# On the Performance of Two-Way Relaying with Multiple-Antenna Relay Stations

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*Abstract*— This paper considers the two-hop relaying case where two nodes $S1$ and $S2$ in a wireless network can communicate with each other via an intermediate relay station (RS). Assuming that an RS can only receive and transmit on orthogonal channel resources, the required resources in one-way amplify-and-forward relaying are doubled compared to a direct communication between two nodes. Recently, two-way relaying has been introduced as a promising protocol in order to compensate for this drawback. In this paper, the two-way relaying protocol is extended to the case of nodes and RSs with multiple antennas where both nodes $S1$ and $S2$ have the same number of antennas and the number of antennas at the RS is at least twice as much. This multiple input multiple output (MIMO) two-way relaying protocol only requires channel state information (CSI) at the RS which reduces the CSI signaling overhead in the two-way relay network compared to previous work. Comparing with other relaying protocols, the performance gain in terms of sum rate of the proposed MIMO two-way relaying protocol is verified. It is shown that the sum rate may be further increased for the case of different channel qualities on the two channels to the RS.

## I. INTRODUCTION

Recently, relaying and in particular cooperative relaying gain much attention in the wireless communications research community [1]. This paper considers the two-hop relaying case where two nodes $S1$ and $S2$ can communicate with each other via an intermediate relay station (RS) assuming that a direct communication between the two nodes is not possible, e.g., due to shadowing or limited transmit power. It is assumed that the RS can apply linear signal processing, i.e., the RS receives a signal on a first hop, filters this signal and retransmits it on a second hop. However, the receive signal at the RS is not decoded and re-encoded. One possible signal processing at the RS could be amplify-and-forward (AF) relaying, but other approaches are also considered in this paper.

Assuming one-way relaying with an AF approach from node $S1$ to node $S2$, where an RS can only receive and transmit on orthogonal channel resources, the required resources are doubled compared to a direct communication between $S1$ and $S2$, i.e., although two-hop relaying provides an increase in spectral efficiency due to the improved receive signal quality the spectral efficiency is also decreased due to the underlying protocol. There exist several protocols which aim at compensating for this degradation [2] [3]. What all these protocols have in common is that it is not possible to improve the spectral efficiency of a single two-hop connection between source and destination, but the overall spectral efficiency of different two-hop connections.

For this paper, the two-way relaying protocol introduced in [3] which is restricted to the case of nodes and RSs with single antennas is of particular interest. The principle of this protocol is based on the framework of network coding [4] in which data packets from different sources in a multi-node computer network are jointly encoded at intermediate network nodes, thus saving network resources. For two-way relaying, $S1$ and $S2$ transmit simultaneously on a first channel resource to the RS which receives a superposition of both signals. On the second channel resource, the RS retransmits this superposition. Due to the broadcast nature of the wireless channel, both nodes receive this superposition and may detect the desired signal from the other node by subtracting their own known signal. In [5], it is shown that the spectral efficiency of two-way relaying with subtraction is significantly increased compared to one-way relaying.

In this paper, two-way relaying is extended to nodes and RSs with multiple antennas leading to multiple input multiple output (MIMO) two-way relaying. Since the RS in the two-way relay channel is a transmitter as well as a receiver, spatial transmit and receive processing may be applied jointly at the RS if channel state information (CSI) is available at the RS. The spatial filter matrix at the RS is termed transceive filter matrix in the following since it consists of a transmit and a receive filter [6]. Its design can be divided into three steps. Firstly, the receive filter matrix separates the signals from $S1$ and $S2$. Secondly, the RS mapping matrix is introduced which ensures that each node is provided with its desired signal after retransmission from the RS. Thirdly, the transmit filter matrix is applied at the RS, which separates the signals designated to $S1$ and $S2$ before retransmission. The overall transceive filter at the RS substitutes receive processing and in particular subtracting the own signal at $S1$ and $S2$ which makes knowledge of CSI unnecessary at $S1$ and $S2$. In contrast to this approach, two-way relaying with subtraction requires CSI at $S1$ and $S2$. In particular, CSI of the own channel to the RS as well as CSI about the channel from the other node to the RS is required. Exchanging this CSI requires signaling effort for a feedback channel. Compared to this signaling effort, it is relatively easy to obtain CSI about both channels at the RS for the proposed MIMO two-way relaying, e.g., by estimating the channel at the RS in a time

division duplex system.

