on the first hop has a better BER performance.

Fig. 4 gives the BER performance at $S1$ and $S2$, respectively, for the linear MMSE transceive filter. There also exist intersection points between the BER at $S1$ and $S2$ for $\mathrm{SNR}^{(1)} = \mathrm{SNR}^{(2)}$. However, the behavior is exactly vice versa. As long as $\mathrm{SNR}^{(2)} < \mathrm{SNR}^{(1)}$, the BER at $S1$ is lower than the BER at $S2$. For $\mathrm{SNR}^{(2)} > \mathrm{SNR}^{(1)}$, the BER at $S2$ is lower than the BER at $S1$. In contrast to the ZF receive filter, the MMSE receive filter balances the different channel qualities since it considers both SNRs, i.e., both data streams have the same SNR after the receive filter. The MMSE transmit filter only considers the sum of the noise at $S1$ and $S2$ which can be seen from Eq.(25). Therefore, it does not distinguish between the good and bad channel which means that the data stream which is transmitted over the good channel will benefit after retransmission. Hence, the linear MMSE transceive filter is more sensitive to the channel quality on the second hop and the data stream which is transmitted over the better channel on the second hop has a better BER performance.

Obviously, the choice of the linear transceive filter influences which direction of communication has a better BER performance for given channel qualities $\mathrm{SNR}^{(1)}$ and $\mathrm{SNR}^{(2)}$.

V. Conclusion

MIMO two-way relaying is a relaying approach which requires only half the channel resources compared to one-way relaying due to the simultaneous transmission from $S1$ to $S2$ and from $S2$ to $S1$ via an intermediate RS. For MIMO two-way relaying, a spatial transceive filter matrix is introduced at the RS which applies both, transmit and receive processing. In this paper, the design of linear transceive filter matrices which fulfill the zero forcing (ZF) and minimum mean square error (MMSE) criteria is given. The matrices for receive processing are derived independently from the matrices for transmit processing. It is shown that the linear MMSE transceive filter outperforms the linear ZF transceive filter in terms of overall BER performance. Additionally, the choice of the linear transceive filter influences which direction of communication has a better BER performance for different channel qualities.

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