In this paper, the performance of the MIMO two-way relaying protocol is verified by means of the sum rate and compared to one-way relaying and two-way relaying with subtraction. It is shown how the sum rate of MIMO two-way relaying may be maximized for the case of different channel qualities on the two channels to the RS.

The paper is organized as follows: In Section II, the system model of MIMO two-way relaying is introduced. Section III gives a discussion about obtaining CSI at the RS. In Section IV, the sum rate of MIMO two-way relaying is given and it is shown how the sum rate may be maximized for different channel qualities on the two channels from the two nodes to the RS. The performance of MIMO two-way relaying is analyzed by means of simulation in Section V. Section VI concludes this work.

## II. System Model

In the following, the communication between two nodes $S1$ and $S2$ is considered which cannot exchange information directly, e.g., due to shadowing conditions, but via an intermediate RS. $S1$ and $S2$ are equipped with $M^{(1)}$ and $M^{(2)}$ antennas, respectively. For the introduced MIMO two-way relaying protocol

$$M^{(1)} = M^{(2)} = M \qquad (1)$$

is required while it is assumed that the RS is equipped with

$$M_{\text{RS}} \geq M^{(1)} + M^{(2)} = 2M \qquad (2)$$

antennas.

Data vector $\mathbf{x}^{(1)} = \left[x_1^{(1)}, \ldots, x_M^{(1)}\right]^T$ of data symbols $x_n^{(1)}$, $n = 1, \ldots, M$, shall be transmitted from $S1$ to $S2$, and data vector $\mathbf{x}^{(2)} = \left[x_1^{(2)}, \ldots, x_M^{(2)}\right]^T$ of data symbols $x_n^{(2)}$, $n = 1, \ldots, M$, shall be transmitted from $S2$ to $S1$, where $[\cdot]^T$ denotes the transpose. For simplicity, but without loss of generality, the wireless channel is assumed to be flat fading. Hence, the channel between $Sk$, $k = 1, 2$, and the RS may be described by the channel matrix

$$\mathbf{H}^{(k)} = \begin{bmatrix} h_{1,1}^{(k)} & \cdots & h_{1,M}^{(k)} \\ \vdots & \ddots & \vdots \\ h_{M_{\text{RS}},1}^{(k)} & \cdots & h_{M_{\text{RS}},M}^{(k)} \end{bmatrix}, \qquad (3)$$

where $h_{m,n}^{(k)}$, $m = 1, \ldots, M_{\text{RS}}$ and $n = 1, \ldots, M$, are complex fading coefficients. In Fig. 1, the described relay network is depicted for the case of $M = 1$ and $M_{\text{RS}} = 2$. In MIMO two-way relaying, the data vectors $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are exchanged between $S1$ and $S2$ during two orthogonal time slots. During the first time slot, $S1$ and $S2$ transmit simultaneously to the RS. The overall data vector $\mathbf{x} = \left[\mathbf{x}^{(1)^T}, \mathbf{x}^{(2)^T}\right]^T$ is defined with covariance matrix $\mathbf{R_x} = \text{E}\left\{\mathbf{x}\mathbf{x}^H\right\}$ where $\text{E}\{\cdot\}$ and $[\cdot]^H$ denote the expectation and the conjugate transpose, respectively. Since spatial filtering shall only be applied at the RS, only scalar transmit filters $\mathbf{Q}^{(1)} = q^{(1)}\mathbf{I}_M$ and $\mathbf{Q}^{(2)} = $
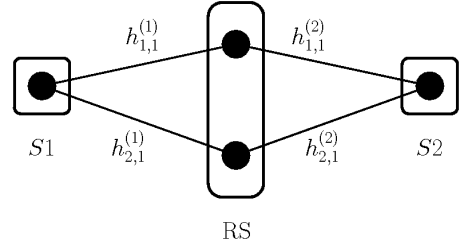


Fig. 1. Relay network for $M^{(1)} = 1$ antenna at $S1$, $M^{(2)} = 1$ antenna at $S2$, and $M_{\text{RS}} = 2$ antennas at the RS

$q^{(2)}\mathbf{I}_M$ are applied at $S1$ and $S2$, where $\mathbf{I}_M$ is an identity matrix of size $M$. These transmit filters are required in order to fulfill the transmit energy constraints. Assuming that $E^{(1)}$ and $E^{(2)}$ are the maximum transmit energies of nodes $S1$ and $S2$, the transmit energy constraints are given by

$$\text{E}\left\{\| q^{(k)}\mathbf{x}^{(k)} \|_2^2\right\} \leq E^{(k)}, \quad k = 1, 2, \qquad (4)$$

where $\| \cdot \|_2^2$ is the Euclidian norm of a vector. The overall transmit filter is given by the block diagonal matrix

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}^{(1)} & \mathbf{I}_M \\ \mathbf{I}_M & \mathbf{Q}^{(2)} \end{bmatrix}. \qquad (5)$$

Defining the overall channel matrix $\mathbf{H} = \left[\mathbf{H}^{(1)}, \mathbf{H}^{(2)}\right]$, the receive vector $\mathbf{y}_{\text{RS}}$ at the RS is given by

$$\mathbf{y}_{\text{RS}} = \mathbf{H}\mathbf{Q}\mathbf{x} + \mathbf{n}_{\text{RS}}, \qquad (6)$$

where $\mathbf{n}_{\text{RS}}$ is an additive white Gaussian noise vector with covariance matrix $\mathbf{R}_{\mathbf{n}_{\text{RS}}} = \text{E}\left\{\mathbf{n}_{\text{RS}}\mathbf{n}_{\text{RS}}^H\right\}$.

In the following, a linear transceive filter $\mathbf{G}$ at the RS shall be designed. This linear transceive filter $\mathbf{G}$ is a combination of a linear receive filter $\mathbf{G}_{\text{R}}$ and a linear transmit filter $\mathbf{G}_{\text{T}}$ where both filters can be determined independently.

The receive vector $\mathbf{y}_{\text{RS}}$ is multiplied with the linear receive filter $\mathbf{G}_{\text{R}}$ resulting in the overall RS estimation vector

$$\hat{\mathbf{x}}_{\text{RS}} = \left[\hat{\mathbf{x}}_{\text{RS}}^{(1)^T}, \hat{\mathbf{x}}_{\text{RS}}^{(2)^T}\right]^T = \mathbf{G}_{\text{R}}\mathbf{y}_{\text{RS}} = \mathbf{G}_{\text{R}}\mathbf{H}\mathbf{Q}\mathbf{x} + \mathbf{G}_{\text{R}}\mathbf{n}_{\text{RS}} \quad (7)$$

with the RS estimation vector $\hat{\mathbf{x}}_{\text{RS}}^{(1)}$ for $\mathbf{x}^{(1)}$ and the RS estimation vector $\hat{\mathbf{x}}_{\text{RS}}^{(2)}$ for $\mathbf{x}^{(2)}$, respectively.

During the second time slot, $S1$ should receive an estimate of data vector $\mathbf{x}^{(2)}$ and $S2$ should receive an estimate of data vector $\mathbf{x}^{(1)}$. Therefore, before applying the transmit filter $\mathbf{G}_{\text{T}}$, the RS estimation vector $\hat{\mathbf{x}}_{\text{RS}}$ is multiplied with the RS mapping matrix

$$\mathbf{G}_{\Pi} = \begin{bmatrix} \emptyset_M & \sqrt{(1-\beta)}\mathbf{I}_M \\ \sqrt{\beta}\mathbf{I}_M & \emptyset_M \end{bmatrix} \qquad (8)$$

where $\emptyset_M$ is a null matrix with $M$ rows and $M$ columns and where the parameter $\beta$ with $0 \leq \beta \leq 1$ is a weight factor which is applied to the RS estimation vectors before retransmission. For $\beta = 0.5$, the RS estimation vectors are equally weighted, for $\beta = 1$ only $\hat{\mathbf{x}}_{\text{RS}}^{(1)}$ is transmitted and for $\beta = 0$ only $\hat{\mathbf{x}}_{\text{RS}}^{(2)}$ is transmitted. Note that the RS mapping

matrix $\mathbf{G}_\Pi$ is an essential part of the introduced MIMO two-way relaying since $\mathbf{G}_\Pi$ ensures that the RS transmits the estimate $\hat{\mathbf{x}}_{RS}^{(2)}$ in the direction of $S1$ and the estimate $\hat{\mathbf{x}}_{RS}^{(1)}$ in the direction of $S2$. With the transmit filter $\mathbf{G}_T$, the overall transceive filter becomes

$$\mathbf{G} = \mathbf{G}_T \mathbf{G}_\Pi \mathbf{G}_R \qquad (9)$$

and the RS transmit vector is given by

$$\mathbf{x}_{RS} = \mathbf{G} \mathbf{y}_{RS} = \mathbf{GHQx} + \mathbf{Gn}_{RS}. \qquad (10)$$

In a linear system, the transceive filter $\mathbf{G}$ of (9) is a multiplication of a receive filter $\mathbf{G}_R$, the RS mapping matrix $\mathbf{G}_\Pi$, and a transmit filter $\mathbf{G}_T$. The RS transmit vector has to fulfill the transmit energy constraint at the RS

$$\mathrm{E}\left\{ \| \mathbf{x}_{RS} \|_2^2 \right\} \le E_{RS}, \qquad (11)$$

where $E_{RS}$ is the maximum transmit energy at the RS. Note that the channel matrix from the RS to nodes $S1$ and $S2$ is the transpose $\mathbf{H}^T$ of channel matrix $\mathbf{H}$ assuming that the channel is constant during two consecutive time slots. In the following, the estimate for data vector $\mathbf{x}^{(2)}$ at $S1$ is termed $\hat{\mathbf{x}}^{(1)}$ and the estimate for data vector $\mathbf{x}^{(1)}$ at $S2$ is termed $\hat{\mathbf{x}}^{(2)}$. For each receiving node $S1$ and $S2$, a scalar receive filter is assumed which results in an overall receive filter matrix

$$\mathbf{P} = \begin{bmatrix} p^{(1)}\mathbf{I}_M & \emptyset_M \\ \emptyset_M & p^{(2)}\mathbf{I}_M \end{bmatrix} \qquad (12)$$

with the filter coefficients $p^{(1)}$ at $S1$ and $p^{(2)}$ at $S2$. The overall estimated data vector $\hat{\mathbf{x}} = \left[ \hat{\mathbf{x}}^{(1)^T}, \hat{\mathbf{x}}^{(2)^T} \right]^T$ is given by

$$\hat{\mathbf{x}} = \mathbf{P}\left( \mathbf{H}^T \mathbf{GHQx} + \mathbf{H}^T \mathbf{Gn}_{RS} + \mathbf{n}_R \right) \qquad (13)$$

where it is assumed that $\mathbf{n}_R = \left[ \mathbf{n}_R^{(1)^T}, \mathbf{n}_R^{(2)^T} \right]^T$ is an additive white Gaussian noise vector. For purposes of further investigations, Eq. (13) may be rewritten as

$$\begin{aligned} \hat{\mathbf{x}} &= \mathbf{A}_{TW}\mathbf{x} &+ [\mathbf{D} \quad \mathbf{P}] & \quad \mathbf{n} \\ &= \begin{bmatrix} \mathbf{A}_{TW}^{(1)} \\ \mathbf{A}_{TW}^{(2)} \end{bmatrix}\mathbf{x} &+ \begin{bmatrix} \mathbf{B}_{TW}^{(1)} \\ \mathbf{B}_{TW}^{(2)} \end{bmatrix} & \quad \mathbf{n} \end{aligned} \qquad (14)$$

with

$$\mathbf{A}_{TW} = \mathbf{PH}^T \mathbf{GHQ} \qquad (15)$$
$$\mathbf{D} = \mathbf{PH}^T \mathbf{G} \qquad (16)$$
$$\mathbf{n} = \left[ \mathbf{n}_{RS}^T, \mathbf{n}_R^T \right]^T \qquad (17)$$

leading to the separated estimates at nodes $S1$ and $S2$

$$\hat{\mathbf{x}}^{(k)} = \mathbf{A}_{TW}^{(k)}\mathbf{x} + \mathbf{B}_{TW}^{(k)}\mathbf{n} \qquad \text{for } k = 1,2 \qquad (18)$$

with $\mathbf{A}_{TW}^{(k)}$ of dimension $M \times 2M$ and $\mathbf{B}_{TW}^{(k)}$ of dimension $M \times (2M + M_{RS})$.

## III. OBTAINING CSI AT THE RS

Since the reduced effort in obtaining CSI is a main advantage of the proposed MIMO two-way relaying this aspect is discussed in the following.

In two-way relaying with subtraction [3], both nodes $S1$ and $S2$ require CSI about their own channel to the RS as well as CSI about the channel of the other node to the RS. Exchanging this CSI requires a feedback channel. In [7], it is proposed that transmit and receive processing can be restricted to the RS for one-way relaying, i.e., CSI is only required at the RS. A similar approach may be applied in MIMO two-way relaying. However, there is one significant difference between the effort for obtaining CSI in [7] and the proposed MIMO two-way relaying protocol. In [7], CSI of the channels from nodes $S1$ and $S2$ to the RS can be achieved by pilot signaling. But CSI of the channels from the RS to the nodes can only be obtained by feedback from the nodes to the RS since up- and downlink are separated on orthogonal channel resources. In MIMO two-way relaying, CSI of the channels from the nodes to the RS as well as from the RS to nodes $S1$ and $S2$ is obtained by only one pilot signal since up- and downlink are processed simultaneously. Since feedback channels require additional radio resources, MIMO two-way relaying is very promising in terms of CSI signaling effort.

## IV. SUM RATE OF MIMO TWO-WAY RELAYING

In the following, the sum rate of a system is defined as the sum of the mutual information values for all transmissions using the same channel resources. In [8], it is shown that for a MIMO system with

$$\mathbf{y} = \mathbf{Ax} + \mathbf{Bn} \qquad (19)$$

the mutual information is given by

$$C_{MIMO} = \log_2\left( \det\left[ \mathbf{I} + \frac{\mathbf{A}\mathbf{R_x}\mathbf{A}^H}{\mathbf{B}\mathbf{R_n}\mathbf{B}^H} \right] \right) \qquad (20)$$

where $\mathbf{A}$ and $\mathbf{B}$ depend on the underlying MIMO system and $\mathbf{R_x}$ and $\mathbf{R_n}$ are the transmit vector and receive noise vector covariance matrices, respectively.

For one-way AF relaying, where the RS can only receive and transmit on orthogonal channel resources, the pre-log factor 1/2 is introduced in order to indicate the increase in required channel resources leading to the mutual information

$$C_{AF} = \frac{1}{2}\log_2\left( \det\left[ \mathbf{I} + \frac{\mathbf{A}_{AF}\mathbf{R_x}\mathbf{A}_{AF}^H}{\mathbf{B}_{AF}\mathbf{R_n}\mathbf{B}_{AF}^H} \right] \right). \qquad (21)$$

For one-way AF relaying, the sum rate equals the mutual information since communication only takes place in one direction, either from $S1$ to $S2$ or from $S2$ to $S1$.

For the introduced MIMO two-way relaying, at each receive node the mutual information is given by

$$C_{TW}^{(k)} = \frac{1}{2}\log_2\left( \det\left[ \mathbf{I}_M + \frac{\mathbf{A}_{TW}^{(k)}\mathbf{R_x}\mathbf{A}_{TW}^{(k)^H}}{\mathbf{B}_{TW}^{(k)}\mathbf{R_n}\mathbf{B}_{TW}^{(k)^H}} \right] \right) \text{ for } k = 1,2. \qquad (22)$$

with $\mathbf{A}_{\mathrm{TW}}^{(k)}$ and $\mathbf{B}_{\mathrm{TW}}^{(k)}$ from Eq. (18). Since communication takes place in two directions by using the same channel resources, the sum rate of MIMO two-way relaying results in

$$C_{\mathrm{TW}} = C_{\mathrm{TW}}^{(1)} + C_{\mathrm{TW}}^{(2)}. \qquad (23)$$

Both mutual information values $C_{\mathrm{TW}}^{(1)}$ and $C_{\mathrm{TW}}^{(2)}$ depend on the quality of both channels, $\mathbf{H}^{(1)}$ between $S1$ and the RS and $\mathbf{H}^{(2)}$ between $S2$ and the RS, i.e., even if one channel is much better than the other channel, both directions of communication are degraded by the bad channel. However, assigning equal weight to both RS estimation vectors at the RS before retransmission may lead to a sub-optimum sum rate if one RS estimation vector is received over a good channel while the other RS estimation vector is received over a bad channel. The sum rate of Eq. (23) can be maximized by optimizing $\beta$ from Eq. (12). The underlying optimization problem is formulated as

$$\beta_{opt} = \arg\max_{\beta} \left\{ C_{\mathrm{TW}}^{(1)} + C_{\mathrm{TW}}^{(2)} \right\} \qquad (24a)$$

$$\text{subject to:} \quad 0 \le \beta \le 1. \qquad (24b)$$

There exists no closed form solution to this optimization problem. However, it may be solved by numeric optimization.

In the following, the optimization problem in (24) is simplified leading to a closed form approximation $\beta_{opt}$. Let us assume a fading channel with an average signal-to-noise ratio (SNR) on the channel from $S1$ to the RS given by $\rho^{(1)}$ and an average SNR on the channel from $S2$ to the RS given by $\rho^{(2)}$. In this case, the overall average SNR for AF relaying at receiving node $S1$ results in

$$\rho_{\mathrm{ov}}^{(1)} = \frac{\beta \rho^{(1)} \rho^{(2)}}{\rho^{(1)} + \beta \rho^{(2)} + 1} \qquad (25)$$

and the overall SNR at receiving node $S2$ results in

$$\rho_{\mathrm{ov}}^{(2)} = \frac{(1-\beta)\rho^{(1)}\rho^{(2)}}{(1-\beta)\rho^{(1)} + \rho^{(2)} + 1}. \qquad (26)$$

Approximating the mutual information of Eq. (23) by the single input single output (SISO) mutual information

$$\tilde{C}_{\mathrm{TW}}^{(k)} = \frac{1}{2}\log_2\left(1 + \rho_{\mathrm{ov}}^{(k)}\right) \quad \text{for } k = 1, 2 \qquad (27)$$

the sum rate may be approximated in the high SNR region by

$$\tilde{C}_{\mathrm{TW}} = \frac{1}{2}\log_2\left(\rho_{\mathrm{ov}}^{(1)}\right) + \frac{1}{2}\log_2\left(\rho_{\mathrm{ov}}^{(2)}\right). \qquad (28)$$

For this approximation, the optimization problem of (24) is solved by

$$\beta_{app} = \frac{\rho^{(1)} + 1 - \sqrt{(\rho^{(1)} + 1)(\rho^{(2)} + 1)}}{\rho^{(1)} - \rho^{(2)}}. \qquad (29)$$

Note that the sum rate which is calculated by Eq. (28) is different from the exact sum rate in Eq. (23). However, in order to determine the optimum parameter $\beta$ this approximation provides reasonable results with low effort, which is confirmed by the following simulations.
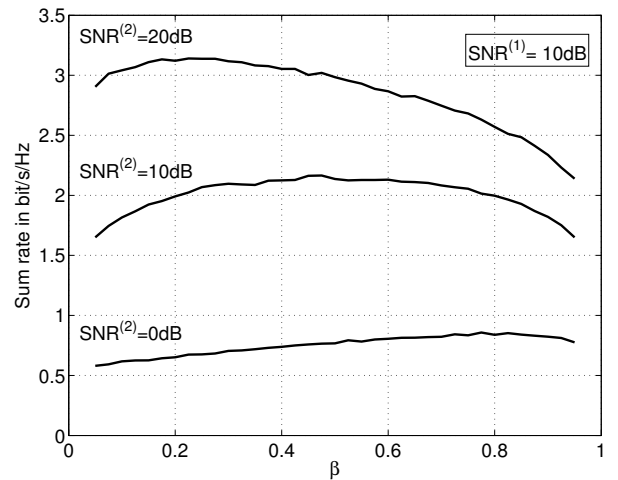


Fig. 2. Sum rate depending on $\beta$ with fixed $\rho^{(1)} = 10$dB and $\rho^{(2)}$ as a parameter

## V. SIMULATION RESULTS

In this section, the sum rate of MIMO two-way relaying is investigated where the filter matrices $\mathbf{Q}$, $\mathbf{G}$, and $\mathbf{P}$ from Section II are chosen according to the linear zero forcing (ZF) constraint introduced in [6]. Firstly, the influence of the weight factor at the RS is considered. Secondly, MIMO two-way relaying is compared to other relaying protocols. It is assumed that nodes $S1$ and $S2$ are each equipped with $M = 1$ antenna and the RS is equipped with $M_{\mathrm{RS}} = 2$ antennas according to the requirement in Eq. (2). The channel coefficients are spatially white and their amplitude is Rayleigh distributed leading to average SNR values of $\rho^{(1)}$ and $\rho^{(2)}$, respectively. In Section IV, maximizing the sum rate by giving different weights to the RS estimation vectors in case of different channel qualities on the two channels is discussed. Fig. 2 gives the sum rate for the linear ZF transceive filter for varying $\beta$ with fixed $\rho^{(1)} = 10$dB and $\rho^{(2)}$ as a parameter. For $\beta = 0.5$, both RS estimation vectors are weighted equally which leads to the maximum sum rate if both channels have the same average channel quality, i.e., $\rho^{(1)} = \rho^{(2)} = 10$dB. If the channels have different SNR, the sum rate may be increased by introducing a higher weight to the RS estimation vector which is received over the better channel on the first hop. This can be explained as follows. The ZF receive filter leads to unbiased estimates at the RS. The noise at the RS is filtered which leads to different SNR for the two receive vectors from $S1$ and $S2$ after the filter. Therefore, the receive vector which comes over the better channel has a higher SNR, provides a higher mutual information, and gets a higher weight before retransmission. The ZF transmit filter also leads to unbiased estimates at $S1$ and $S2$. However, the noise is not filtered at the receivers and both receive vectors have the same SNR after the filter.

In Fig. 3, for fixed $\rho^{(1)} = 5$dB the average sum rate depending on $\rho^{(2)}$ from the numeric optimization is compared to the value achieved by the approximation of Eq.(29) and for fixed $\beta = 0.5$. For equal channel qualities on both channels ($\rho^{(1)} = \rho^{(2)}$), all approaches provide the same sum rate. However, for increasing difference of the channel qualities on both channels
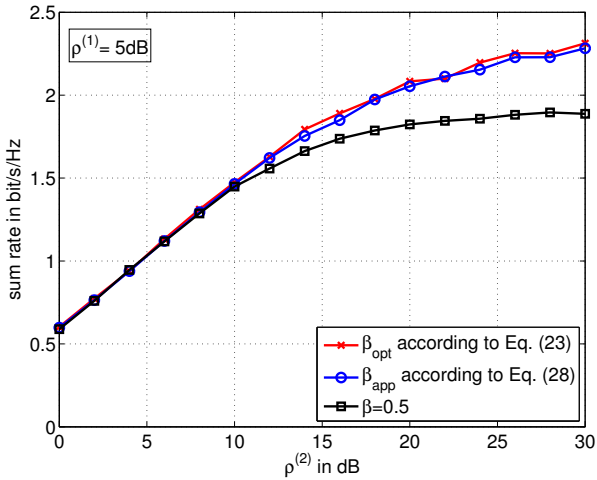
Fig. 3. Average sum capacity depending on $\rho^{(2)}$ for fixed $\rho^{(1)} = 5$dB: comparison of results from Eq. (24), Eq. (29), and $\beta = 0.5$
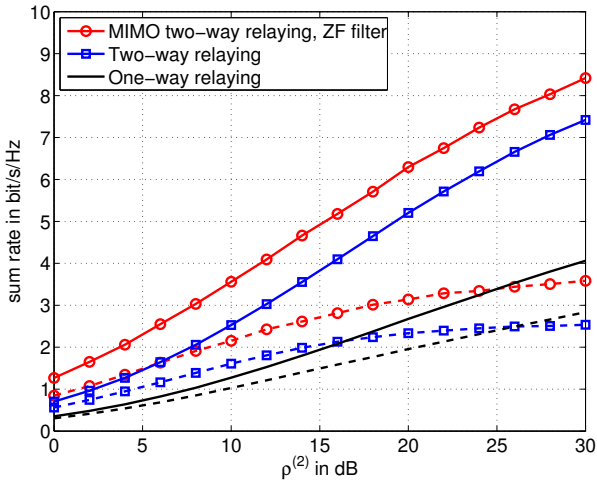


Fig. 4. Sum rate of different relaying protocols depending on $\rho^{(2)}$ with $\rho^{(1)}$ as a parameter; dashed lines: $\rho^{(1)} = 10$dB, solid lines: $\rho^{(1)} = 30$dB

the sum rates diverge. The exact solution provides the largest sum rate and the approximation comes close to this sum rate while $\beta = 0.5$ clearly provides the worst performance.

In the following, the maximized sum rate, i.e., $\beta = \beta_{opt}$, of MIMO two-way relaying for a ZF transceive filter is compared to two other relaying approaches. Firstly, one-way relaying is considered where orthogonal channel resources are used for the transmission from $S1$ and $S2$ and for the transmission from $S2$ to $S1$. Secondly, two-way relaying with subtraction is considered [3]. Note that for both approaches $M_{RS} = 2$ antennas at the RS are assumed in order to guarantee a fair comparison. In this case, the filter matrix at the RS is a simple diagonal matrix with a constant amplification factor on the main diagonal which fulfills the transmit energy constraint at the RS. Since the two receive signals at the RS cannot be separated for both approaches, there is no possibility of assigning different energies to both transmit signals if the two channels have different qualities. Fig. 4 gives the sum rate for the considered relaying approaches. It is depicted depending on $\rho^{(2)}$ with $\rho^{(1)}$ as a parameter. The sum rate strongly depends on $\rho^{(1)}$. For $\rho^{(1)} = 10$dB, the sum rate

converges to a constant maximum for increasing $\rho^{(2)}$ while it monotonically increases for $\rho^{(1)} = 30$dB in the considered region of $\rho^{(2)}$. Obviously, two-way relaying with subtraction and MIMO two-way relaying outperform one-way relaying in terms of sum rate. Especially in the high SNR region, there exists an increase of the sum rate for MIMO two-way relaying by a factor higher than 2 compared to one-way relaying. The proposed MIMO two-way relaying with the linear ZF transceive filter outperforms two-way relaying with subtraction. Since no transmit or receive beamforming is applied for two-way relaying with subtraction, no antenna array gain can be exploited. However, for the linear ZF transceive filter the antenna array gain can be exploited at the RS which leads to an increase of the sum rate. Note that MIMO two-way relaying does not achieve the mutual information of a system with two transmit and two receive antennas since the transmission and reception at $S1$ and $S2$ cannot be encoded and decoded jointly, respectively.

## VI. CONCLUSION

In this paper, MIMO two-way relaying has been introduced which compensates for the degradation of the spectral efficiency by the factor of 2 in one-way AF relay networks. Compared to previous work on the two-way relay channel, MIMO two-way relaying only requires CSI at the RS which significantly reduces the CSI signaling overhead. Furthermore, the introduced protocol outperforms previous protocols in terms of sum rate. It is shown how the overall energy at the RS has to be distributed in case of different channel qualities on the two different channels from the communicating nodes to the RS in order to maximize the sum rate. The derived approximation of the energy distribution provides reasonable results for practical applications.

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